Post-Quantum Cryptography #2

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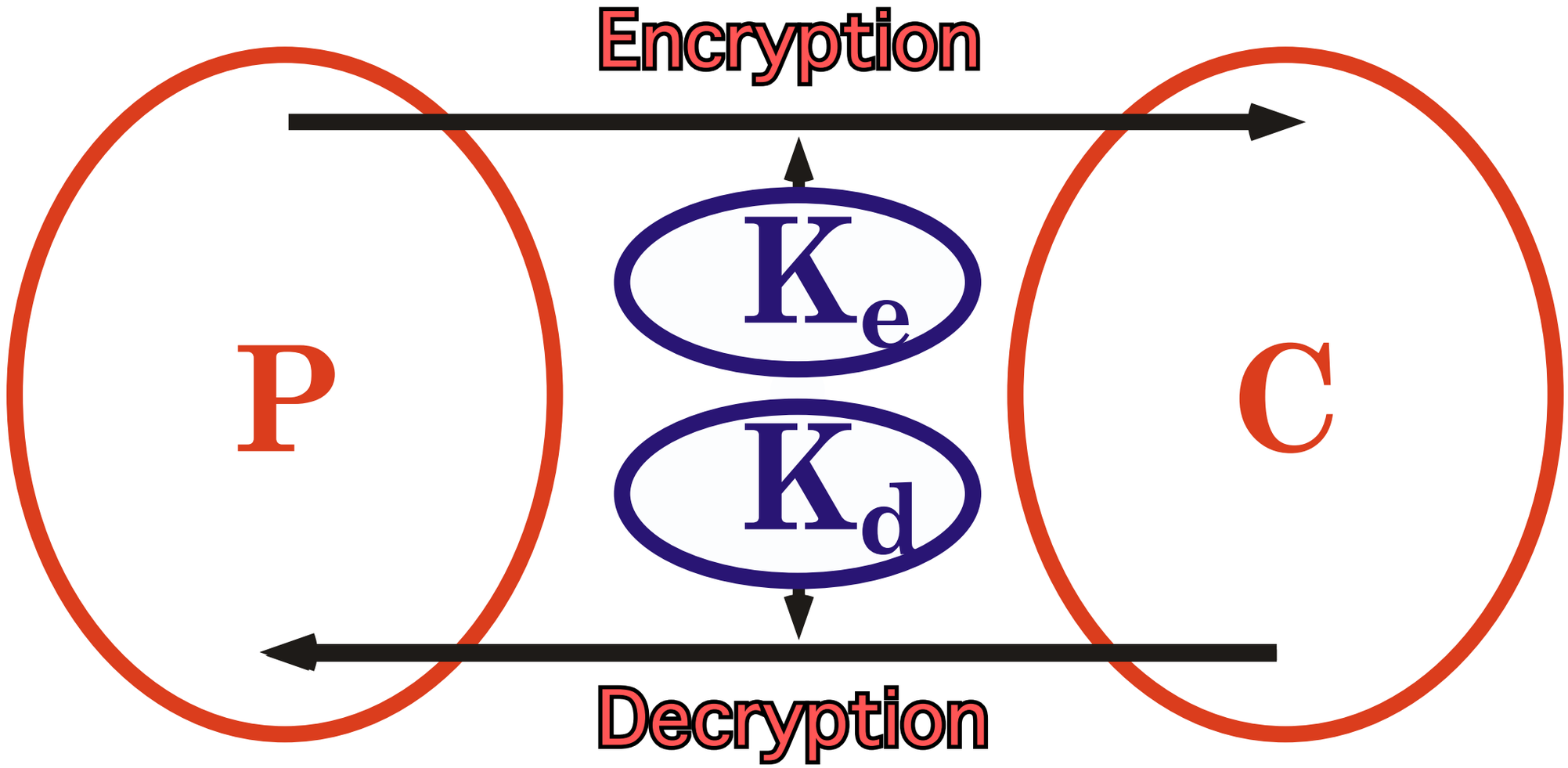
McGill University

Post-Quantum Cryptography

- Finite Fields based cryptography
  - Codes
  - Multi-variate Polynomials
- Integers based cryptography
  - Approximate Integer GCD
  - Lattices

