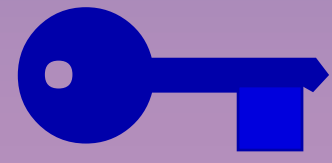
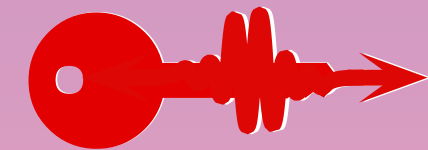


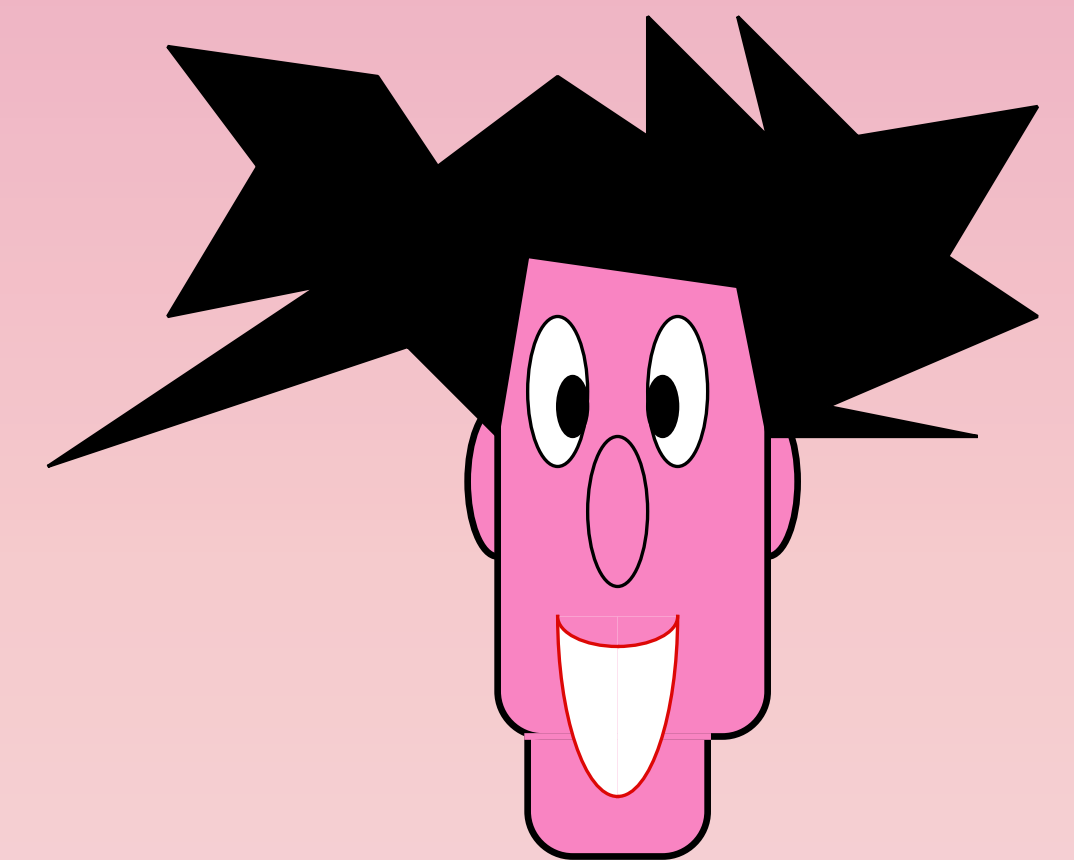
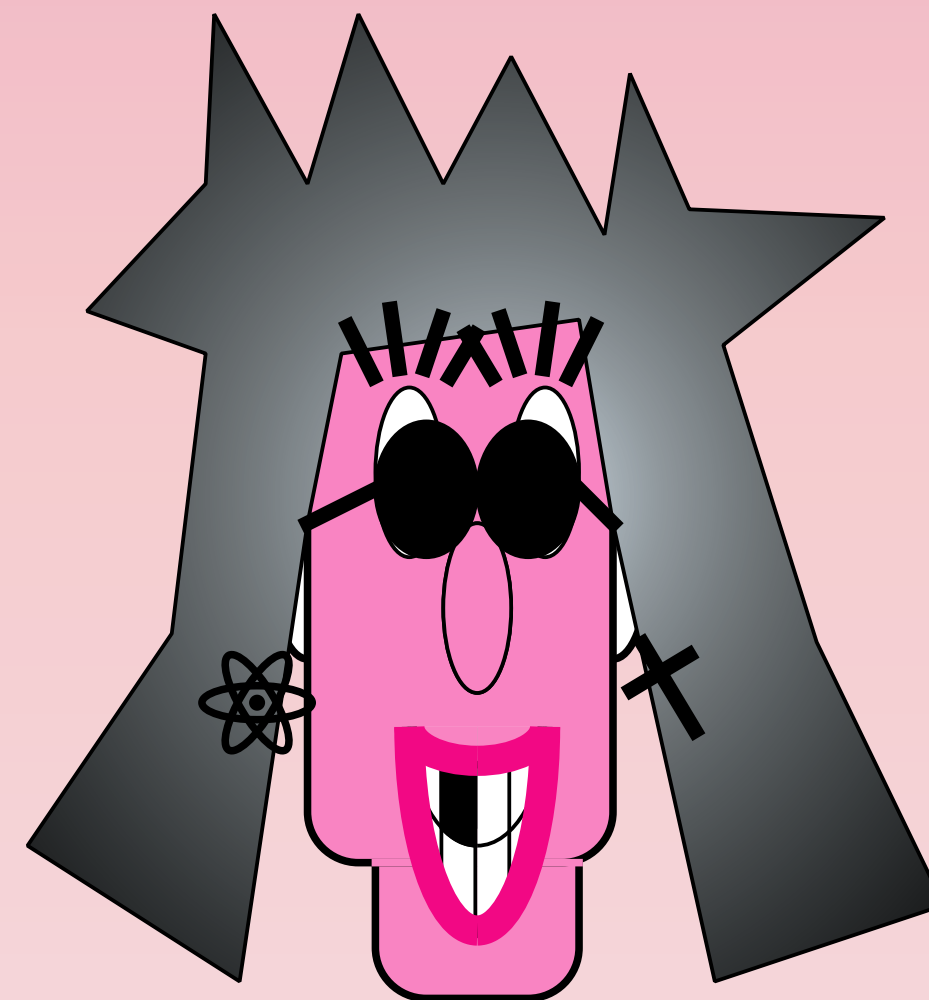
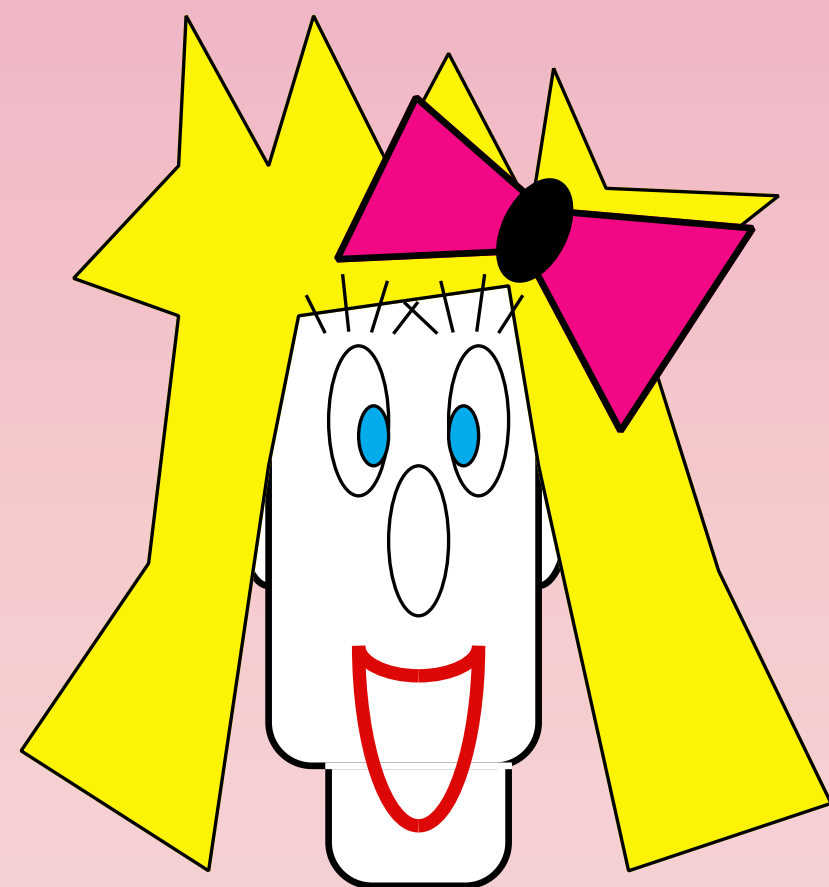
## (3.1.2) One-time pad



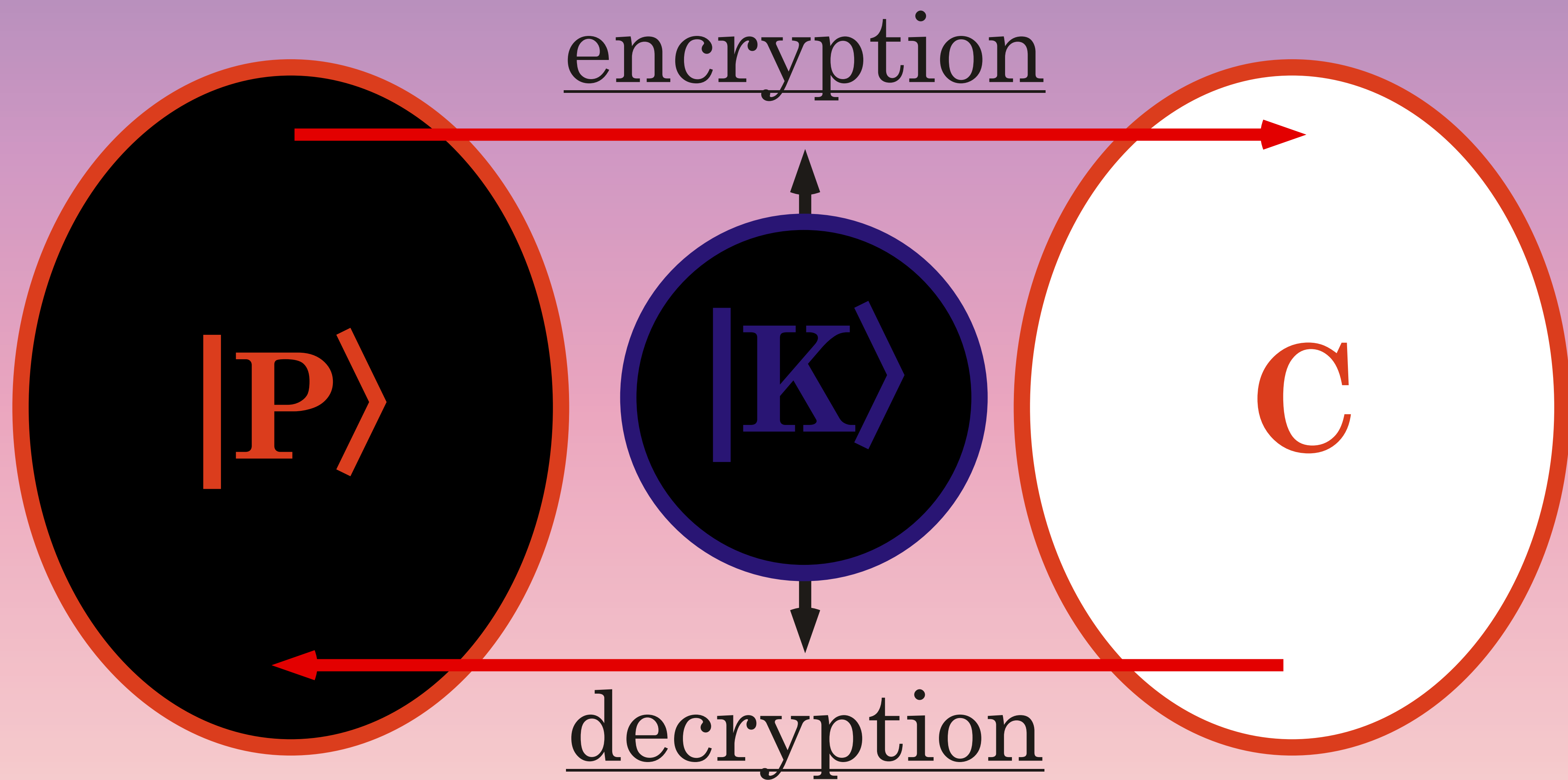
**Classical key** : Vernam **Q**-cipher (various sources)  
**Quantum Ciphertext**



**Quantum key** : one-time **Q**-pad (**Q**-teleportation)  
**Classical Ciphertext** (BBCJPW)

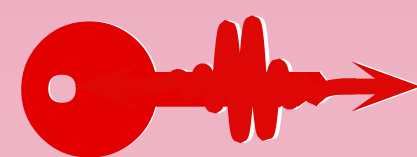
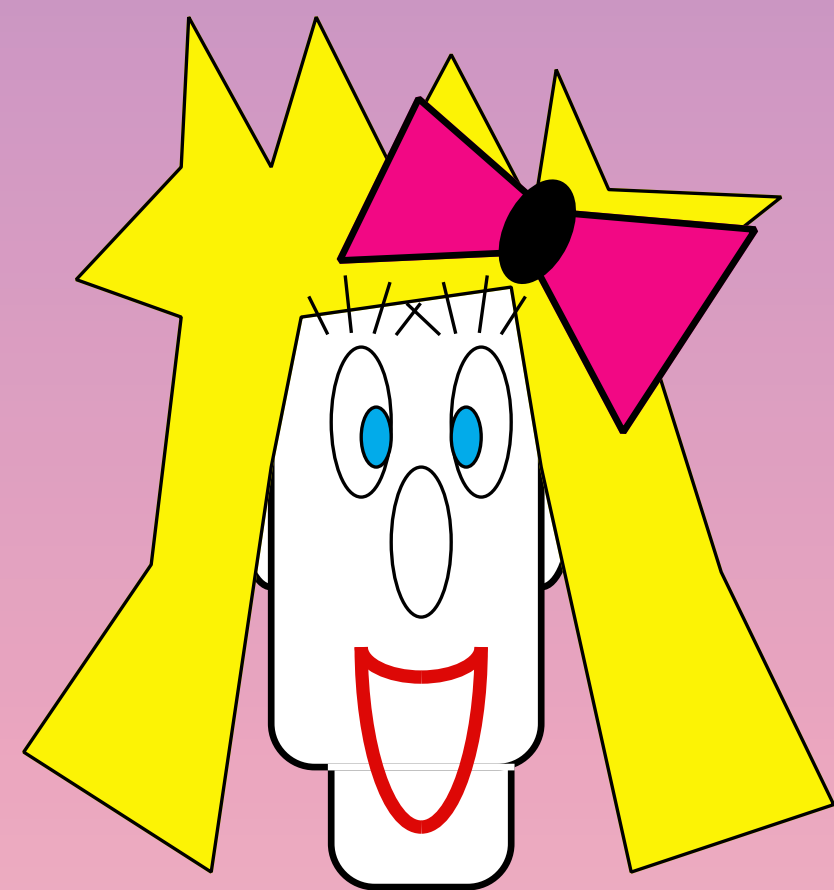
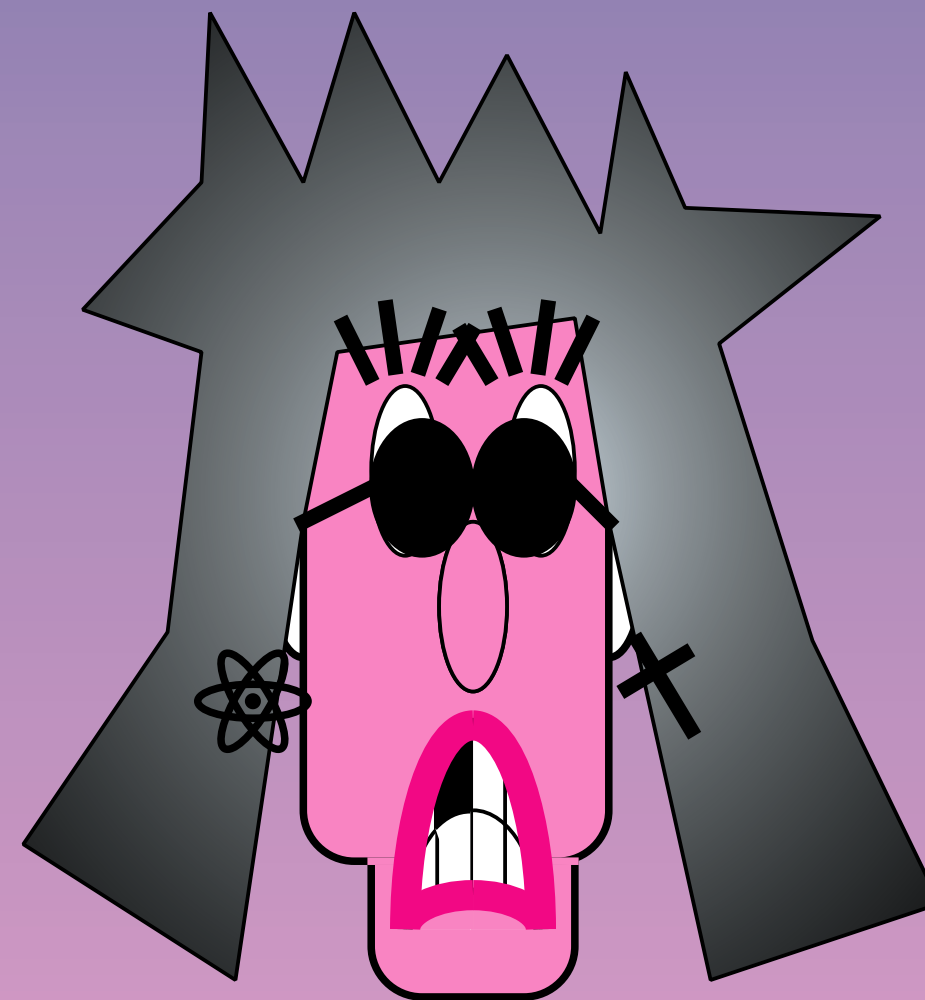


symmetric encryption  
of Quantum messages

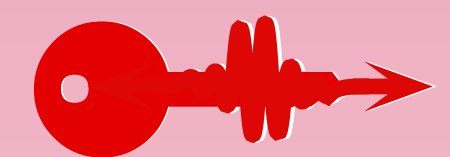
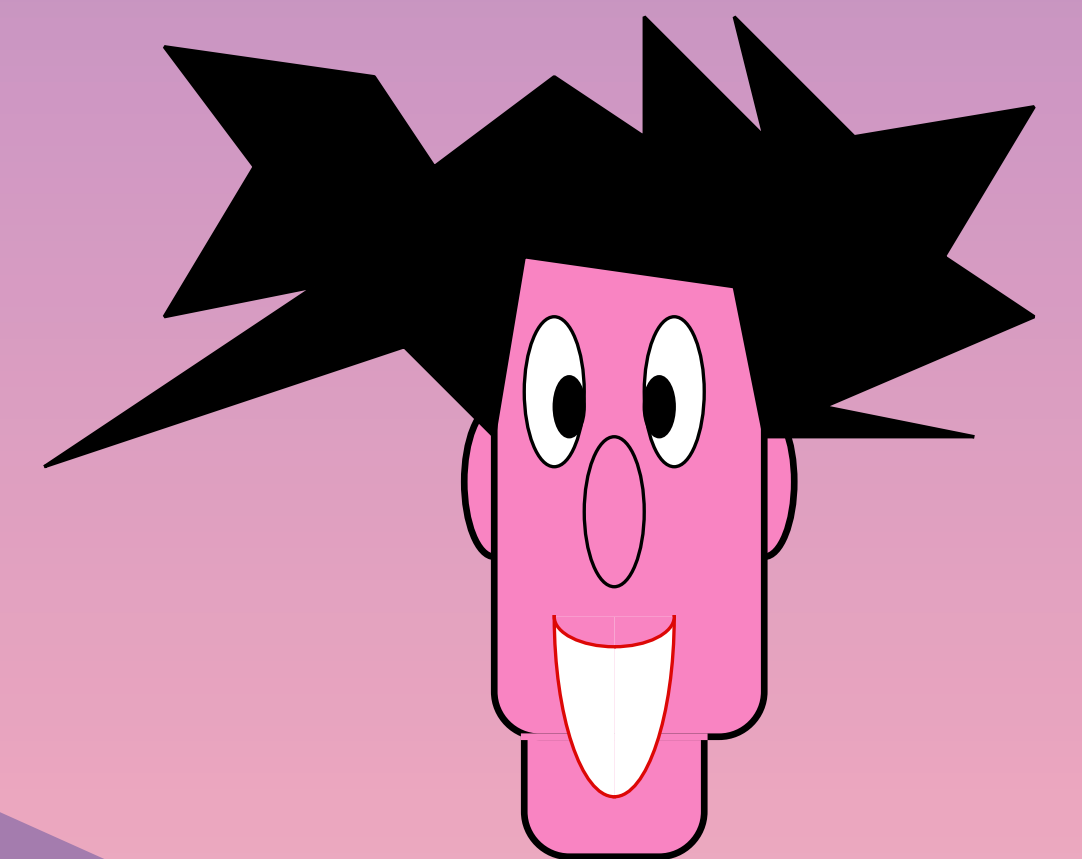


**Information Theoretical Security**

# One-time Q-pad



8RdewtU5qkLa\$es!T9@

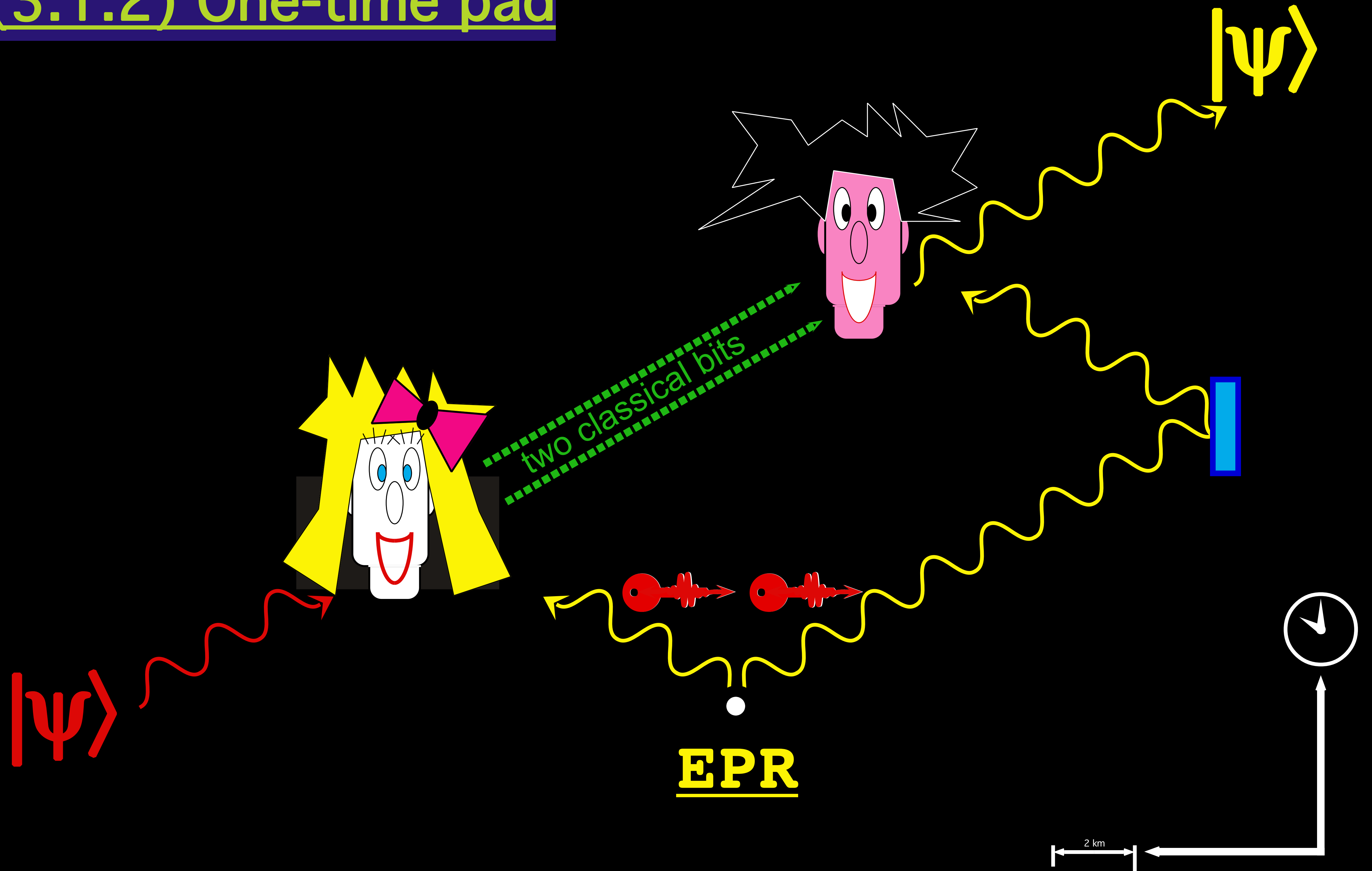


I(D%eXhDqliykl#2cV7dEwnMs

H&fs@tyHvFGhaOKpTrGbl.Z/rUih\*

B7B3tdsjUila

## (3.1.2) One-time pad

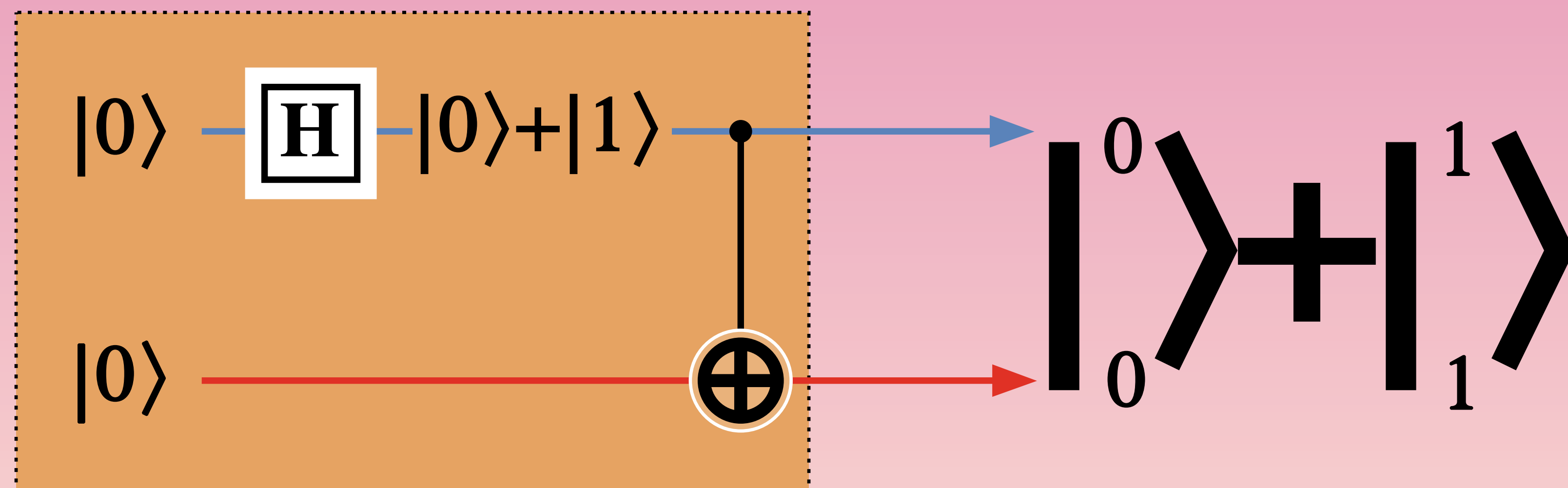
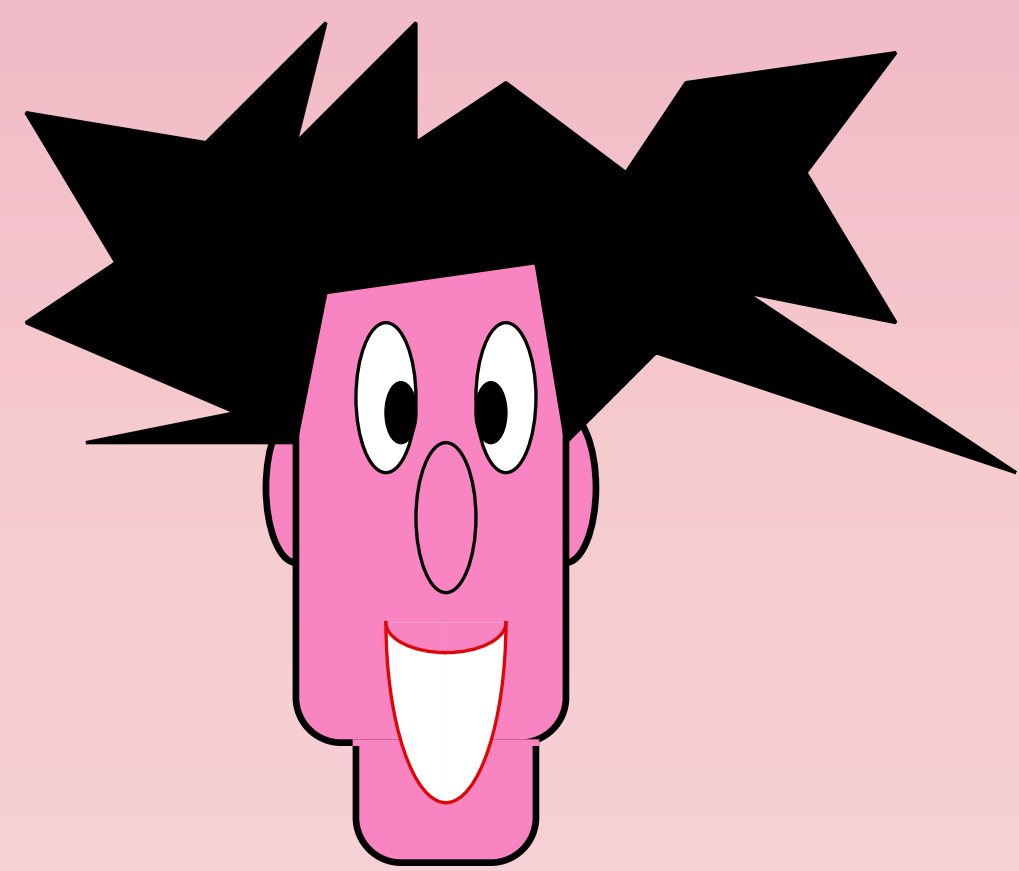
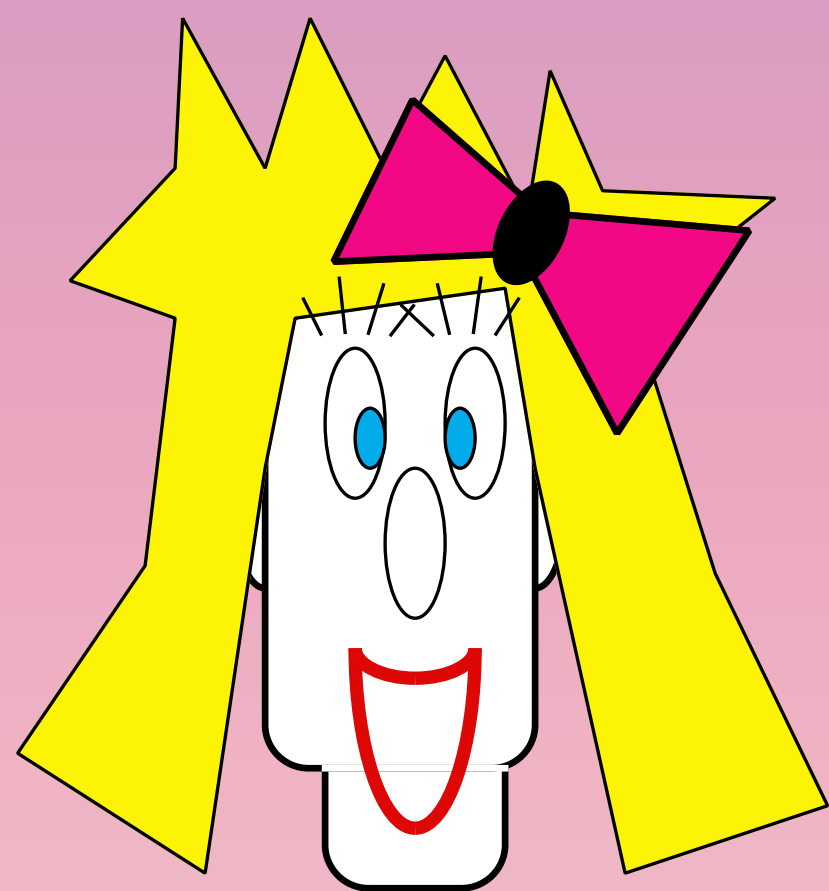


$$|0\rangle \xrightarrow{\mathbf{H}} |0\rangle + |1\rangle$$

$$|1\rangle \xrightarrow{\mathbf{H}} |0\rangle - |1\rangle$$

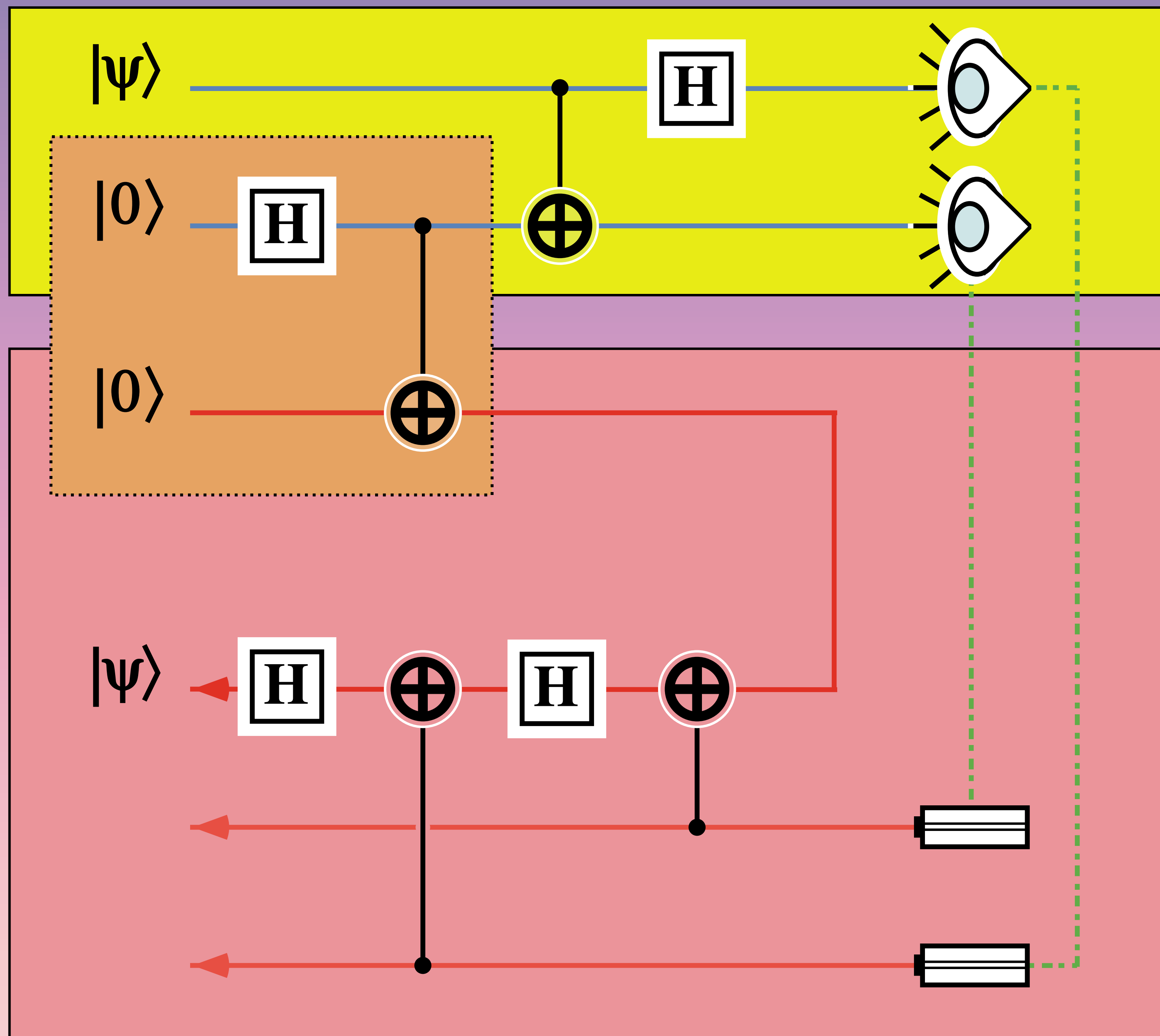
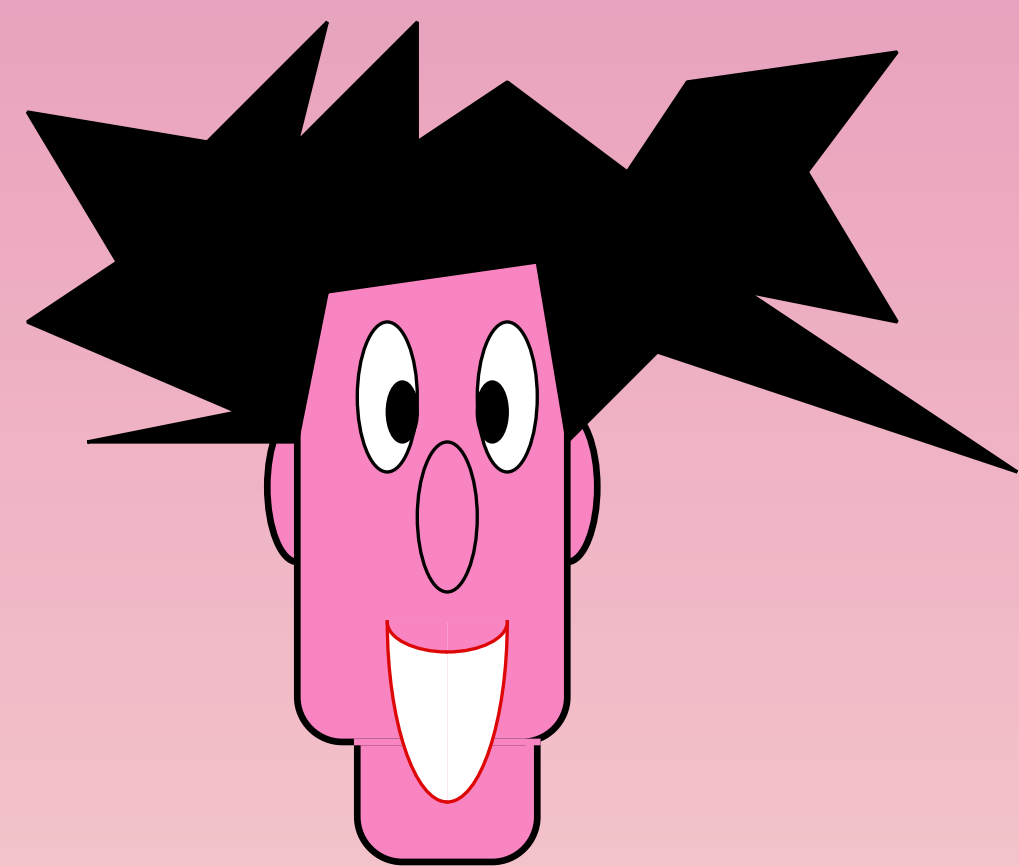
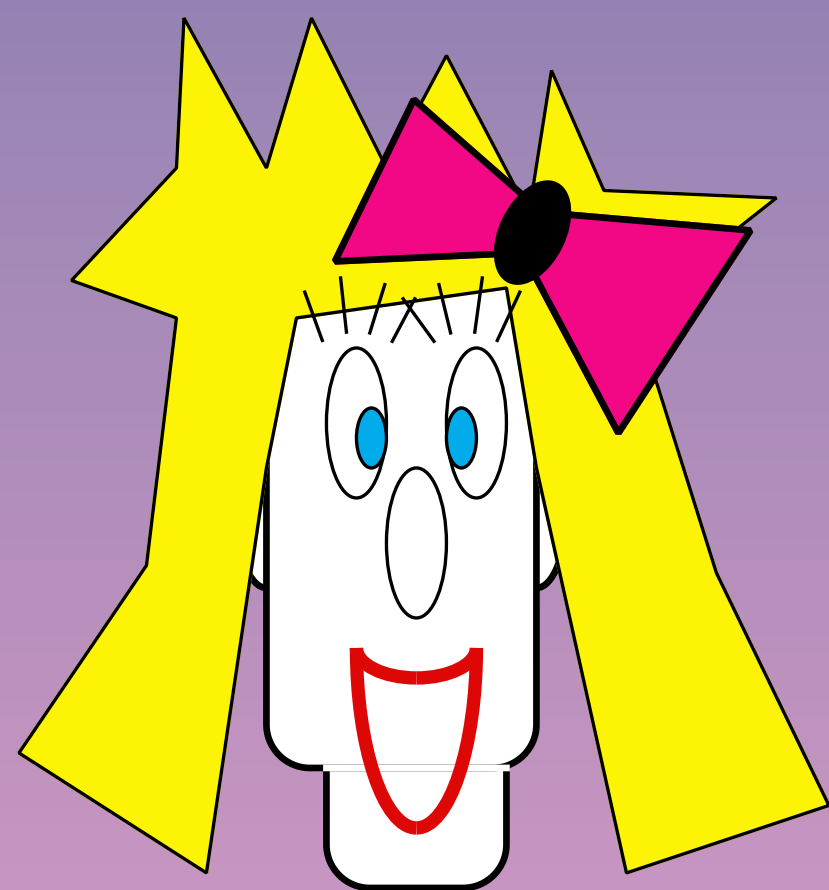
$$|x\rangle \xrightarrow{\text{Control}} |x\rangle$$

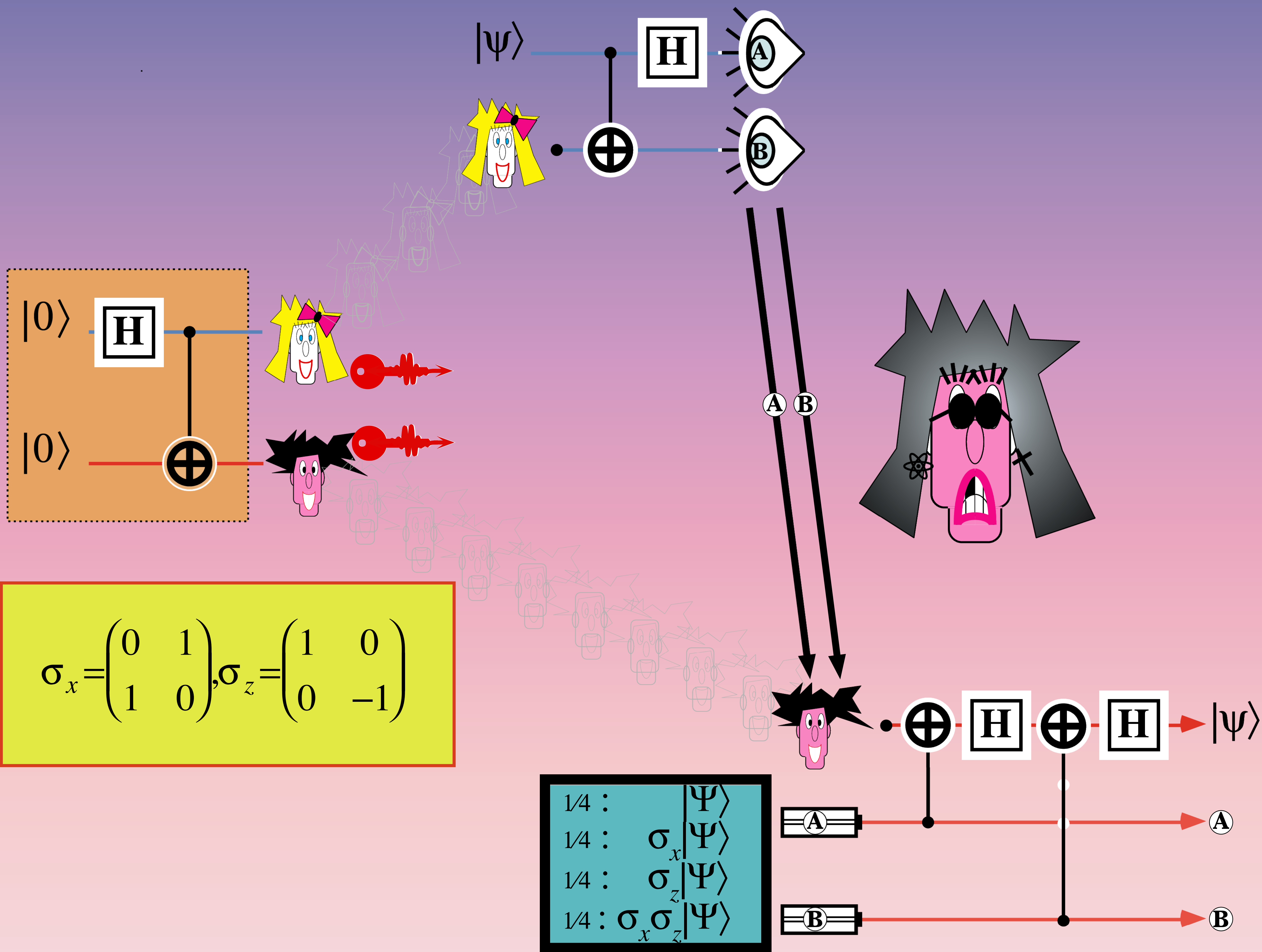
$$|y\rangle \xrightarrow{\oplus} |y \oplus x\rangle$$