Public Key Encryption

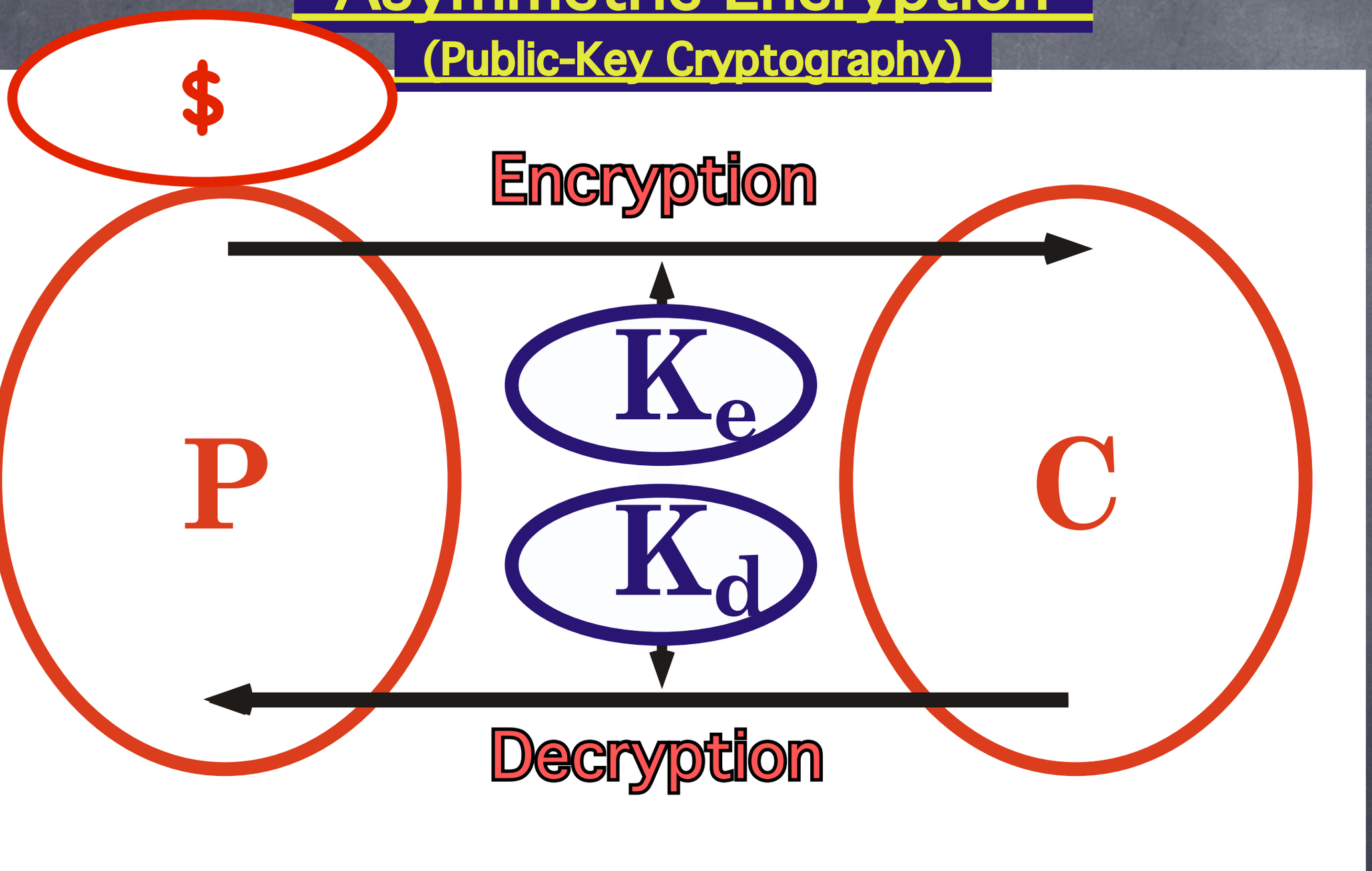
Asymmetric Encryption
(Public-Key Cryptography)



Complexity Theoretical Security

Asymmetric Encryption

(Public-Key Cryptography)



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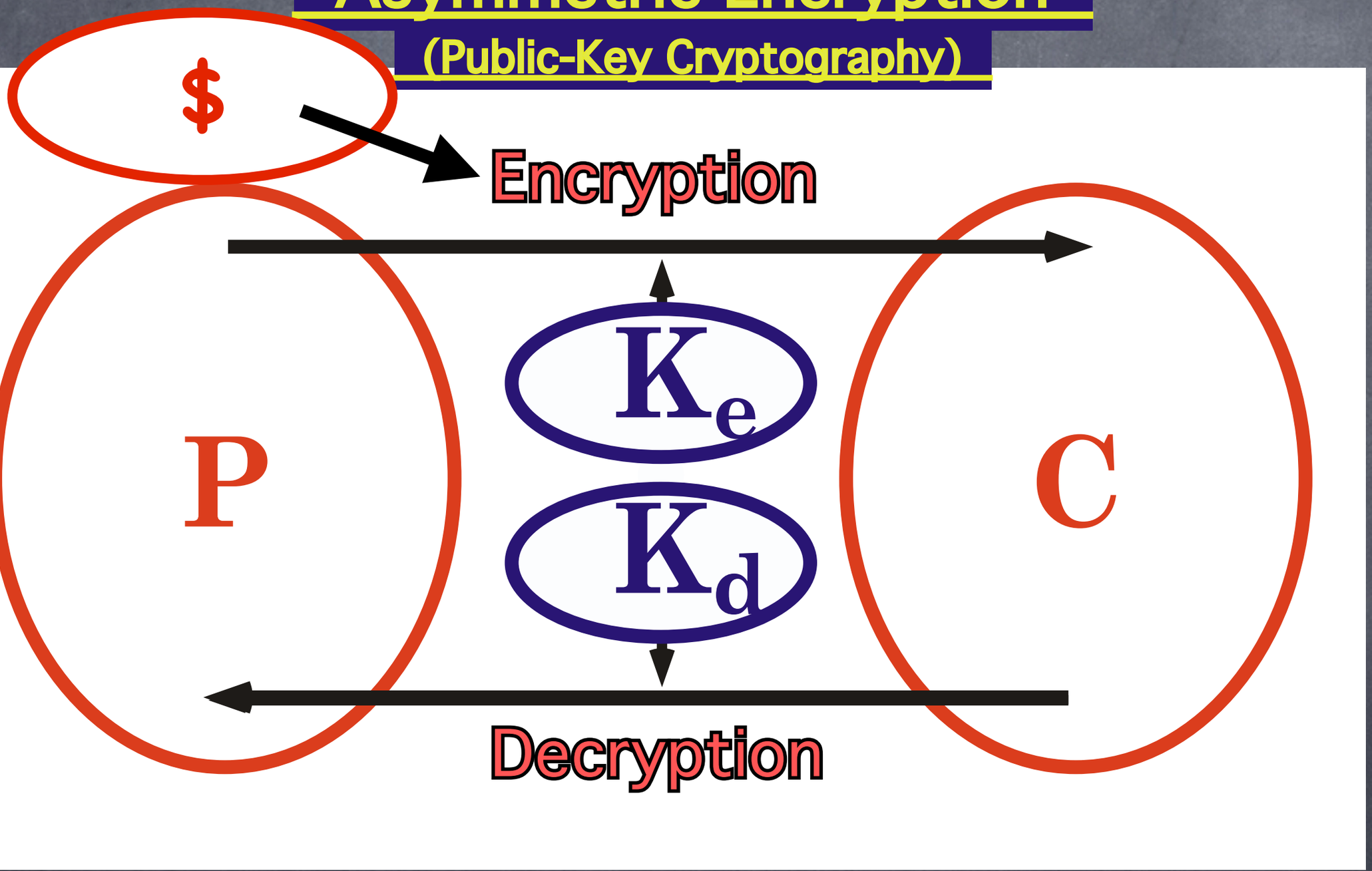

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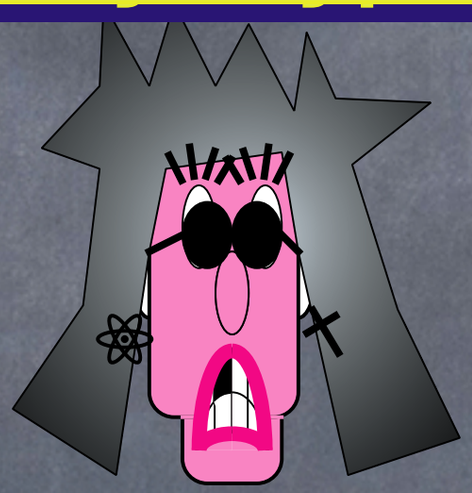
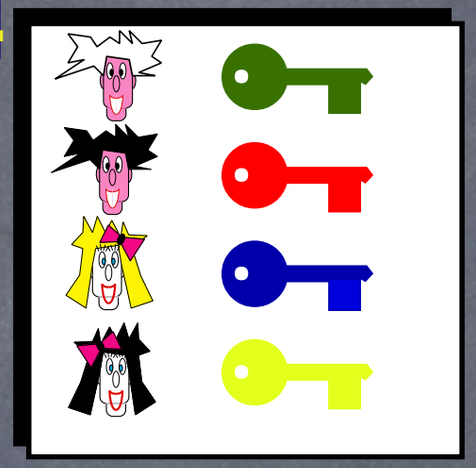
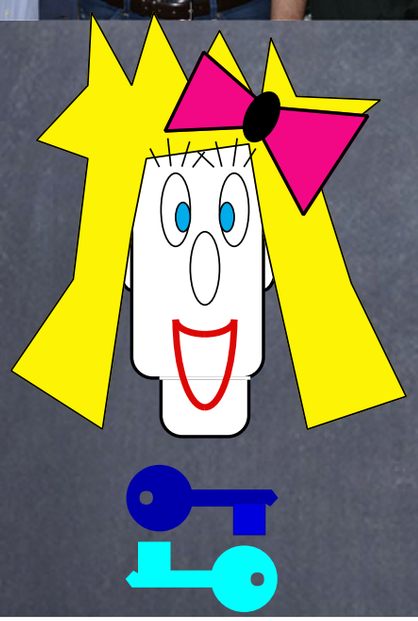
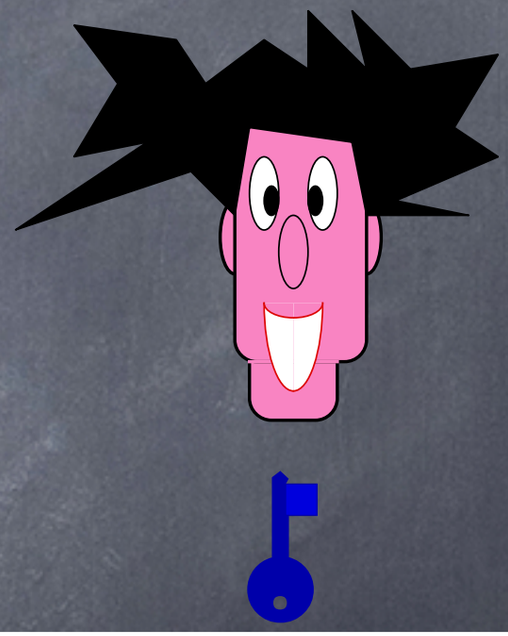
Public-Key Cryptography