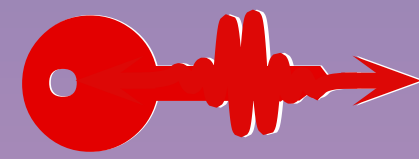
$|??\rangle$



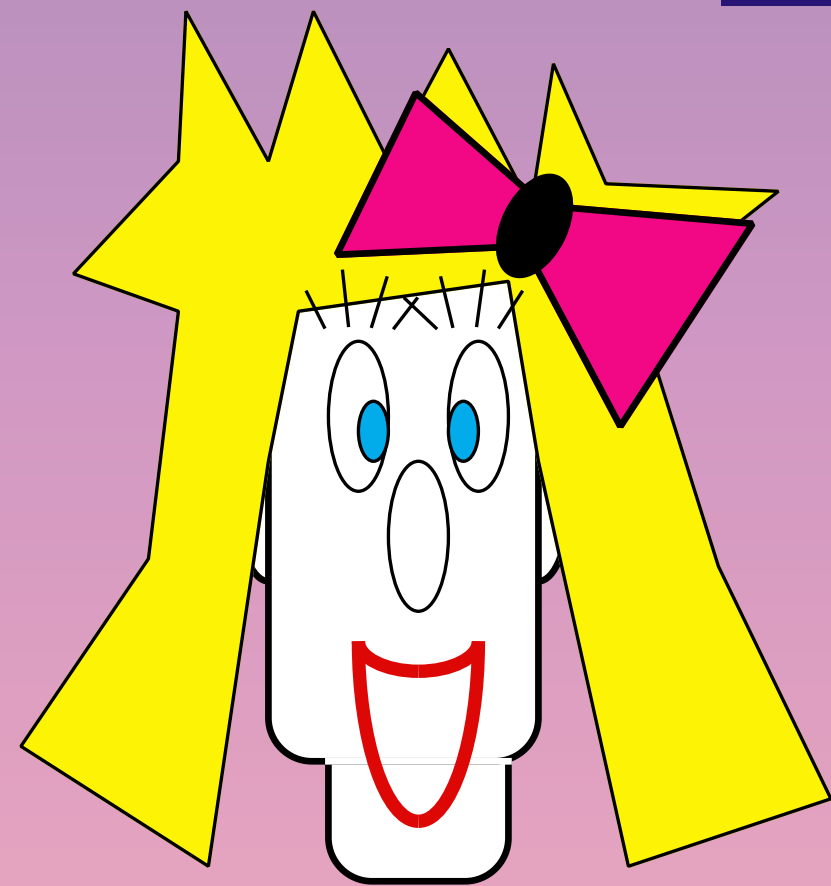




# (3.1.2b) one-time pad



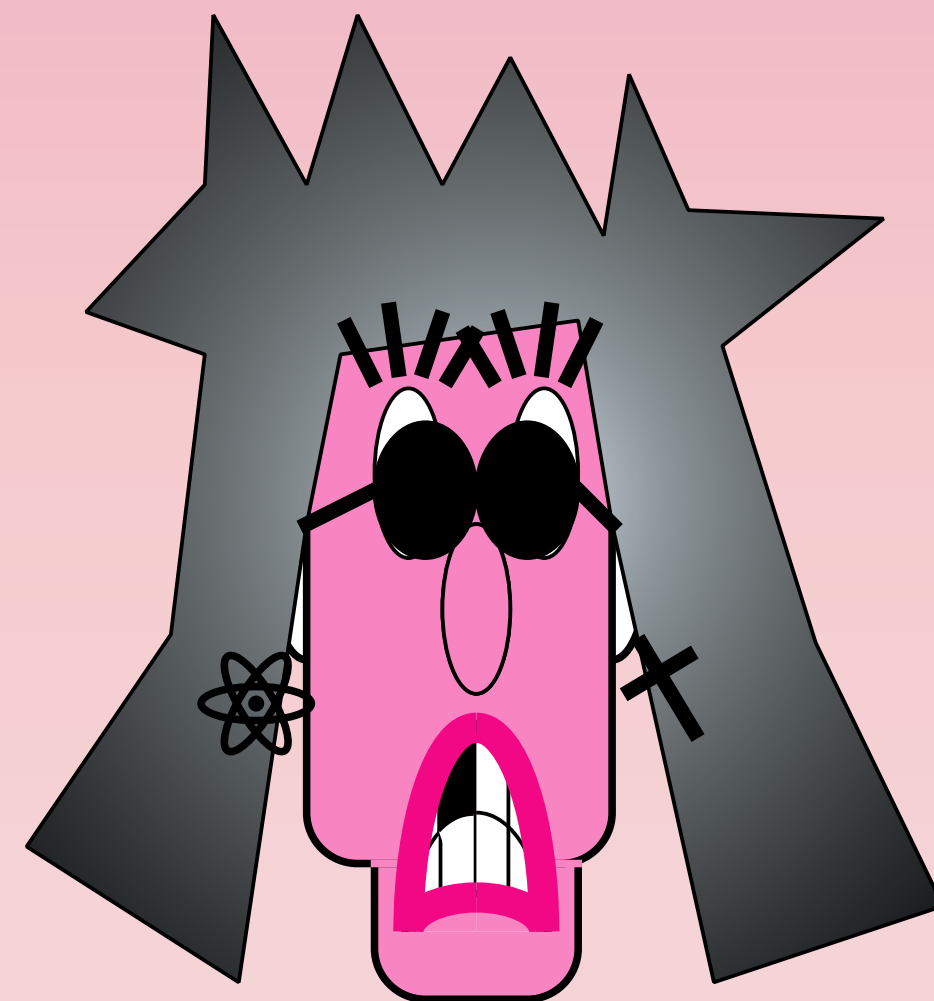
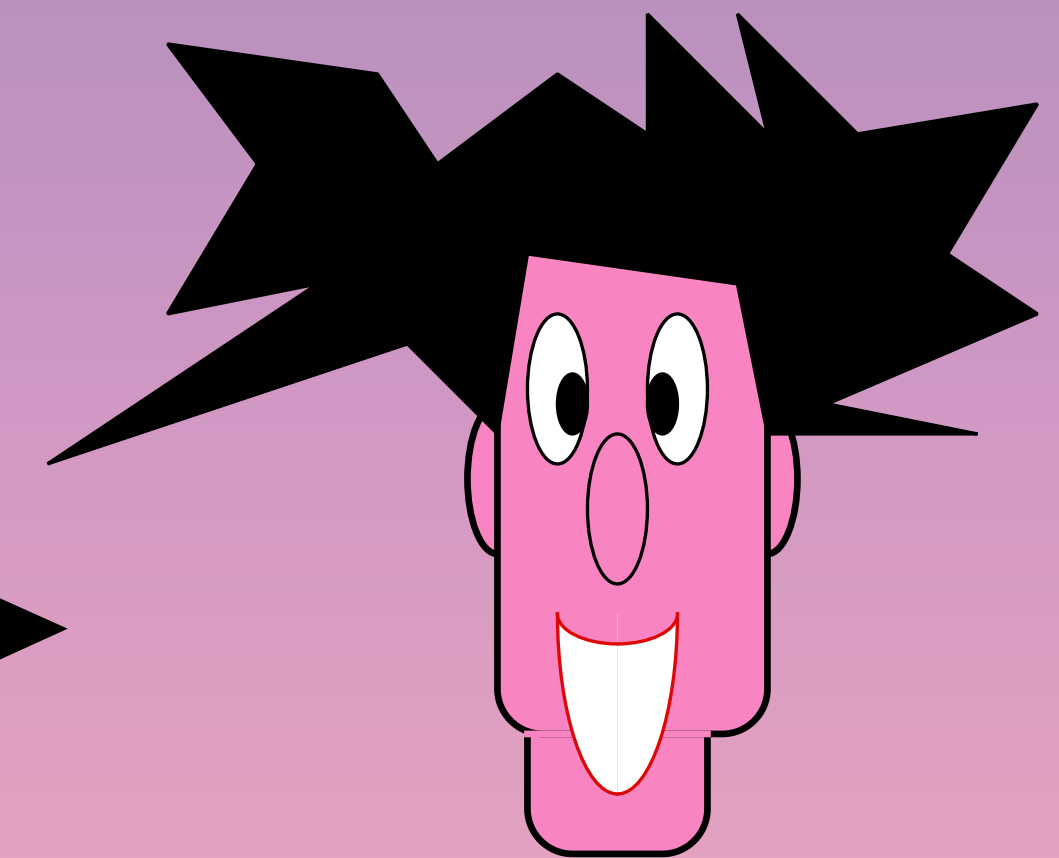
Quantum key : one-time **Q**-pad  
Classical Ciphertext



$|\Psi\rangle$

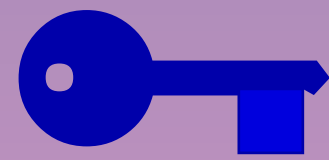


two random bits

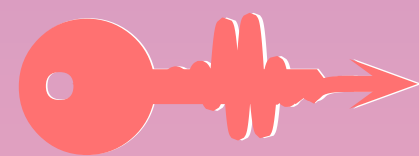




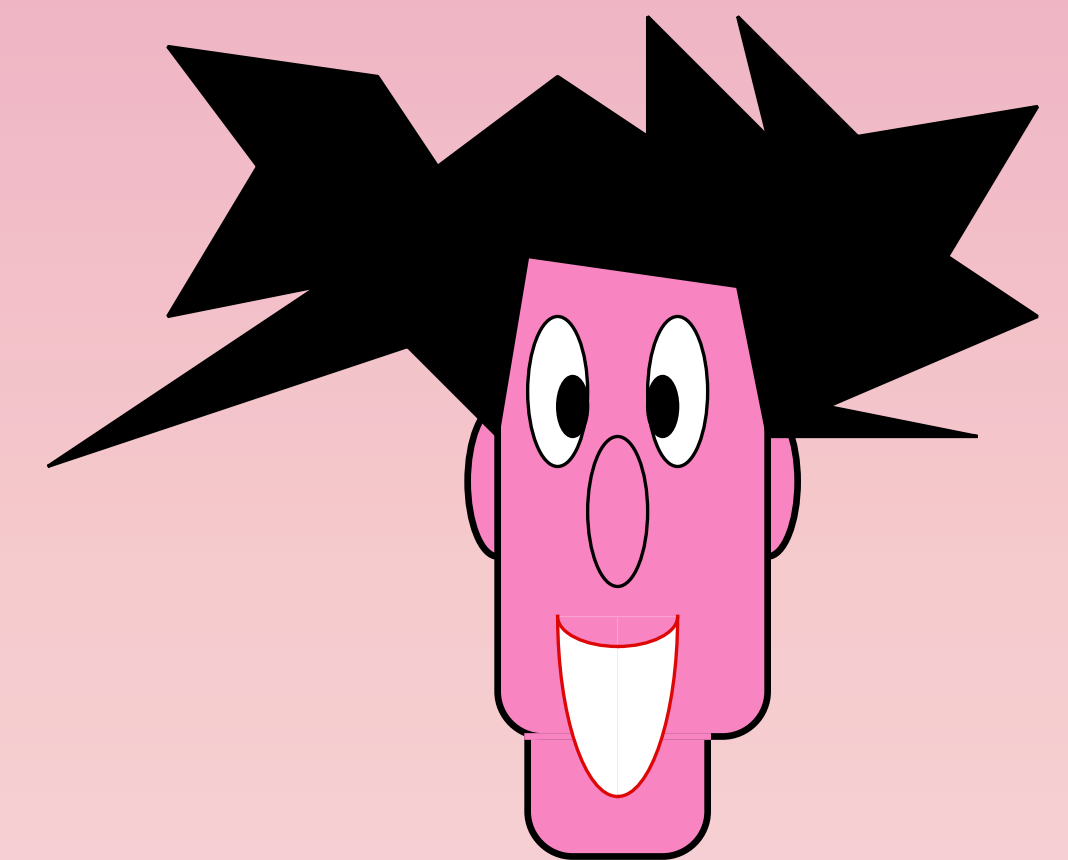
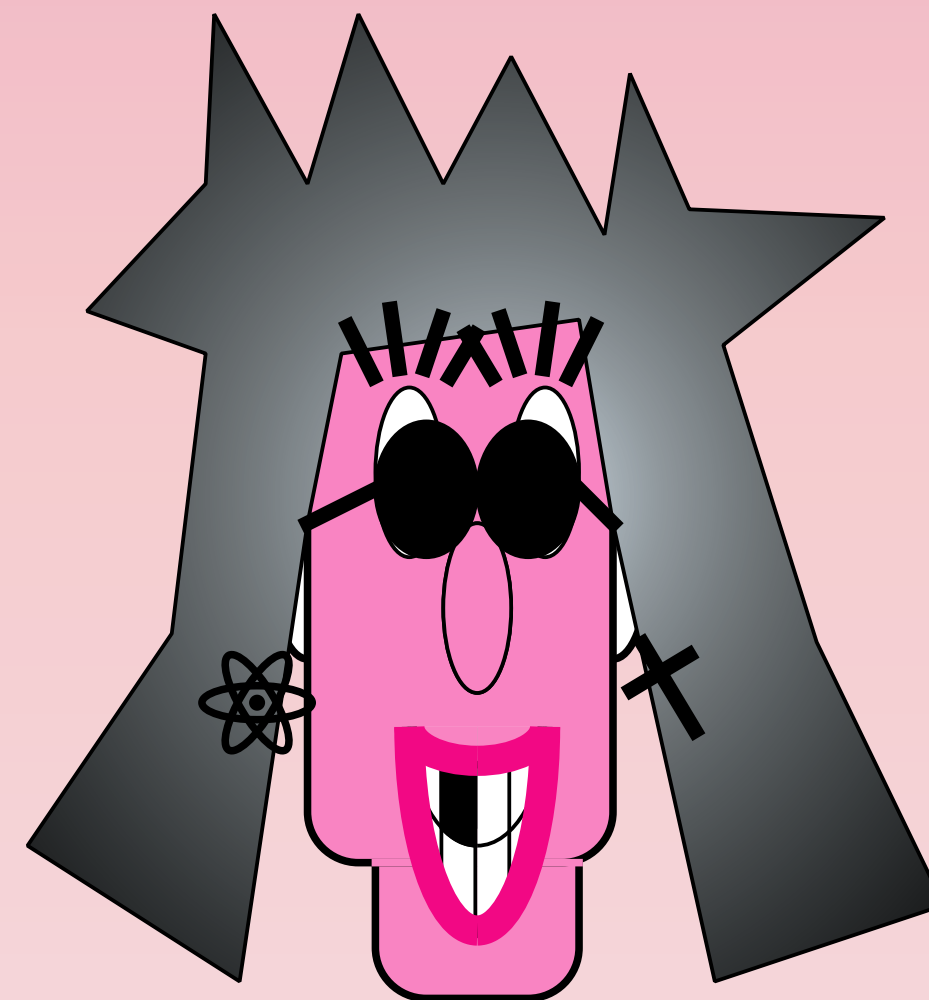
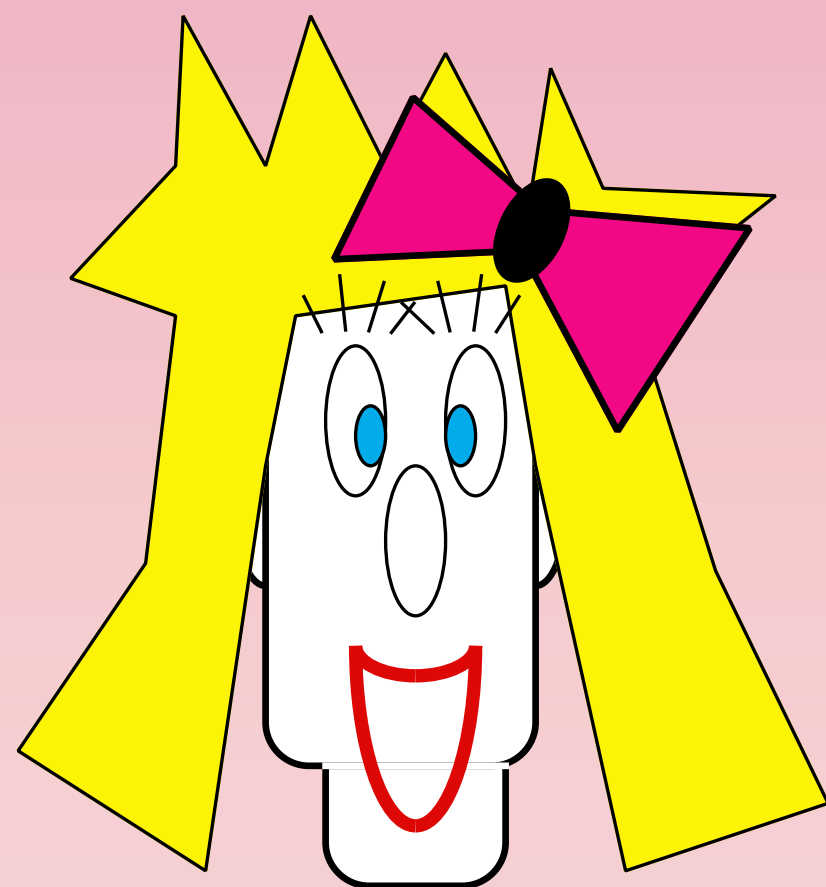
## (3.1.2a) One-time pad



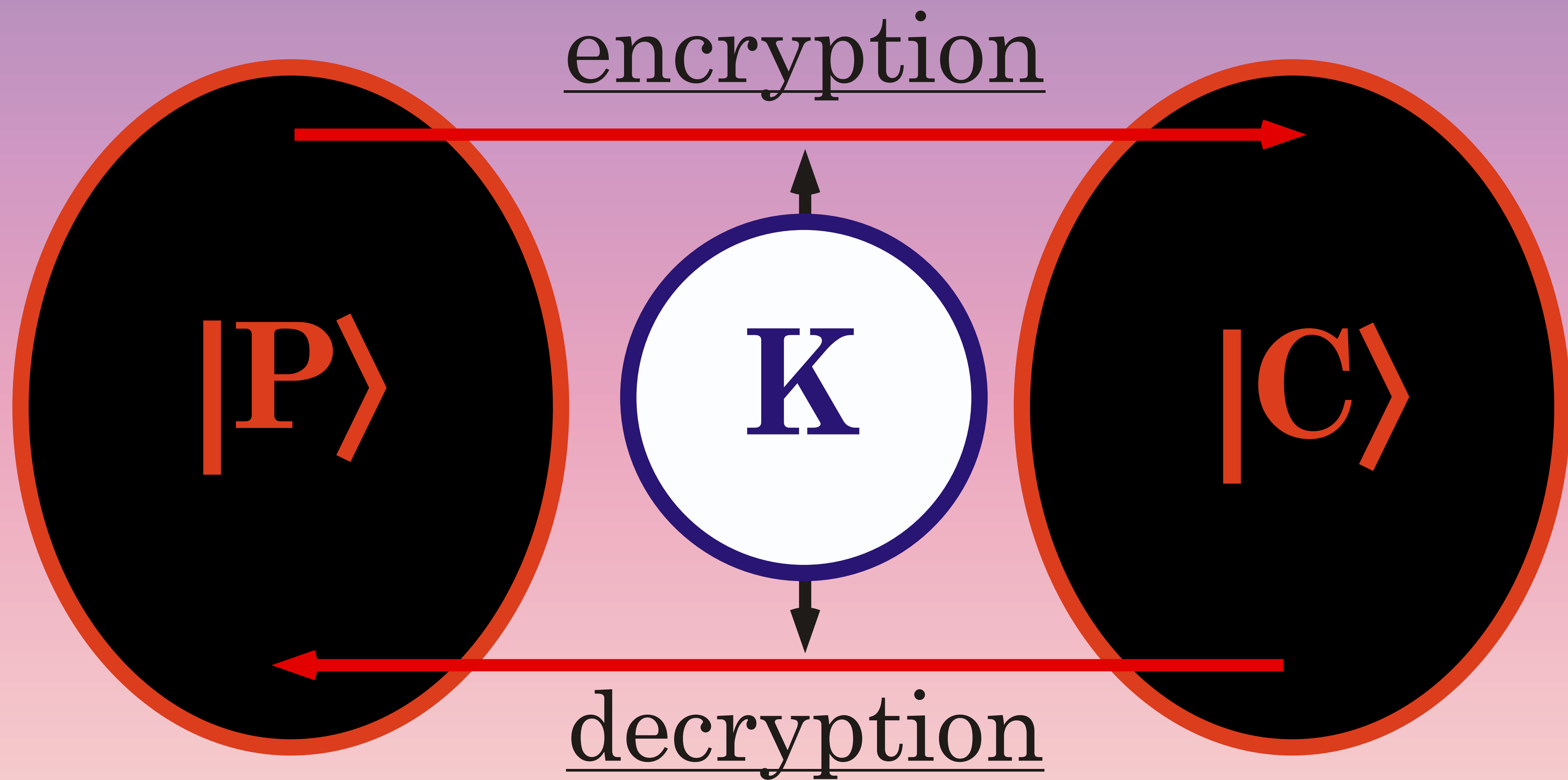
**Classical key** : Vernam **Q**-cipher (various sources)  
**Quantum Ciphertext**



**Quantum key** : one-time **Q**-pad (BBCJPW)  
**Classical Ciphertext**

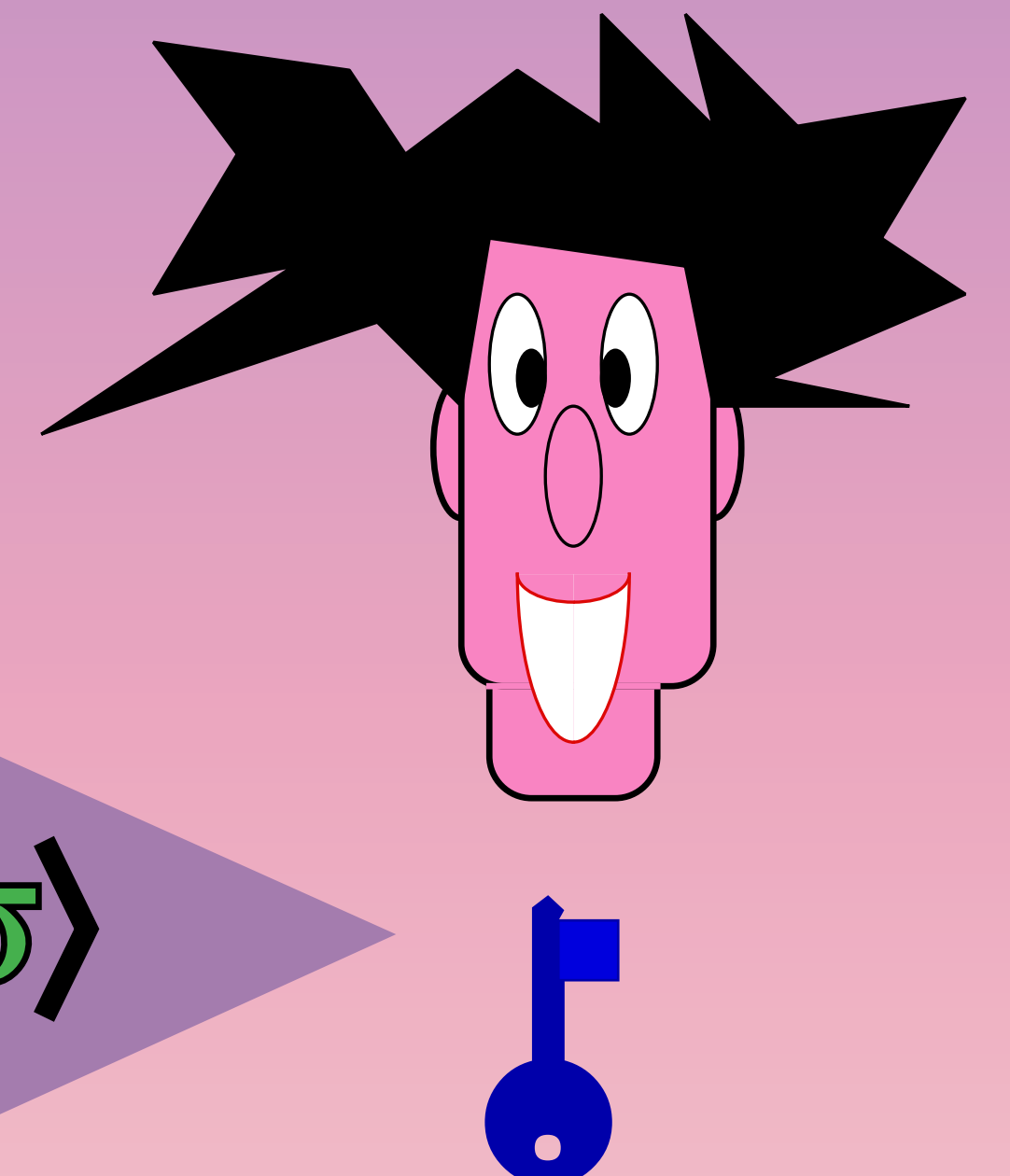
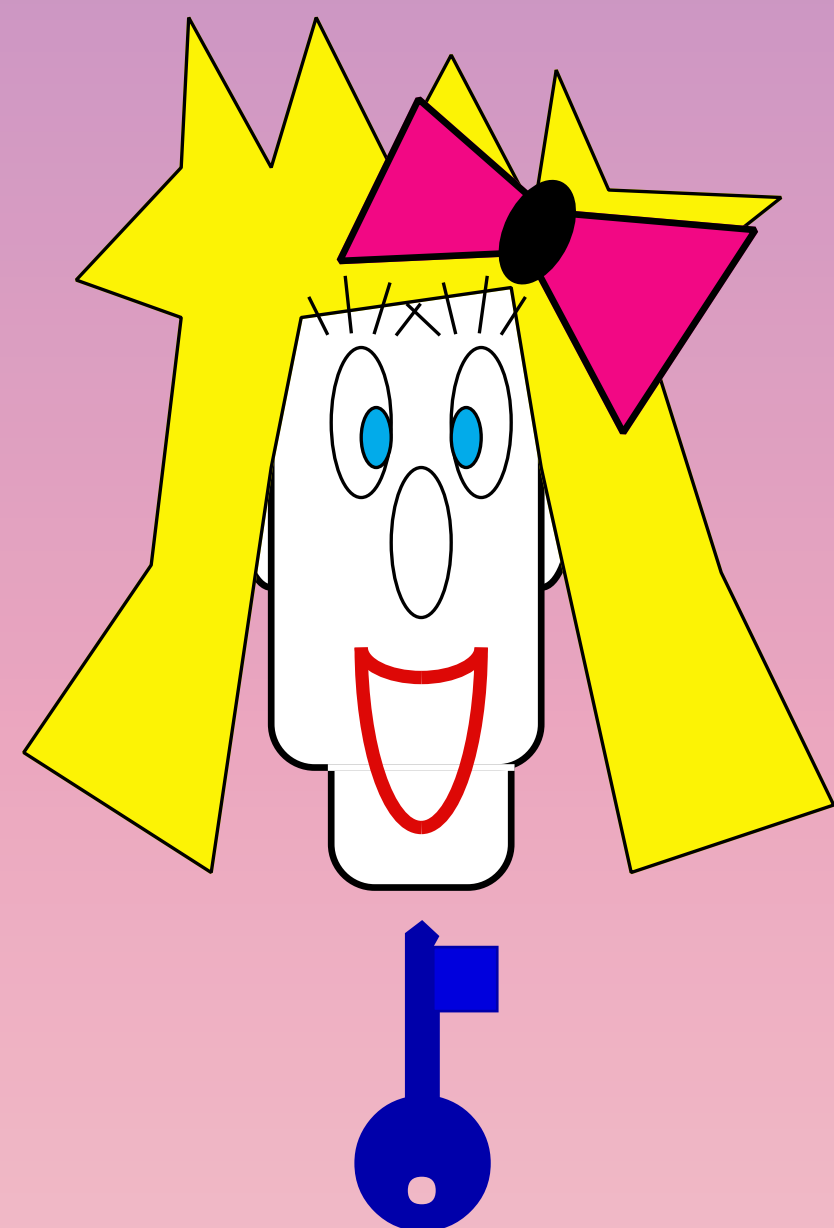
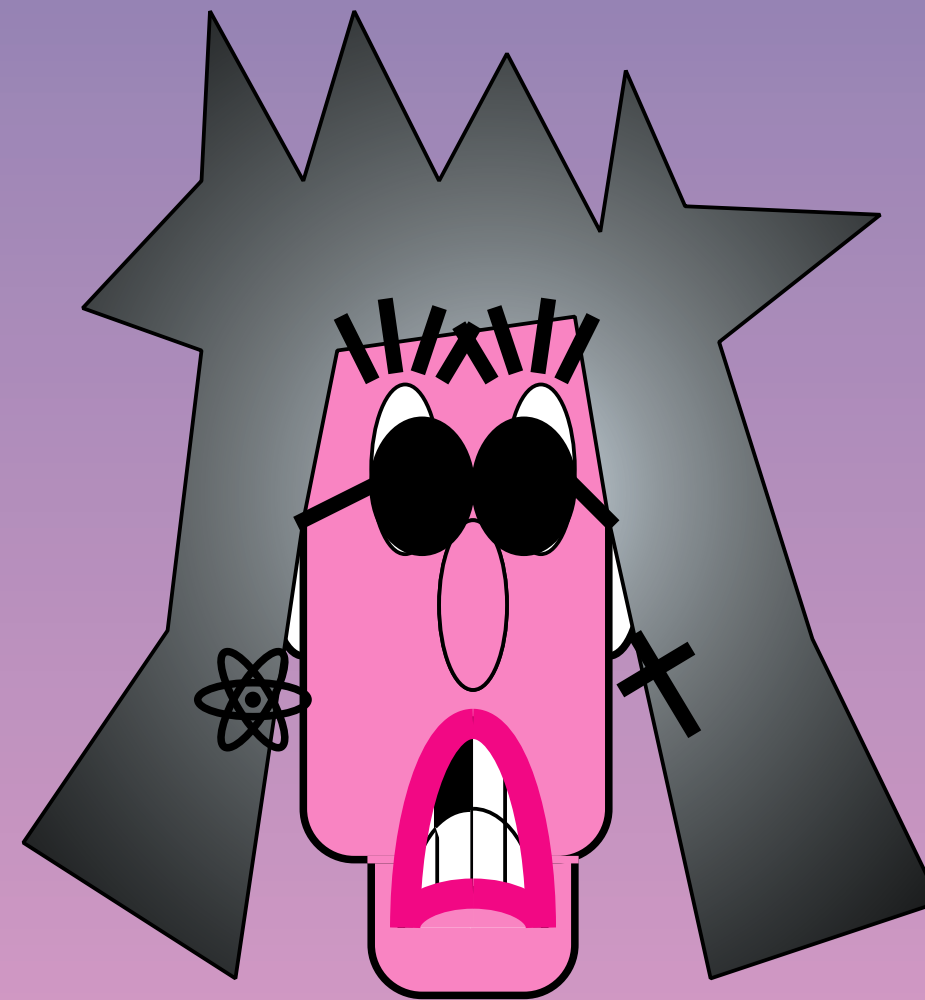


symmetric encryption  
of Quantum messages



**Information Theoretical Security**

# Vernam Q-cipher



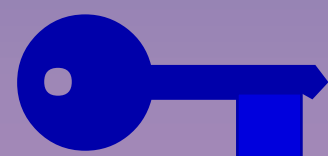
|8PδεωτΥ5θκλαΞεσ!Τ9≅|

|Ι(Δ%εΞηΔθΙιψκλ#2χς7δΕωνΜσ)|

|Η&φσ≅τψωΦηαΟΚπΤρΓβλ.Ζ/ρΥιη\*|

|Β7Β3τδσφΥιλα|

# (3.1.2a) one-time pad

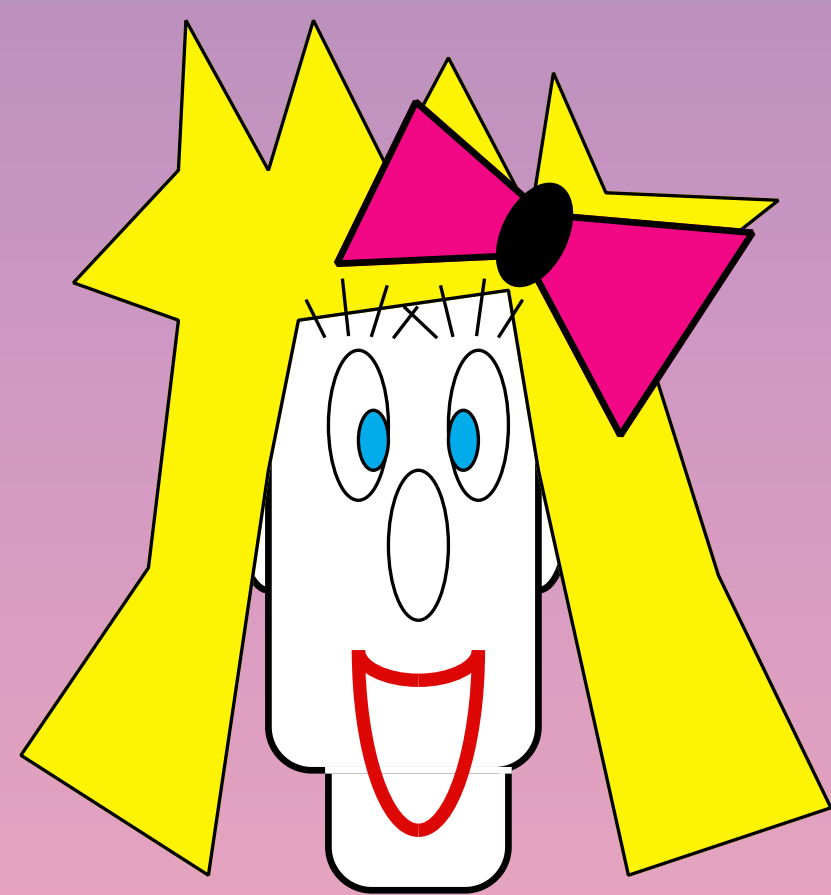


Classical key : Vernam Q-cipher

Quantum Ciphertext

Quantum key : one-time Q-pad

Classical Ciphertext

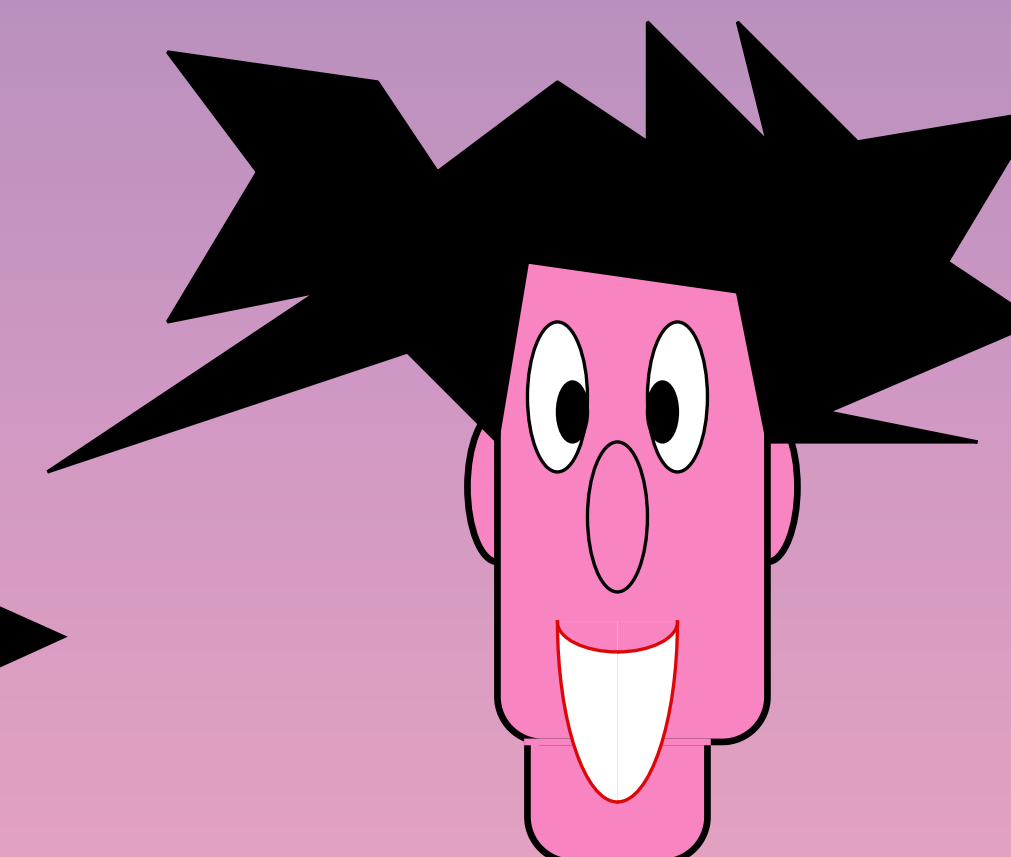


$|\Psi\rangle$

a,b random bit key

$$|\Psi'\rangle = \sigma_x^a \sigma_z^b |\Psi\rangle$$

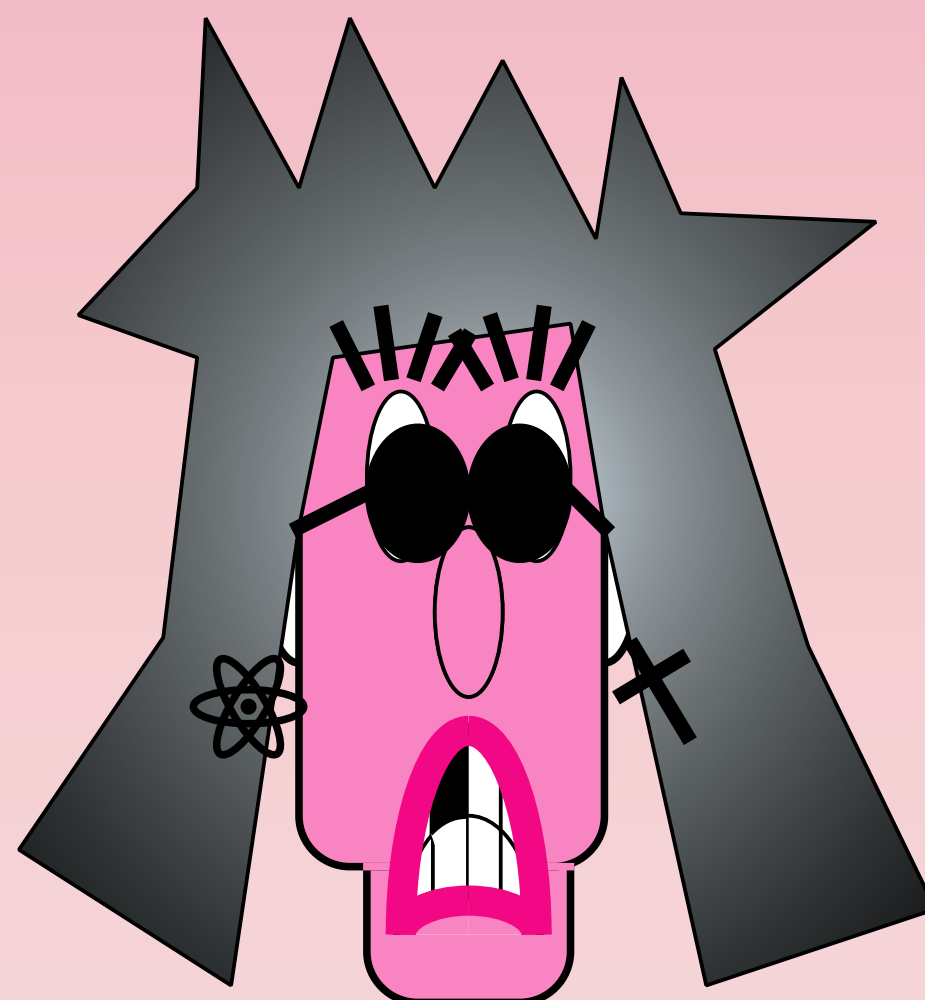
$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



1/4 :		$ \Psi\rangle$
1/4 :	$\sigma_x$	$ \Psi\rangle$
1/4 :	$\sigma_z$	$ \Psi\rangle$
1/4 :	$\sigma_x \sigma_z$	$ \Psi\rangle$

a,b random bit key

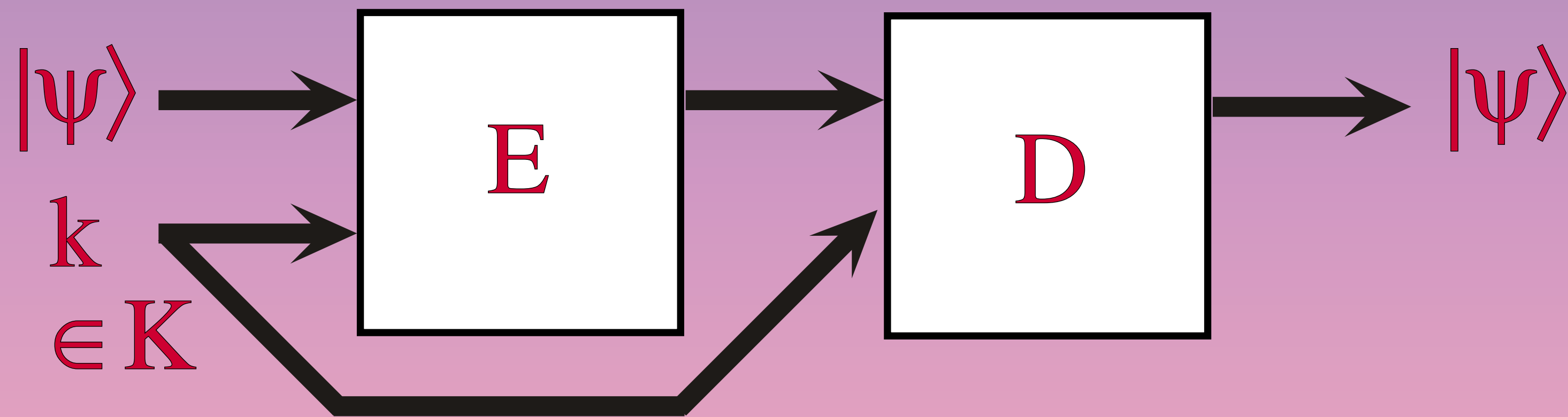
$$|\Psi\rangle = \sigma_z^b \sigma_x^a |\Psi'\rangle$$



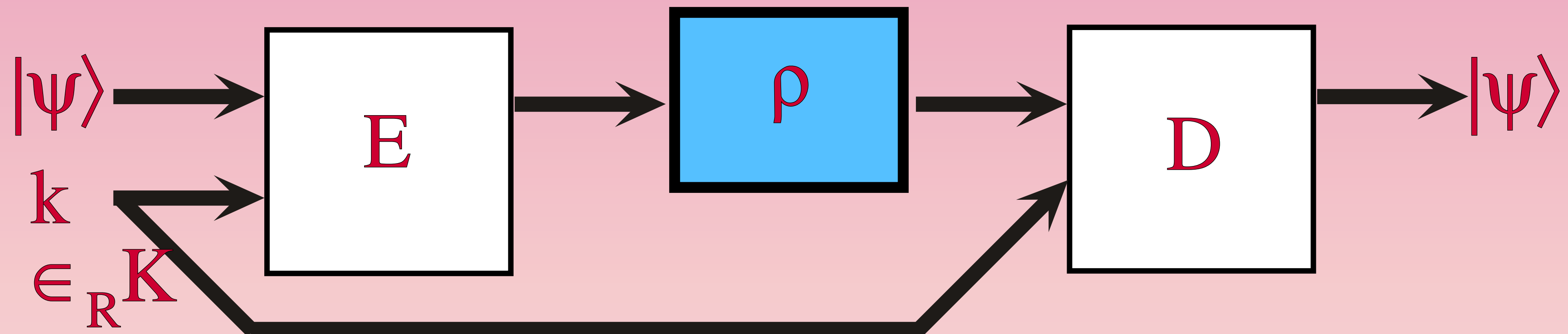


# One-time Q-encryption with error $\epsilon$

Completeness:



Secrecy:



$$\forall |\psi_0\rangle, |\psi_1\rangle \quad D(\rho_0, \rho_1) = \text{Tr}(|\rho_0 - \rho_1|) < \epsilon$$



# One-time Q-encryption with error $\epsilon > 0$

## Lower bounds:

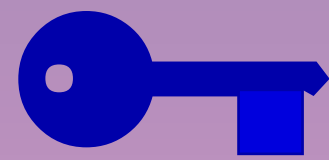
[MTW00]

Arbitrary quantum state = 2 bits / qubit

[HLSW03]

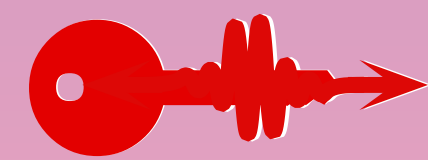
Arbitrary quantum state but not  
entangled with eavesdropper  $\sim$  1 bit / qubit

## (3.1.3) One-time Q-Authentication



Classical key : Q-Authentication (BCGST)

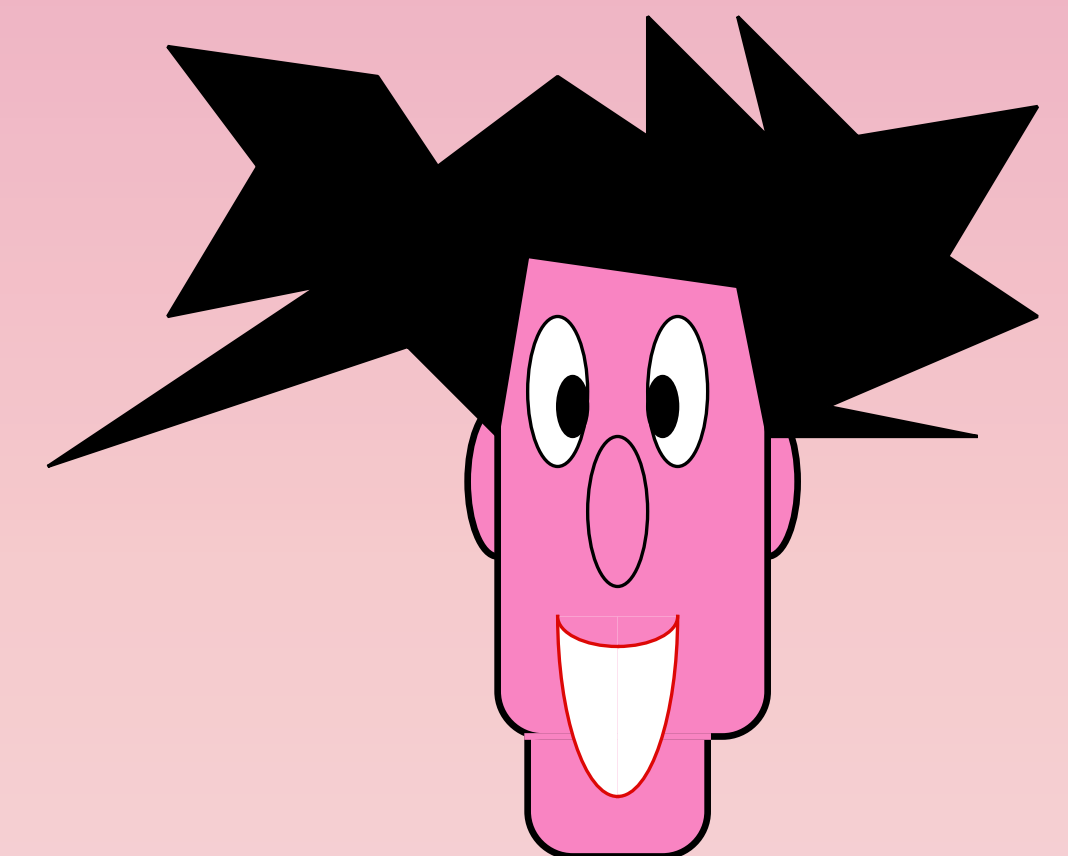
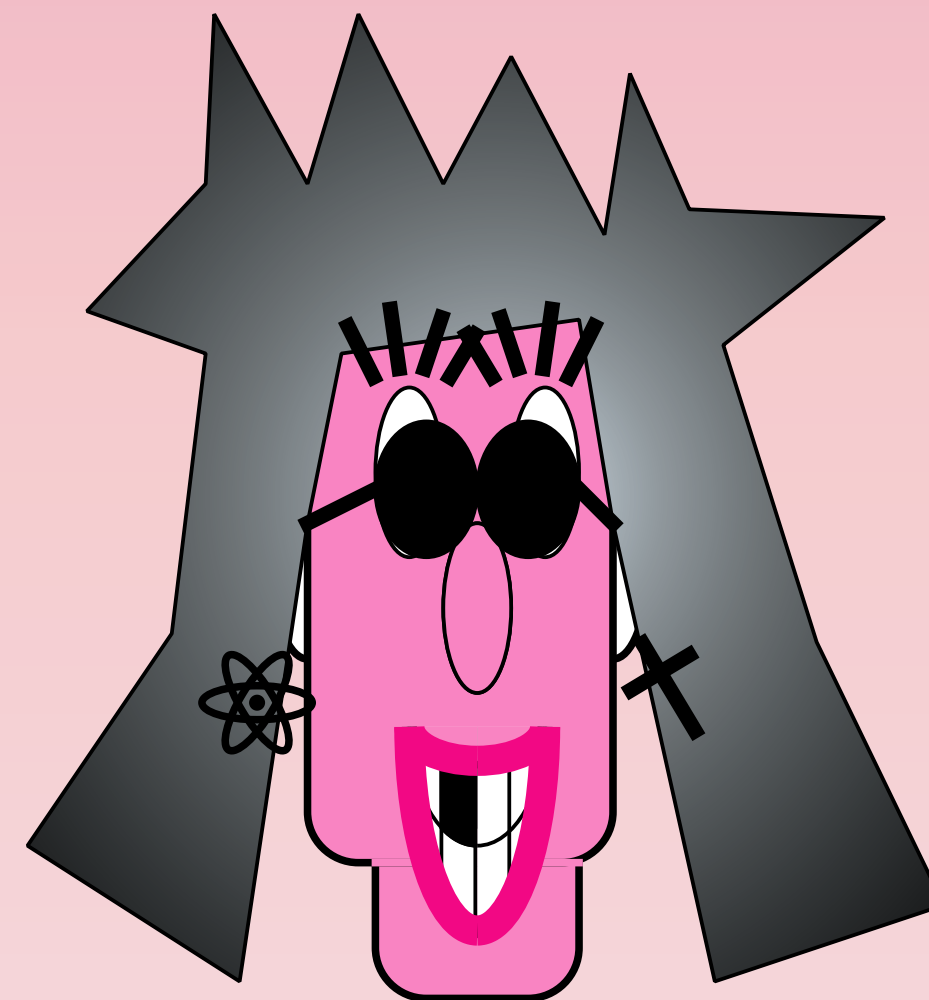
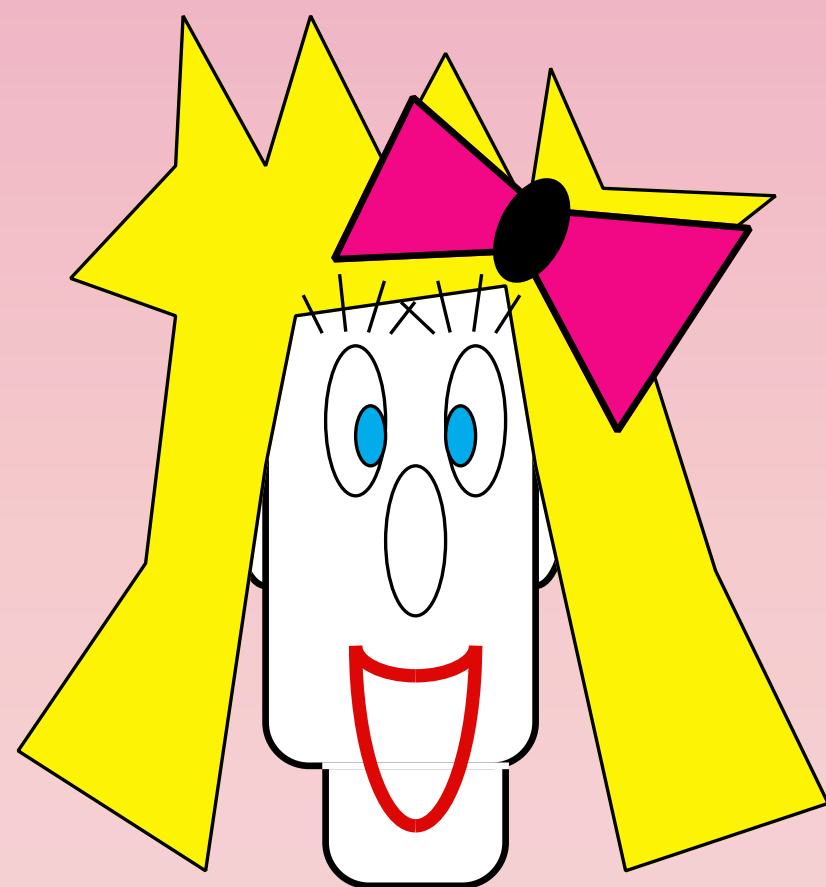
Quantum message+tag



Quantum key : Authenticated Q-teleportation

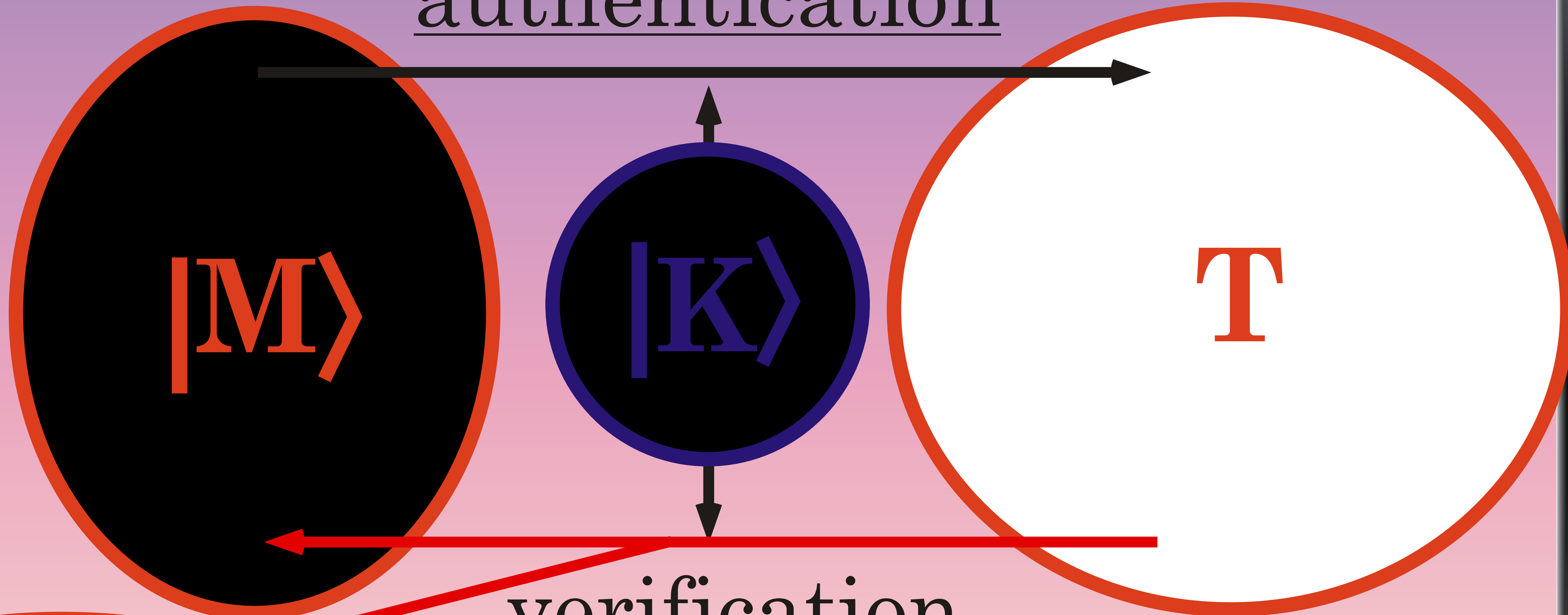
Classical message+tag

(BBCJPW)



# symmetric authentication

authentication

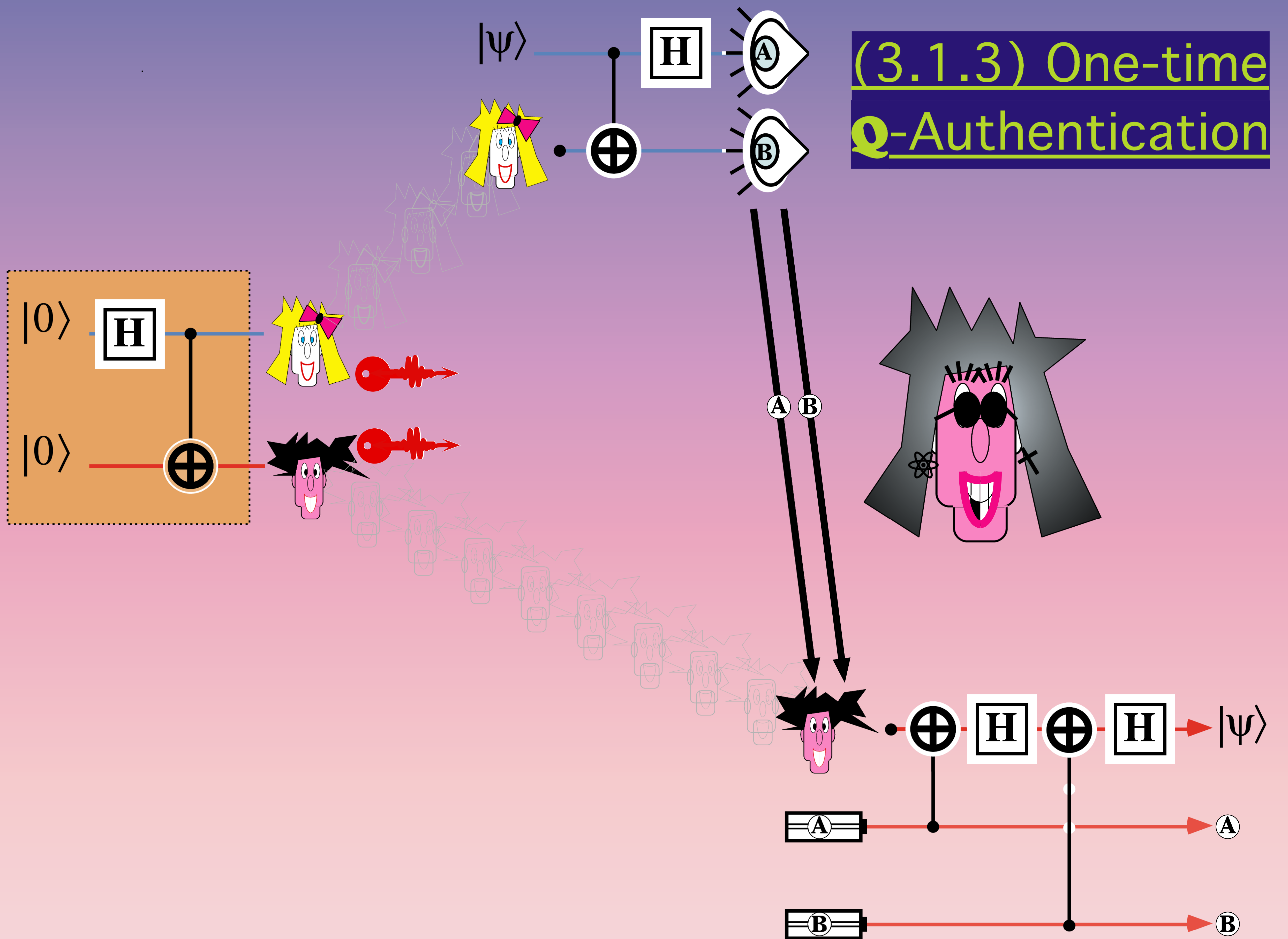


verification

$\{ACC, REJ\}$

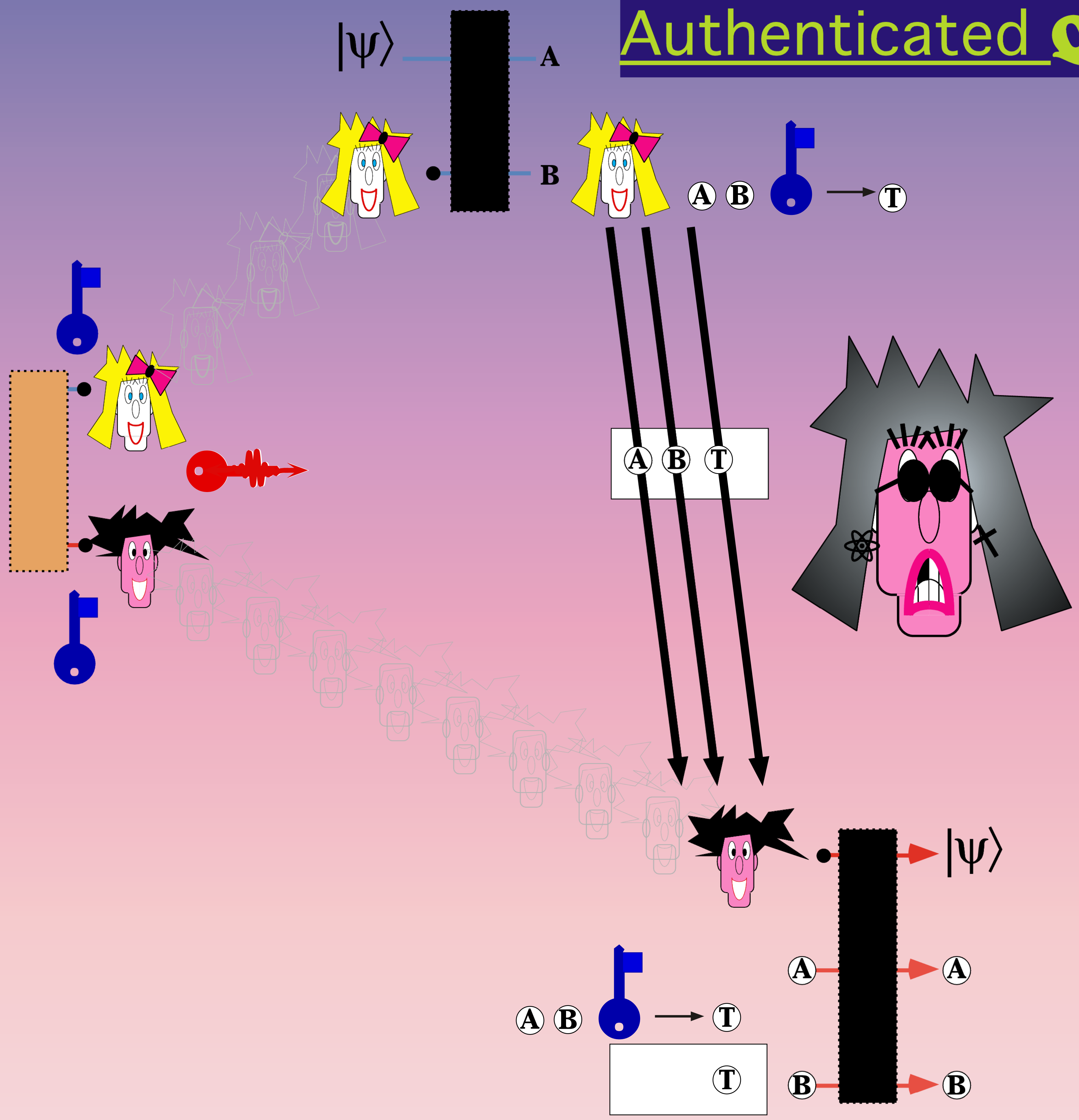
Information Theoretical Security

# (3.1.3) One-time Q-Authentication



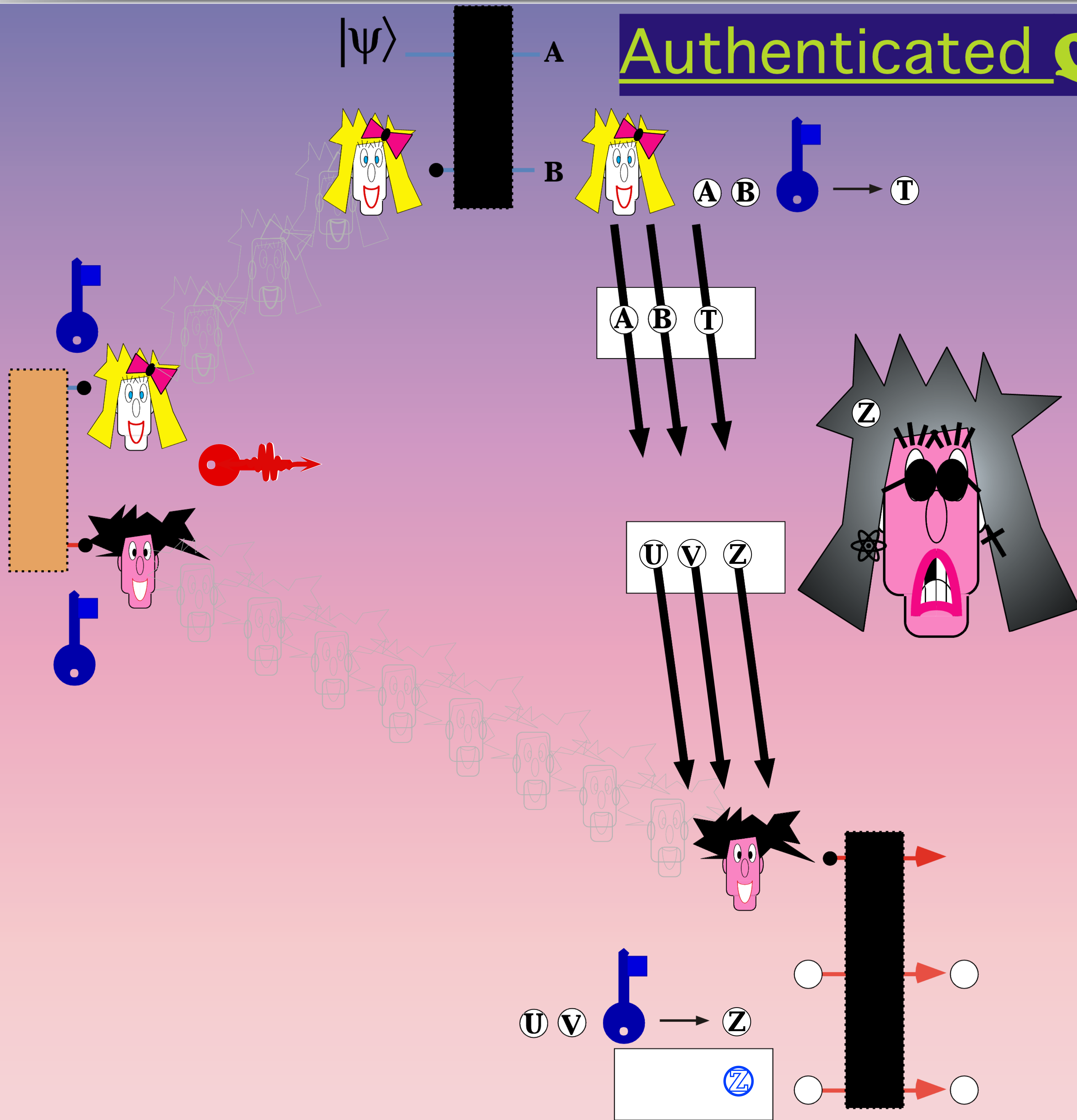


# Authenticated Q-Teleportation

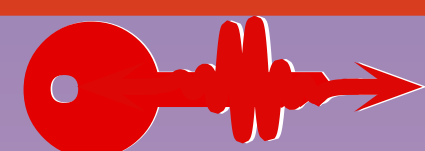




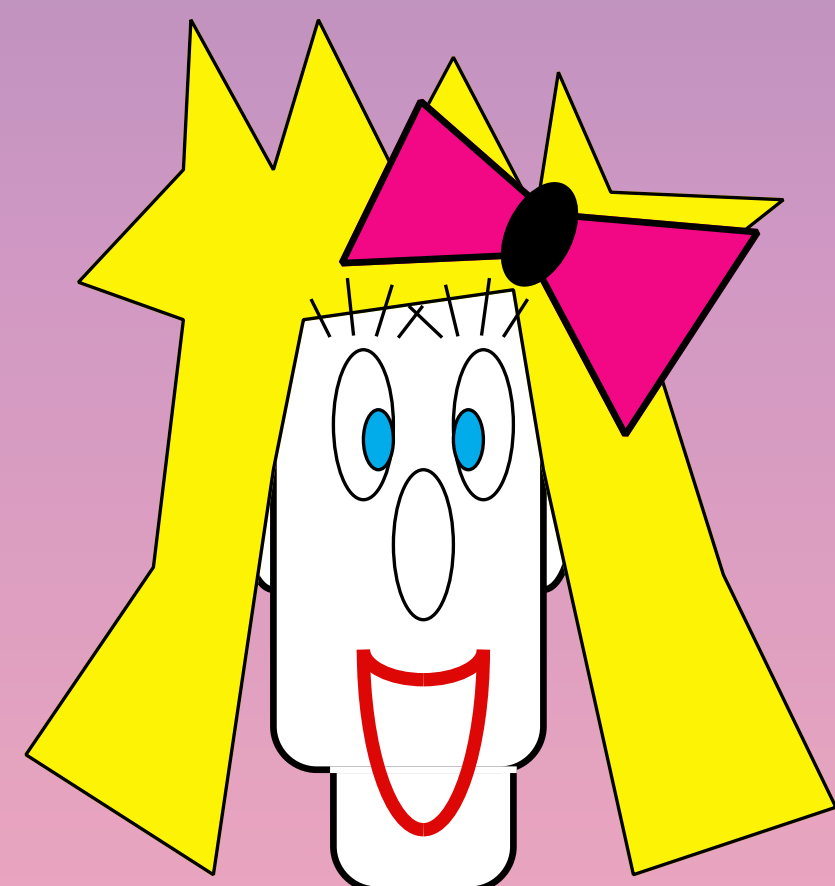
# Authenticated Q-Teleportation



# (3.1.3b) One-time Q-Authentication



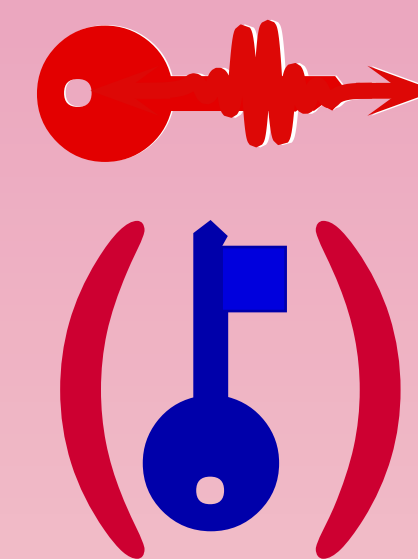
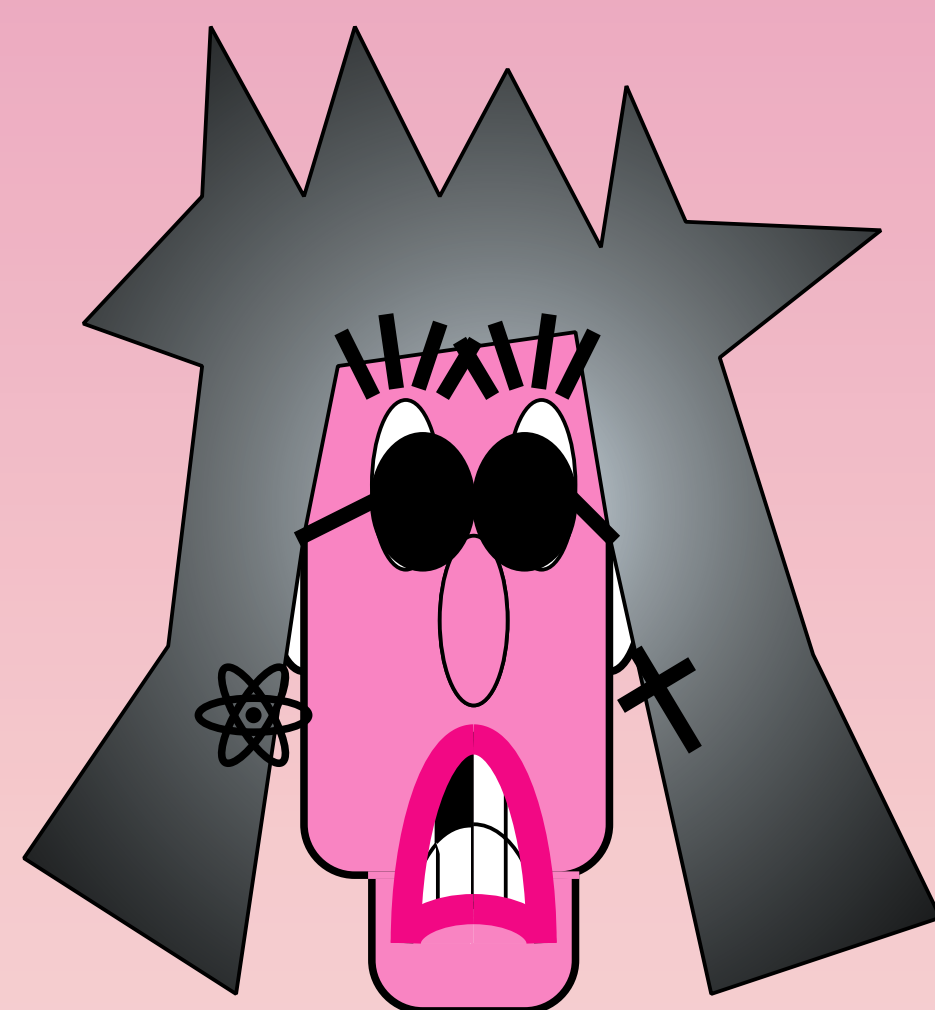
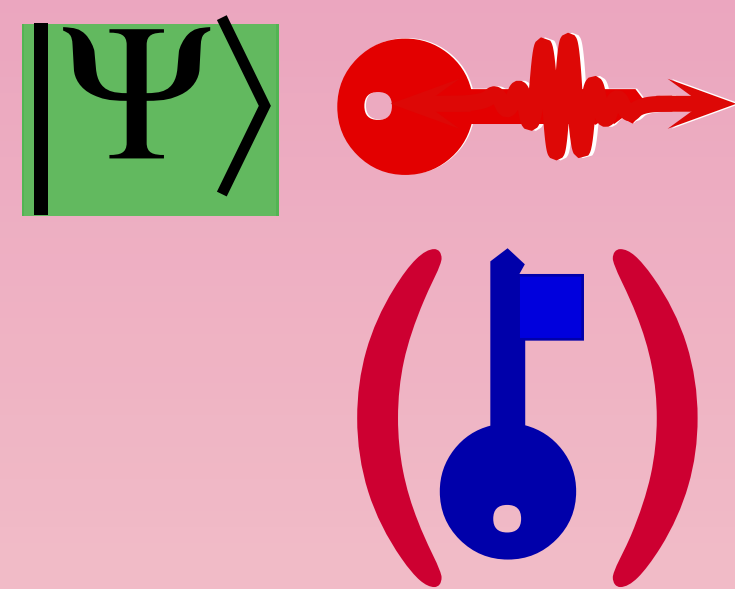
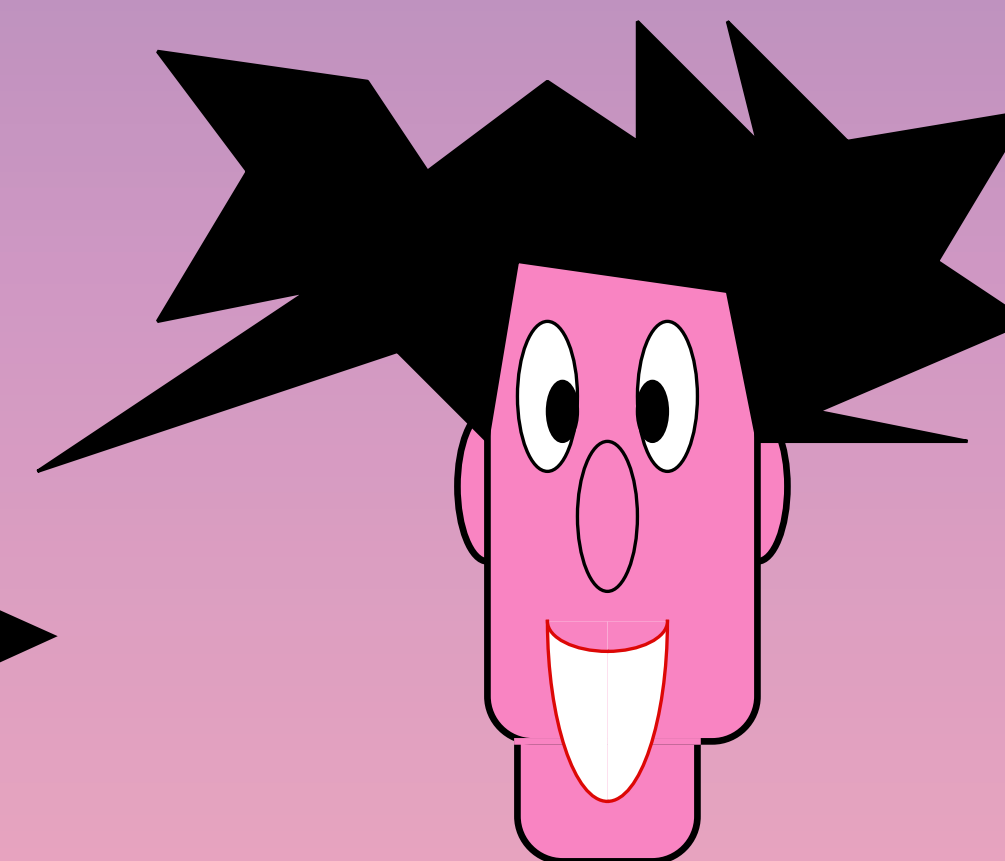
Quantum key : 1x Authenticated Q-pad  
Classical message+tag



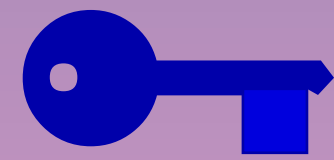
A B T



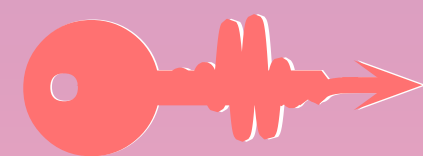
two authenticated random bits



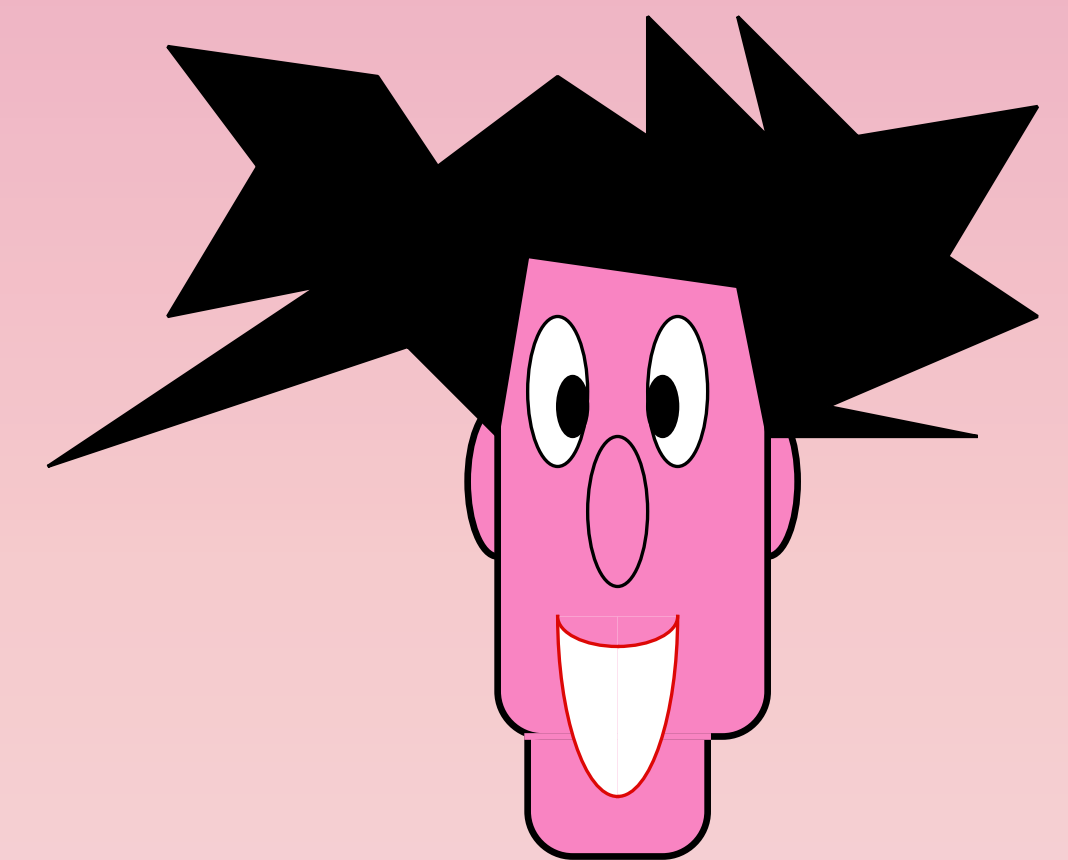
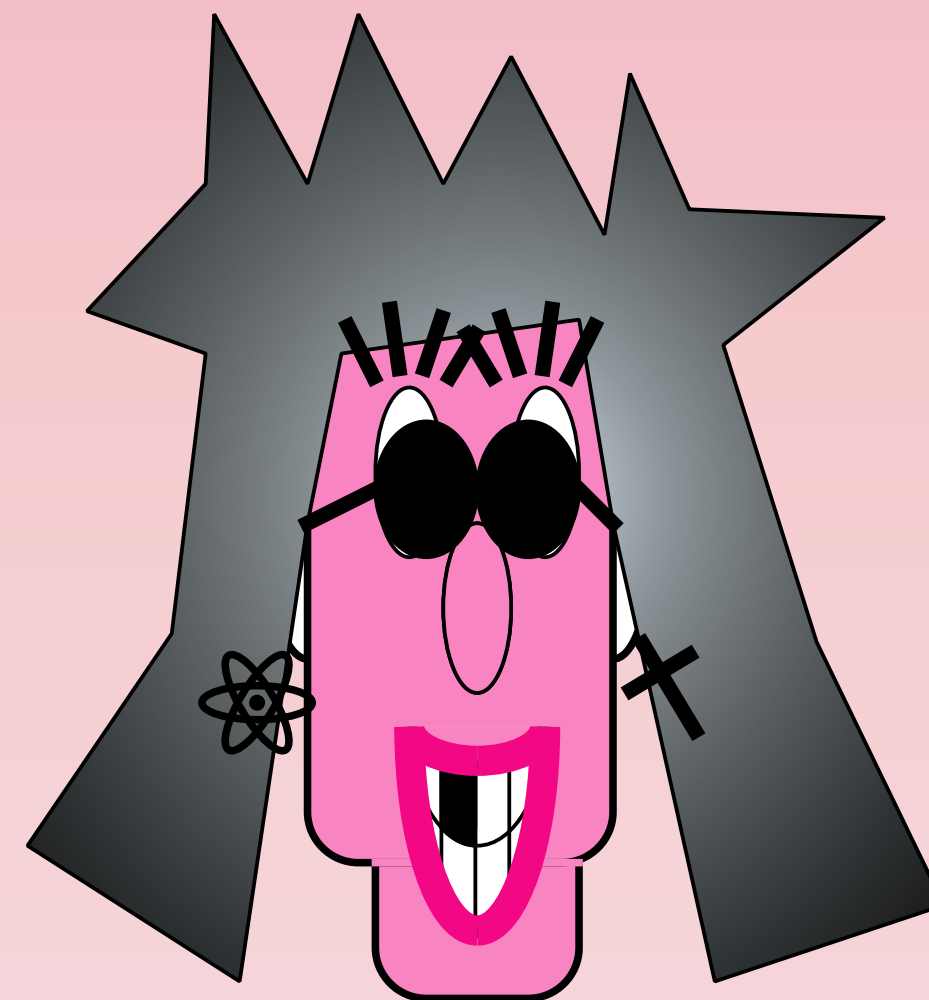
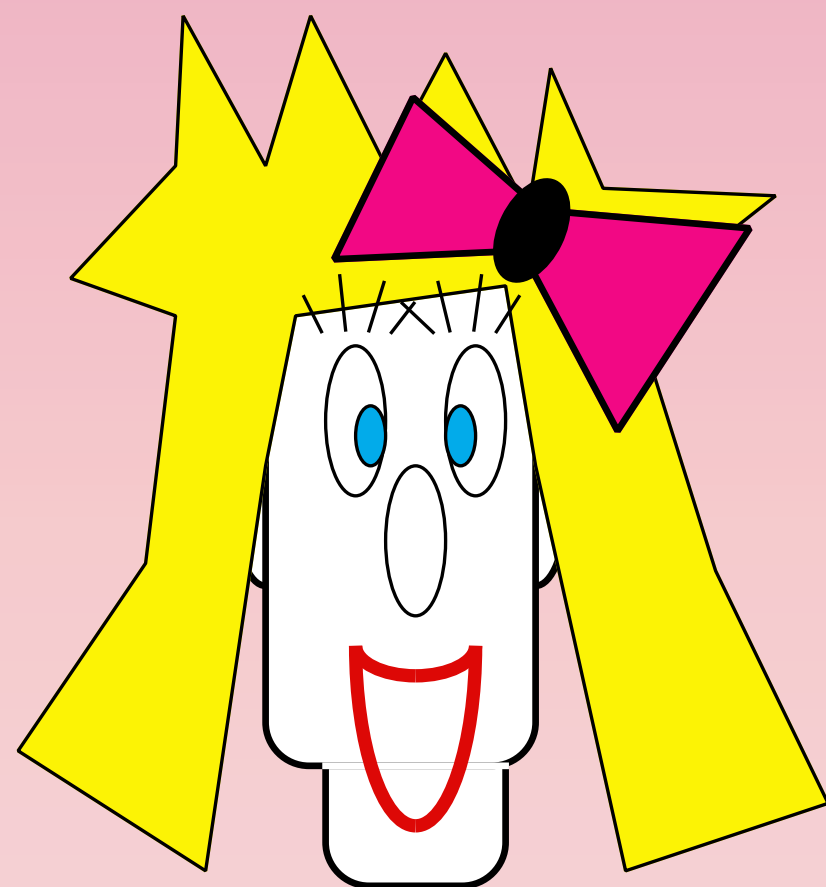
## (3.1.3a) One-time Q-Authentication



Classical key : Q-Authentication (BCGST)  
Quantum message+tag

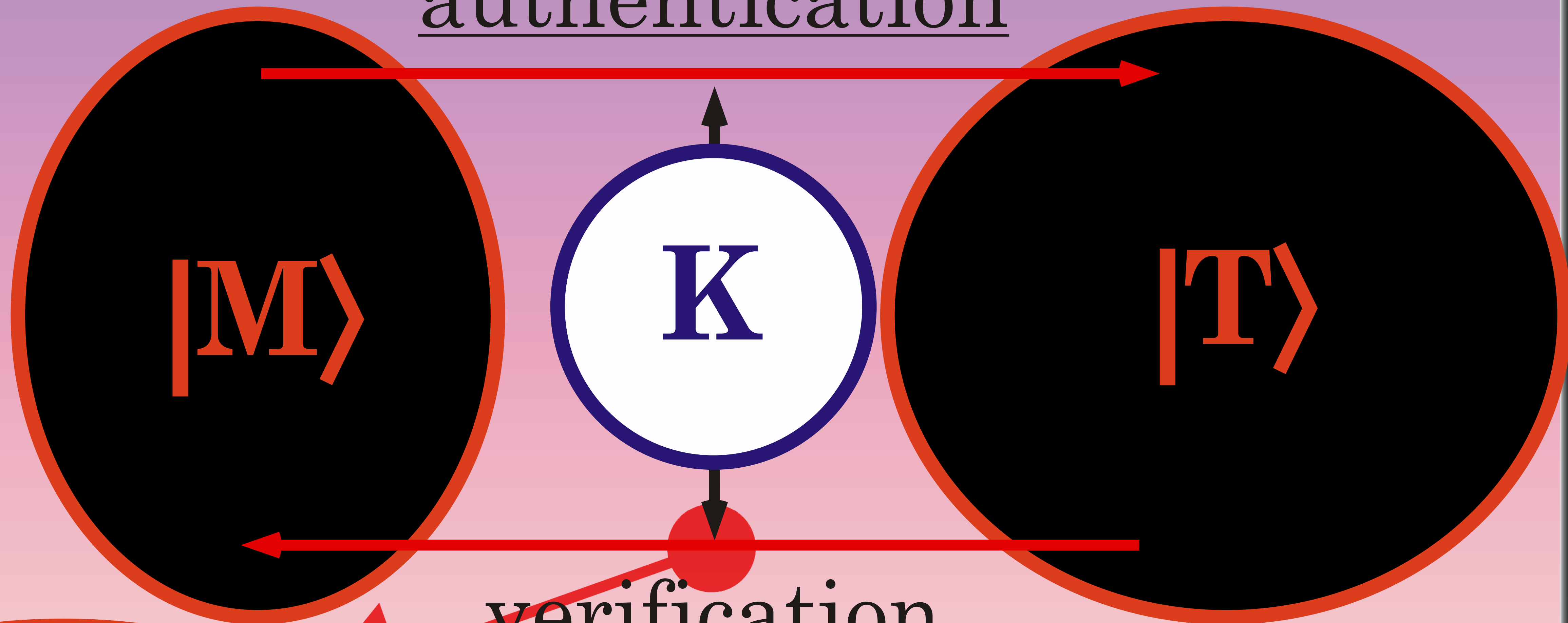


Quantum key : Authenticated Q-teleportation  
Classical message+tag (BBCJPW)



symmetric authentication  
of Quantum Messages

authentication



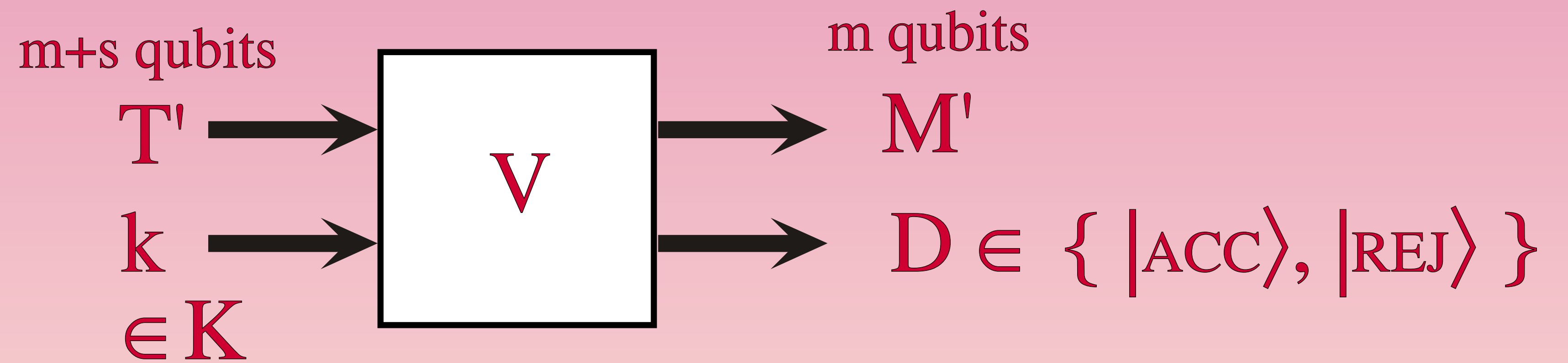
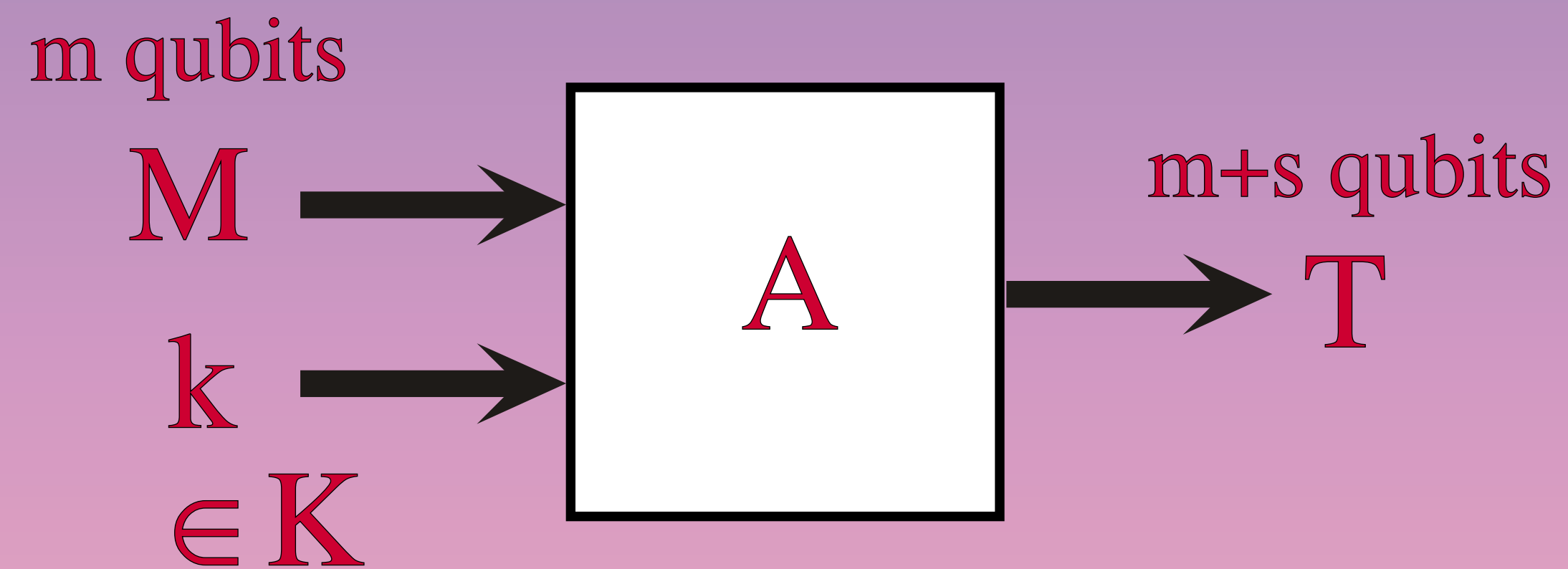
verification

$\{|ACC\rangle, |REJ\rangle\}$

Information Theoretical Security



# One-time Q-Authentication





# One-time Q-Authentication

For any pure state  $|\psi\rangle$  consider the measurement on  $(M',D)$  such that

- output Right if  $M'=|\psi\rangle$  or if  $D=|\text{REJ}\rangle$
- output Wrong otherwise



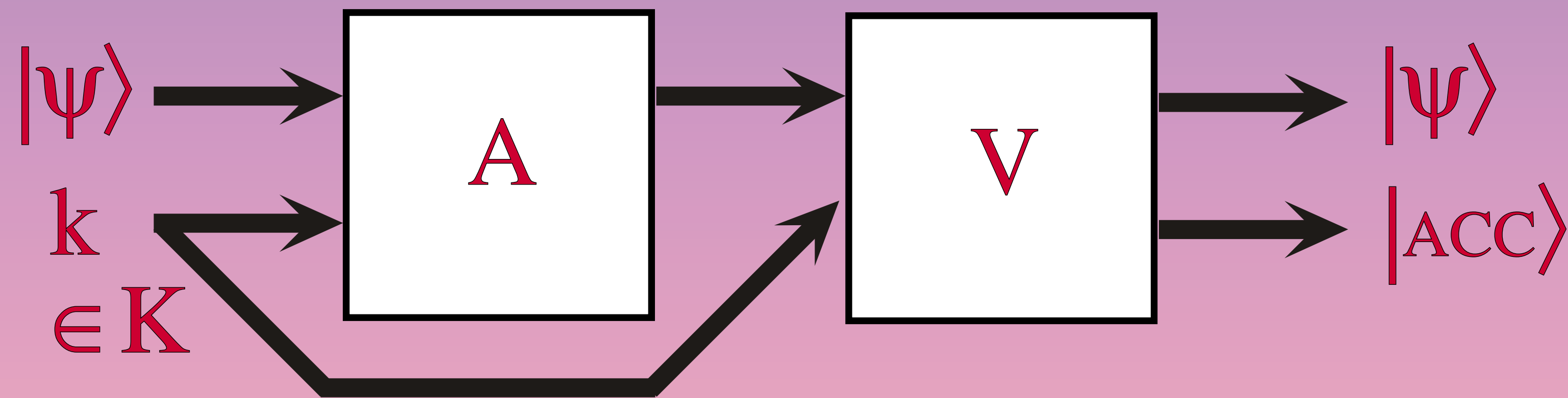
The corresponding projectors are

$$R_{|\psi\rangle} = |\psi\rangle\langle\psi| \otimes I_D + I_{M'} \otimes |\text{REJ}\rangle\langle\text{REJ}| - |\psi\rangle\langle\psi| \otimes |\text{REJ}\rangle\langle\text{REJ}|$$

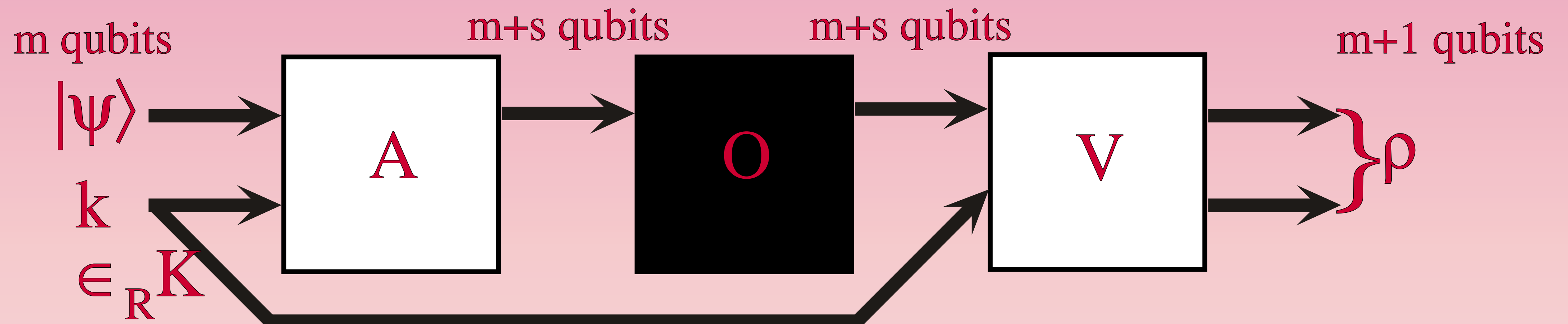
$$W_{|\psi\rangle} = (I_{M'} - |\psi\rangle\langle\psi|) \otimes |\text{ACC}\rangle\langle\text{ACC}|$$

# One-time Q-Authentication

## Completeness:

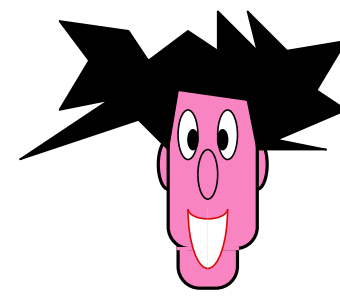
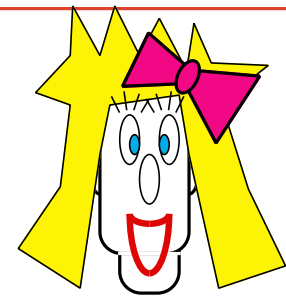


## Soundness:



$$\forall |\psi\rangle \text{Tr}(R_{|\psi\rangle} \rho) \geq 1 - 2^{-\Omega(s)}$$

# (3.1Q) Quantum-Key distribution



A: 1 ? ? 1 ? 0 ? ? 0 ? 1 ? ? ? ? 0 0 ? ? 0 ? 1 1 1  
 × + + + + + × + + + + × × + + × +  
 B: \ i i | i - ? i / i | i i i i / / i i - i | \ |

A: × + + + + × + + + + × × + + × +  
 B: 1 1 0 0 1 0 1 0 1 1 1

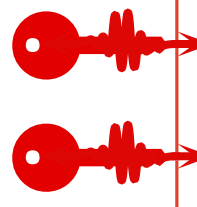
A: 1 1 0 0 1 0 0 0 1 1 1

A: 1 1 0 0 1 0 0 0 1 1 1

B: = = = = = = = = = ≠ = = =

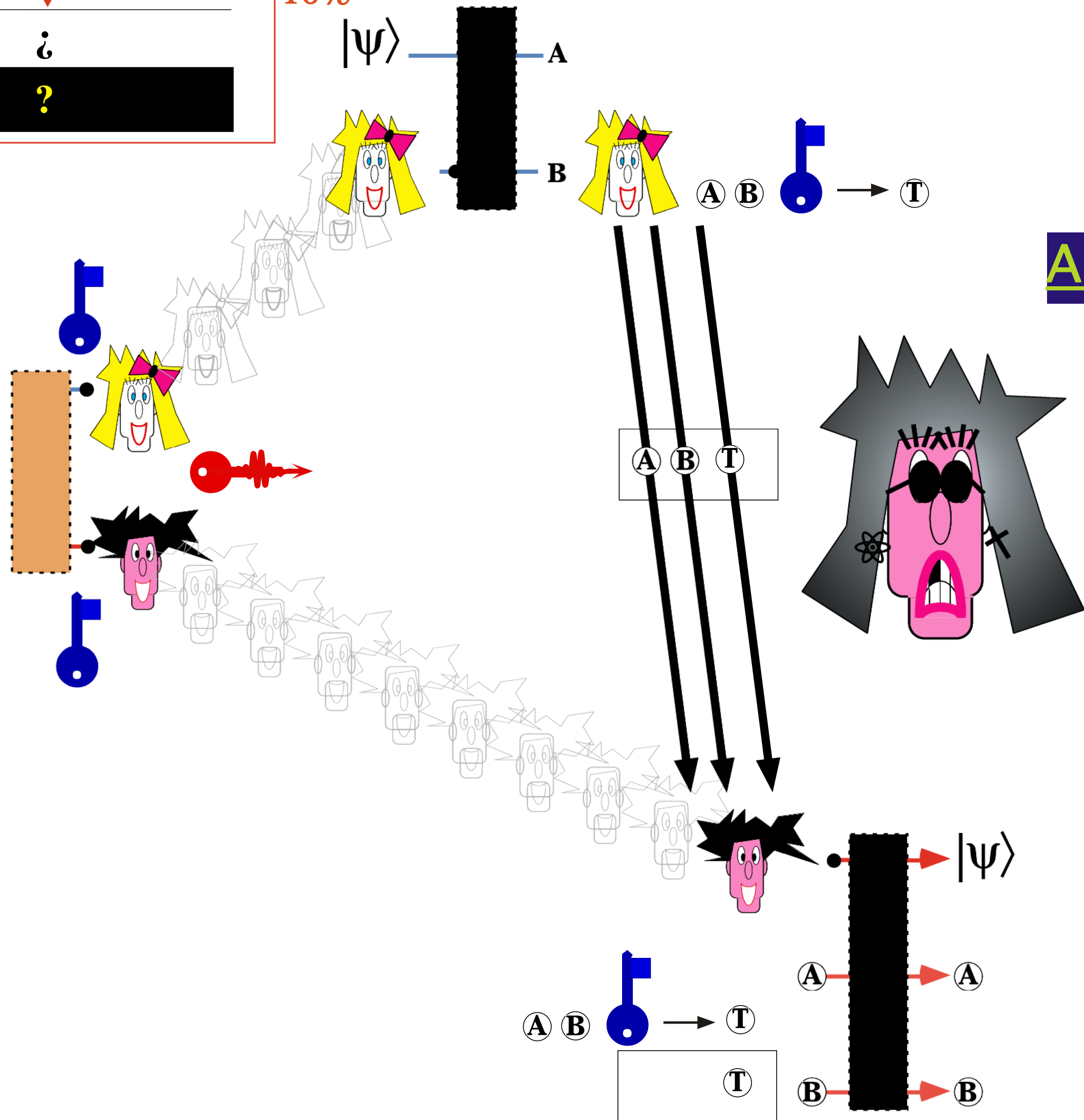
B: i i i ? i i i i i i i i

A: ? ? ? ? ? ? ? ? ? ?