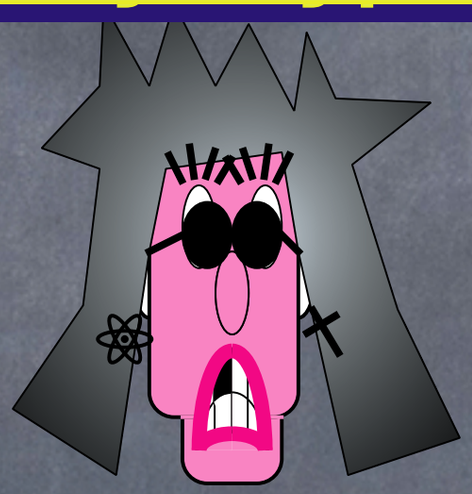


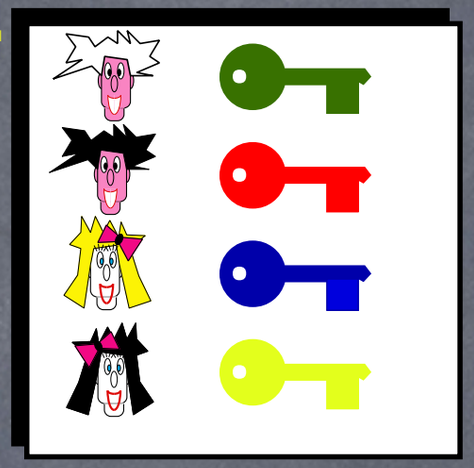


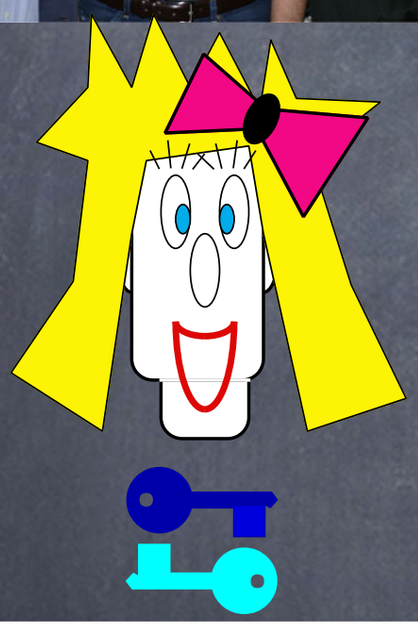
8RdewtU5qkLa$es!T9@



Public-Key Cryptography







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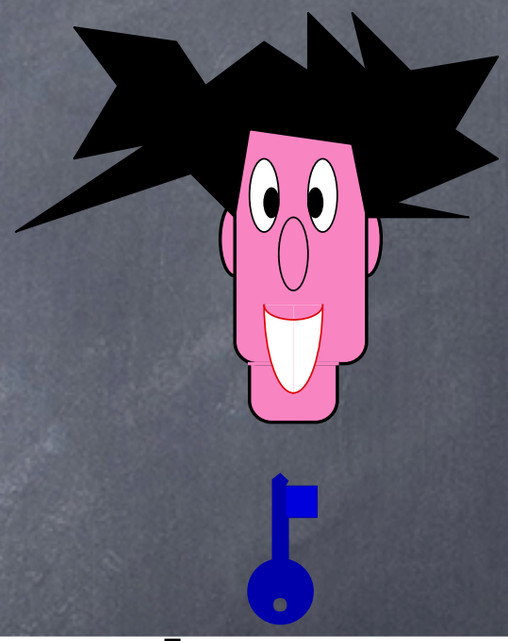
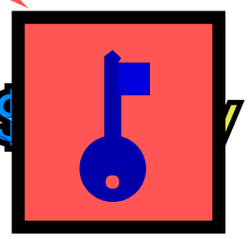
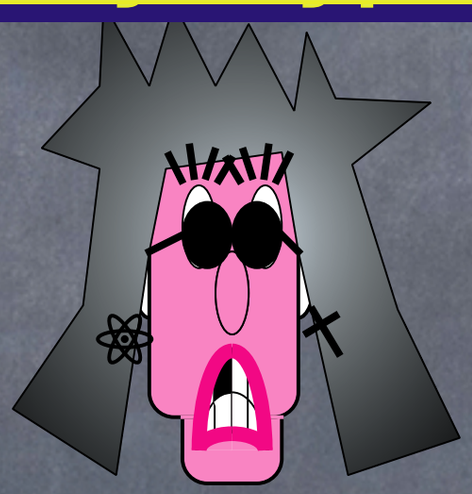


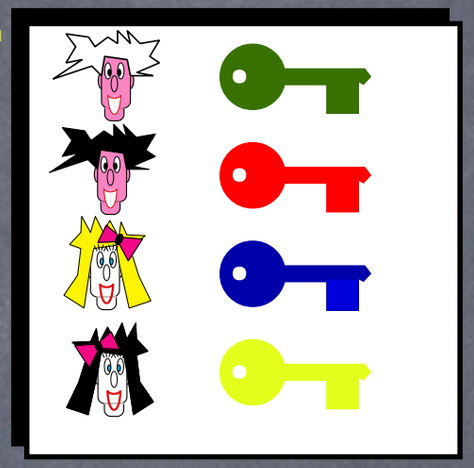
Encryption

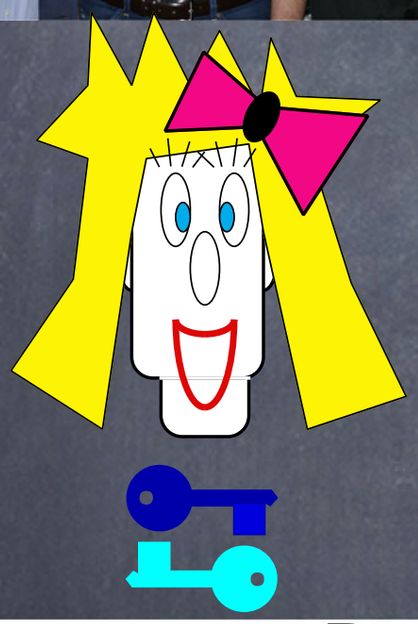


8RdewtU5qkLa$ [illegible] me ?

Public-Key Cryptography







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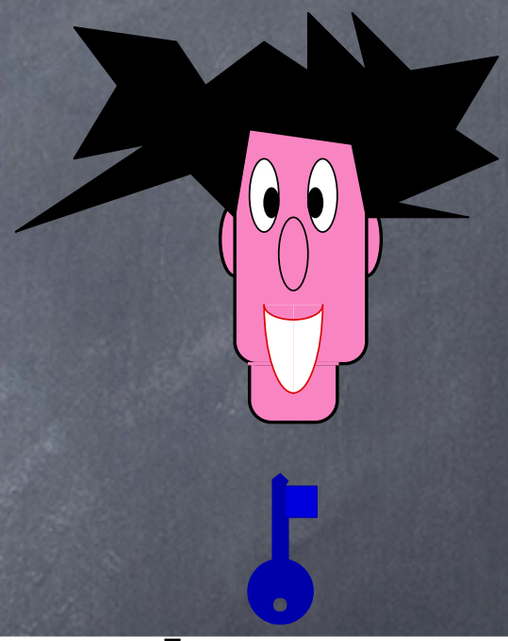


Decryption

Will you marry m es!T9@

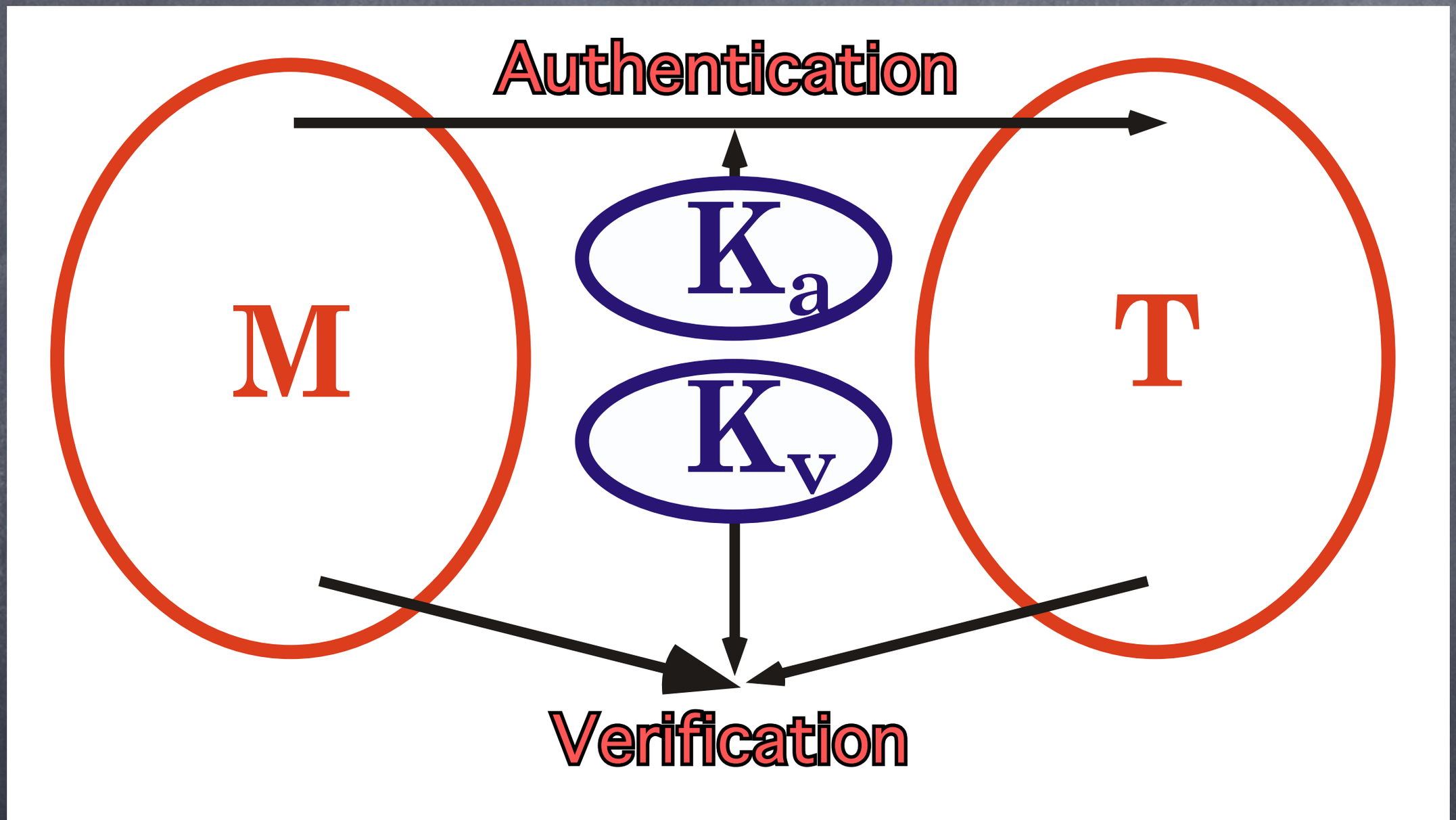
Encryption

8RdewtU5qkLa$ y me ?

Digital Signatures

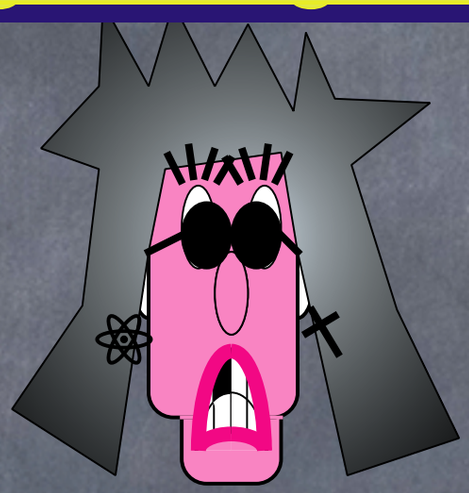
Asymmetric Authentication

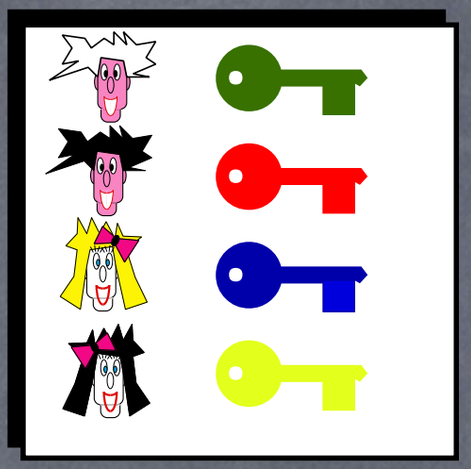
(Digital Signature Scheme)