## Shor-Preskill

10%



## (3.3Q) One-time Authenticated Q-pad

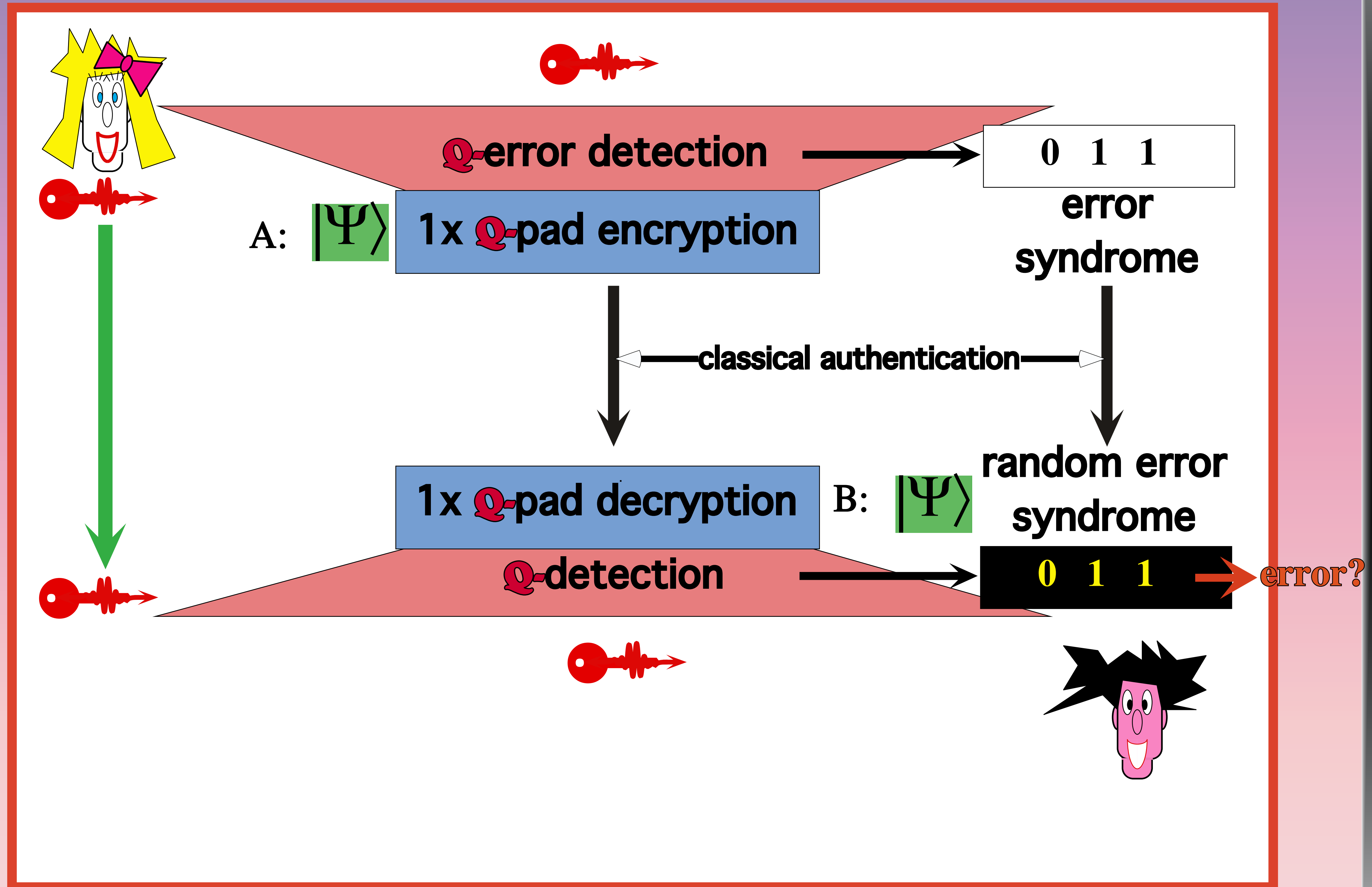
## (3.3C) One-time interactive Q-Authentication

• • • • •

- Transmit quantum key (EPR states)
- Quantum error-correction is used to purify (or test purity of) EPR states to form a smaller pure set
- one-time Authenticated Quantum pad is used to send message

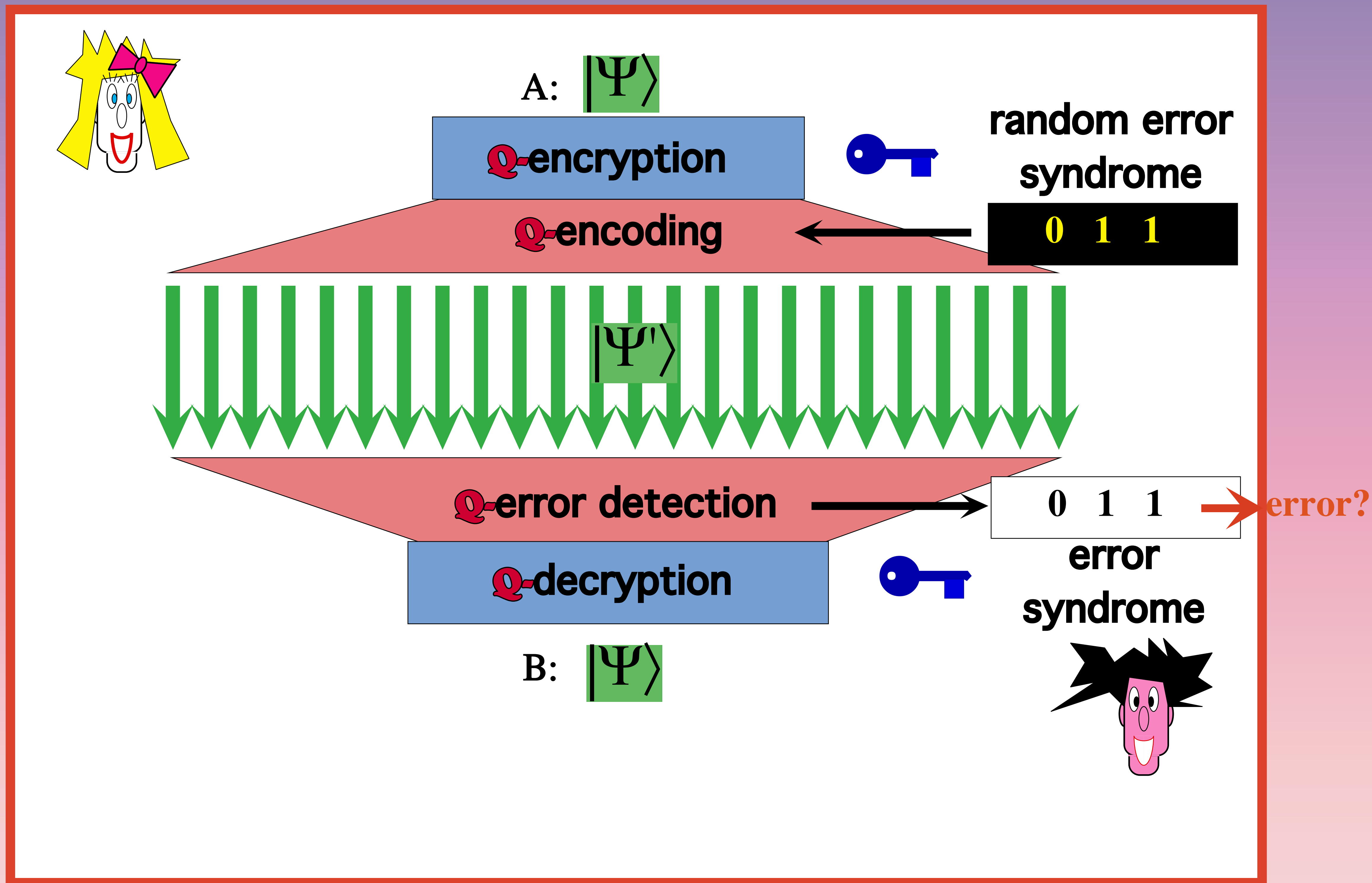
• • • • •

# (3.3C) One-time interactive Q-Authentication





# (3.1.3a) One-time Q-Authentication



Barnum-Crépeau-Gottesman-Smith-Tapp

# (3.1.3a) One-time Q-Authentication

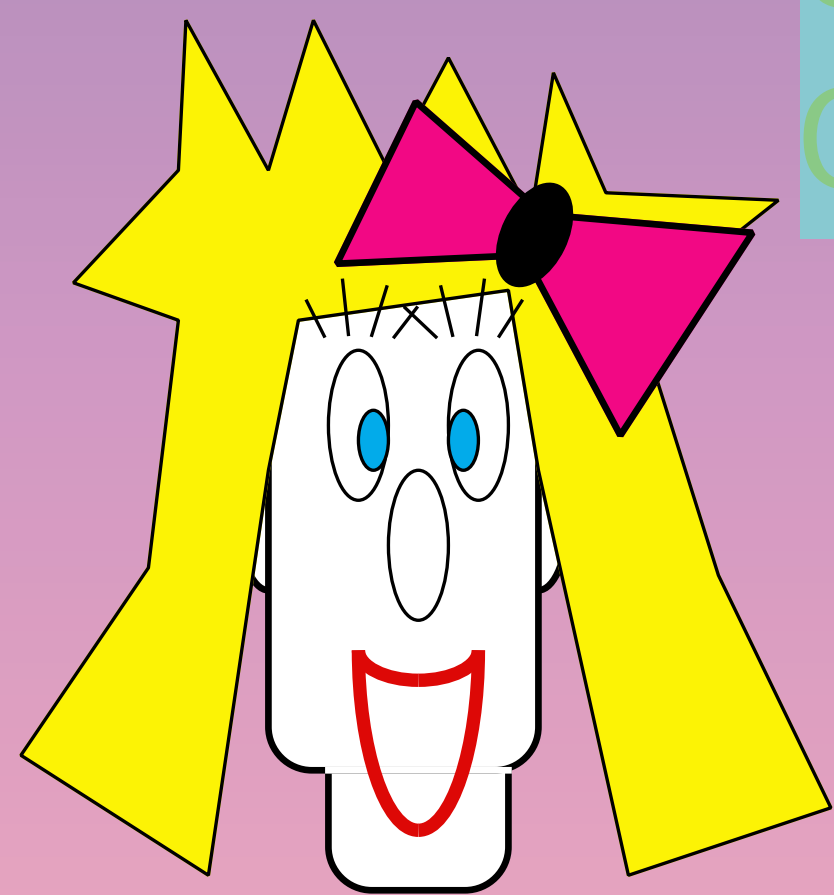


Classical key : one-time Q-authentication

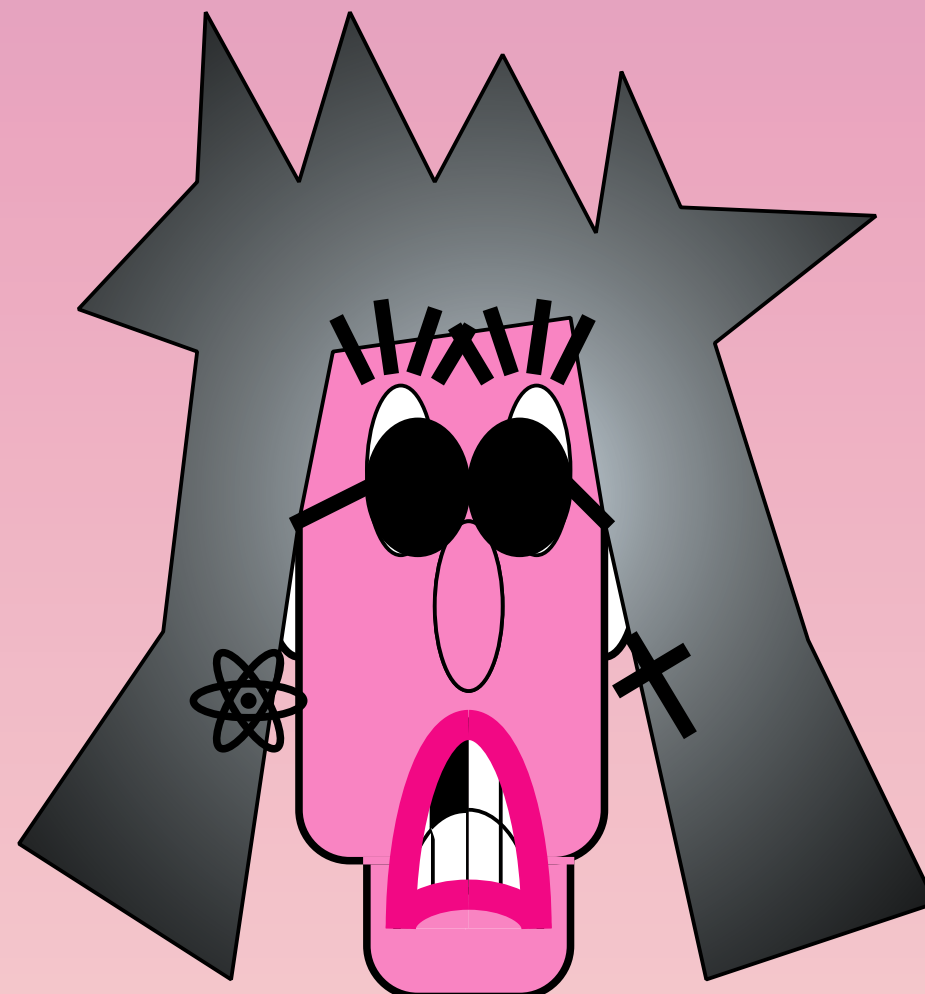
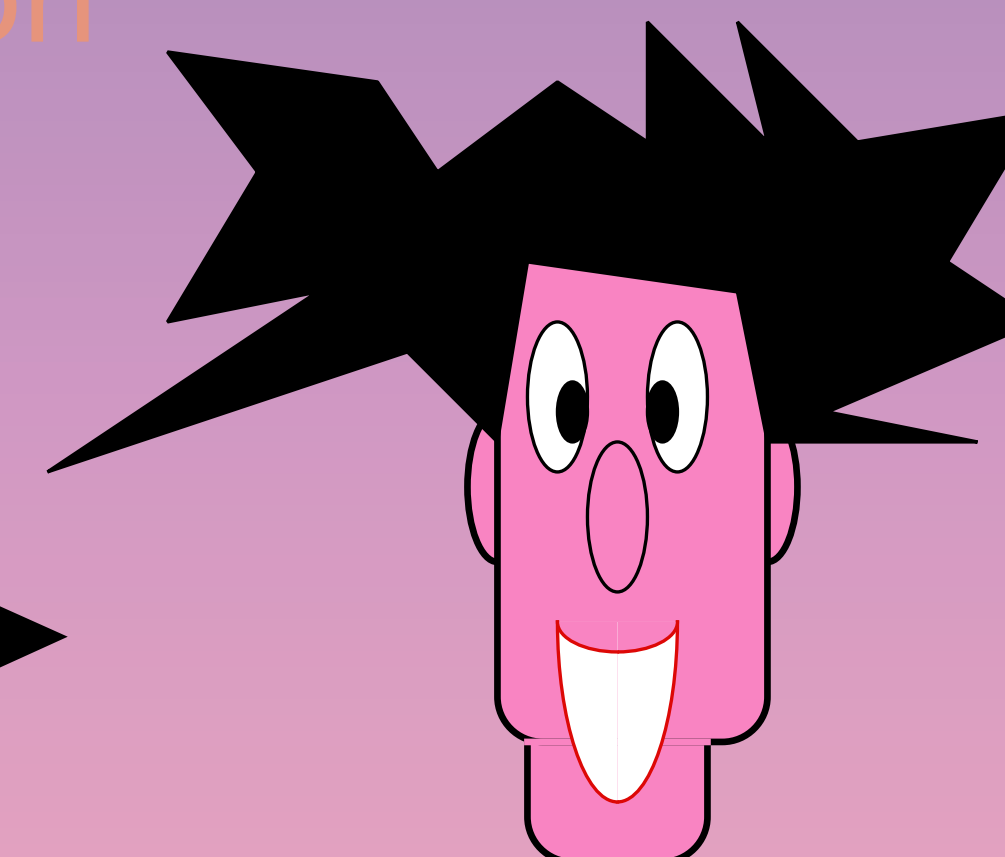
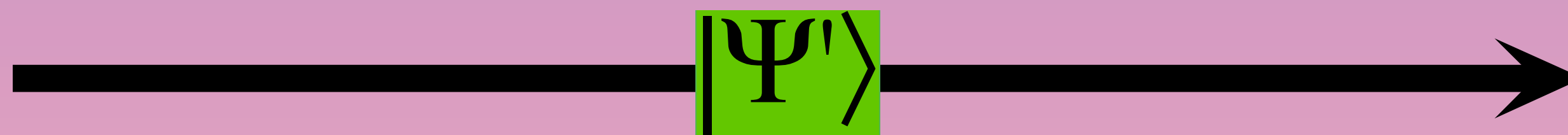
Quantum message+tag

Quantum key : Authenticated Q-teleportation

Classical message+tag

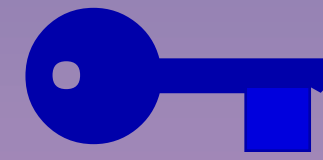


$|\Psi\rangle$



- public Q-error-correcting code

- secret key for encryption & syndrome



one-time **Q**-authentication



Vernam **Q**-cipher

(authenticated quantum messages must be encrypted  
which is false for classical messages! )

## Main Lower Bound

A Quantum Authentication Scheme  
with error probability  $\varepsilon$

is

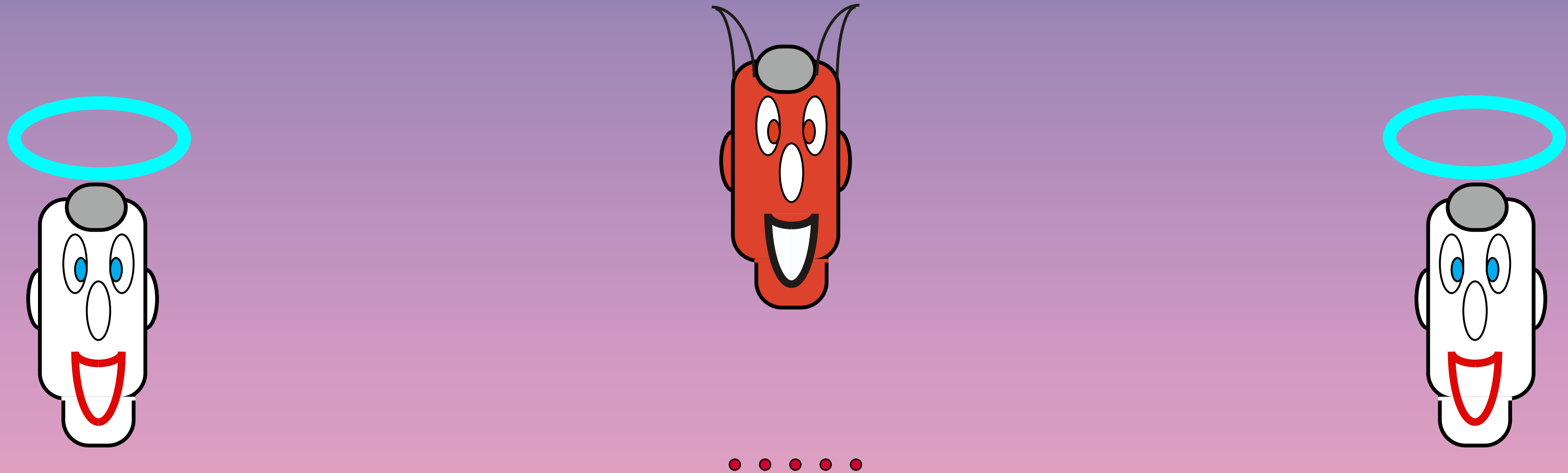
A Quantum Encryption Scheme  
with error probability  $4\varepsilon^{1/6}$ .



**(3.2)**

**Complexity Theoretical  
Quantum Cryptography**

## (3.2) Complexity Theoretical Cryptography



**(3.2.1) Public key cryptosystem** : public-key  $\mathcal{Q}$ -cryptosystem

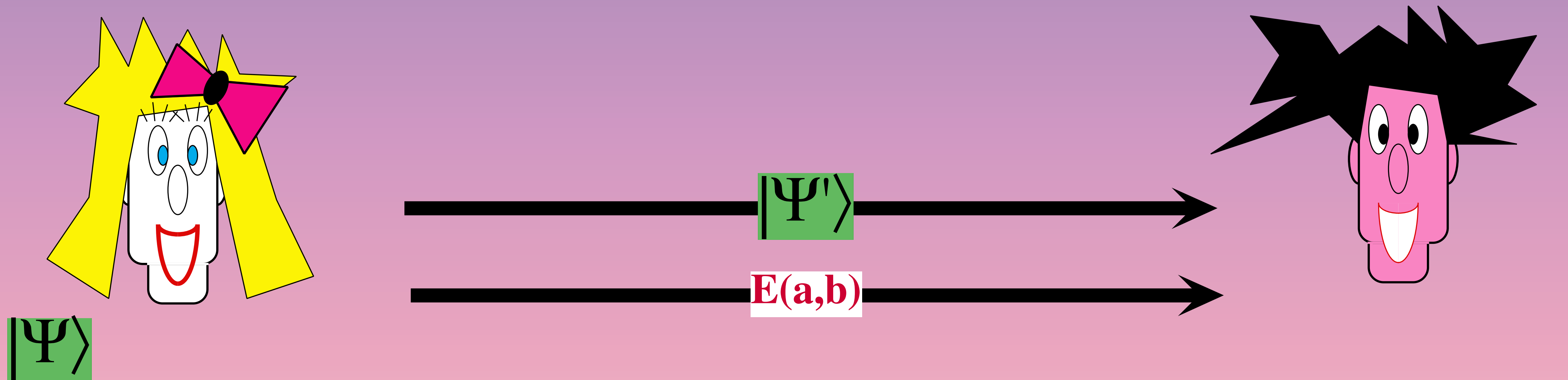
**(3.2.2) Digital signature scheme** : public-key  $\mathcal{Q}$ -Authentication  
 $\mathcal{Q}$ -digital signature scheme

**(3.2.3) (trapdoor) one-way functions** :  $\mathcal{Q}$ -cryptanalysis  
(trapdoor)  $\mathcal{Q}$ -one-way functions



# (3.2.1) Public-Key Q-Cryptosystem

Assuming Classical Public Key Cryptography

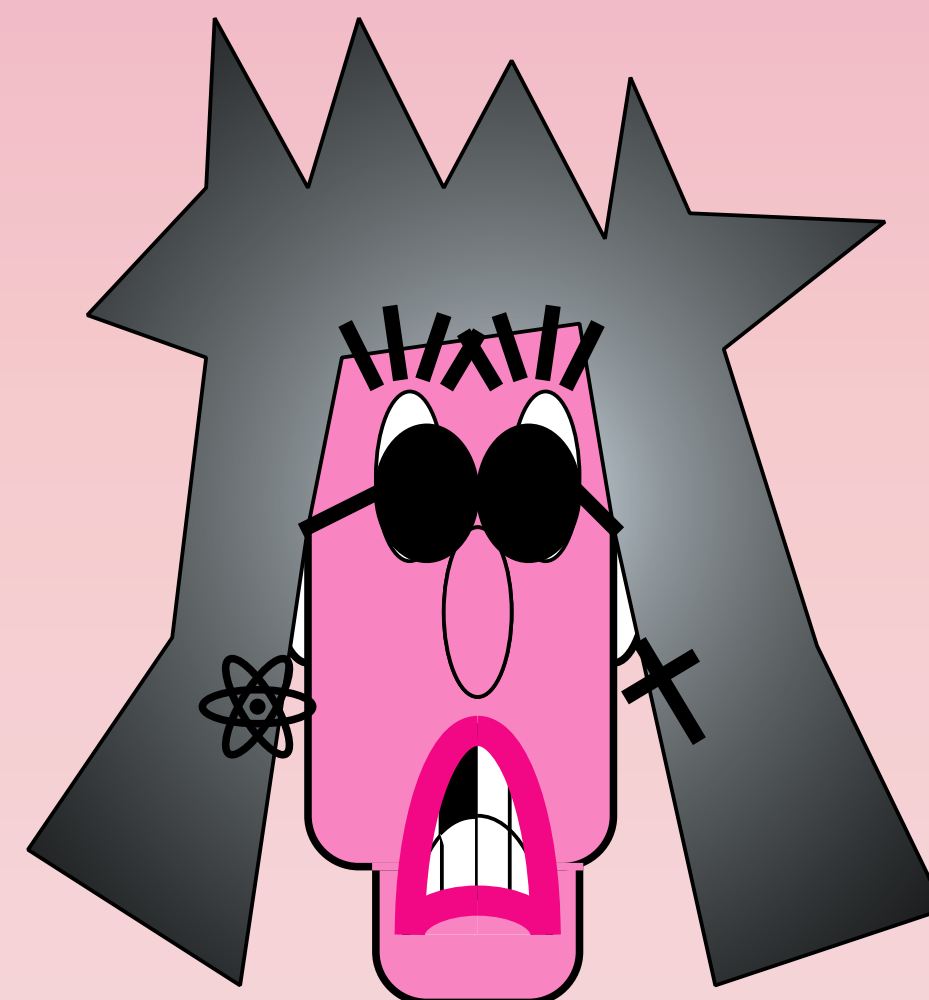


$a, b$  random bits

$(a, b) := D(E(a, b))$

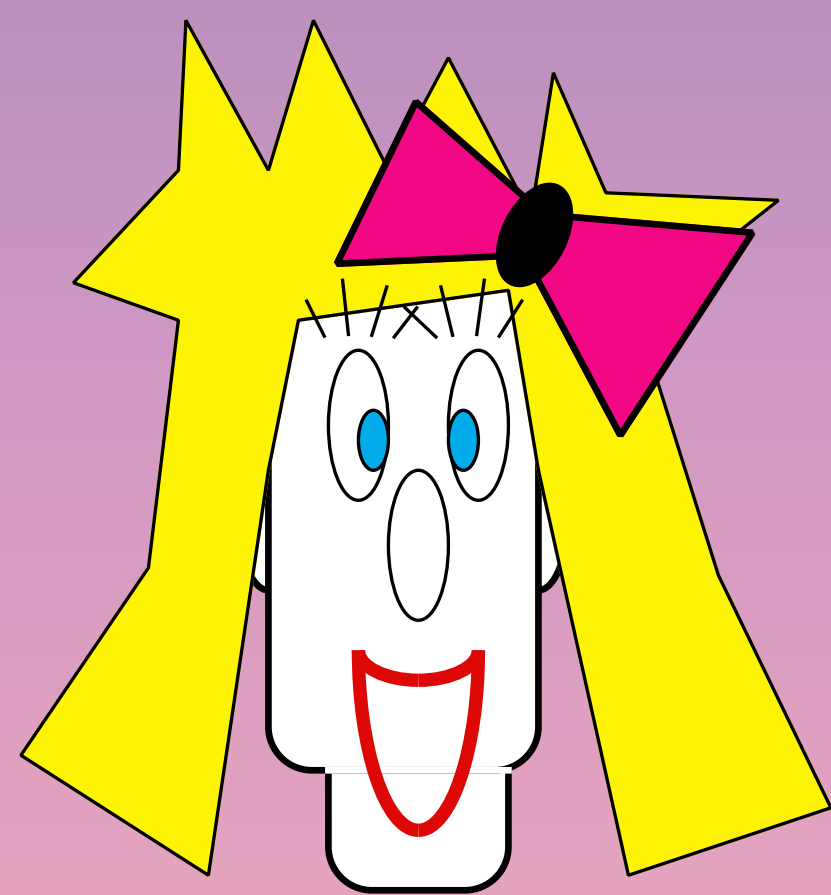
$$|\Psi'\rangle = \sigma_x^a \sigma_z^b |\Psi\rangle$$

$$|\Psi\rangle = \sigma_z^b \sigma_x^a |\Psi'\rangle$$

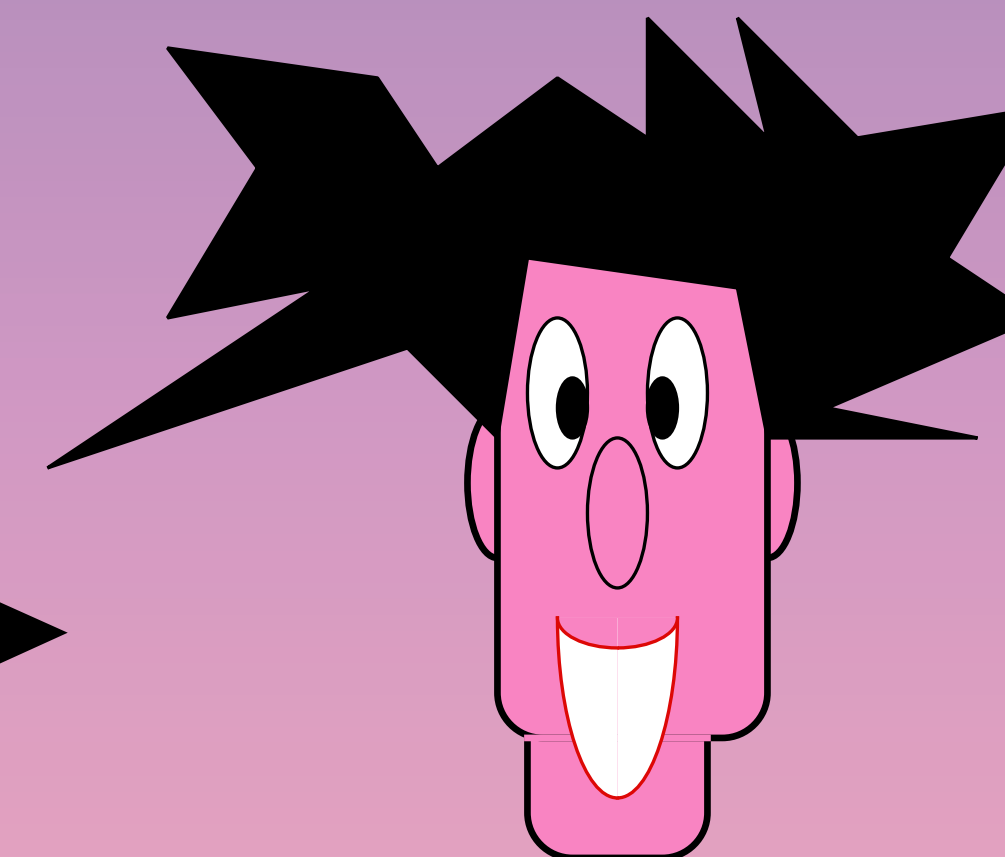
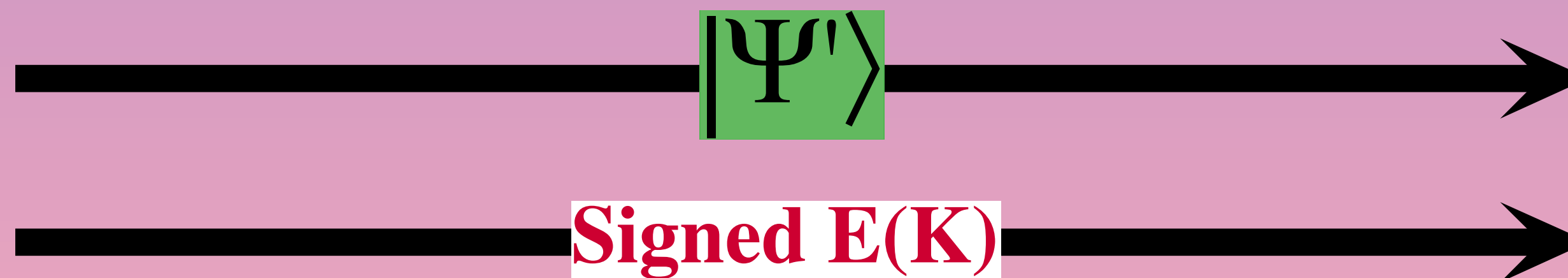


# (3.2.2a) Public-Key Q-Authentication

Assuming Classical Public Key Cryptography  
Assuming Classical Digital Signature



$|\Psi\rangle$

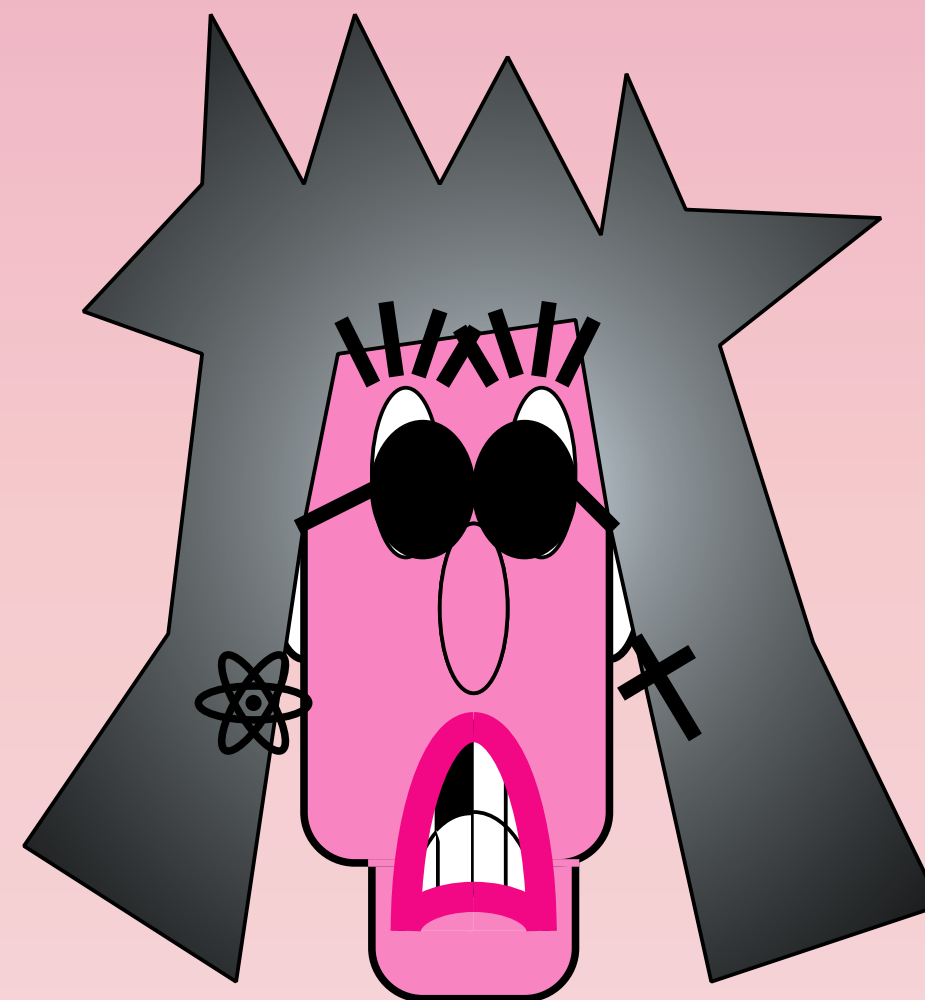


$K$  random authentication key

verify signed  $E(K)$   
 $K := D(E(K))$

$|\Psi'\rangle := \text{Auth}_K(|\Psi\rangle)$

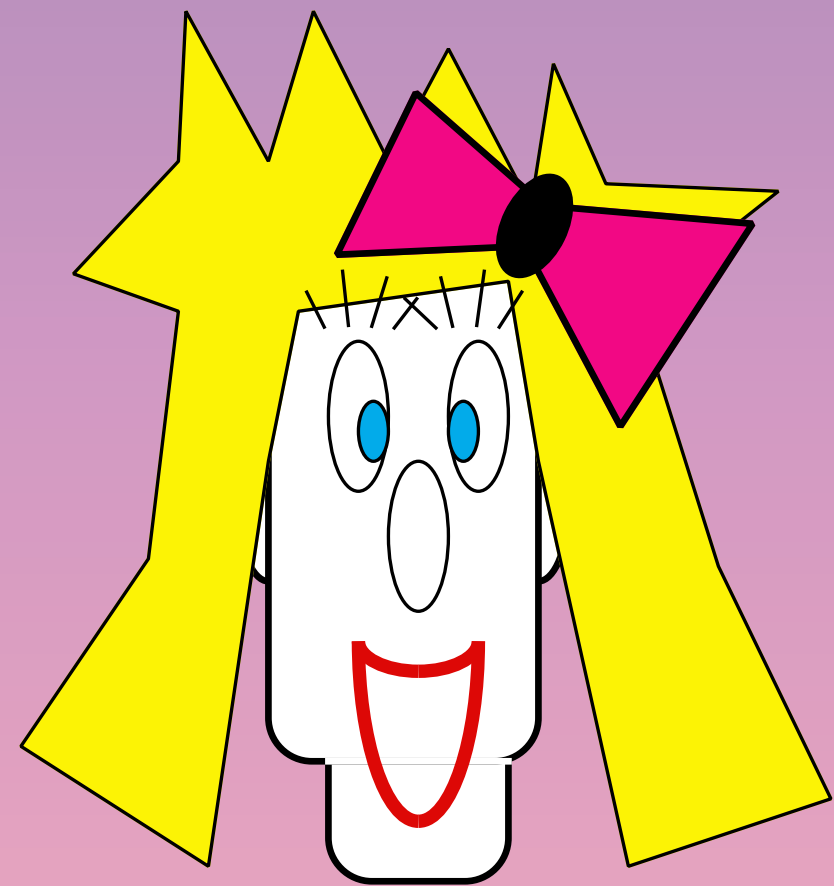
$|\Psi\rangle := \text{Auth}_K^{-1}(|\Psi'\rangle)$



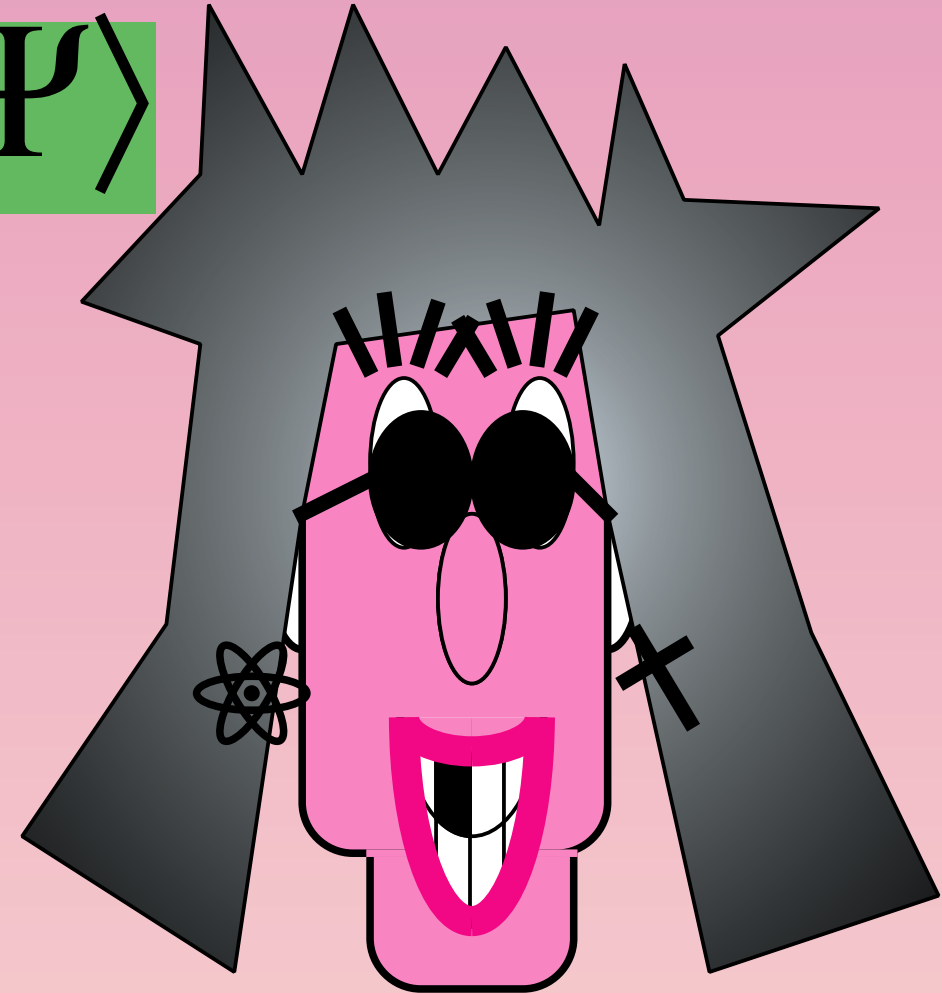


# (3.2.2b) Q-Digital Signature Scheme

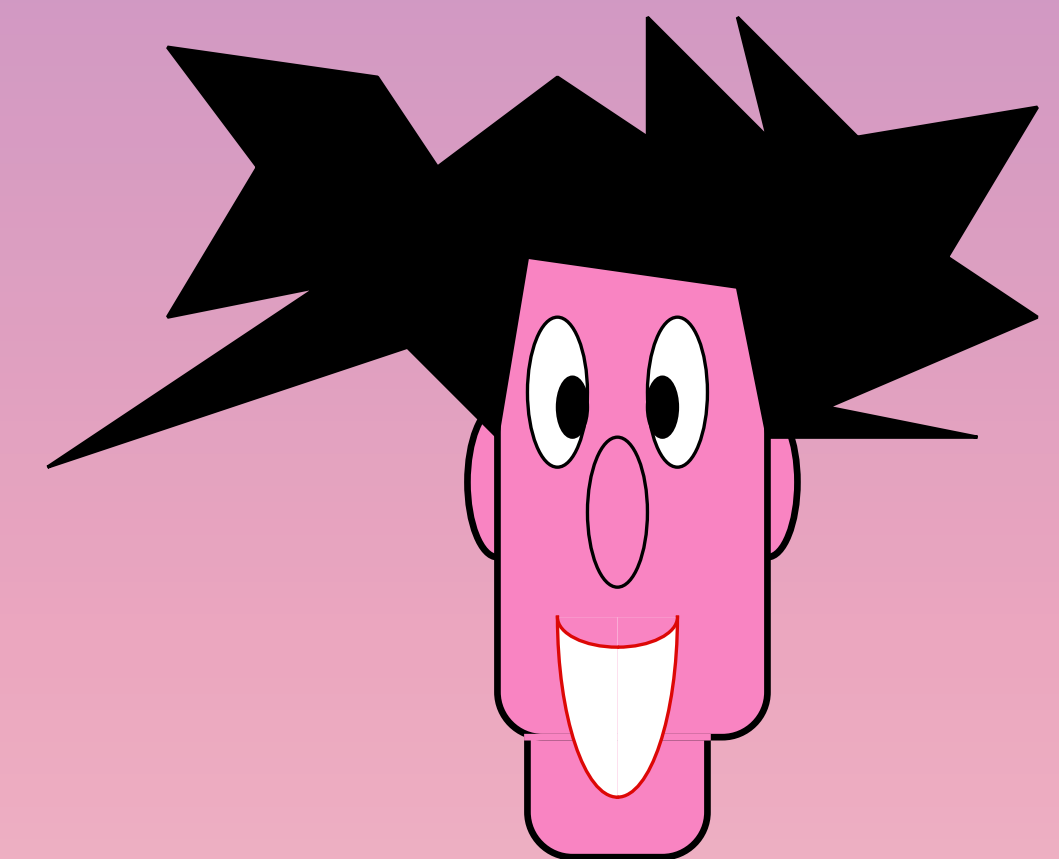
Assuming Classical Public Key Cryptography  
Assuming Classical Digital Signature



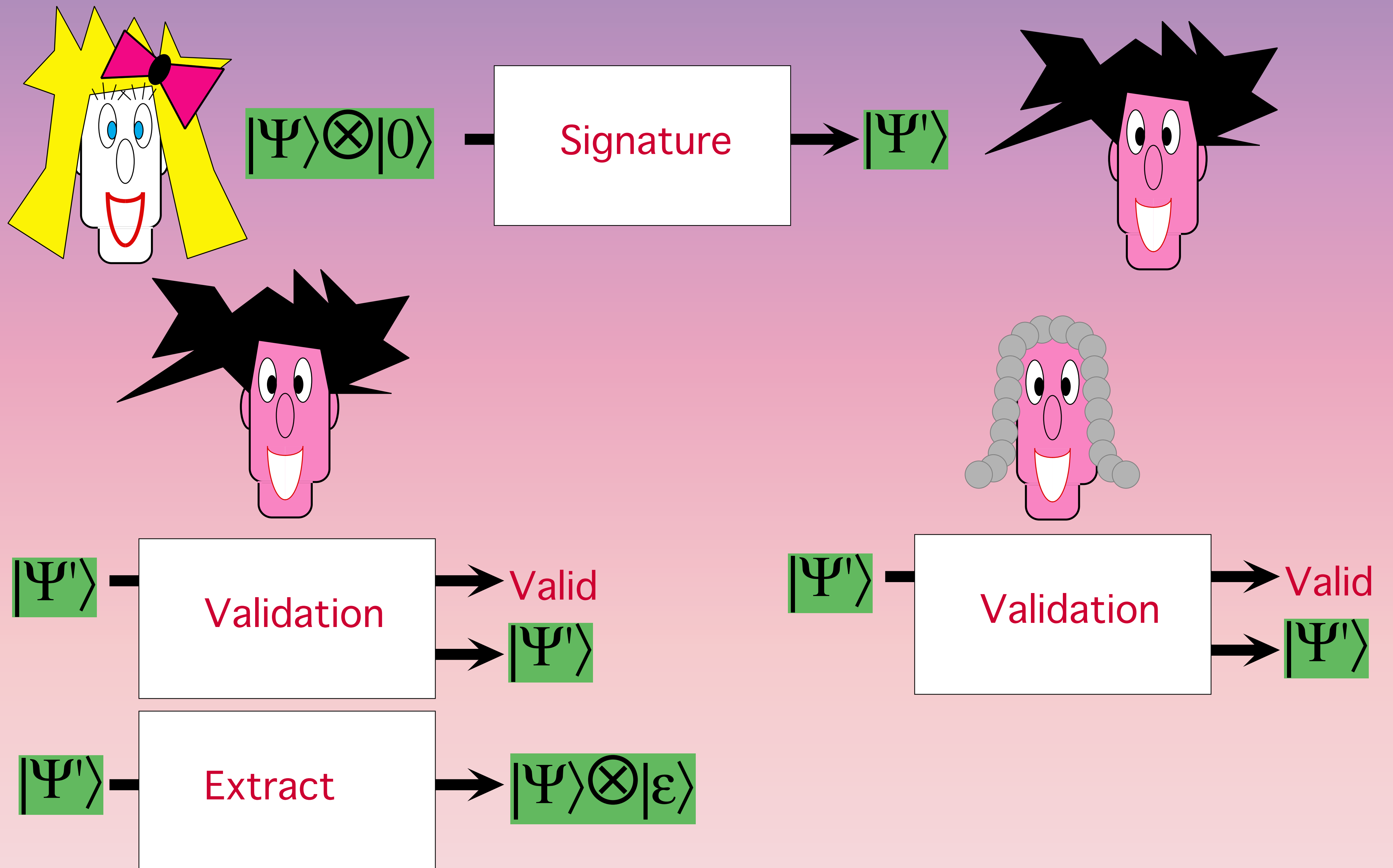
$|\Psi\rangle$



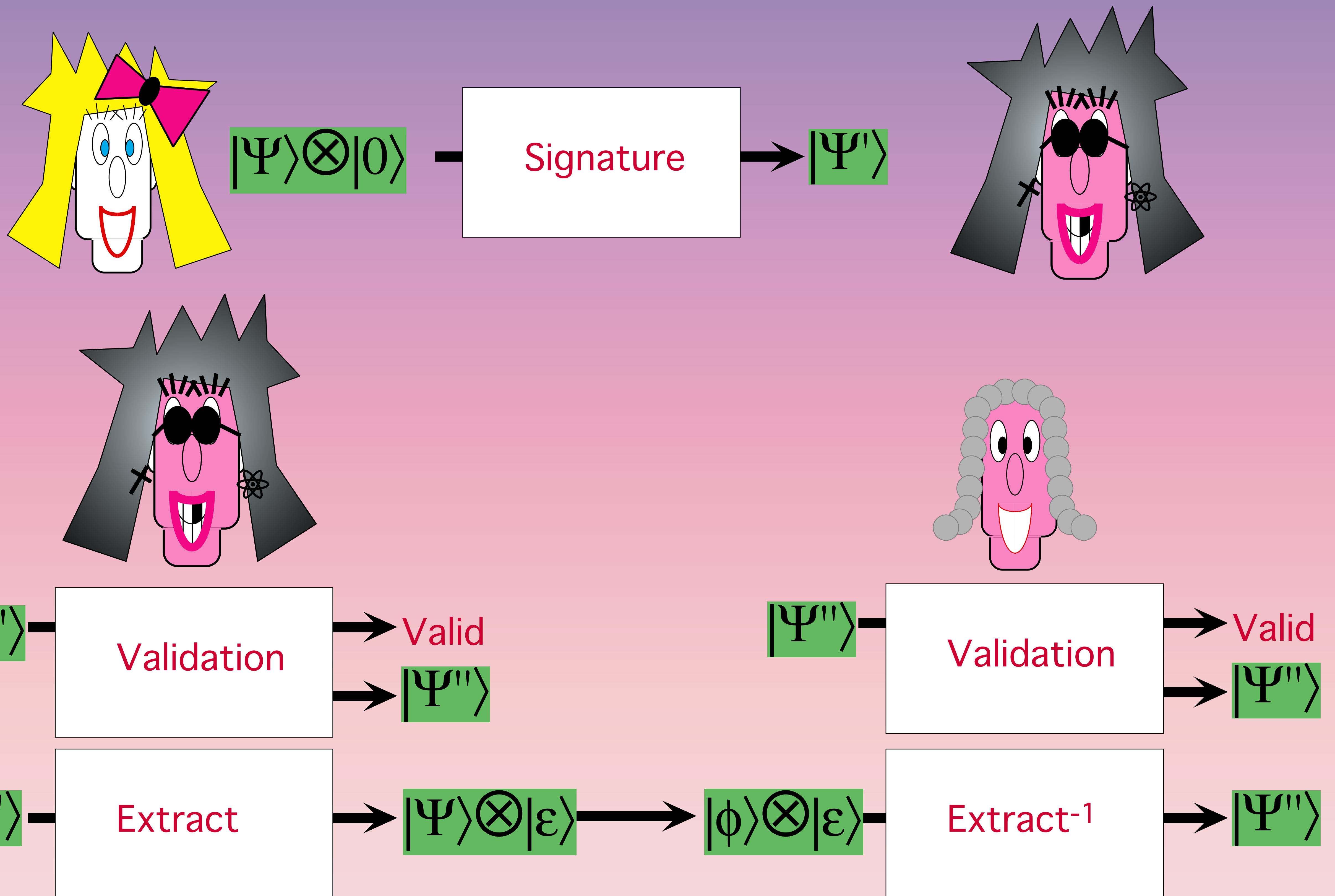
**IMPOSSIBLE**



# (3.2.2b) Q-Digital Signature Scheme



# (3.2.2b) Q-Digital Signature Scheme



### (3.2.3) (Trapdoor) Q-One-way functions

- generate a function  $f$  (and trapdoor) s.t.
- computing  $f(x)$  is easy
- finding  $x$  s.t.  $f(x)=y$  is hard
- finding  $x$  s.t.  $f(x)=y$  is easy with trapdoor

**Q-cryptanalysis** : Shor



# Q-One-way function

Fischer-Stern

one-way function

(error correction code based)

generation : classical easy

computing  $f$  : classical easy

inverting  $f$  : classical / quantum hard ???

# Trapdoor $\mathcal{Q}$ -One-way function

Okamoto-Tanaka-Uchiyama

trapdoor one-way permutation  
(subset products problem based)

generation : quantum easy

computing  $f$  : classical easy

inverting  $f$  : classical / quantum hard ???

trapdooring  $f$  : classical easy

# Trapdoor $\mathcal{Q}$ -One-way function

## McEliece

trapdoor one-way permutation  
(error correction code based)

generation : classical easy

computing  $f$  : classical easy

inverting  $f$  : classical / quantum hard ???

trapdooring  $f$  : classical easy

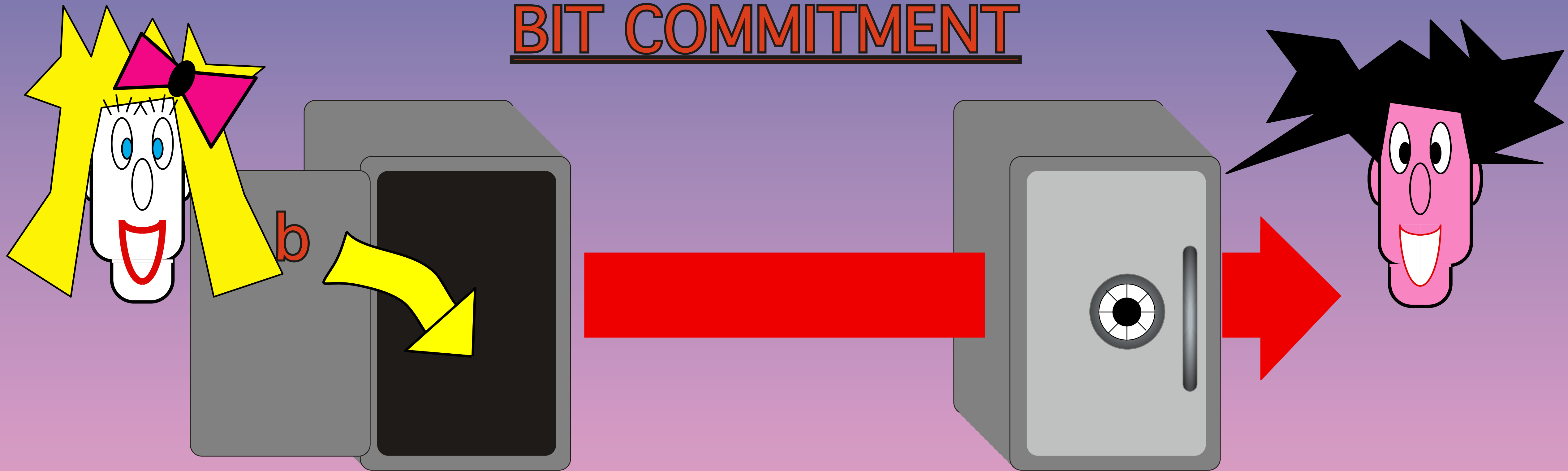
**(4)**

**two-party**

**Cryptographic Protocols**



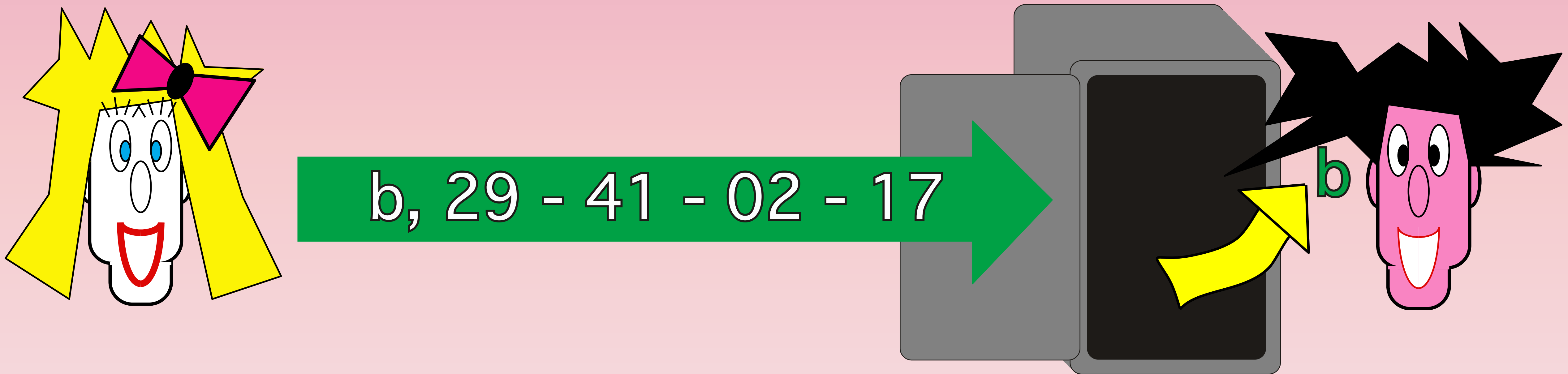
# BIT COMMITMENT



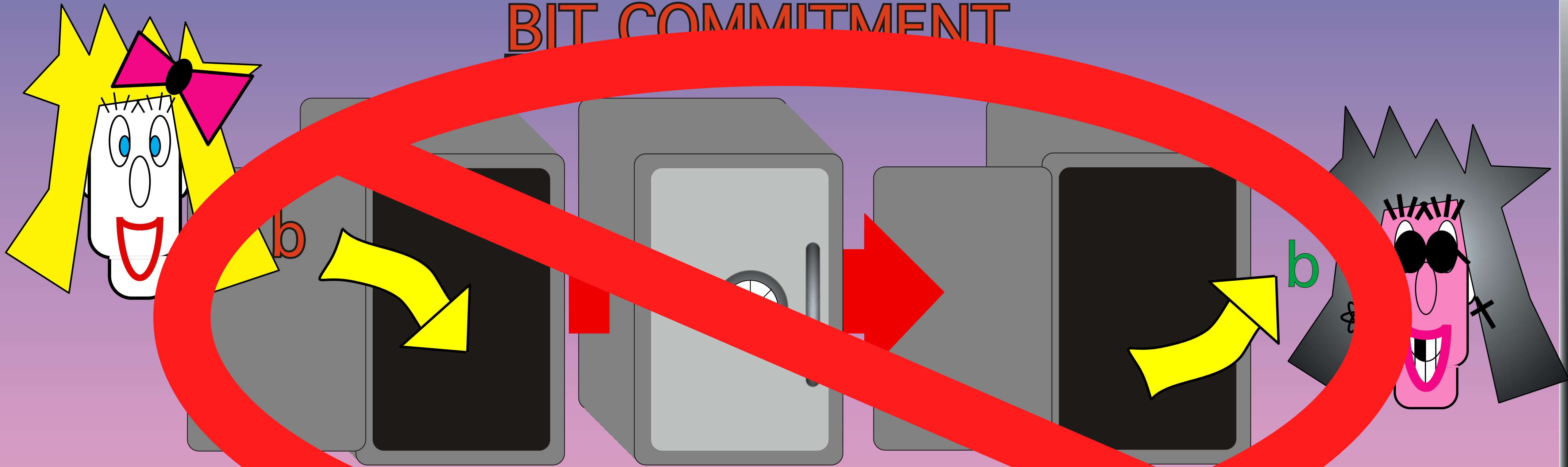
COMMIT

---

UNVEIL



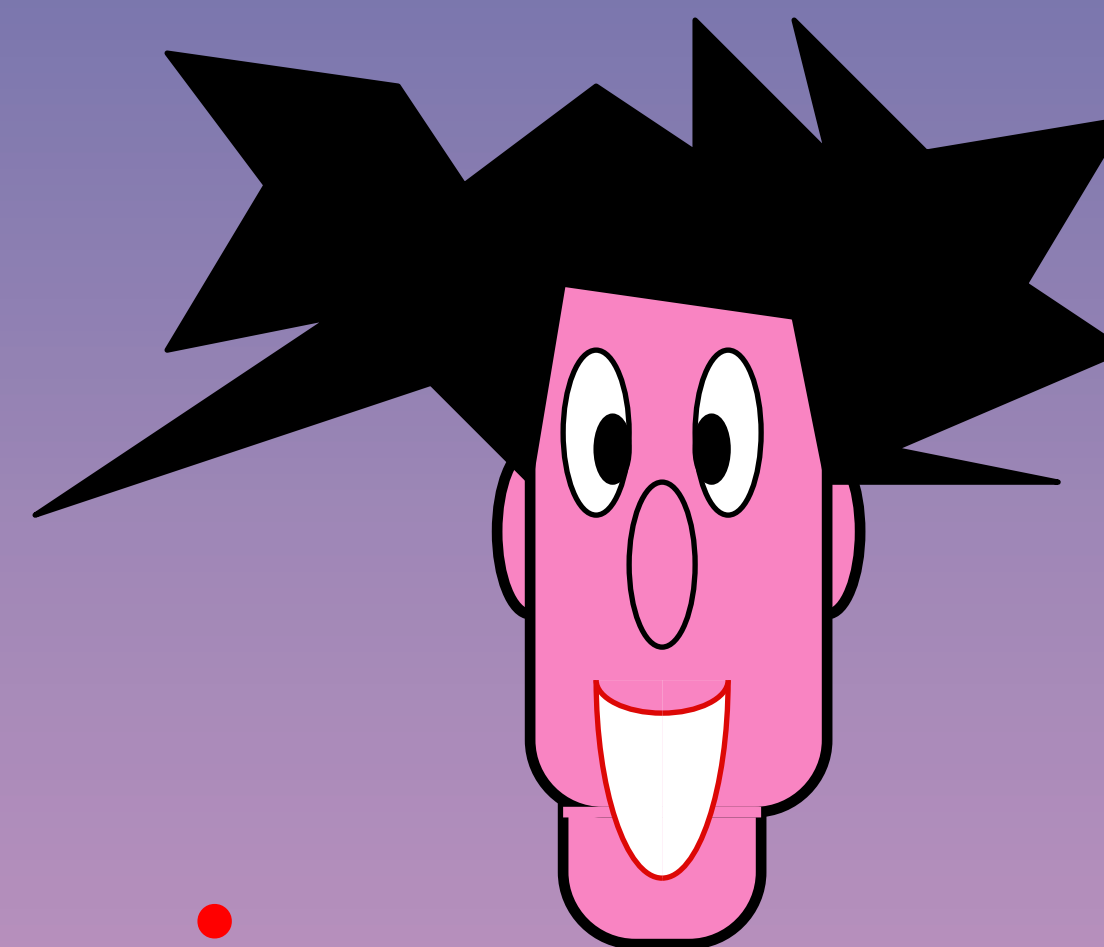
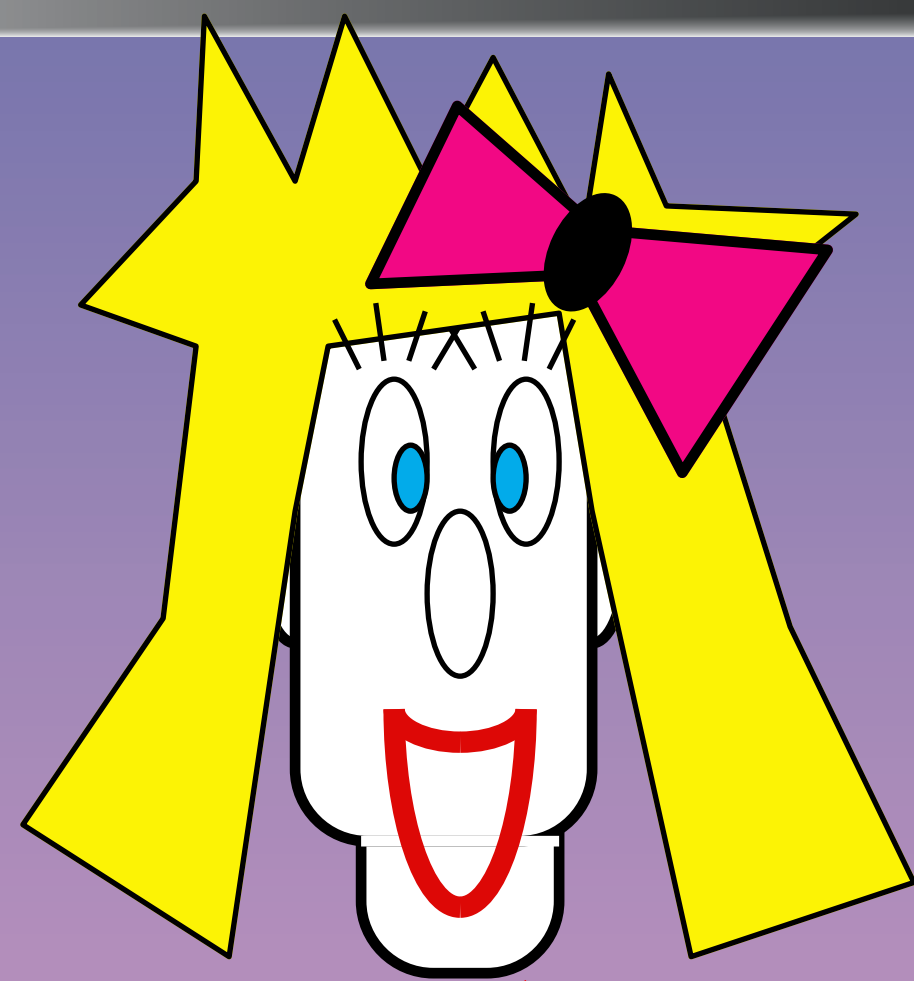
# BIT COMMITMENT



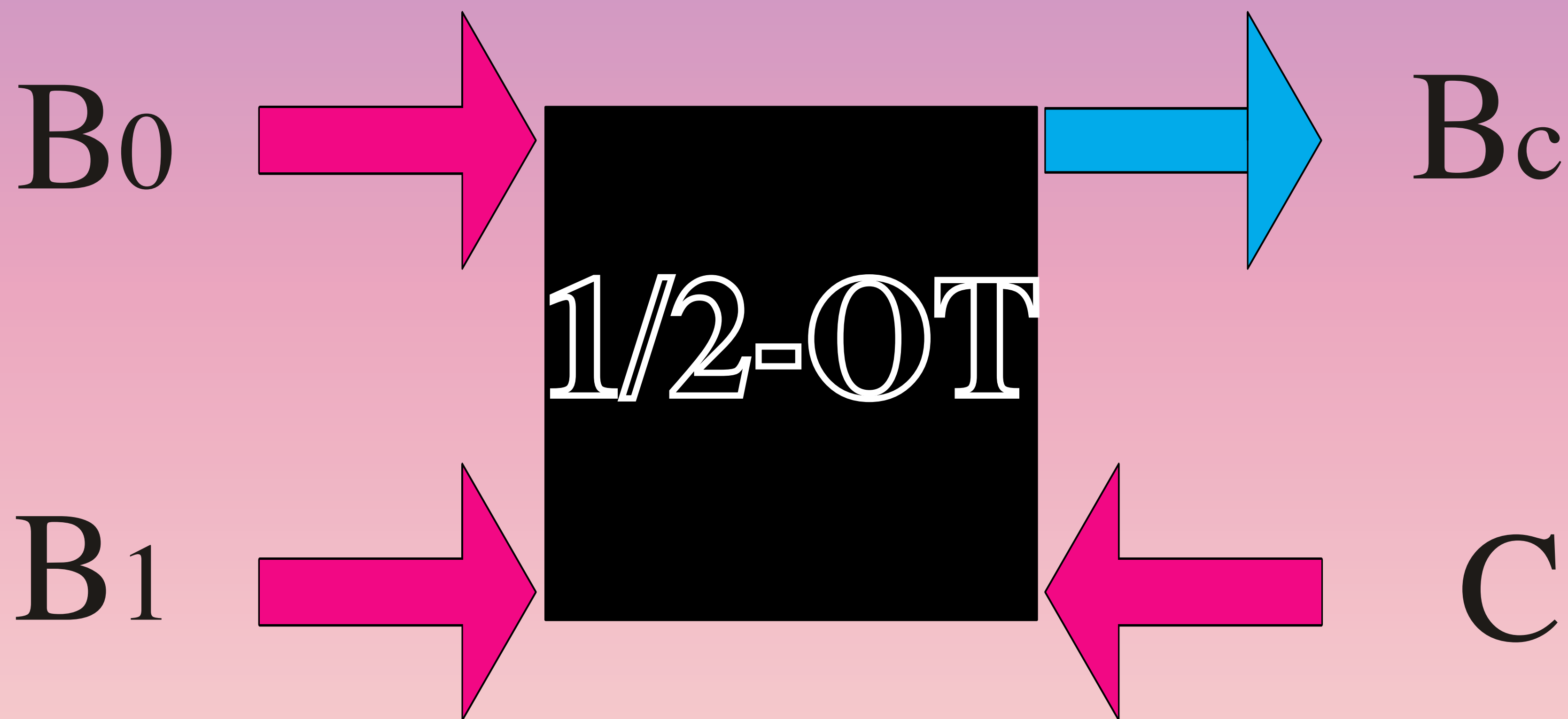
CONCEALING

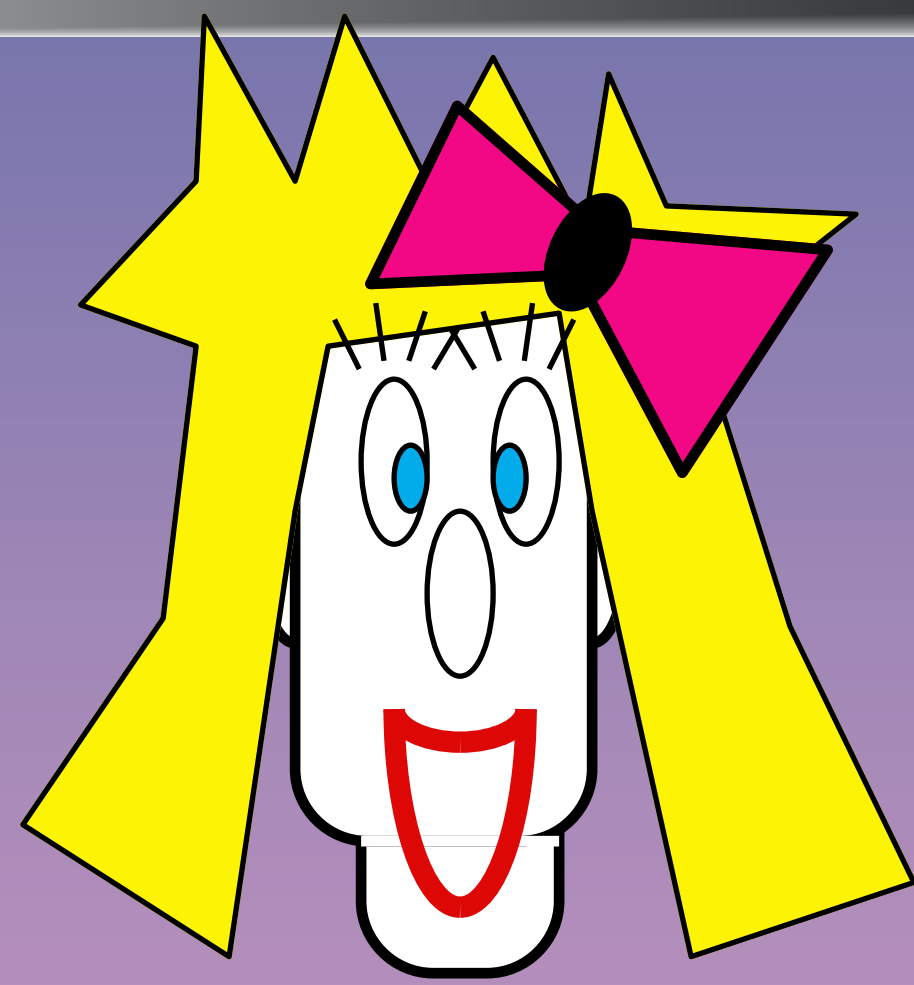
BINDING



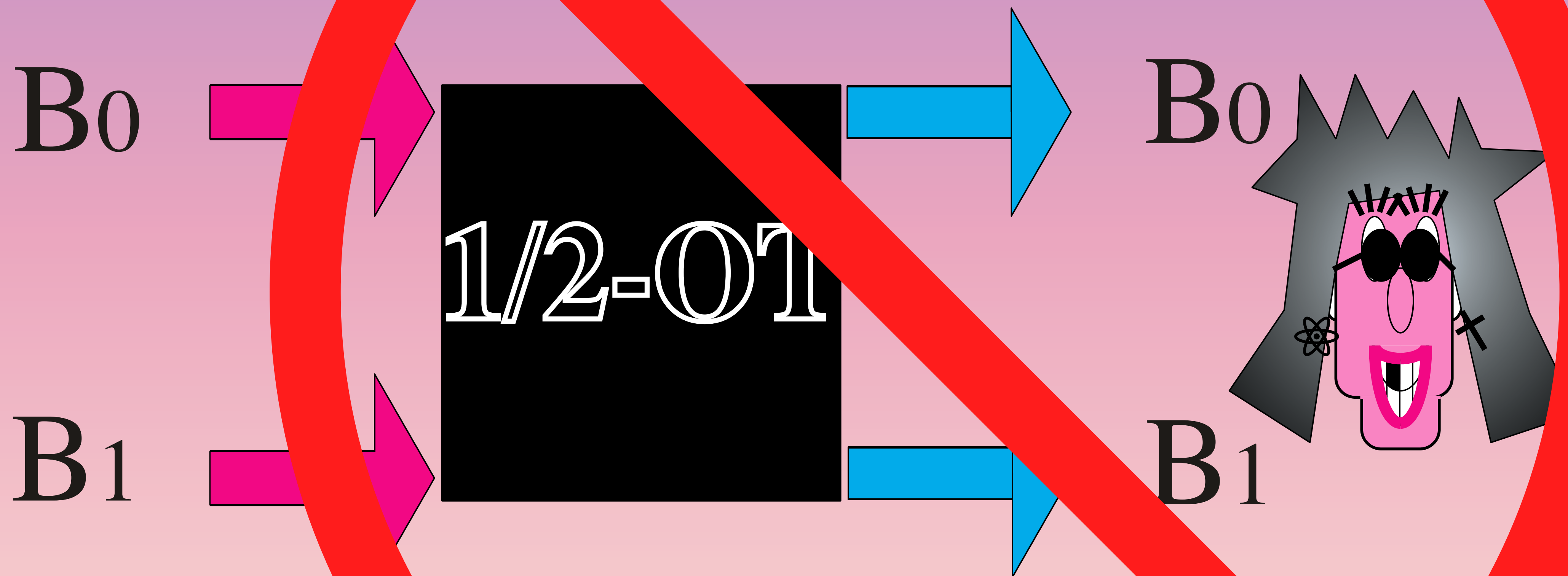


Oblivious  
Transfer  
(message multiplexing)



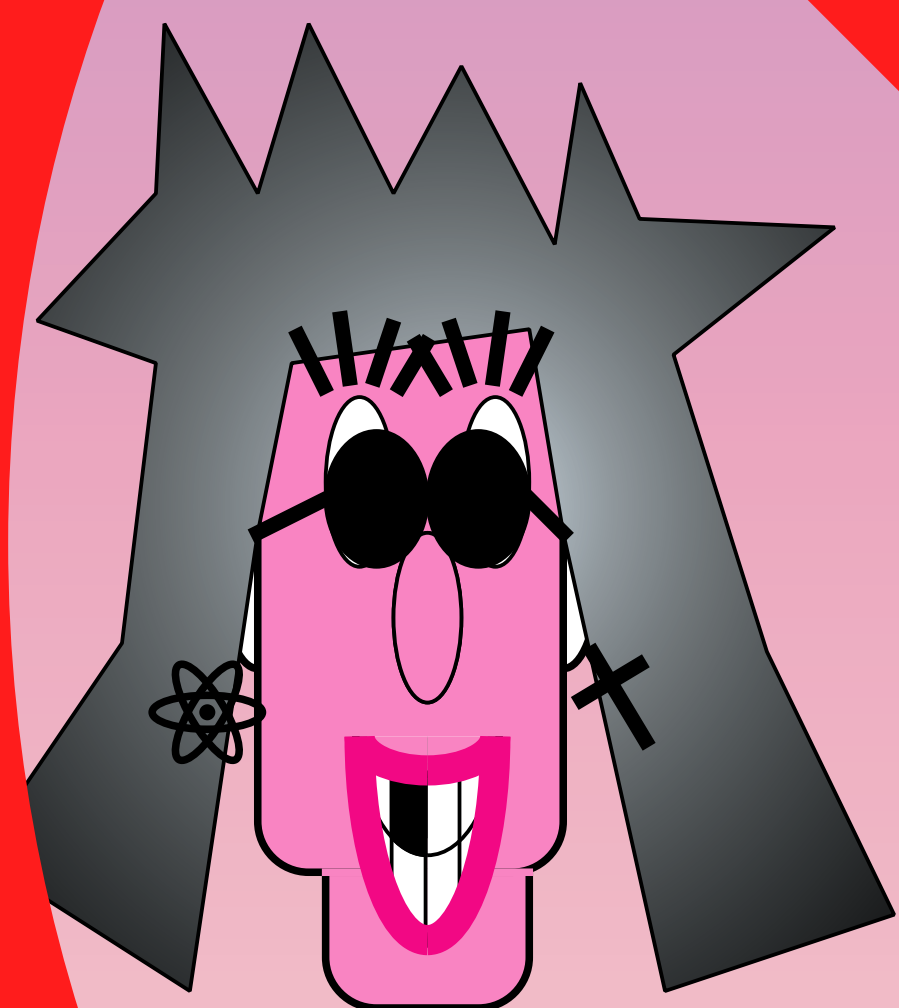
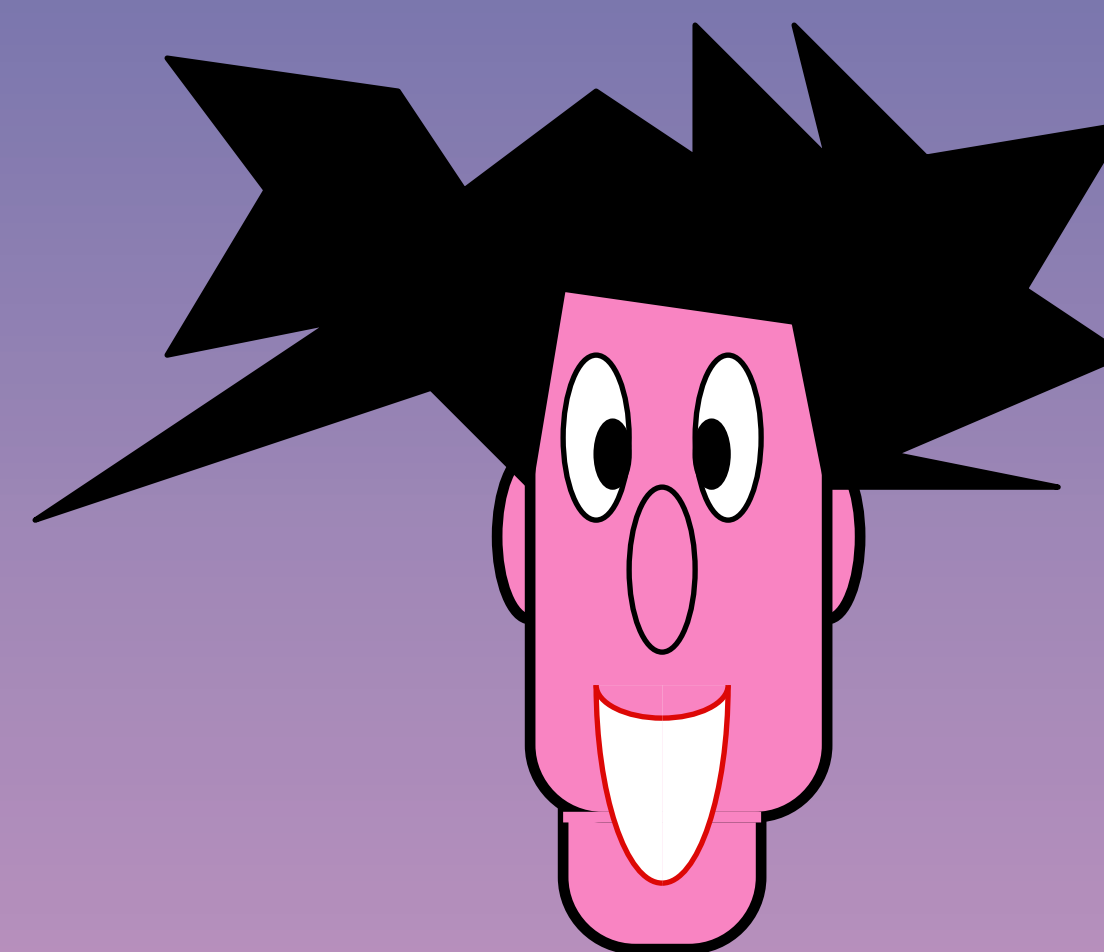


# Oblivious Transfer

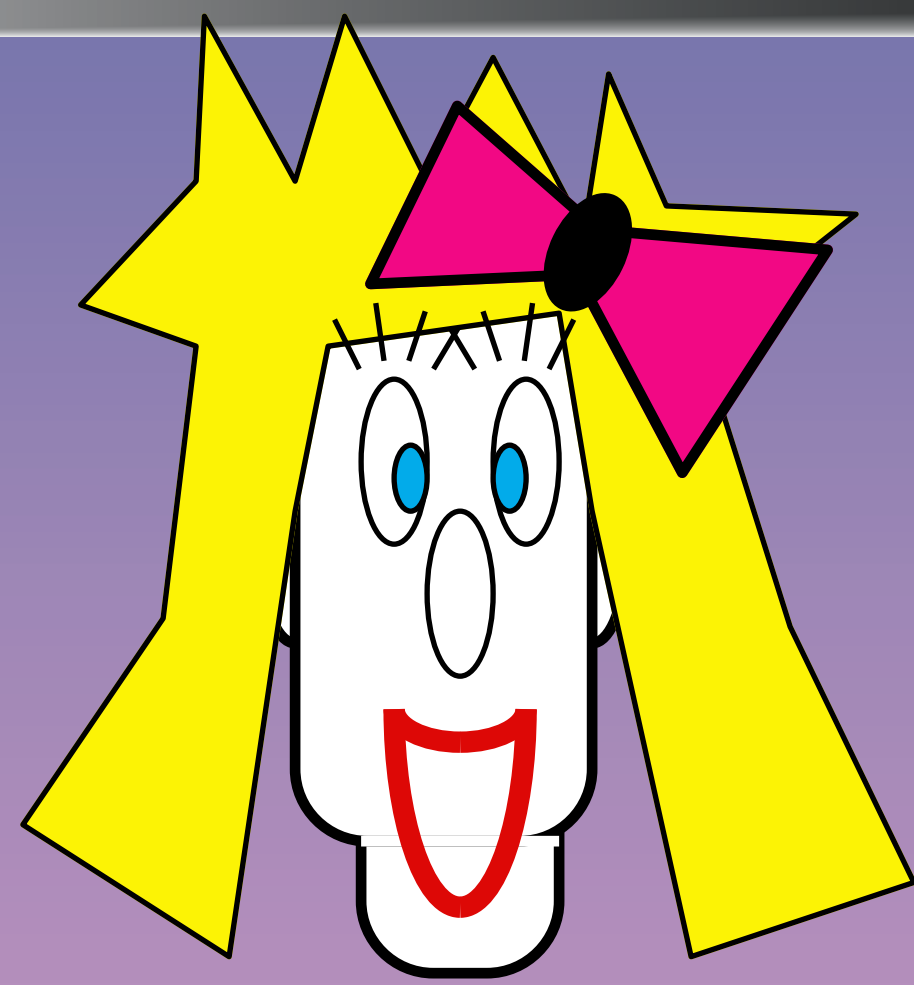




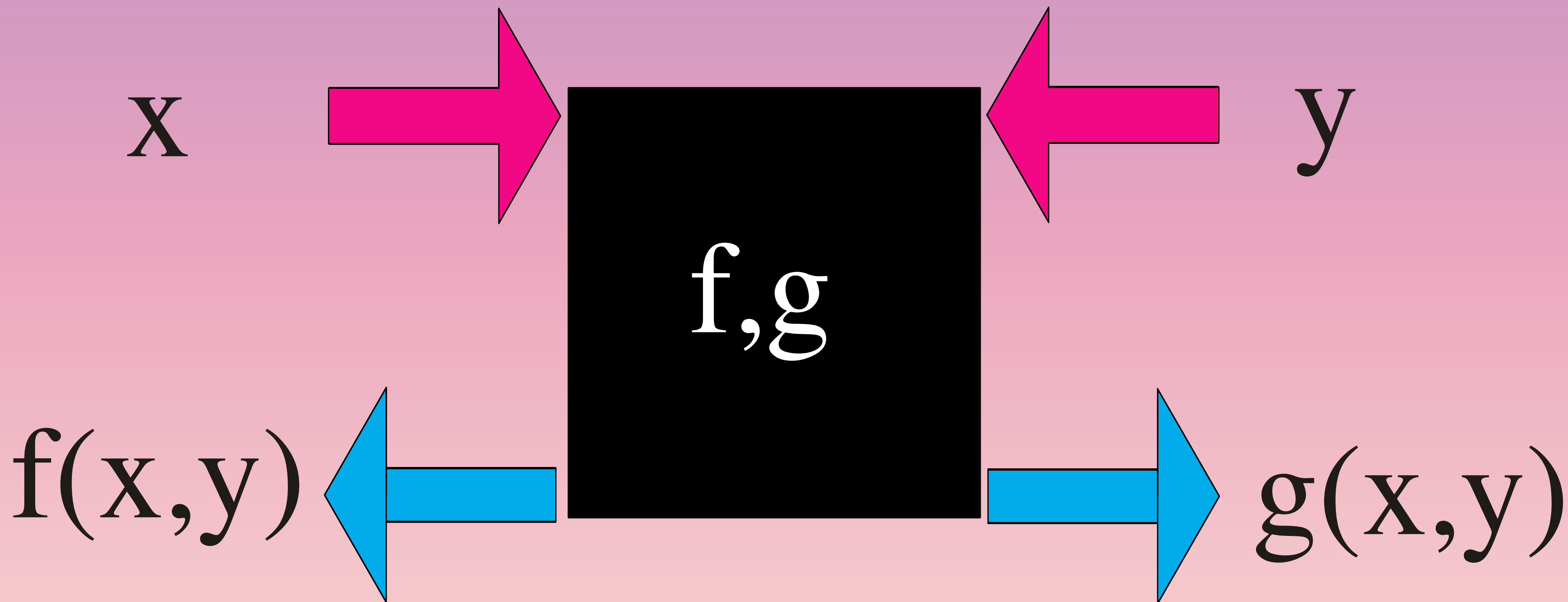
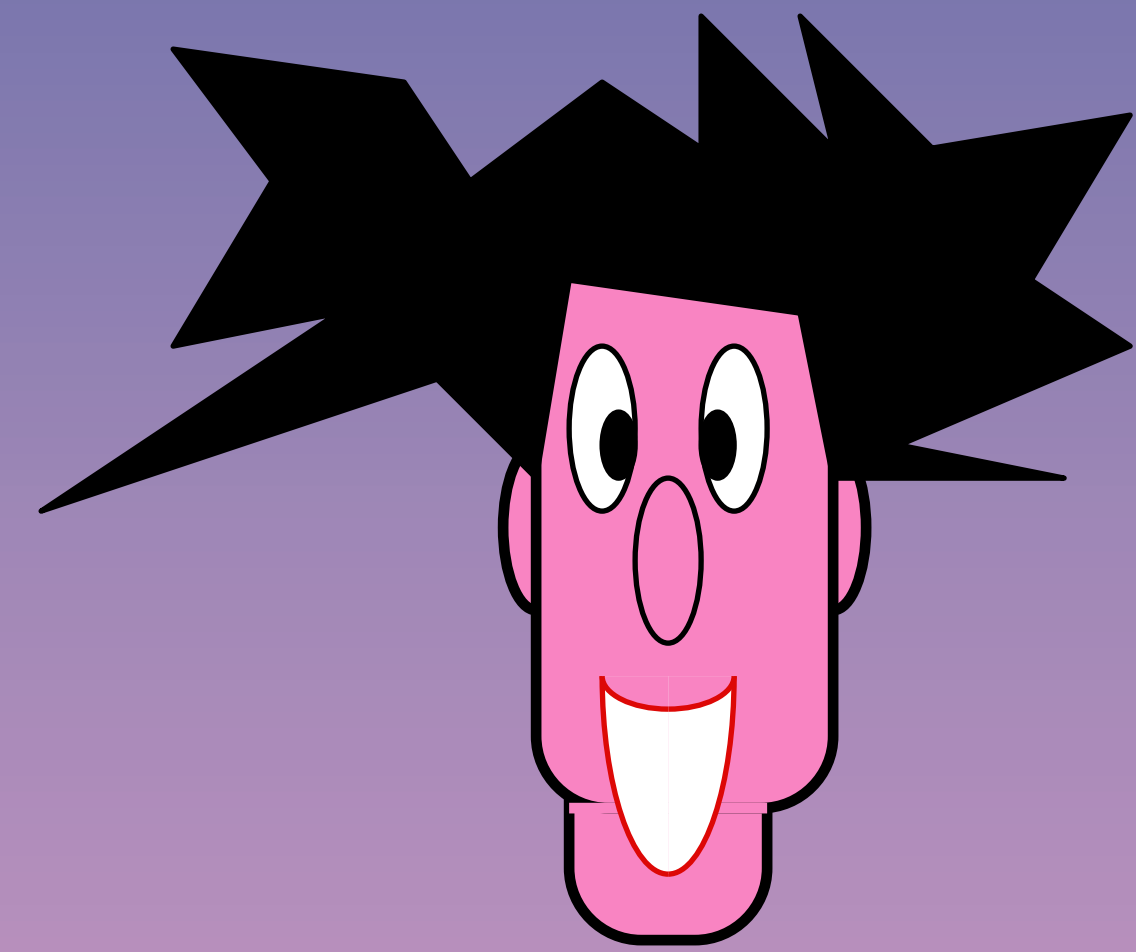
# Oblivious Transfer