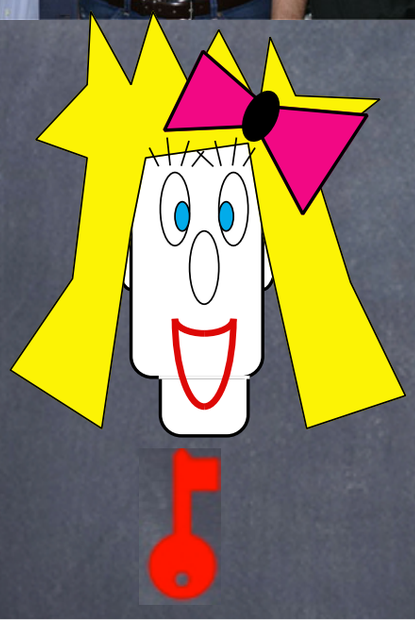


Complexity Theoretical Security

Digital Signature







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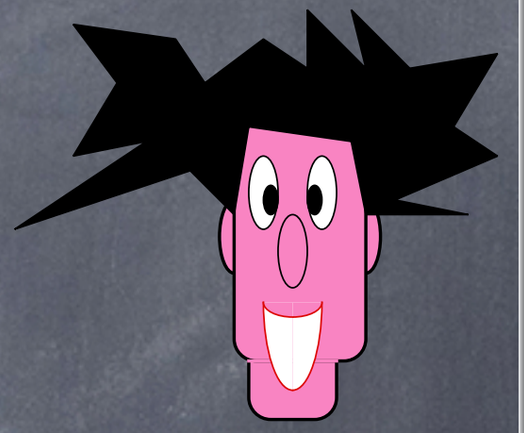
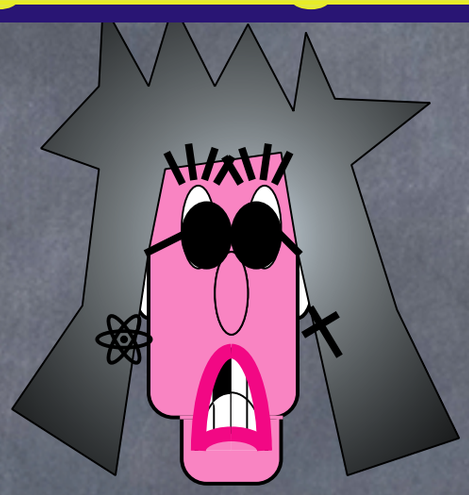
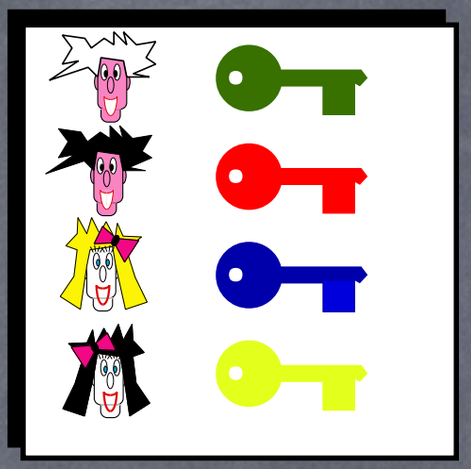
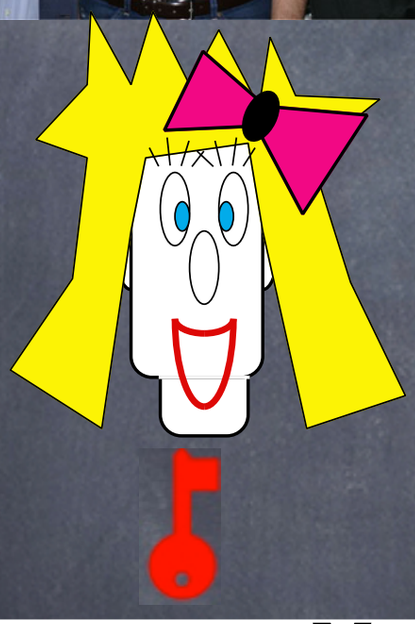


Digital Signature







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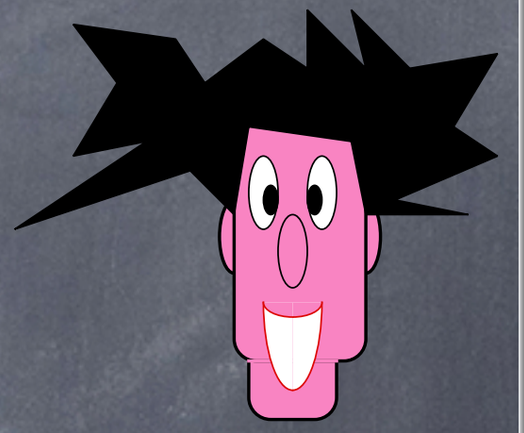
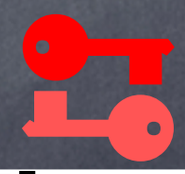
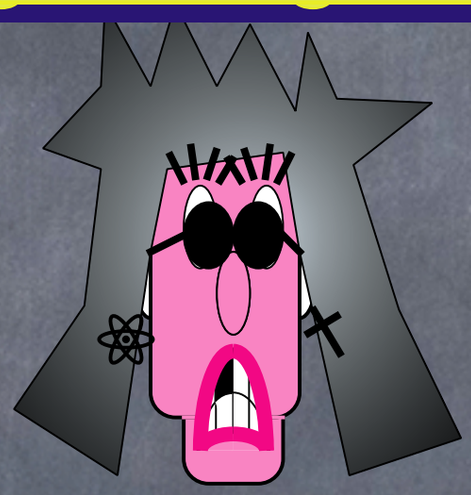


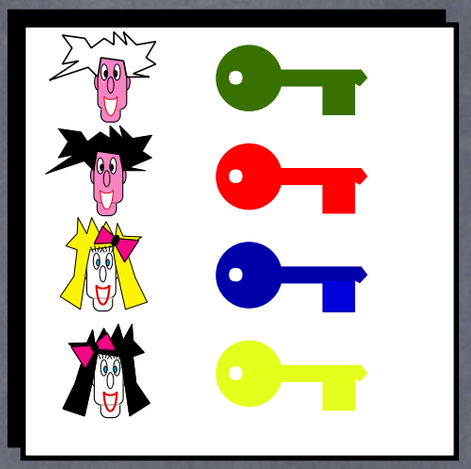


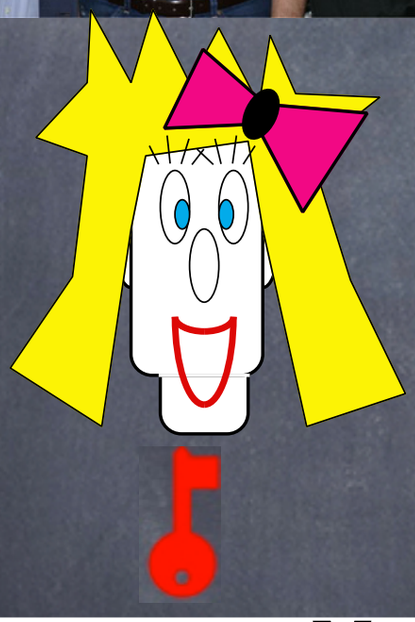
Authentication

8RdewtU5qkLa$ me ?