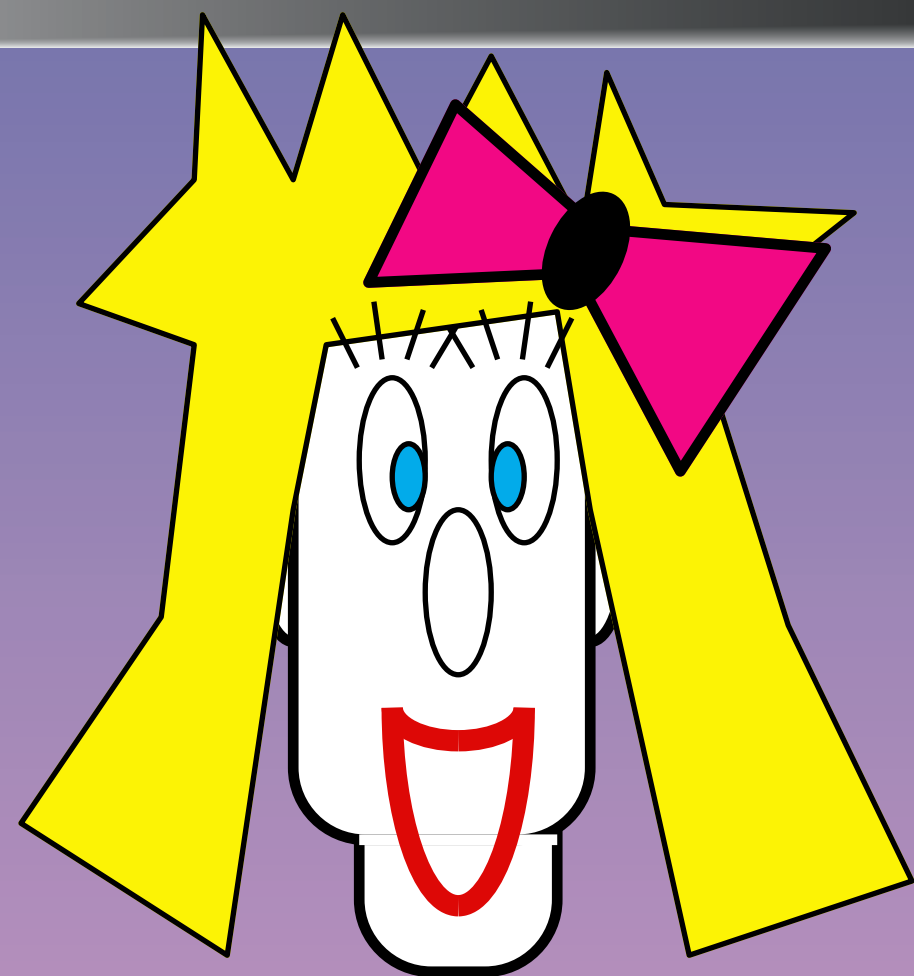




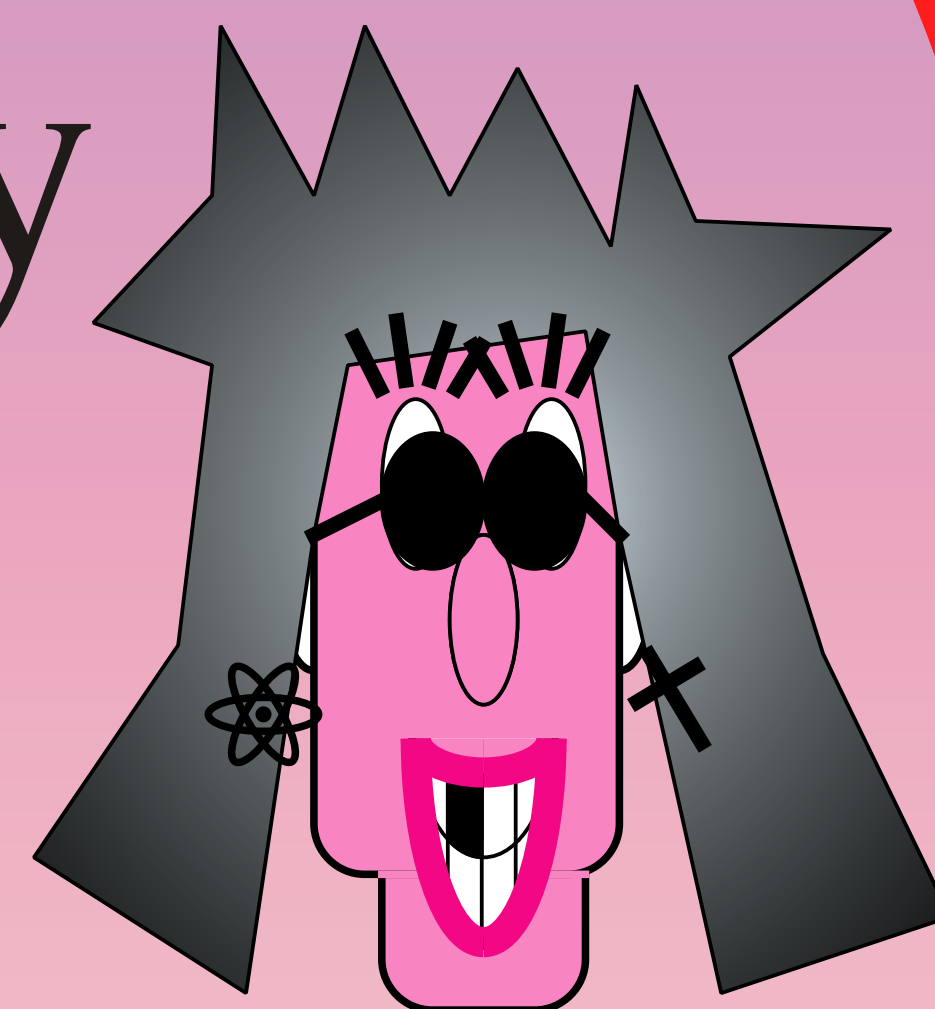
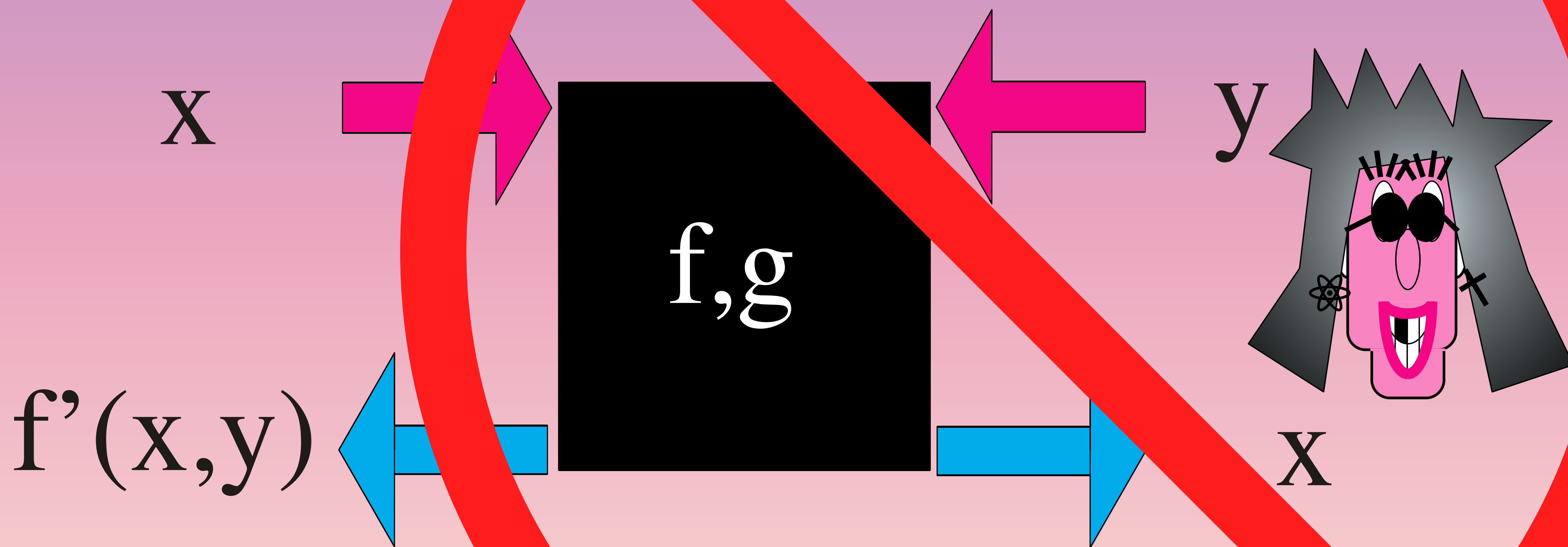


# Oblivious Function Evaluation

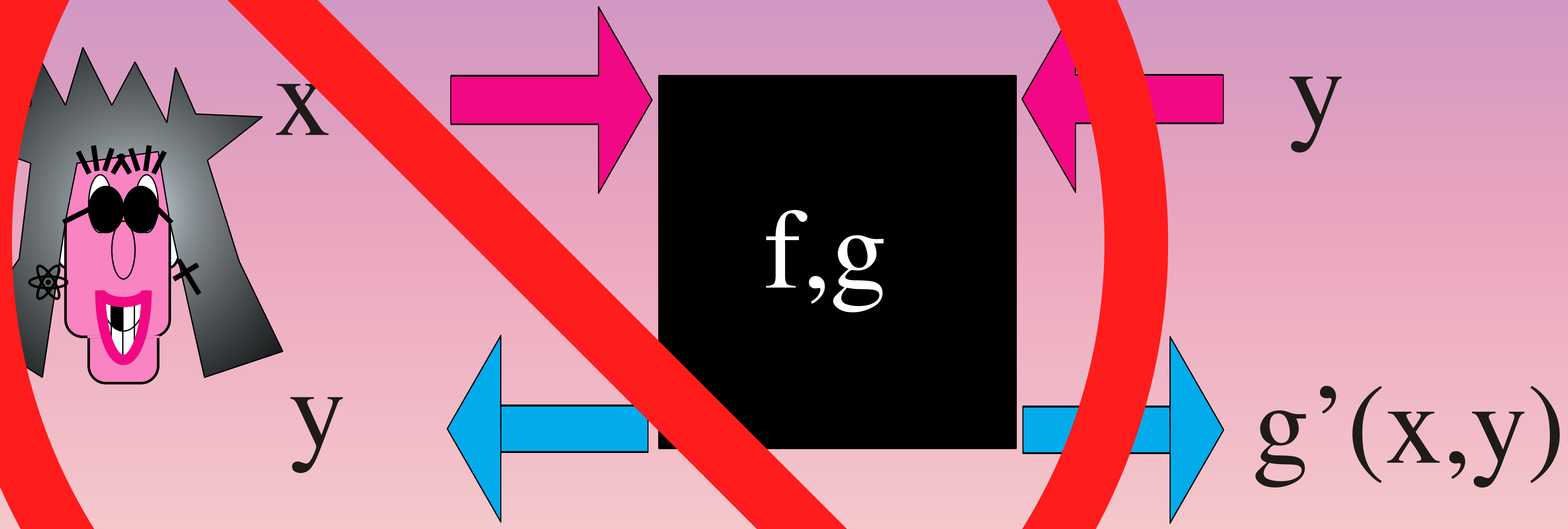
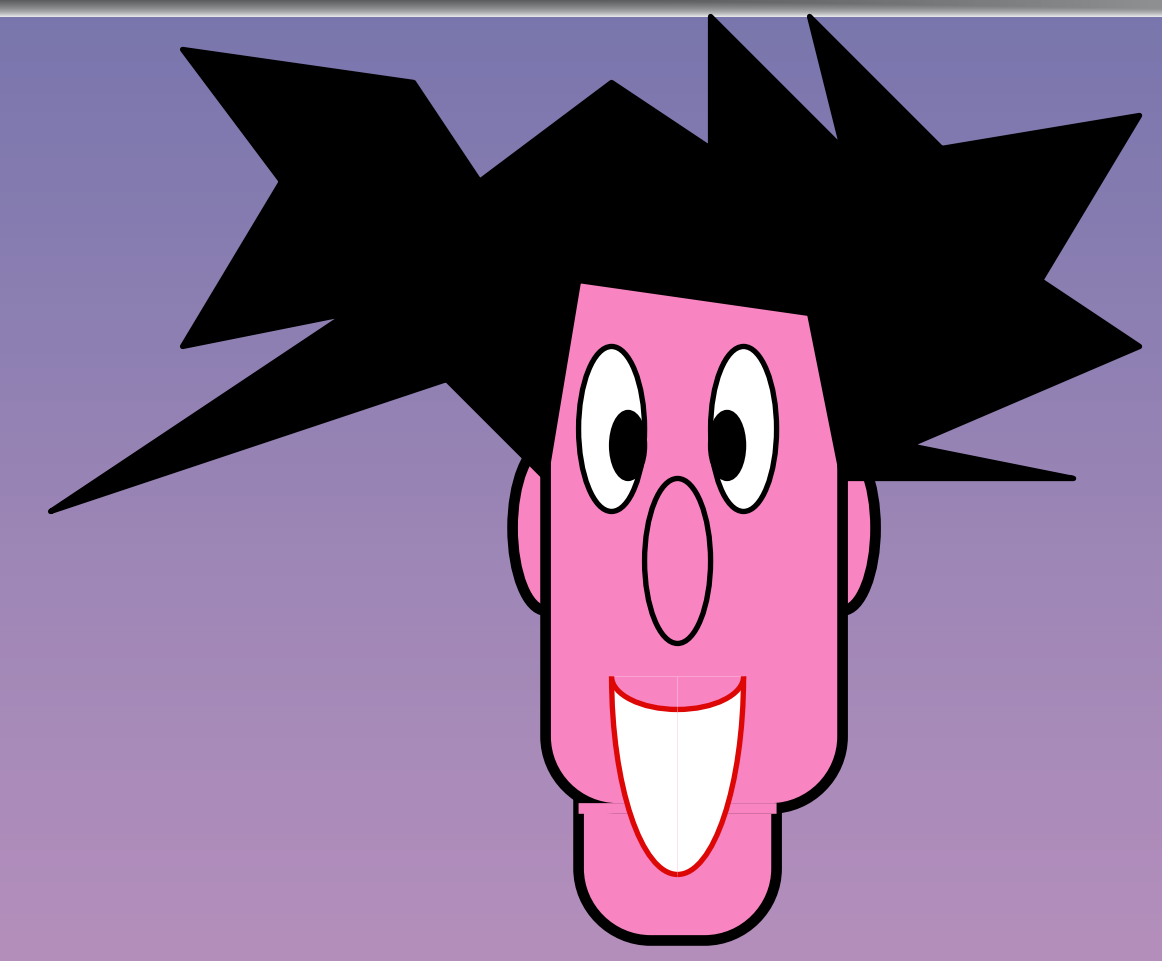


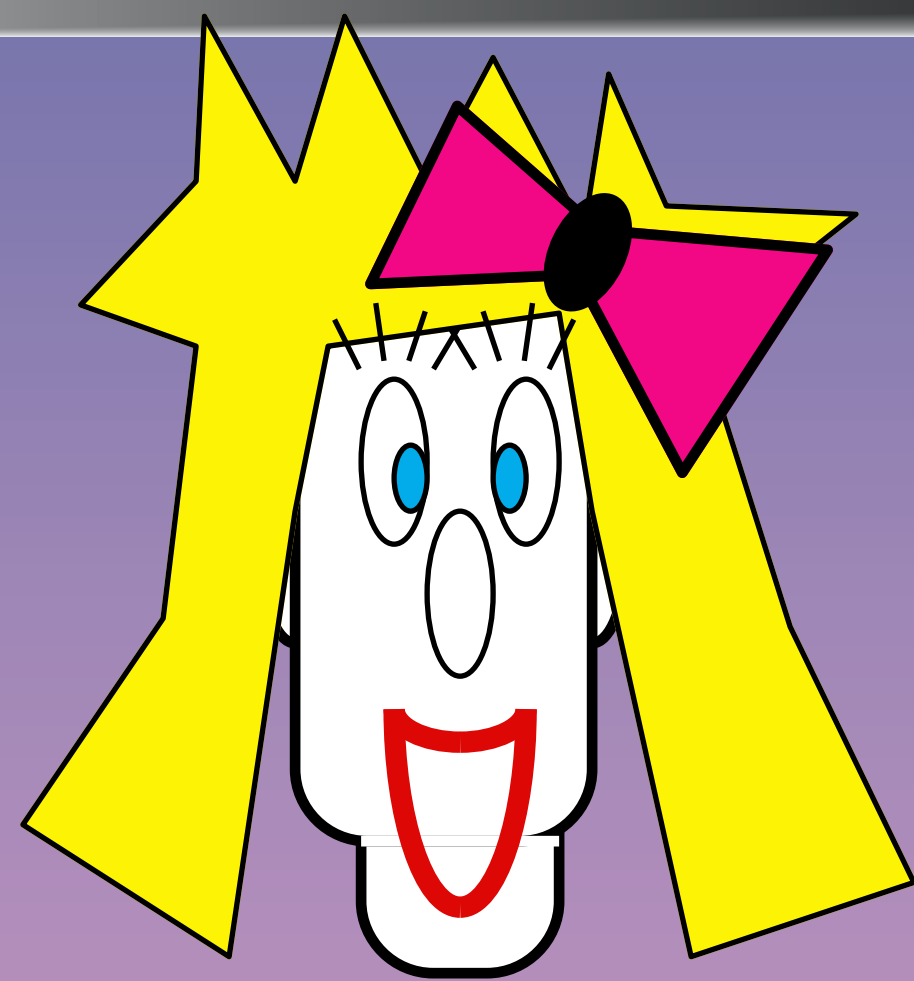


# Oblivious Function Evaluation

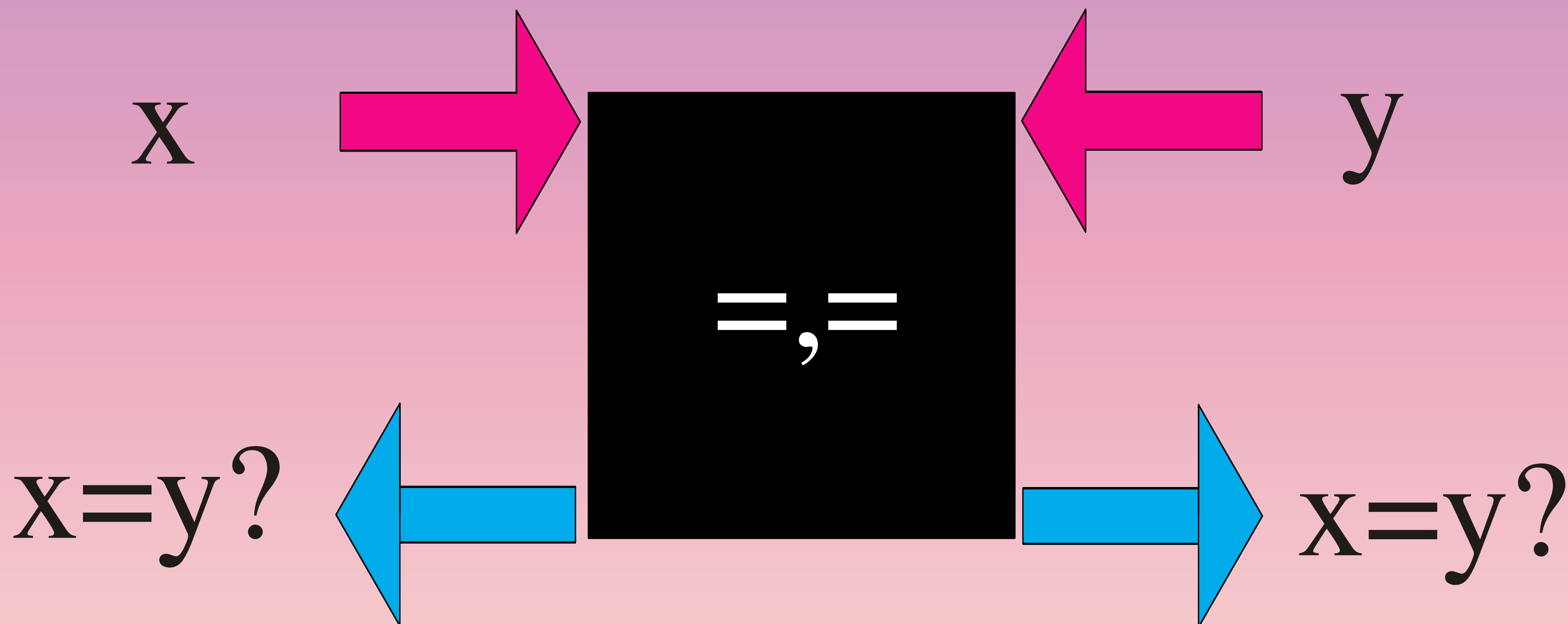
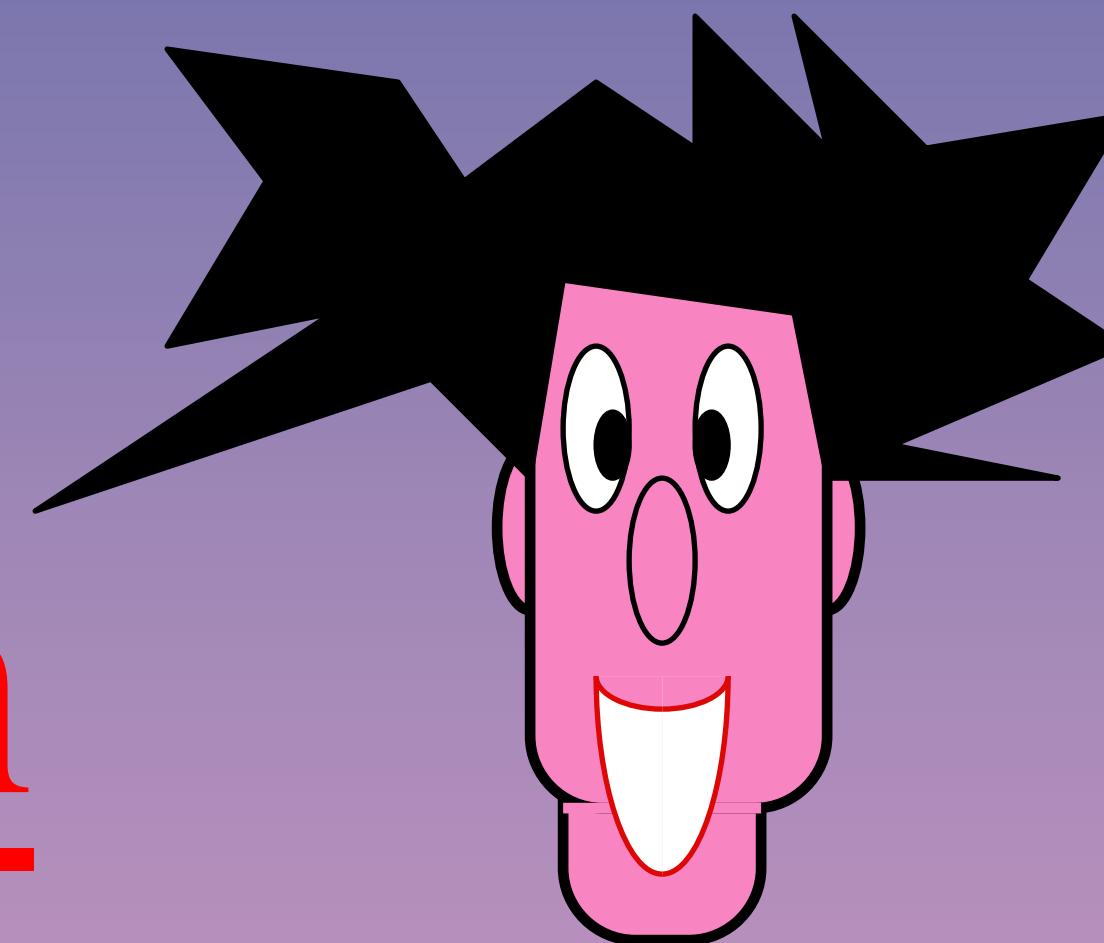


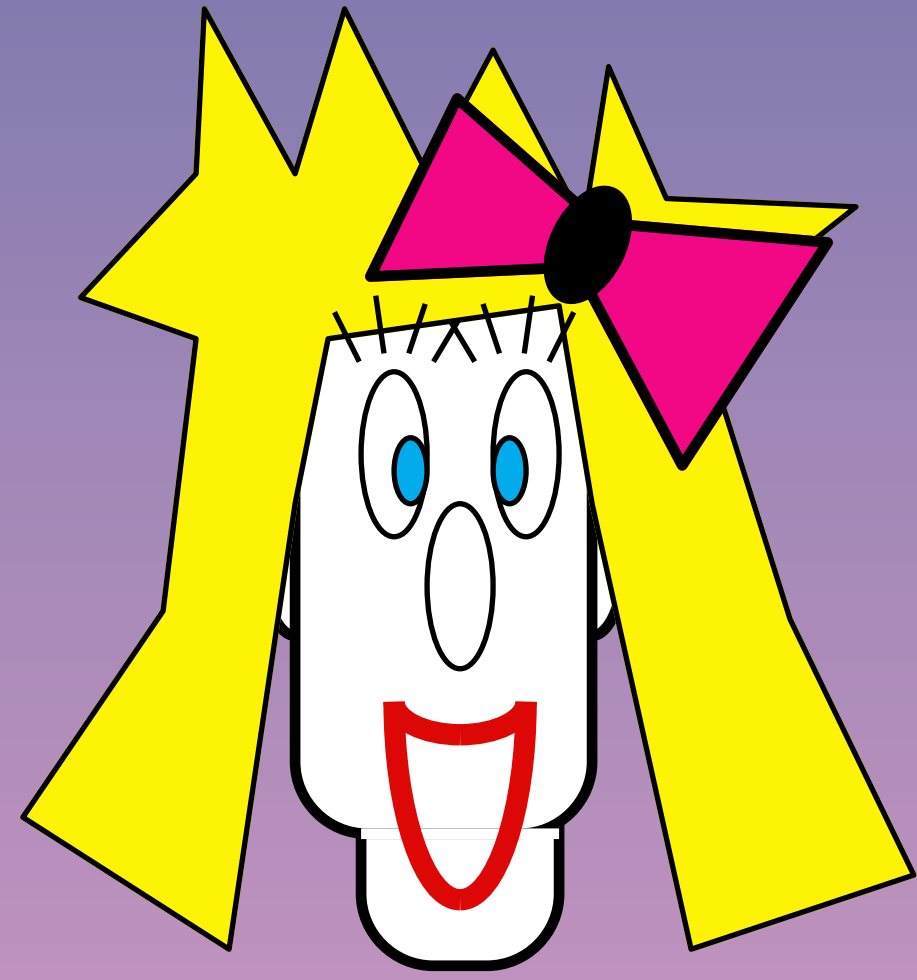
# Oblivious Function Evaluation



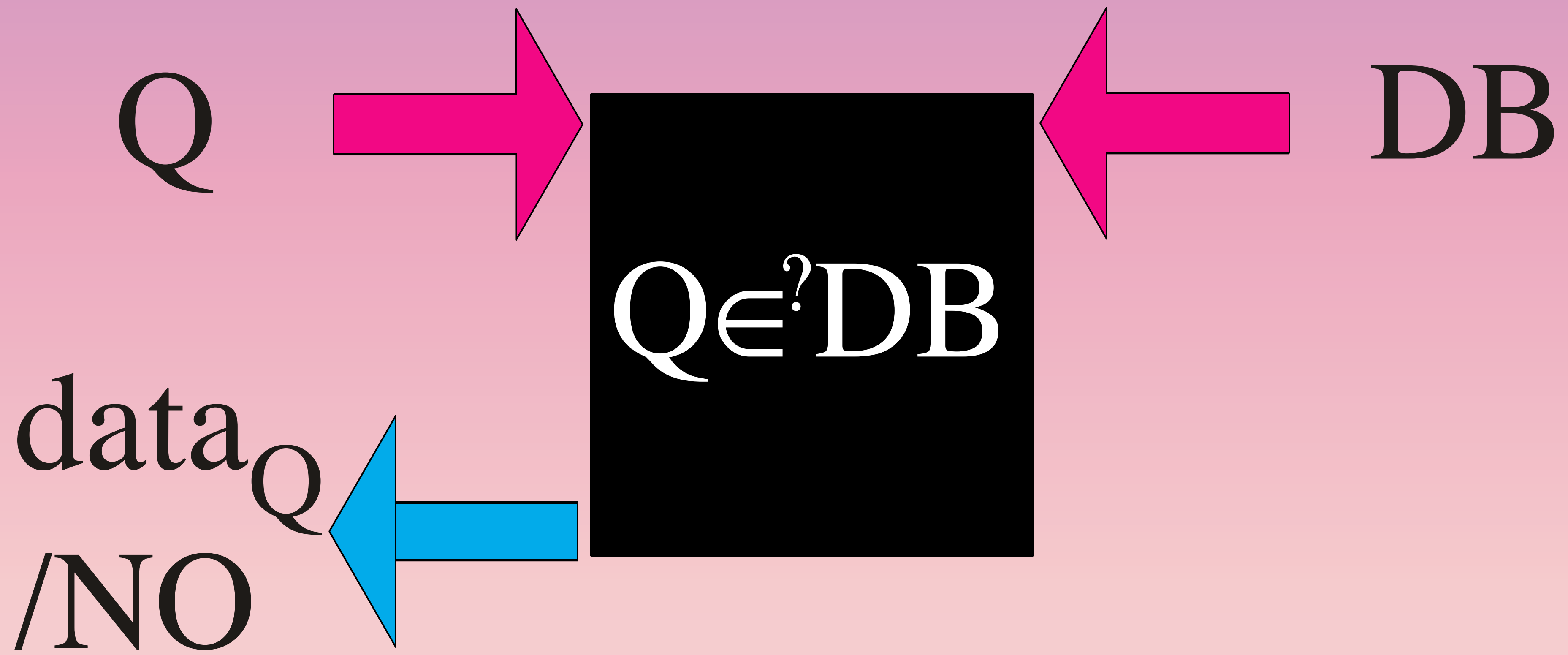
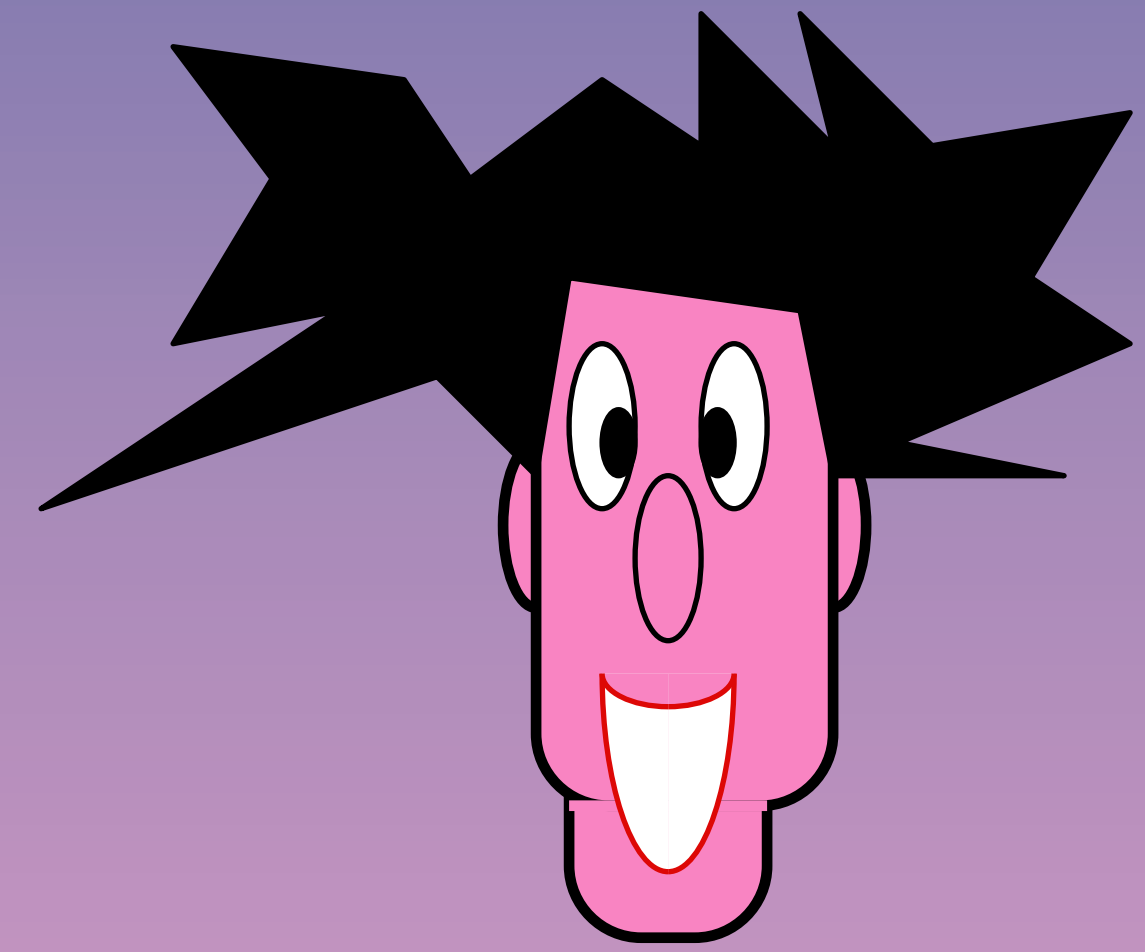


# Mutual Identification

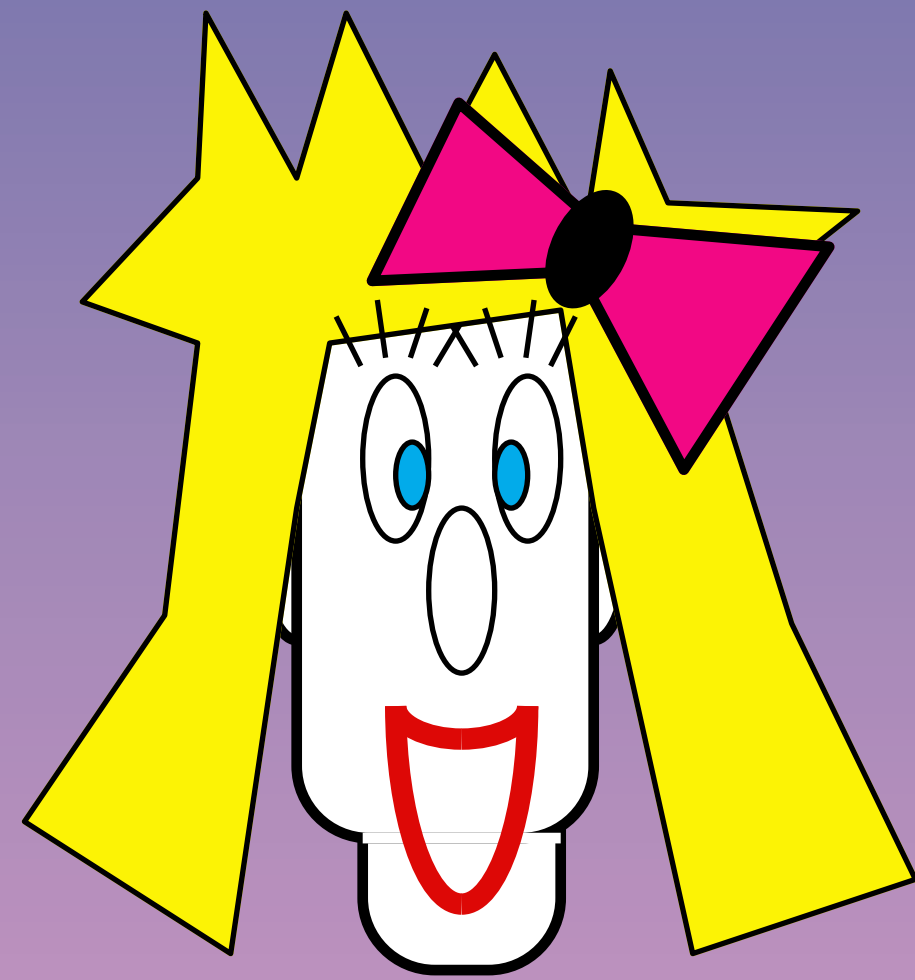




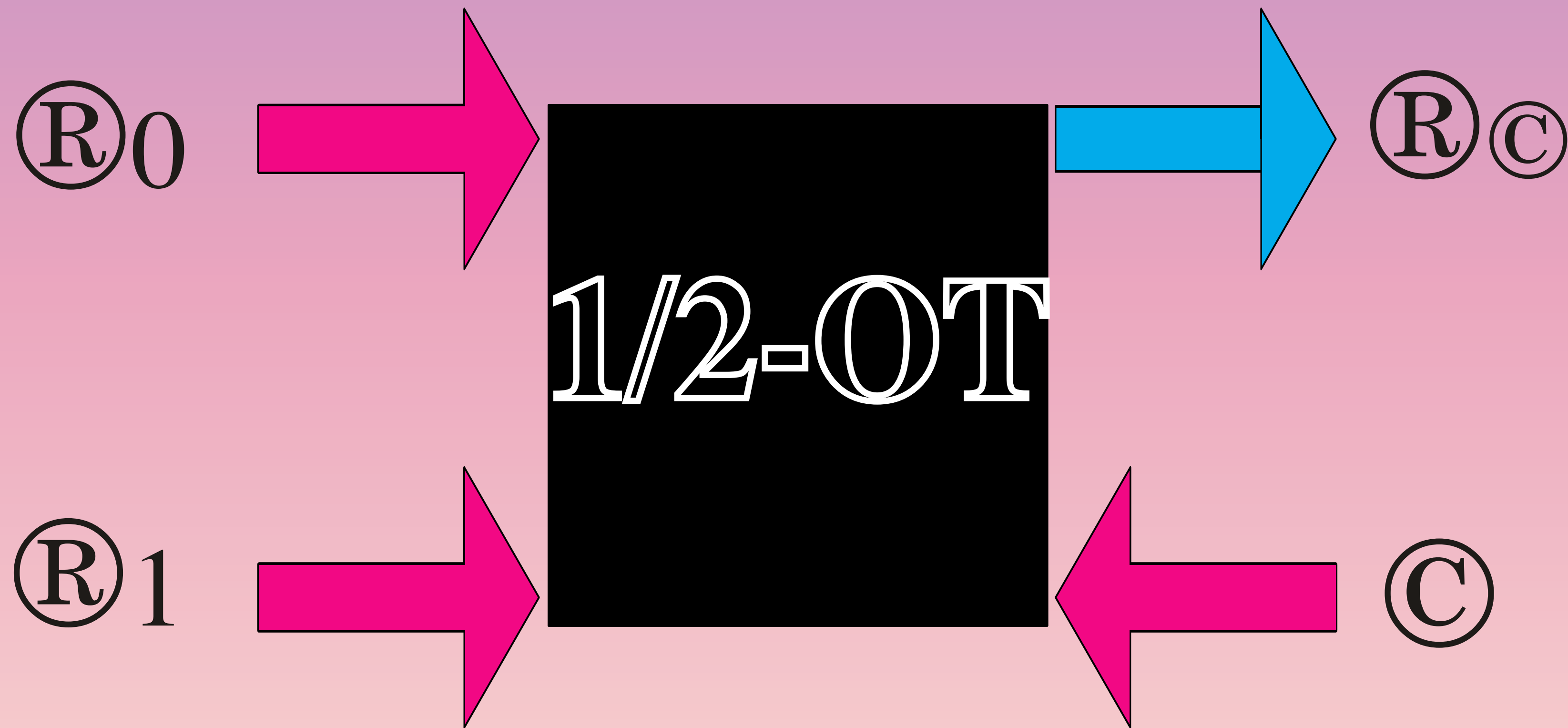
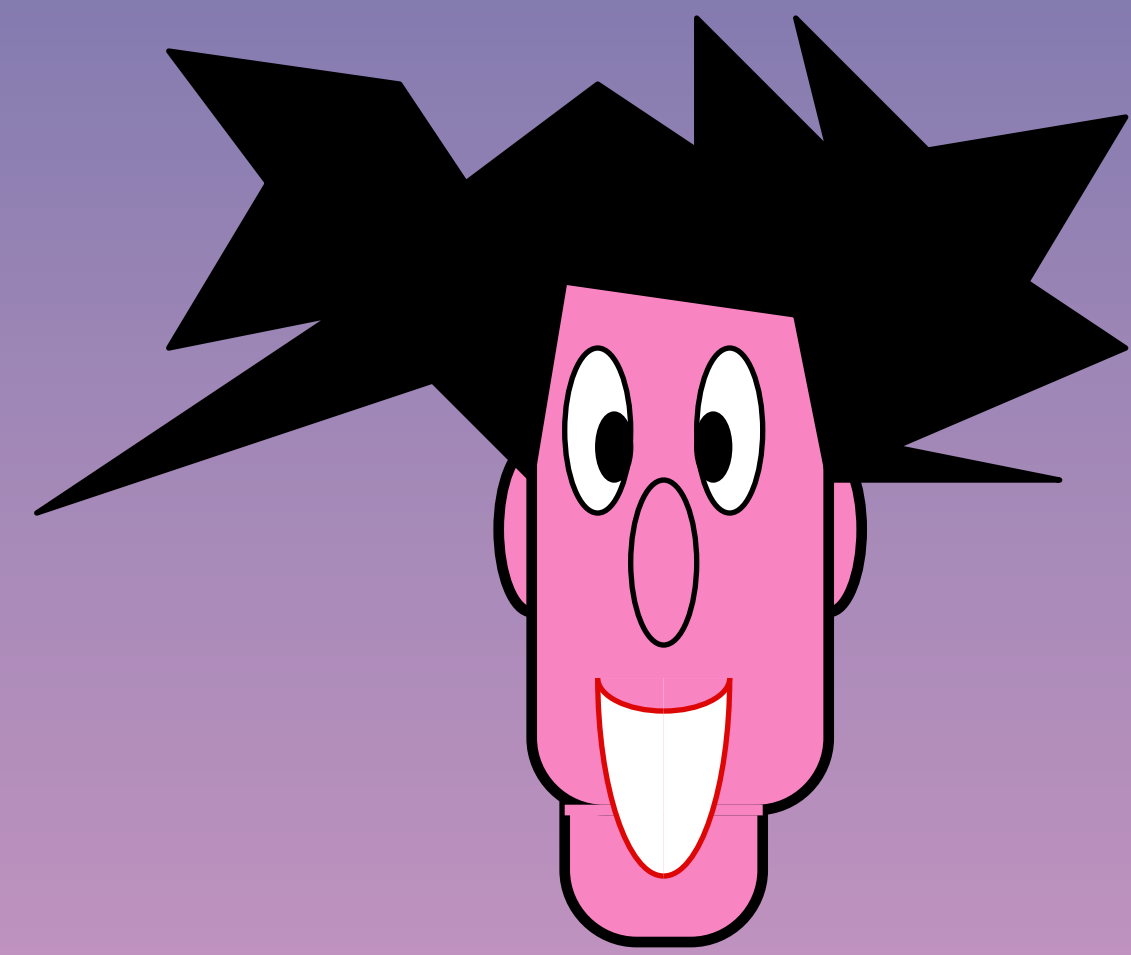
# Oblivious DB query