Digital Signature







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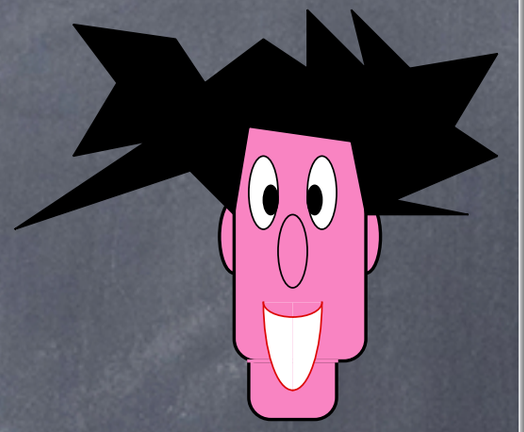

Verification

VALID es!T9@
ry me ?

Authentication

8RdewtU5qkLa$ y me ?

Code Equivalence

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- Two [n,k,d] linear codes C,C' are (permutation) equivalent if there exists a kxk non-singular matrix S & an nxn permutation matrix P s.t.

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- the codewords of C and C' have exactly all the same weights

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- Define $C'or(w) := Cor(wP^{-1})P$,
- then $C'or:\{0,1\}^n \to C'$ is an efficient nearest codeword error-correcting procedure for C′ (upto $\frac{d-1}{2}$ errors)

McEliece Cryptosystem

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- Let $G \in_R Goppa_t$, $S \in_R \mathbb{F}_2^{k \times k}$, & $P \in_R Perm$ be the private-key, & $G' = SGP$ be the public-key,

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$$c' = C'or(w) \text{ is difficult.}$$

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- Given G, S, P, w finding $c = Cor(w)$ and $e = w - c$ is easy.

	(n,t)	(2048,32)	(2048,40)	(4096,22)	(4096,45)
McEliece scheme	plaintext size[1]	1928	1888	4024	3904
	ciphertext size[1]	2048	2048	4096	4096
	encryption rate[2]	176	222	145	192
	decryption rate[2]	1780	2260	600	1650
Niederreiter scheme	plaintext size[1]	232	280	192	352
	ciphertext size[1]	352	440	264	540
	encryption rate[2]	360	370	320	340
	decryption rate[2]	13600	16700	9800	16900
	public key size[3]	73 KB	86 KB	123 KB	234 KB
	key generation[3]	$6.70\,10^7$	$9.55\,10^7$	$7.93\,10^7$	$23.1\,10^7$
	security bits[3][4]	91	98	93	140

[1] plaintext and ciphertext sizes in bits
[2] in cycles per plaintext bytes, Intel Core 2
[3] common to both schemes (number of cycles on a processor Intel Core 2)
[4] $\log_2$ of the non-quantum binary workfactor

Table 4. McEliece and Niederreiter encryption scheme

Families of Codes

Nicolas Sendrier

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Binary Goppa codes seem safe, but not

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Nicolas Sendrier

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- concatenated codes,
- elliptic codes,
- Reed-Muller codes,
- Convolutional codes

Nicolas Sendrier

Code based cryptography

Code based cryptography

- Courtois, Finiasz and Sendrier signature scheme

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- Courtois, Finiasz and Sendrier signature scheme
- Stern's identification scheme

Code based cryptography

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- Stern's identification scheme
- Code based PRNG
- Code based hash function

Code based cryptography

§

Post-Quantum Cryptography

- Finite Fields based cryptography
  - ~~Codes~~
  - Multi-variate Polynomials
- Integers based cryptography
  - Approximate Integer GCD
  - Lattices

Multi-variate Poly based cryptography

§

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Multi-variate Poly based cryptography

- $P = (p_1(x_1,\dots,x_n),\dots,p_m(x_1,\dots,x_n))$ for $x=(x_1,\dots,x_n)$ over $\mathbb{F}^n$.

Multi-variate Poly based cryptography

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- $z_k = p_k(x) := \sum_i P_{ik}x_i + \sum_i Q_{ik}x_i^2 + \sum_{i<j} R_{ijk}x_ix_j$

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- We seek more :
  finding x from $z=P(x)$ being hard on average.

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- Public-key: P

Multi-variate Poly based cryptography

- $P = (p_1(x_1,\dots,x_n),\dots,p_m(x_1,\dots,x_n))$ for $x=(x_1,\dots,x_n)$ over $\mathbb{F}_2^n$.
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- $Dec(z)$= find x s.t. $z=P(x)$ (specific to P's design)

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  where $c'_T := M_T^{-1}\, c_T$ and $c'_S := M_S^{-1}\, c_S$

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- Private-key: (M_T^{-1}, c'_T), (M_S^{-1}, c'_S), Q^{-1}

$$Dec(y) = M_S^{-1}\, Q^{-1}(\, M_T^{-1}\, y - c'_T\,) - c'_S$$

where $c'_T := M_T^{-1}\, c_T$ and $c'_S := M_S^{-1}\, c_S$

Matsumoto-Inai

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- Then there exists $h := (2^a+1)^{-1} \bmod 2^n-1$ such that $Q^{-1}(y)=y^h$ over $\mathbb{F}_{2^n}$