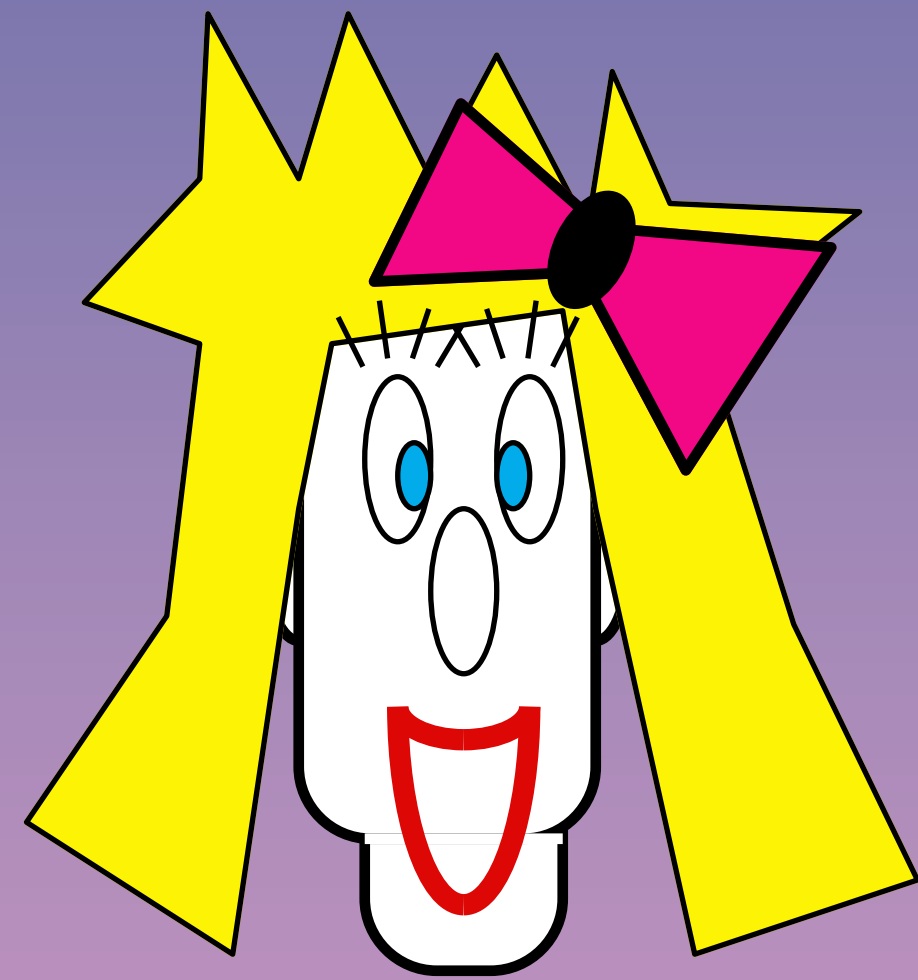




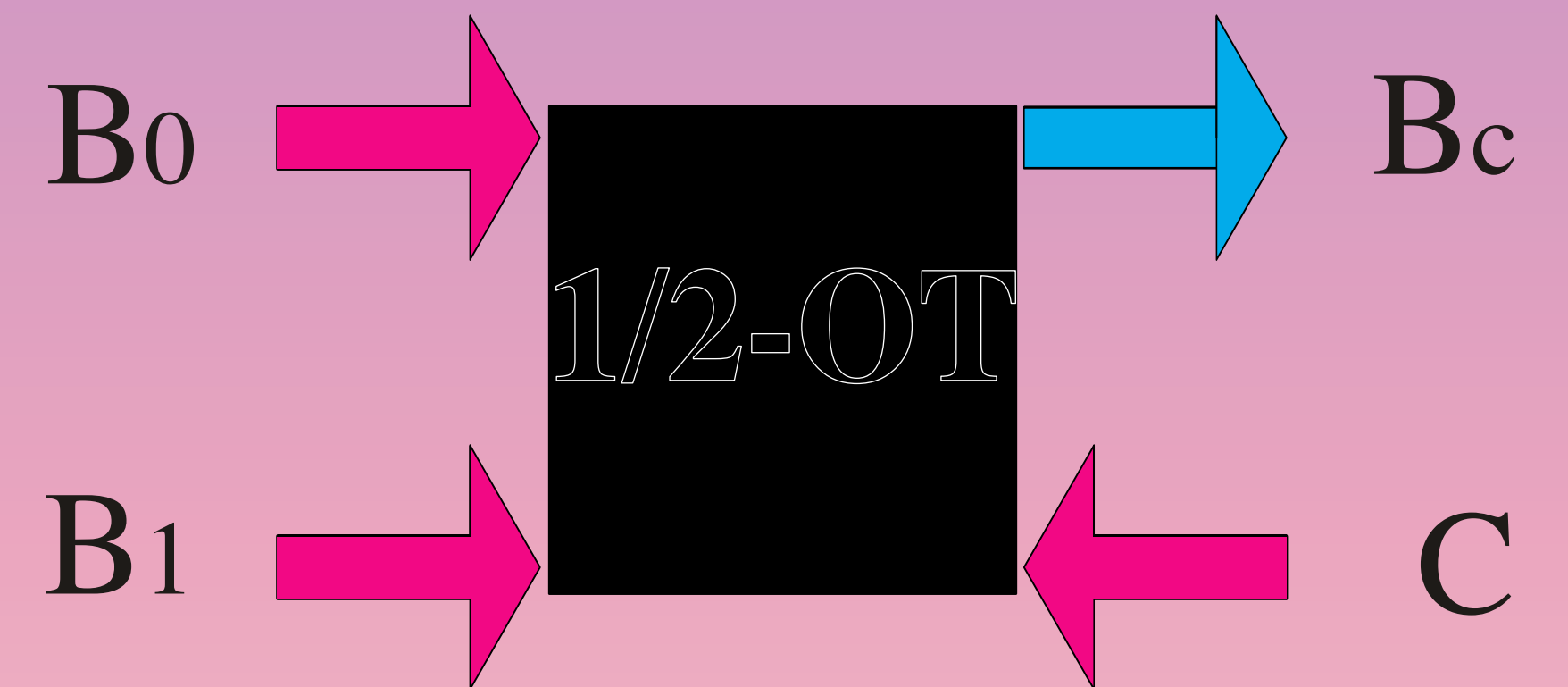
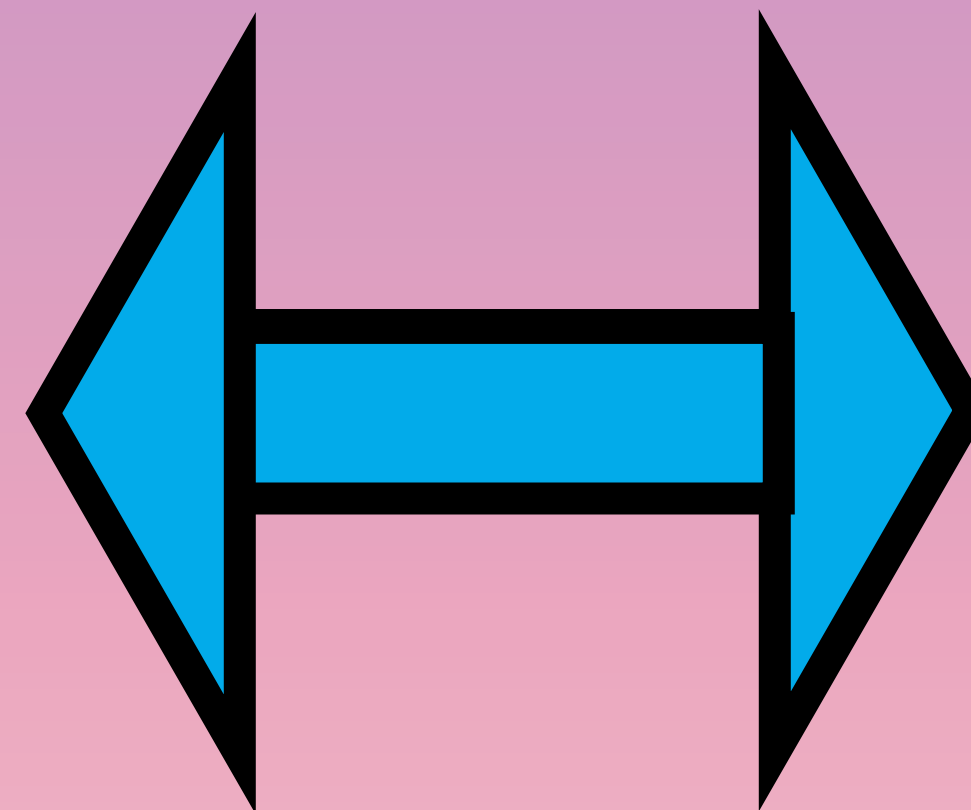
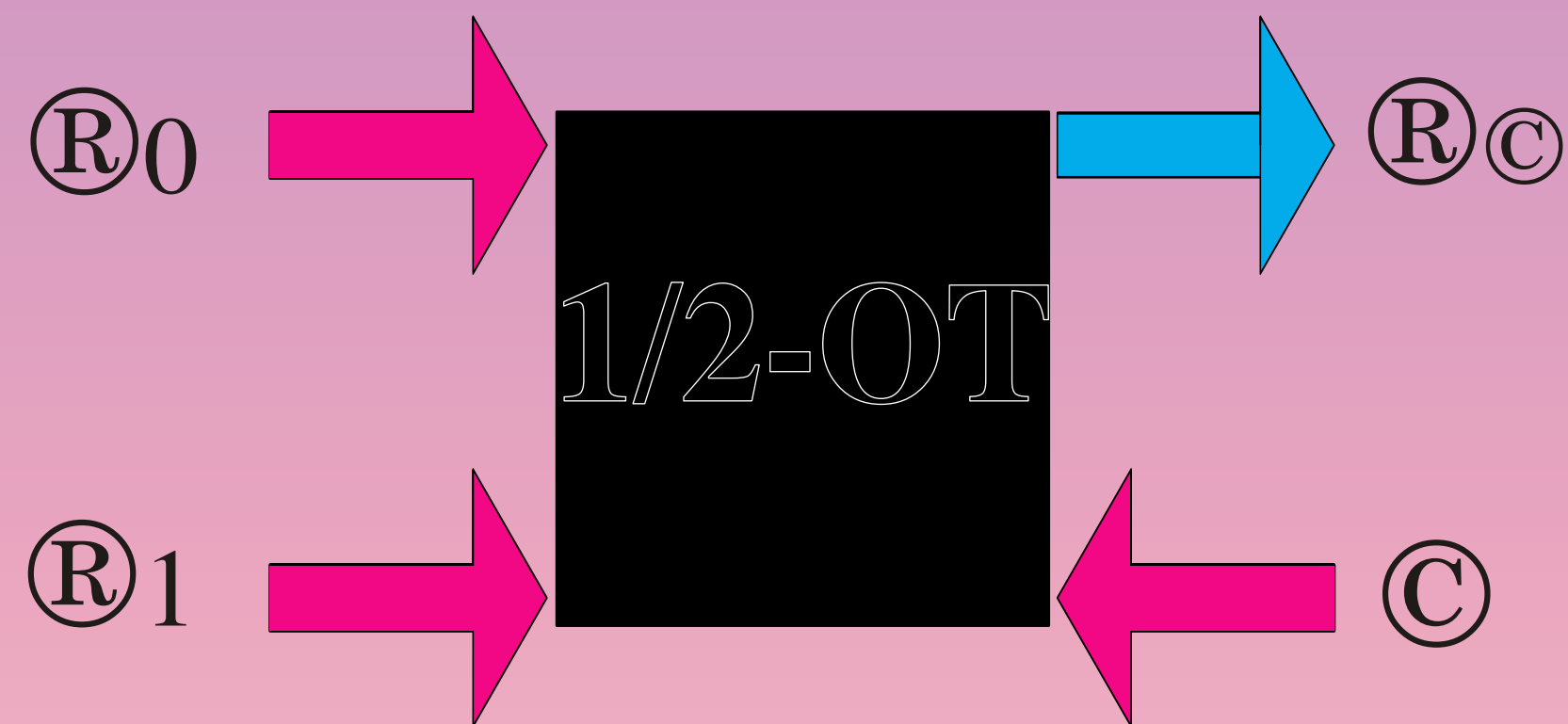
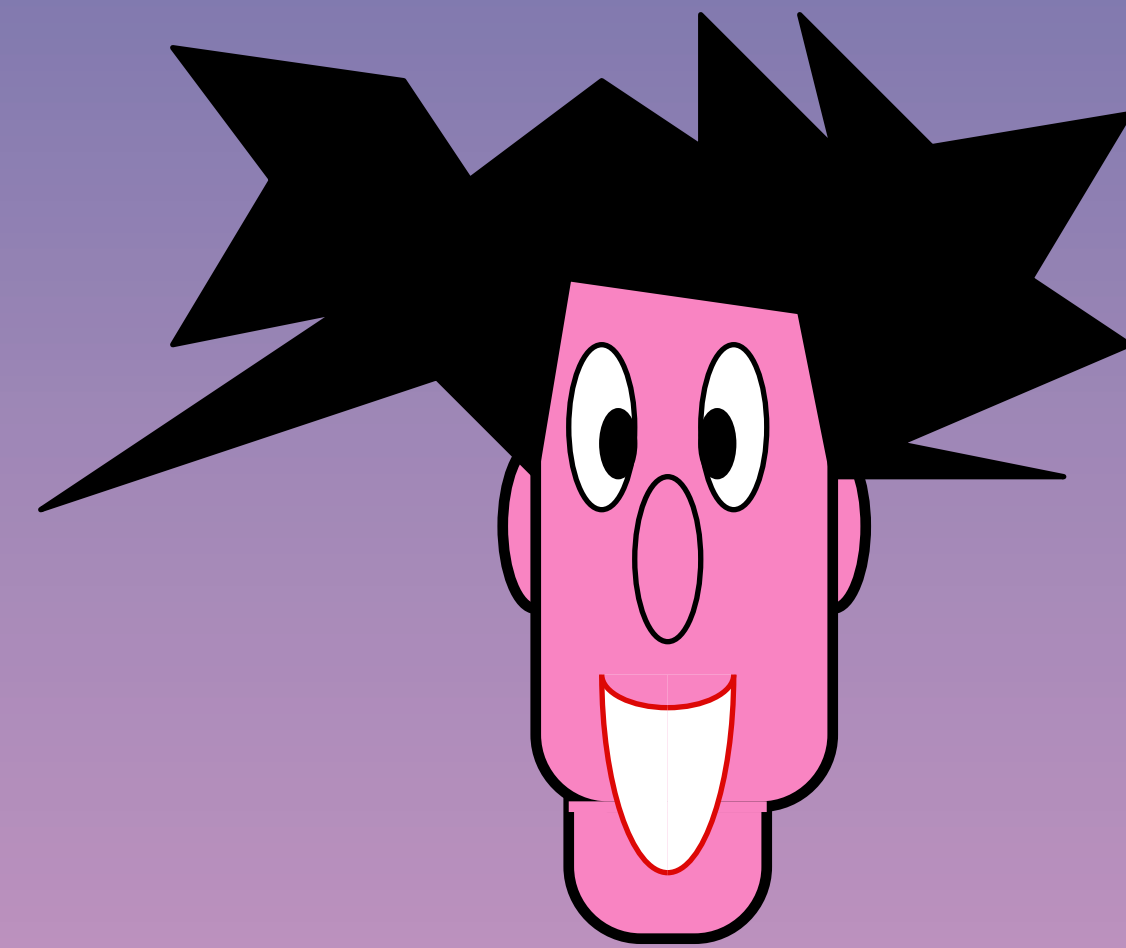


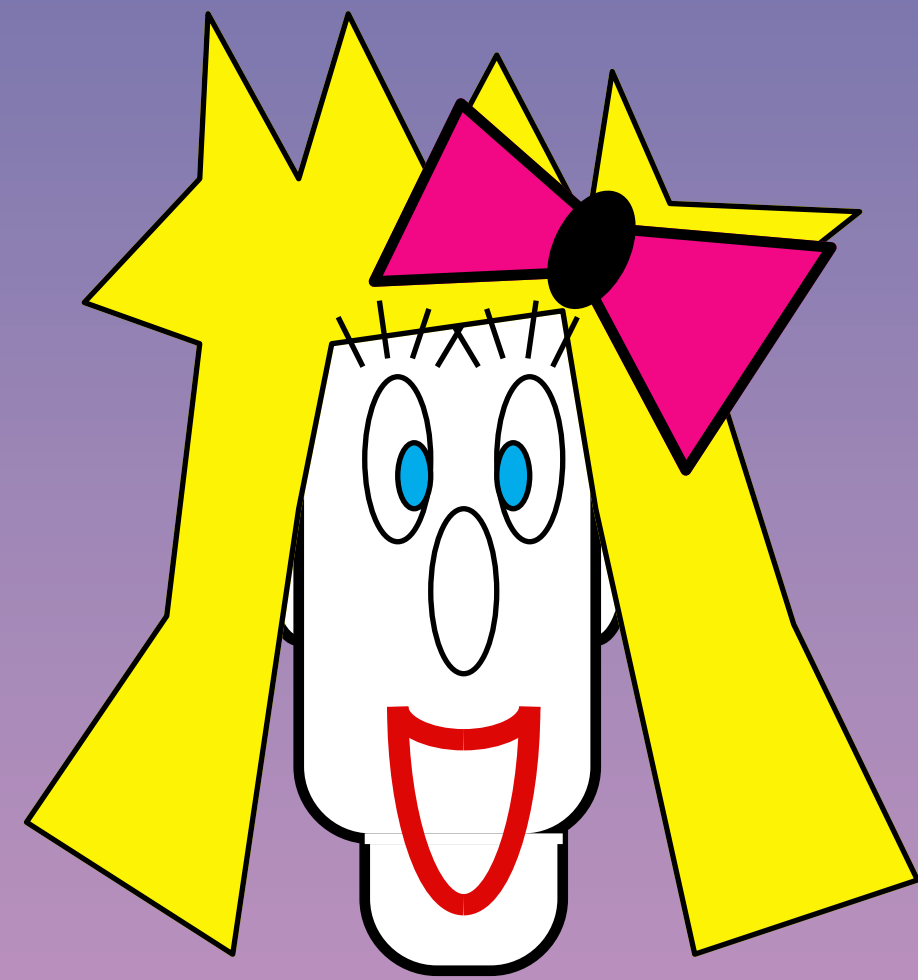
# Randomized Oblivious Transfer



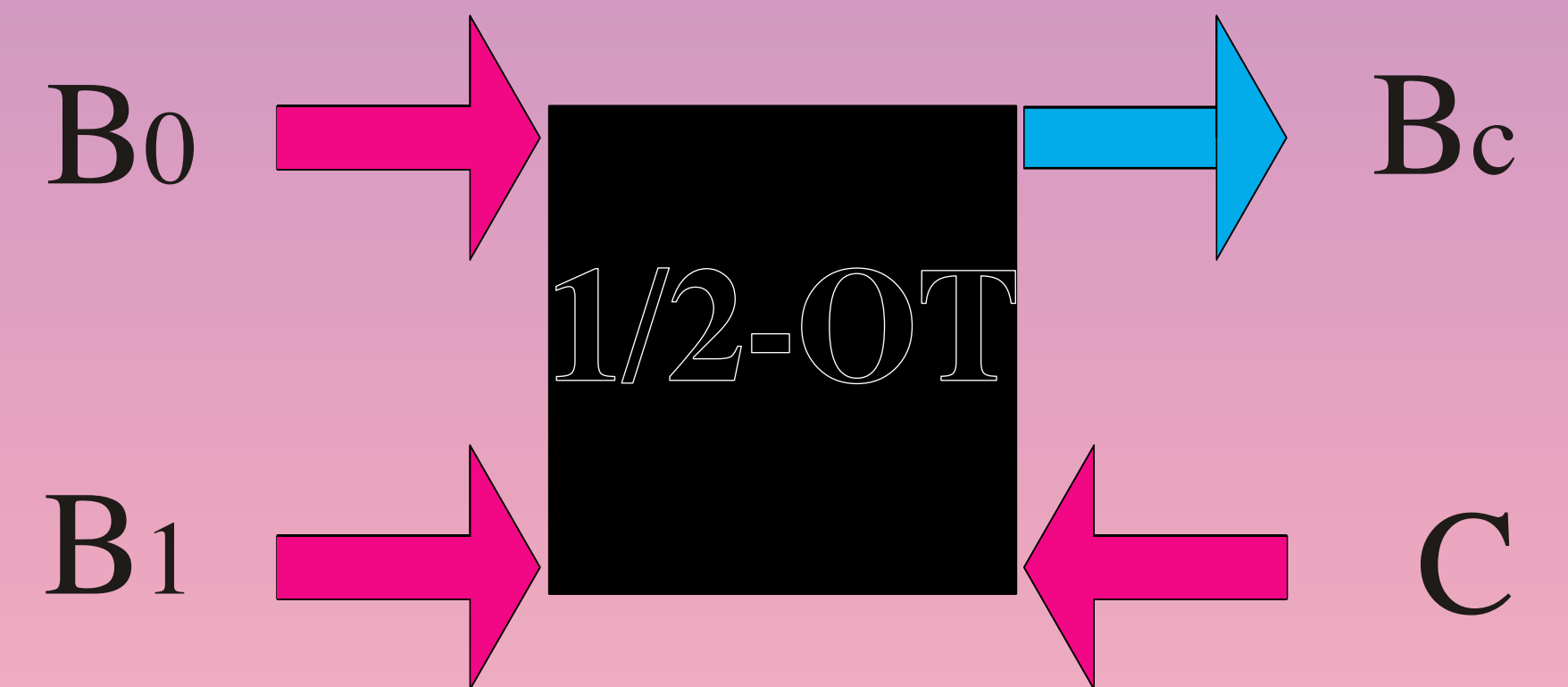
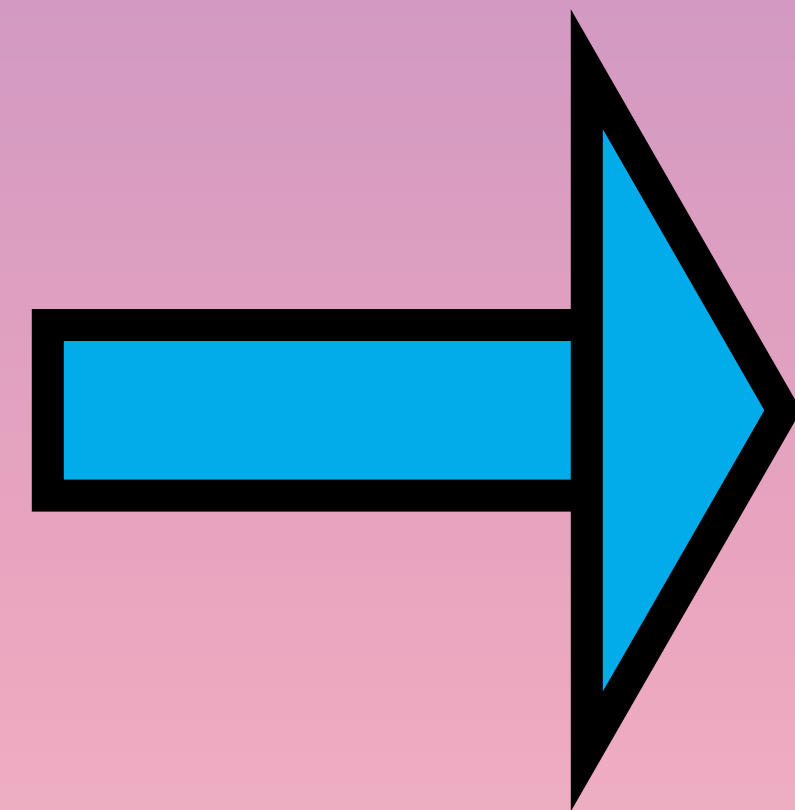
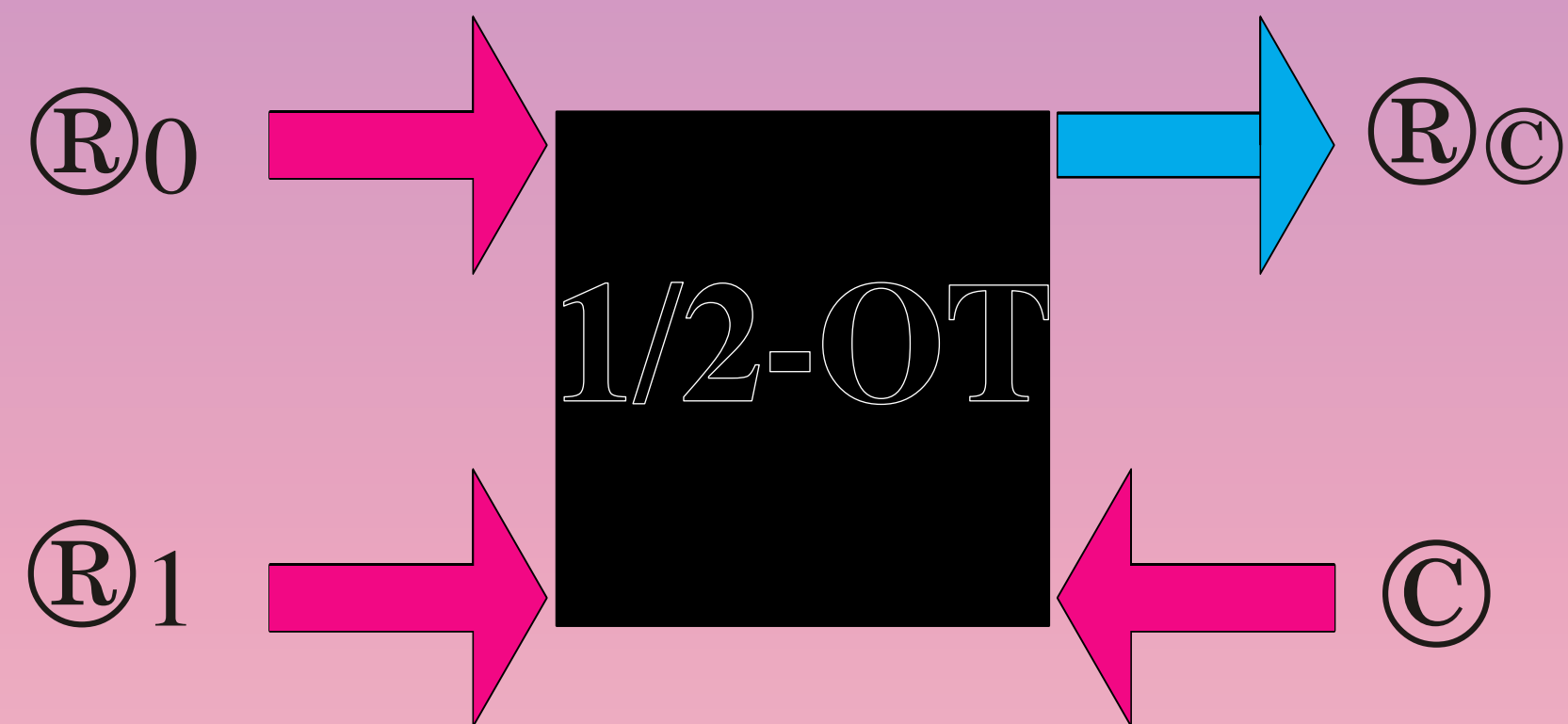
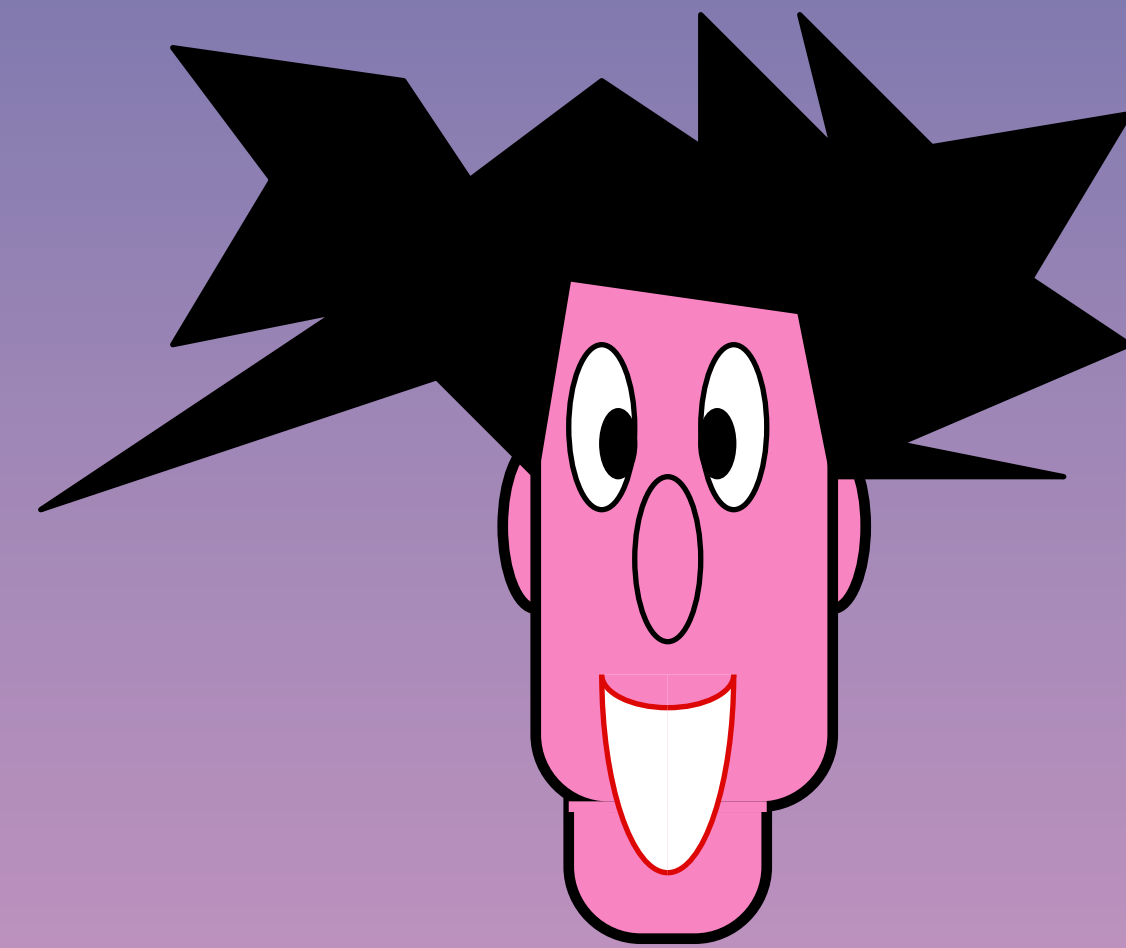


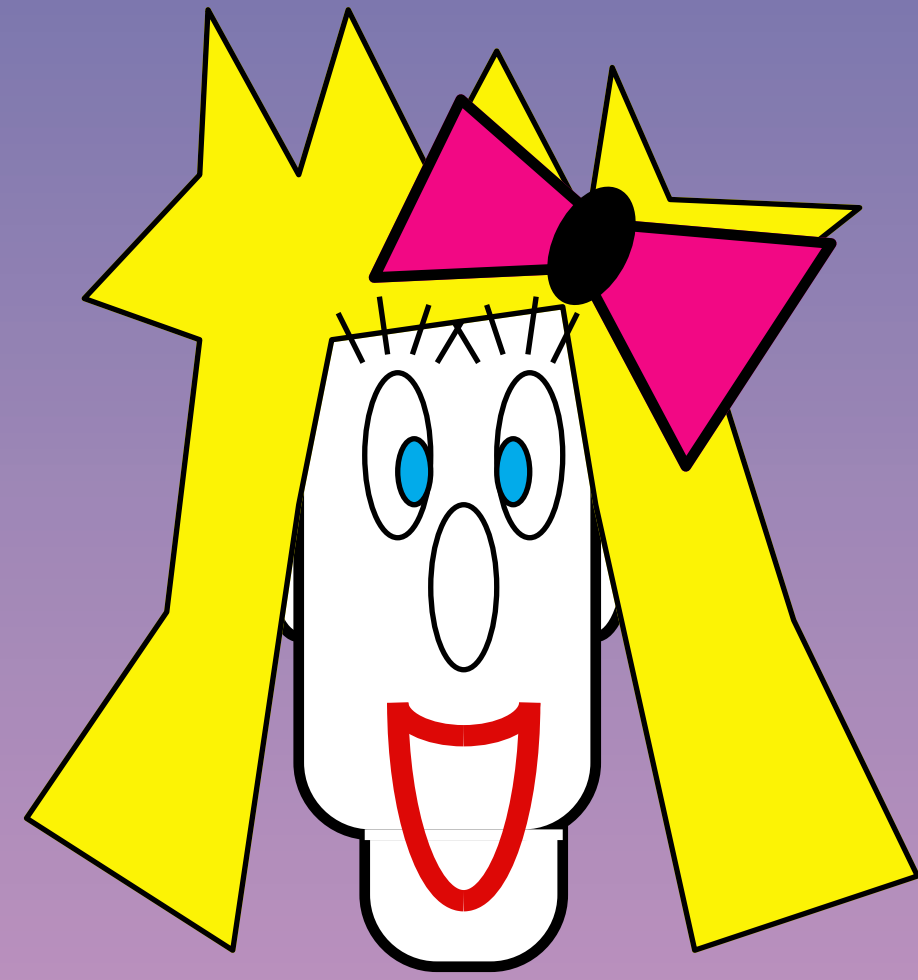
# Randomized Oblivious Transfer



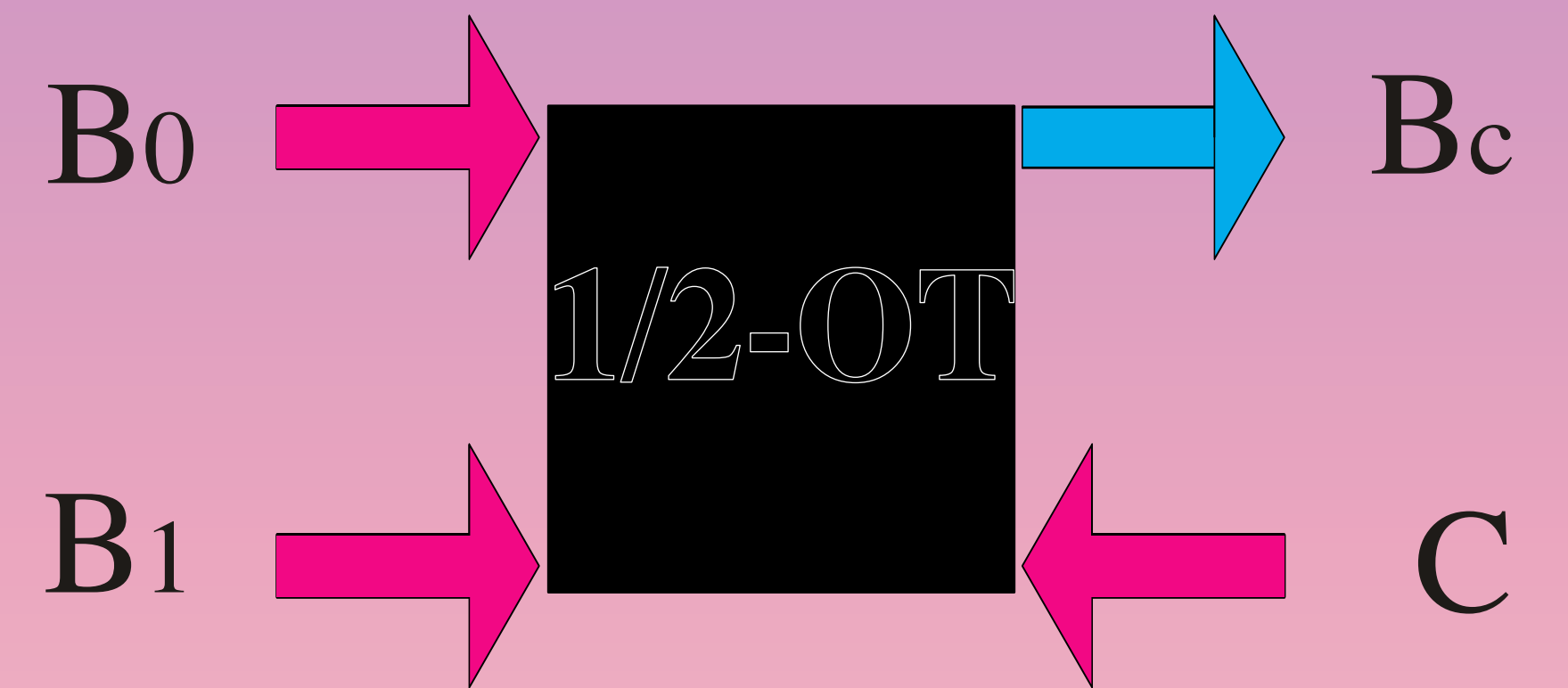
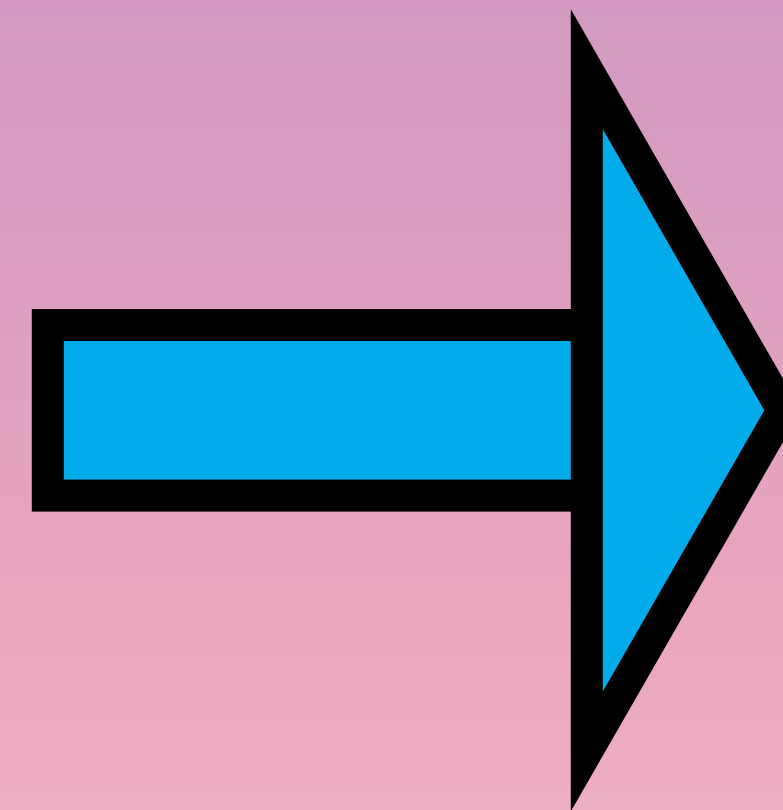
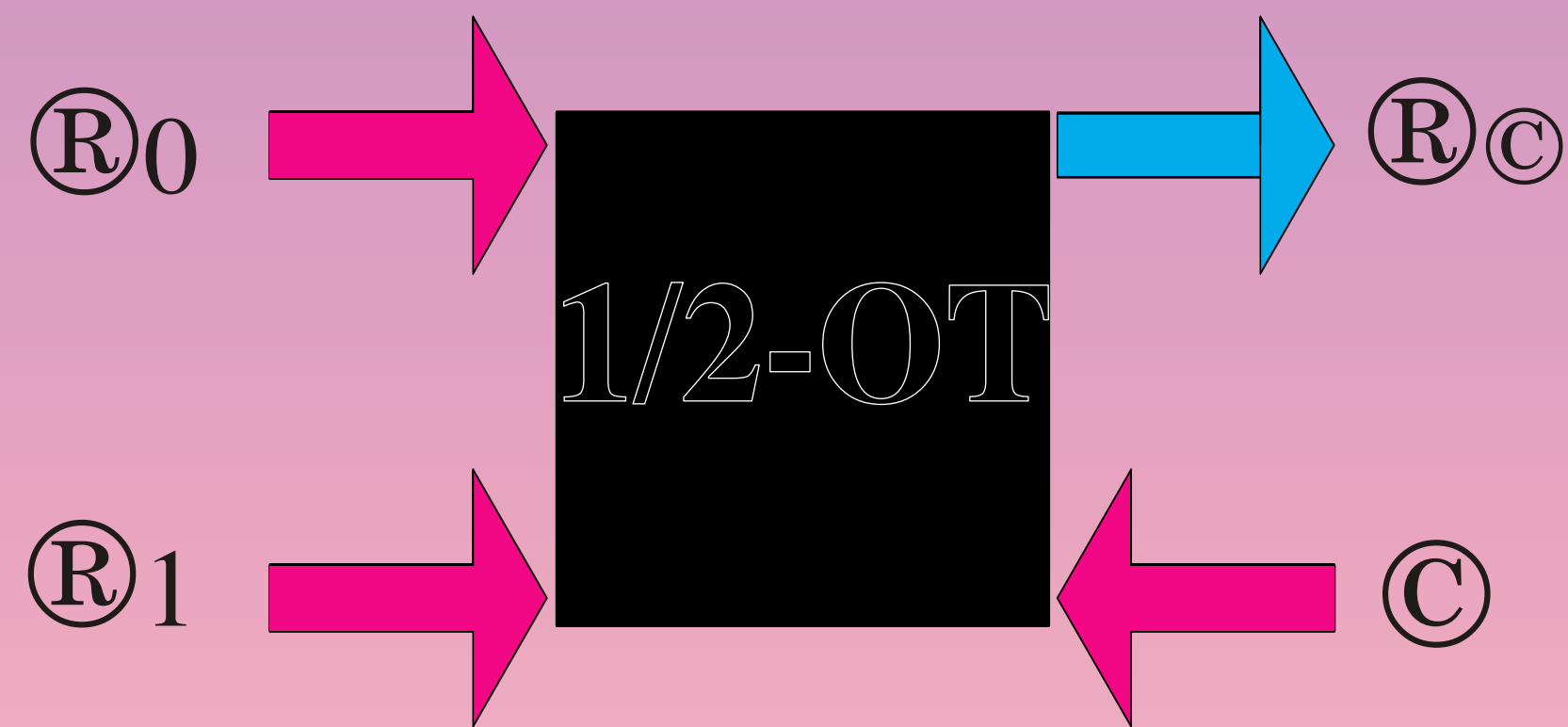
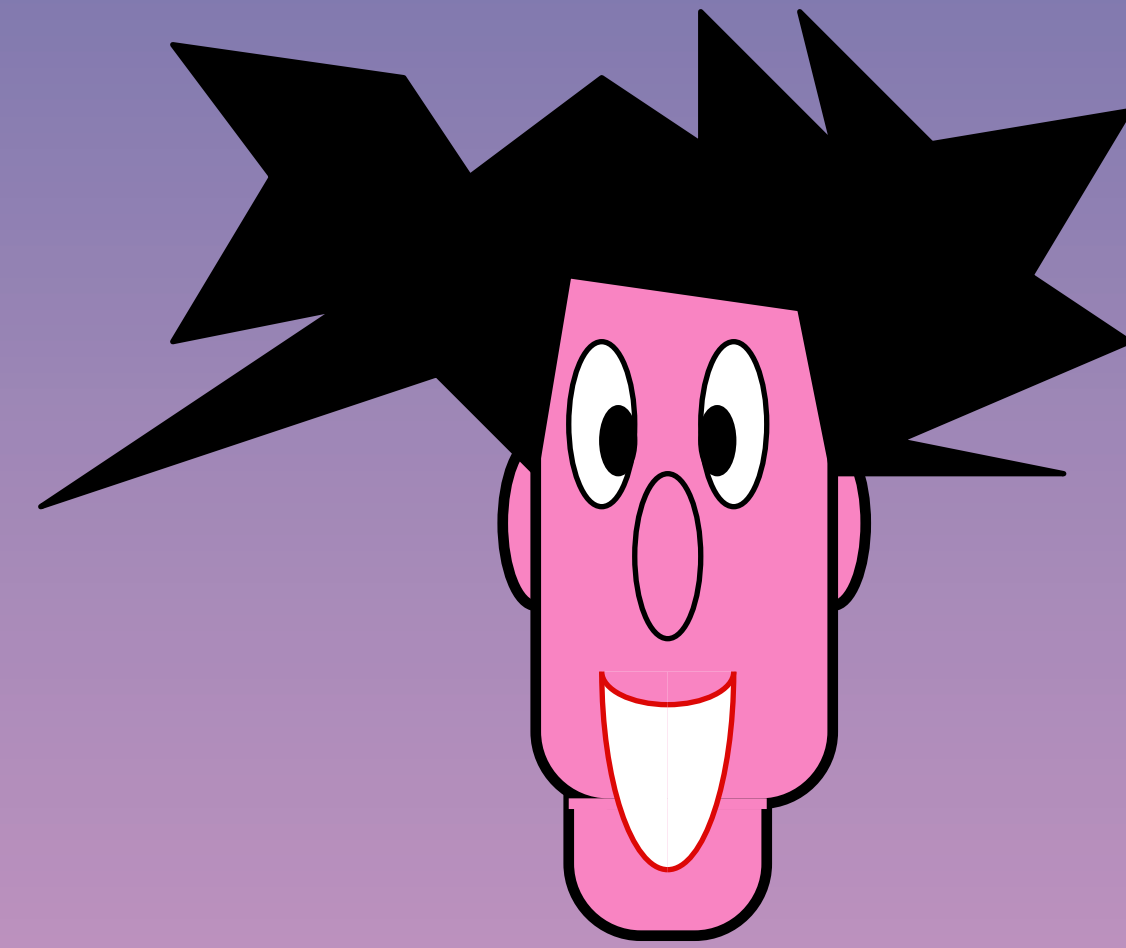


# Randomized Oblivious Transfer

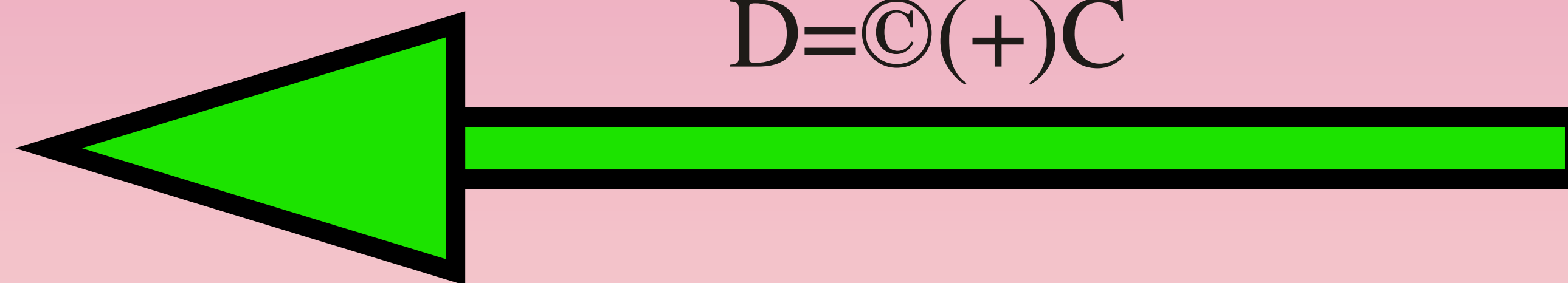


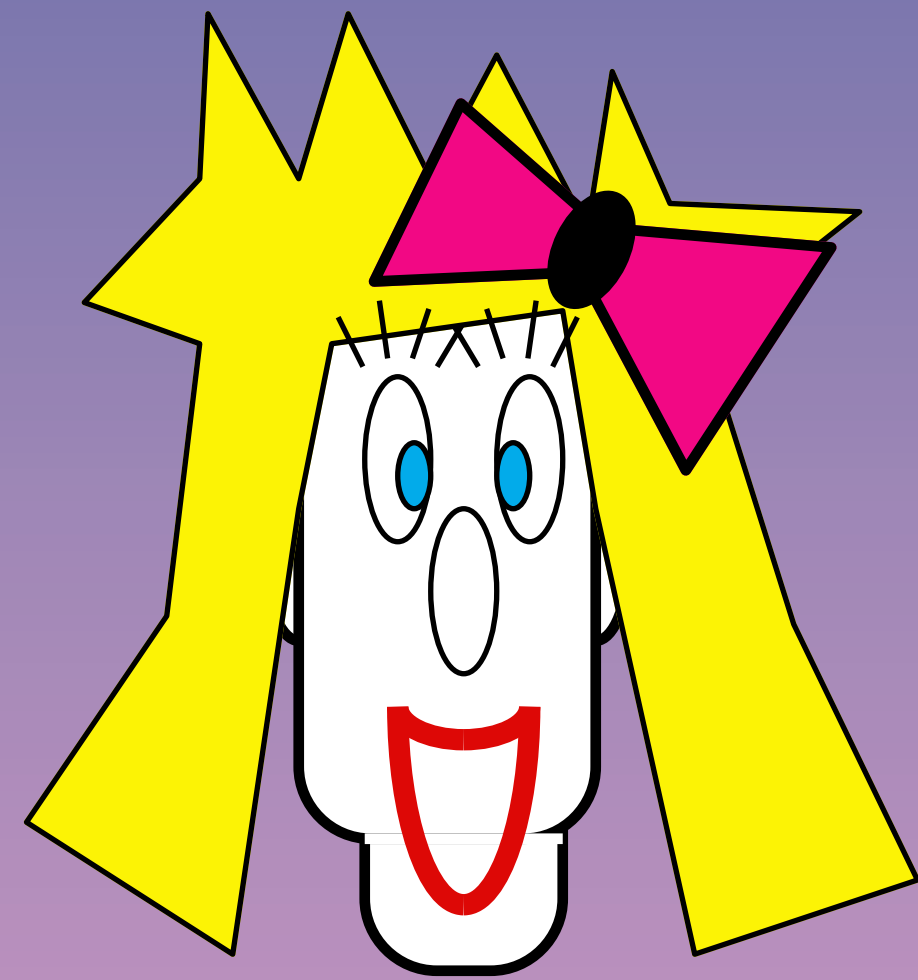


# Randomized Oblivious Transfer

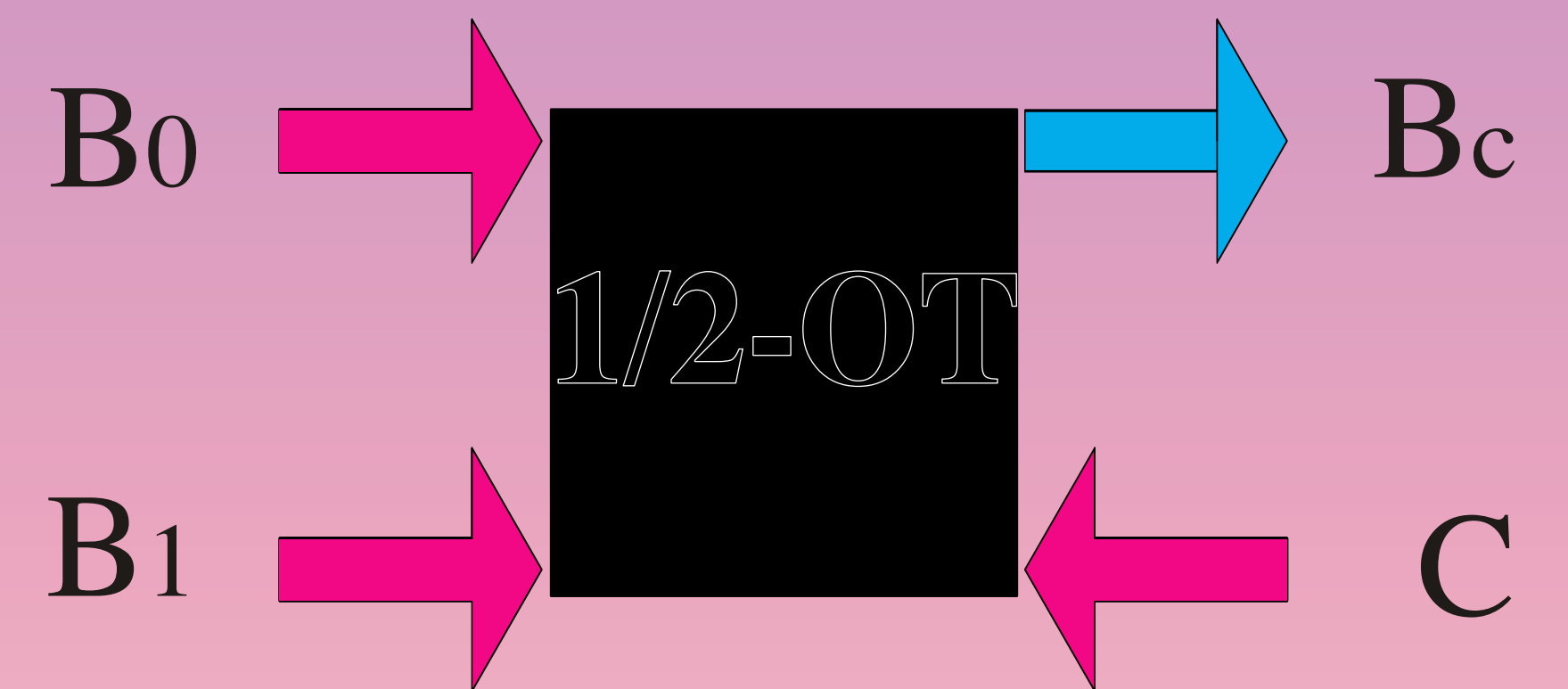
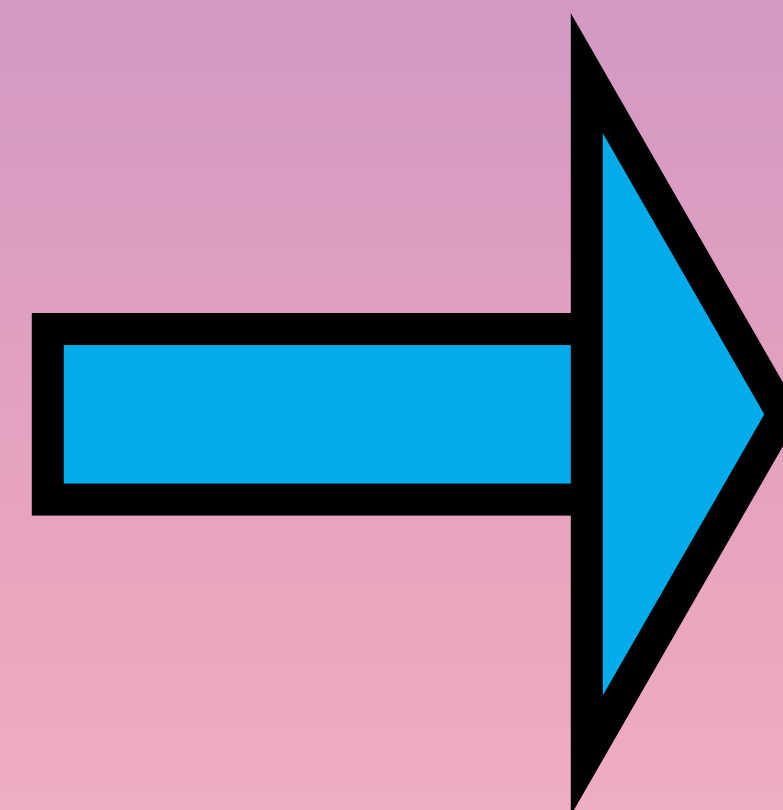
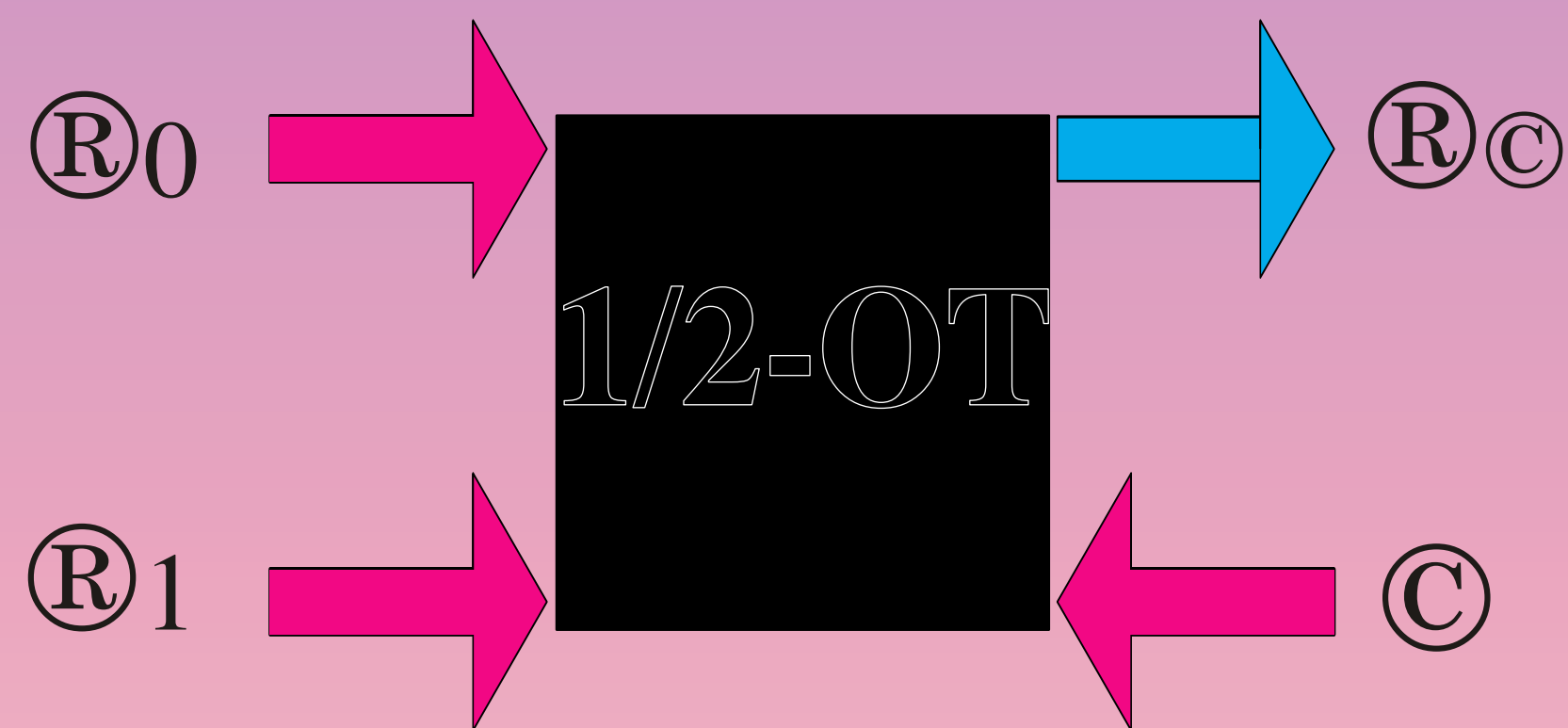
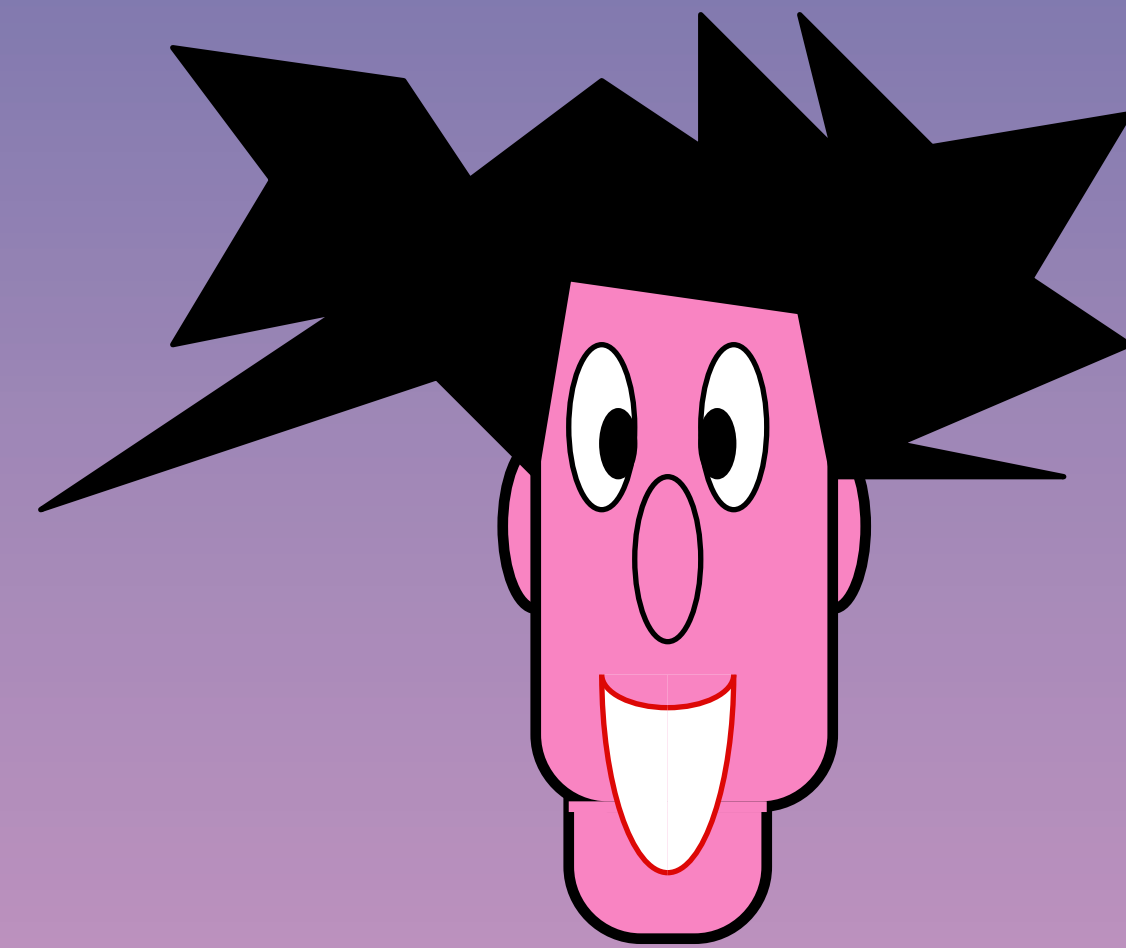