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$$= \begin{pmatrix} 1 & x^2 & \dots & x^{2n-2} \\ \mid & \mid & \dots & \mid \\ \mathrm{mod} & \mathrm{mod} & \dots & \mathrm{mod} \\ \mid & \mid & \dots & \mid \\ P & P & \dots & P \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ \dots \\ x_{n-1} \end{pmatrix}$$

$$= M_{sq}\, x$$

x^{2^i} over $\mathbb{F}_{2^n}$ is linear over $\mathbb{F}_2$

- $(y_{n-1},\dots,y_1,y_0) = (x_{n-1},\dots,x_1,x_0)^{2^i} = M^i_{sq}\ x$

- is a system of n degree 1 equations

$$y_0 = (M^i_{sq})_0\ x \qquad y_1 = (M^i_{sq})_1\ x$$
$$y_2 = (M^i_{sq})_2\ x \quad \dots \quad y_{n-1} = (M^i_{sq})_{n-1}\ x$$

x^{2i+1} over $\mathbb{F}_{2^n}$ is quadratic over $\mathbb{F}_2$

- $(z_{n-1},\dots,z_1,z_0) = (x_{n-1},\dots,x_1,x_0)^{2i+1}$
  $= (y_{n-1},\dots,y_1,y_0)*(x_{n-1},\dots,x_1,x_0)$

is a system of n degree 2 equations

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- MI thus based the security of the scheme on the different principle of mapping obfuscation. (à la McEliece)

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- Based on an idea of Shamir from 1993, Patarin et al proposed to avoid their own attack by deleting r out of the n equations from the MI public key, and called the resulting scheme SFLASH.

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SFLASH

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- Such truncated keys can be used in signature schemes but not in encryption schemes, since they cannot be inverted uniquely.

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- They differ only in their security parameters:
  for SFLASH^{v2} : $q = 2^7$, $n = 37$, $a = 11$ and $r = 11$
  for SFLASH^{v3} : $q = 2^7$, $n = 67$, $a = 33$ and $r = 11$

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- Dubois, Fouque, Shamir, Stern broke $SFLASH^{v2}$ & $v3$ in 2007.

Variations

Var.		Meaning	Slows
Plus	+	extra polynomials in the central map	Slows Signatures
Minus	-	remove central or public polynomials	Slows Encryption
Perturb	i	internal perturbation	Slows All
Project	p	Fix a central variable to be 0	Slows Signatures
Vinegar	v	extra variables that can be set arbitrarily	Slows Encryption
Sparse	s	make single-field central map sparse	*General Speedup*

Table 3. A Summary of Major Modifications in MPKCs, cf. [104]

Variations

Scheme	result	SecrKey	PublKey	KeyGen	SecrMap	PublMap
RSA-1024	1024b	128 B	320 B	2.7 sec	84 ms	2.0 ms
ECDSA-$\mathbb{F}_{2^{163}}$	320b	48 B	24 B	1.6 ms	1.9 ms	5.1 ms
PMI+$(136, 6, 18, 8)$	144b	5.5 kB	165 kB	1.1 sec	1.23 ms	0.18 ms
Rainbow $(2^8, 18, 12, 12)$	336b	24.8 kB	22.5 kB	0.3 sec	0.43 ms	0.40 ms
Rainbow $(2^4, 24, 20, 20)$	256b	91.5 kB	83 kB	1.6 sec	0.93 ms	0.74 ms
QUARTZ	128b	71.0 kB	3.9 kB	3.1 sec	11 sec	0.24 ms

Table 2. Current* Multivariate PKCs Compared on a Pentium III 500

*as of 2008

Multi-variate Poly based cryptography

§

Post-Quantum Cryptography

- Finite Fields based cryptography
  - ~~Codes~~
  - ~~Multi-variate Polynomials~~
- Integers based cryptography
  - Approximate Integer GCD
  - Lattices

Cryptographic Money based on hidden codes *(hidden sub-spaces)*

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* For simplicity, assume we use $\mathbb{F}=\mathbb{F}_2$.

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  Fix $C \subset \mathbb{F}_2^n$ and $\beta > 1$, and choose βn independent uniformly-random samples from $I_{D,C}$.
  With probability $1 - 2^{-\Omega(n)}$, the set of points on which they are all zero is exactly C.

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* $|\$\rangle = \Sigma_{c\in C} |c\rangle, \qquad [H]^{\otimes n}|\$\rangle = \Sigma_{c'\in C^{\perp}} |c'\rangle$
* checking $|\$\rangle$: using $P_1(x),\ldots,P_{\beta n}(x)$, validate that $|\$\rangle$ is made only of states from C and using $Q_1(x),\ldots,Q_{\beta n}(x)$, validate that $[H]|\$\rangle$ is made only of states from $C^{\perp}$.

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* In Q-Money C or $C^{\perp}$ may be sampled once.
* Weakens the security. Degree D=4 with sample is as hard as degree 3 without a sample. So they choose D=4.

Hidden Code

* Let $Z_{D,C,\varepsilon}$ be the distribution which sets

$$Z_{D,C,\varepsilon} = \begin{cases} I_{D,C} \text{ with probability } 1-\varepsilon \\ I_{D,\mathcal{R}} \text{ with probability } \varepsilon \end{cases}$$

where $\mathcal{R}$ is a random code of dimension k.

* **Lemma C**

Fix $C \subset \mathbb{F}_2^n$ and $\varepsilon < 1$, let $\beta = 32/(1-\varepsilon)^2$, and choose βn independent samples from $Z_{D,C,\varepsilon}$. Let $\delta = 1/2 + (1-\varepsilon)/4$. With probability $1 - 2^{-\Omega(n)}$ the set of points on which at least $\delta\beta n$ polynomials are zero is exactly C.

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* $\forall c\in C, c'\in C^{\perp}$ $P_i(c)=0$ and $Q_j(c')=0$ with probability $\geq \delta$.
* Adding misleading polynomials may only make the assumption harder to break...

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