$$D = C(+ )C$$

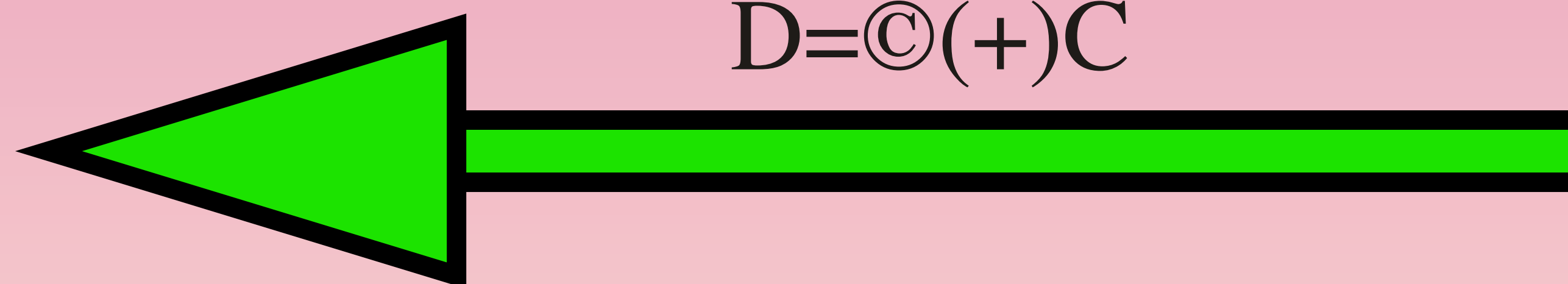




# Randomized Oblivious Transfer

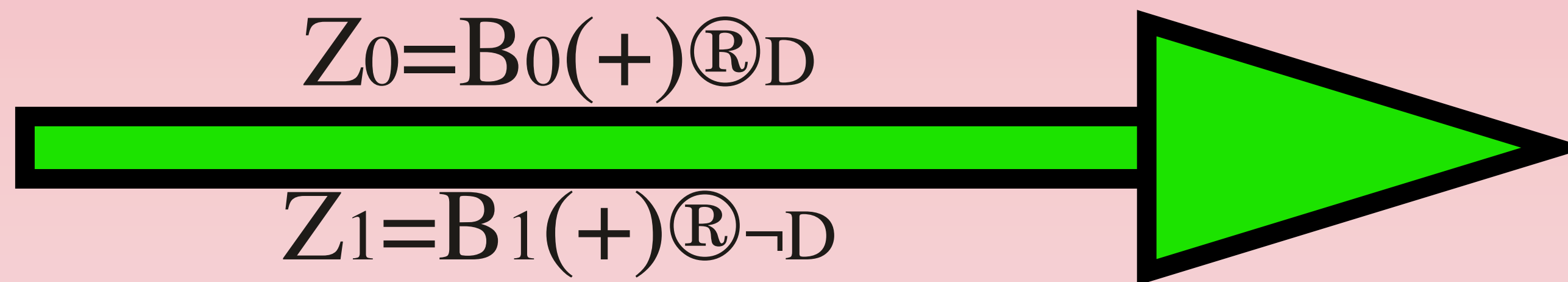


$$D = C(+ )C$$



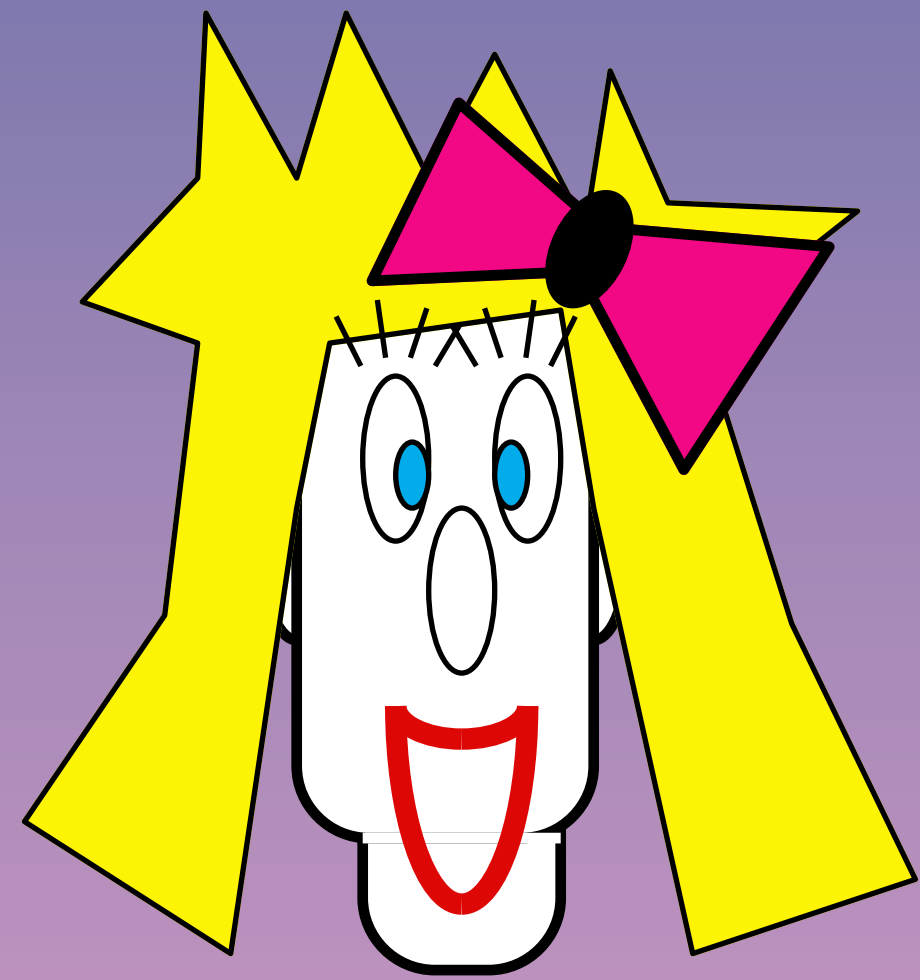
$$Z_0 = B_0(+ )R_D$$

$$Z_1 = B_1(+ )R_{\neg D}$$

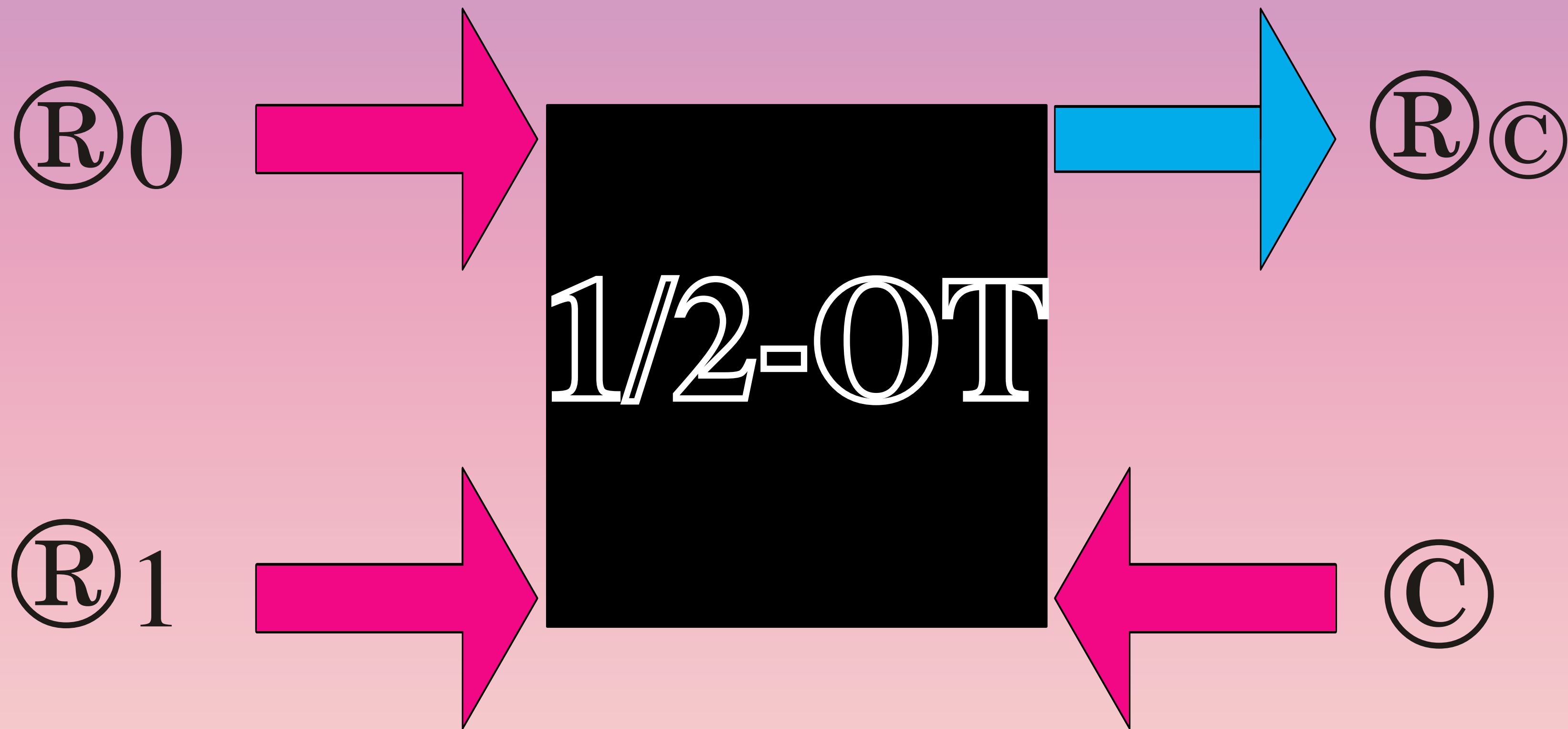
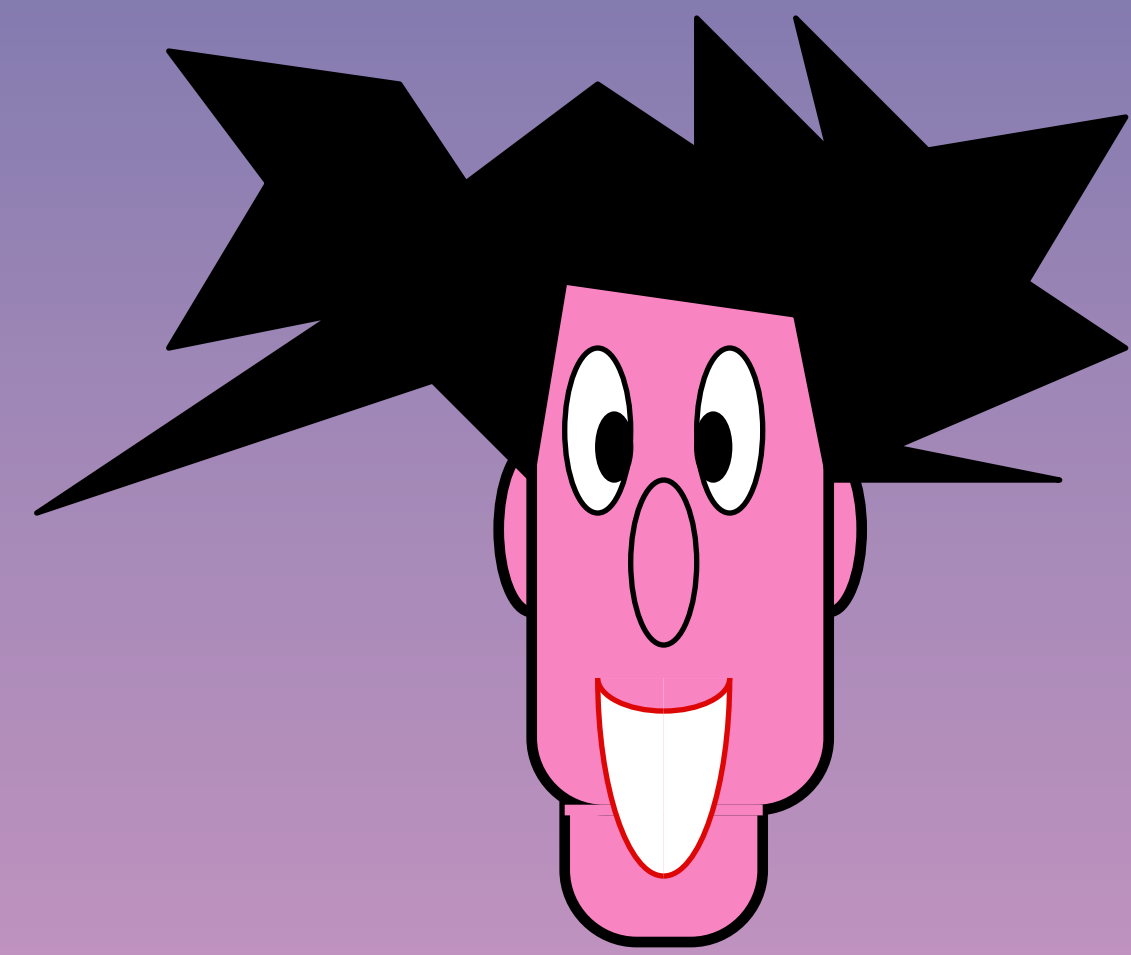


$$B_C = Z_C(+ )R_C$$



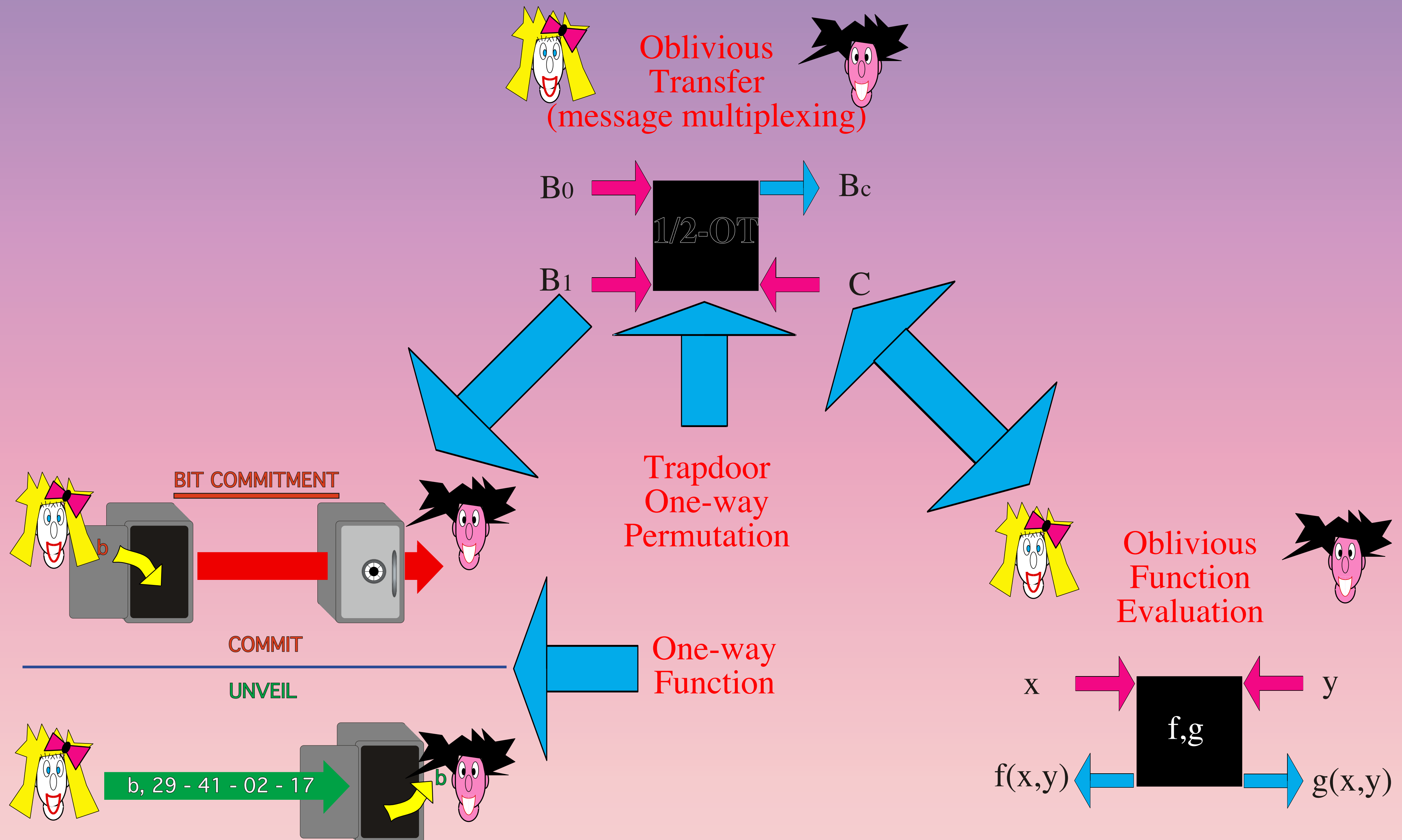


# Randomized Oblivious Transfer



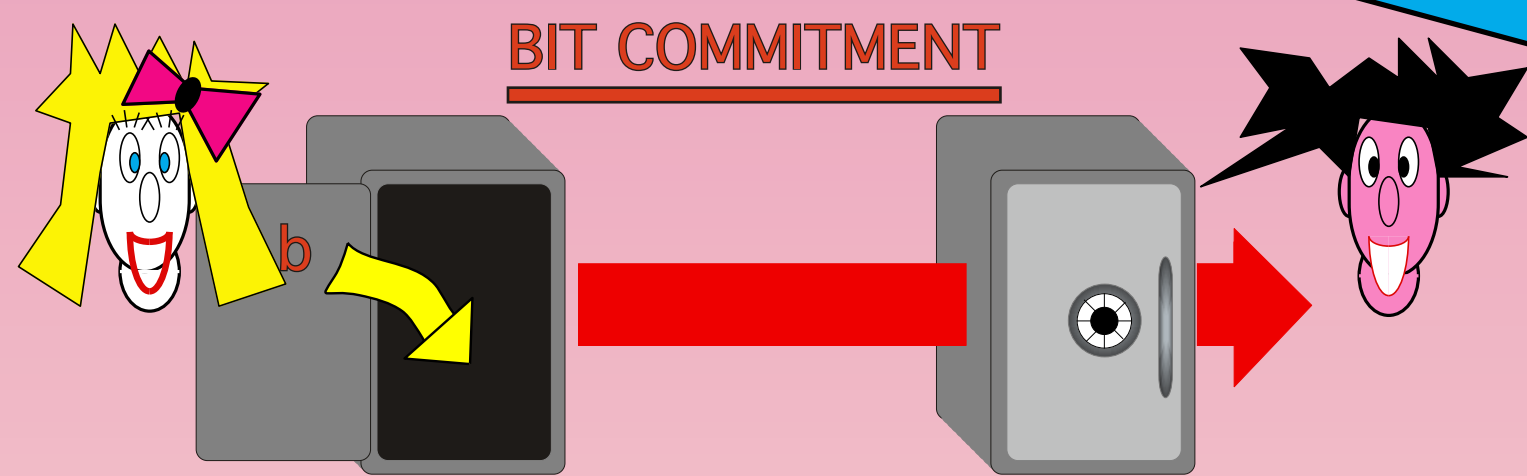
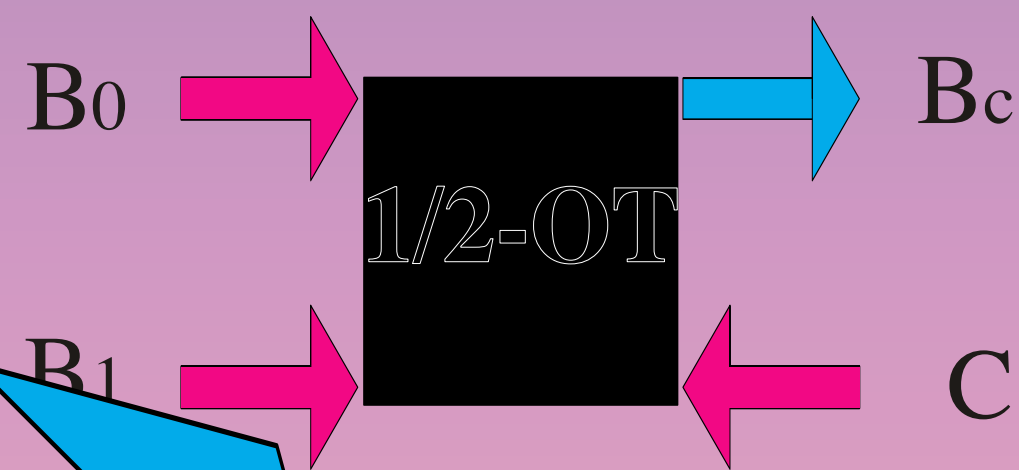
***IS AN INVESTMENT  
IN THE FUTURE***

# Classically



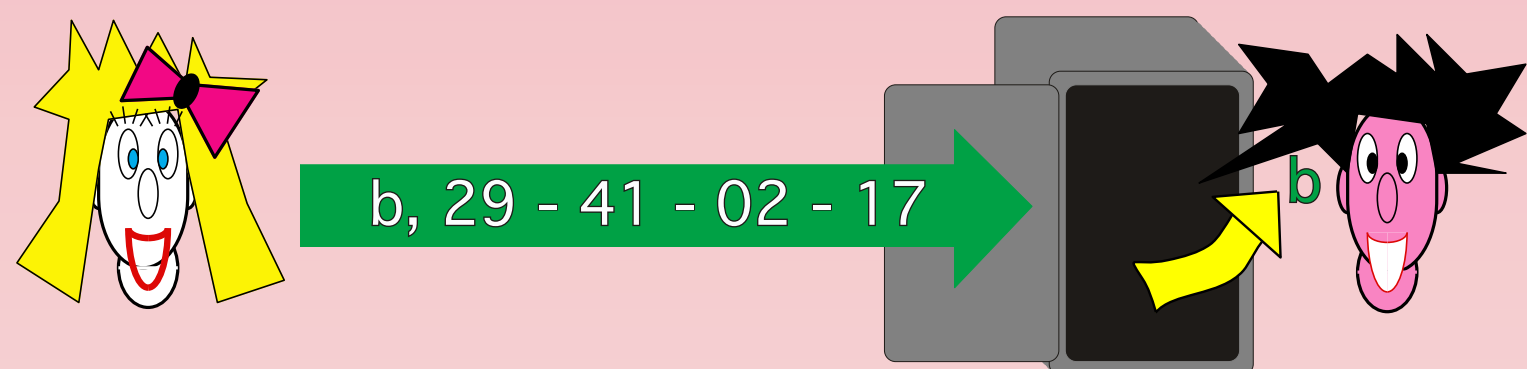
# Quantumly

Oblivious Transfer  
(message multiplexing)



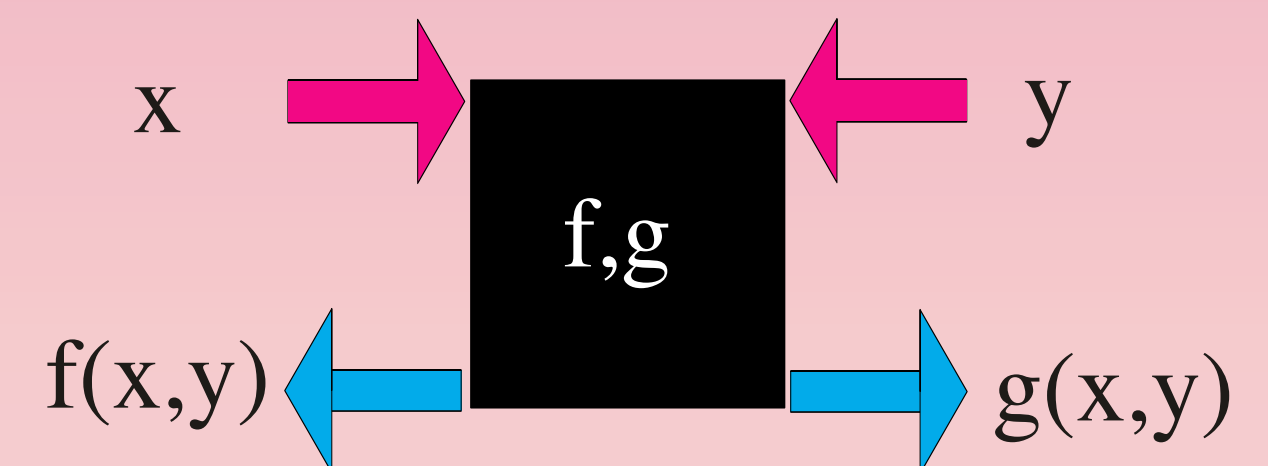
COMMIT

UNVEIL

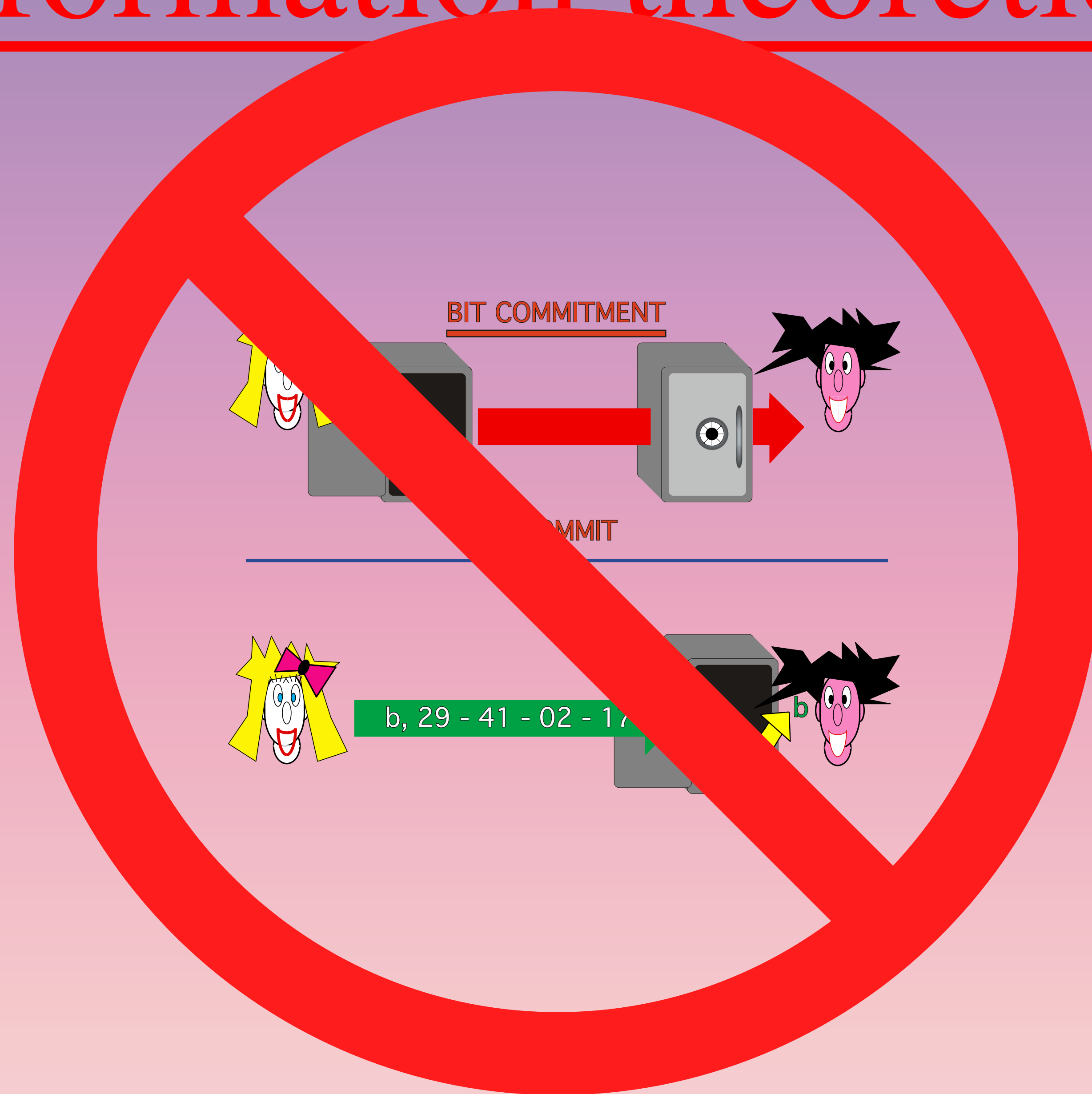


One-way  
Function

Oblivious  
Function  
Evaluation

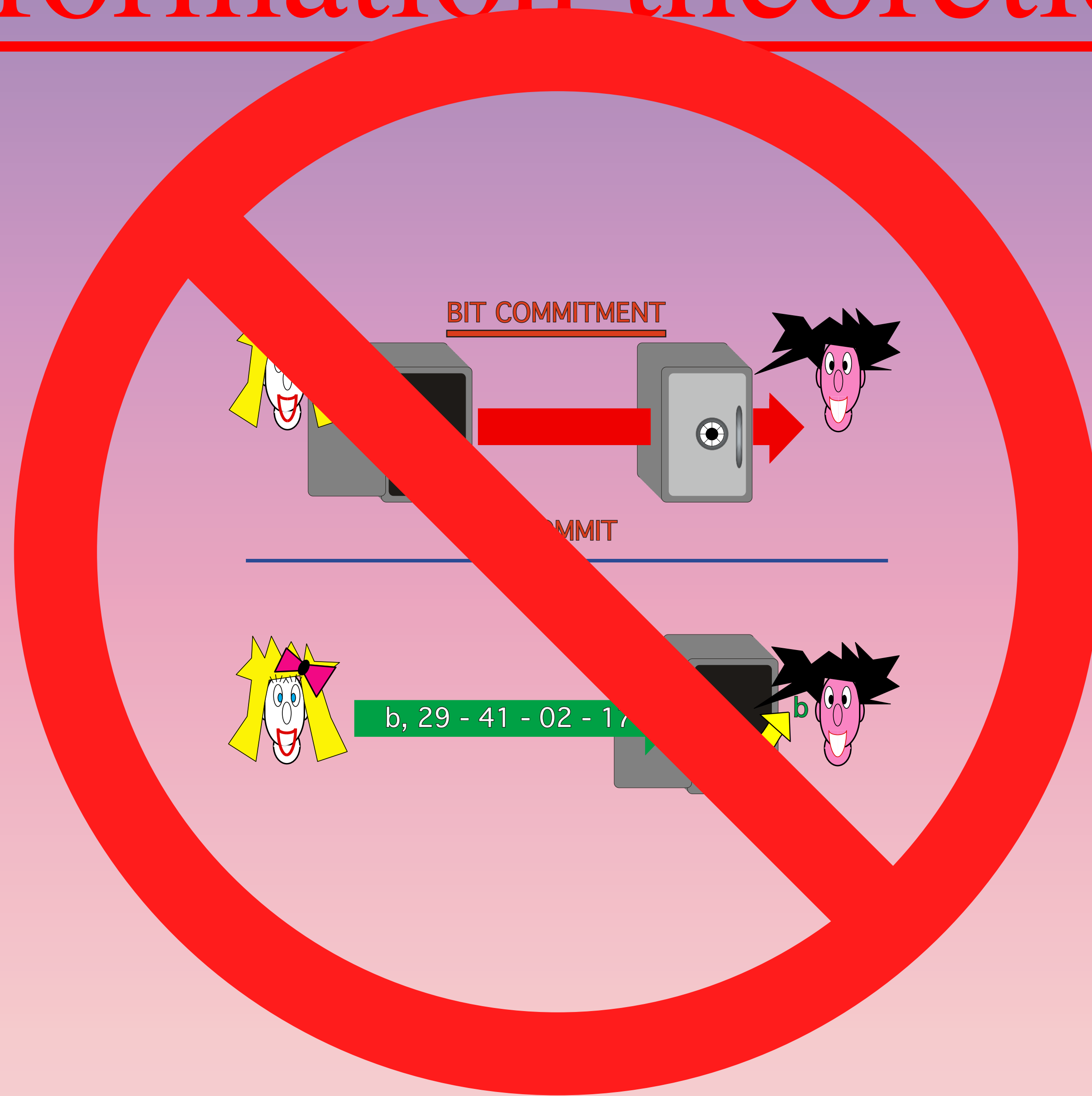


# Classically (information theoretical)



# Folklore

# Quantumly (information theoretical)

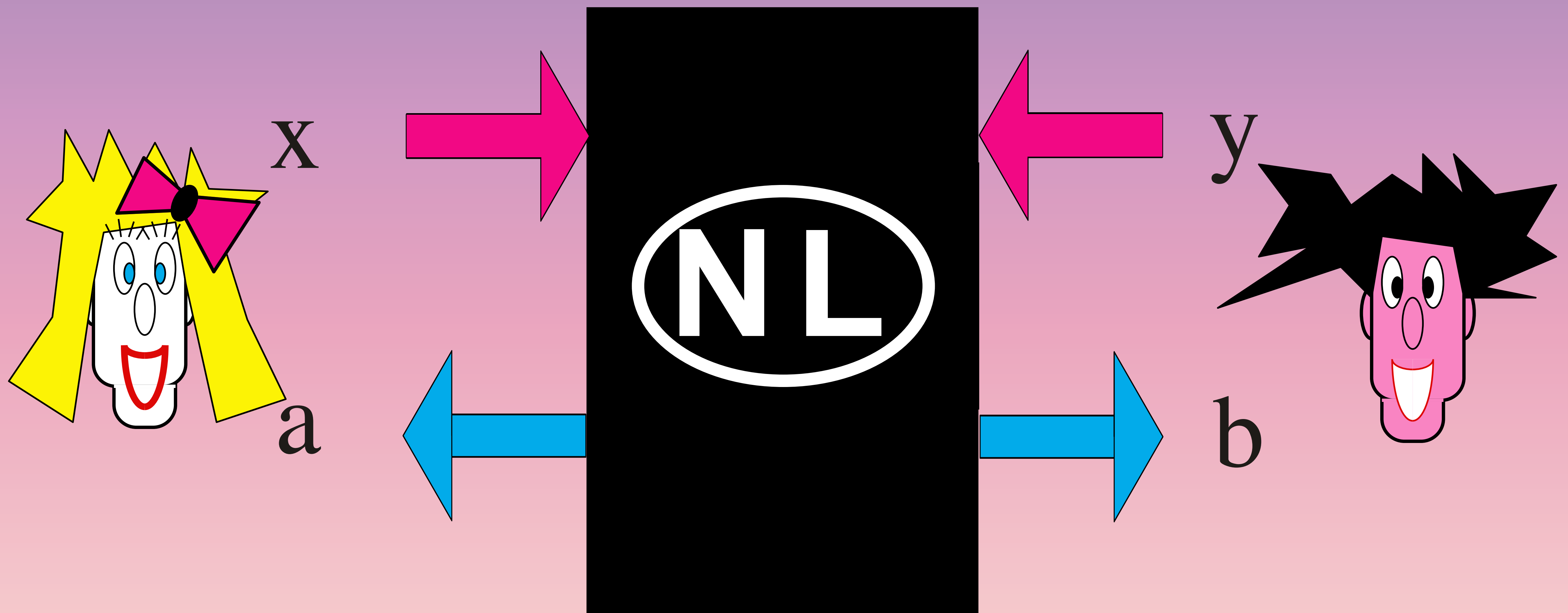


Mayers, Lo-Chau



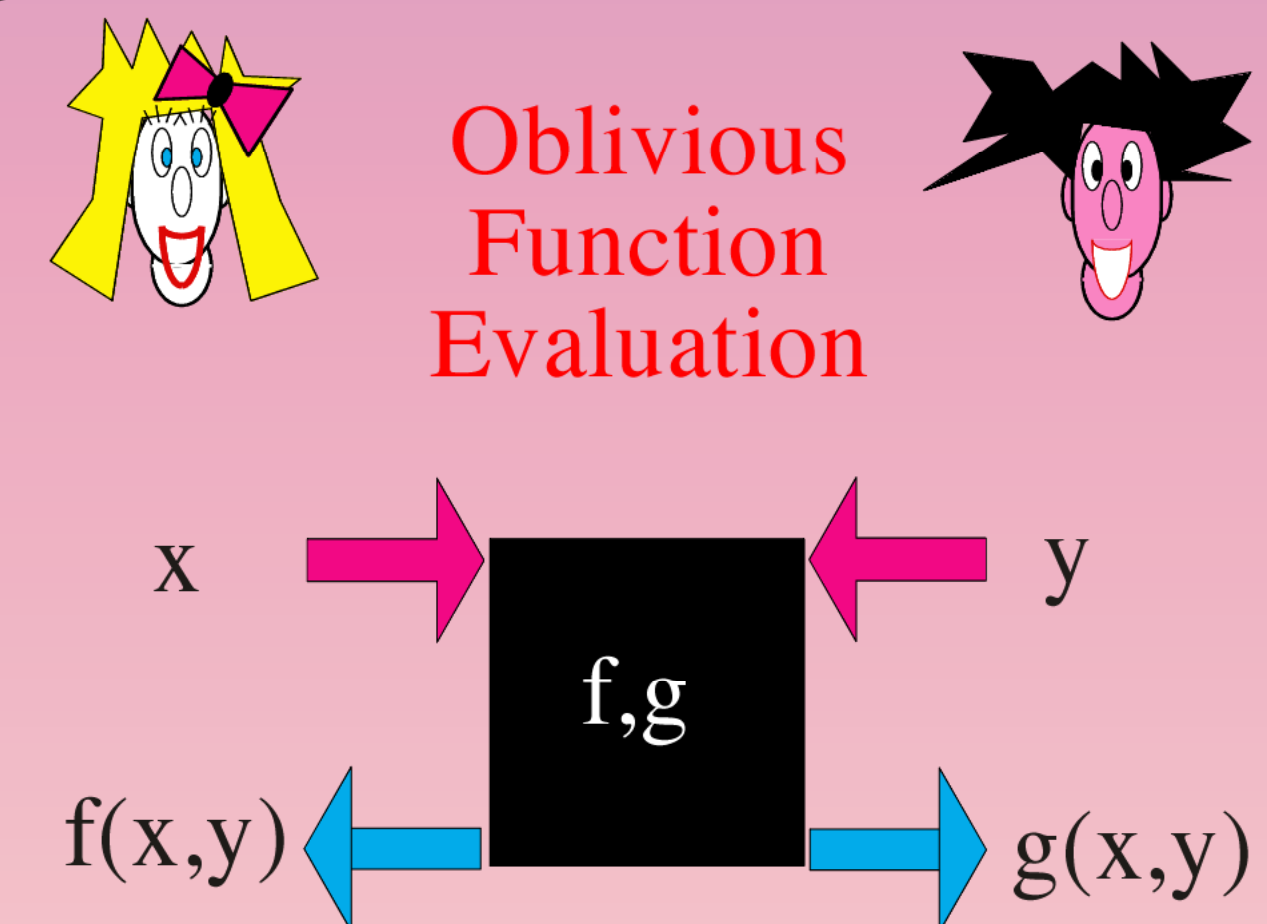
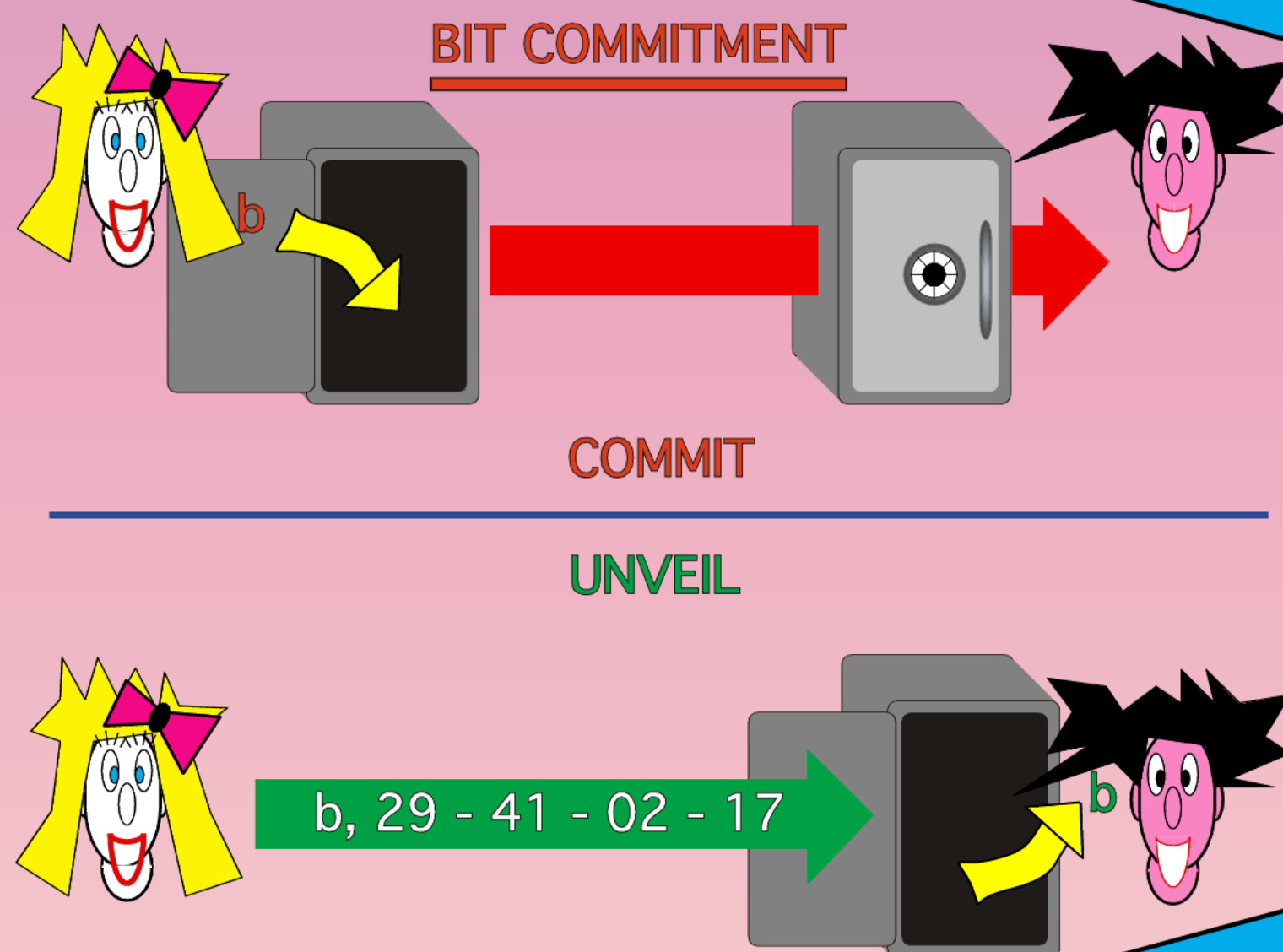
# Non-Locality Box

$$a \oplus b = x \otimes y$$

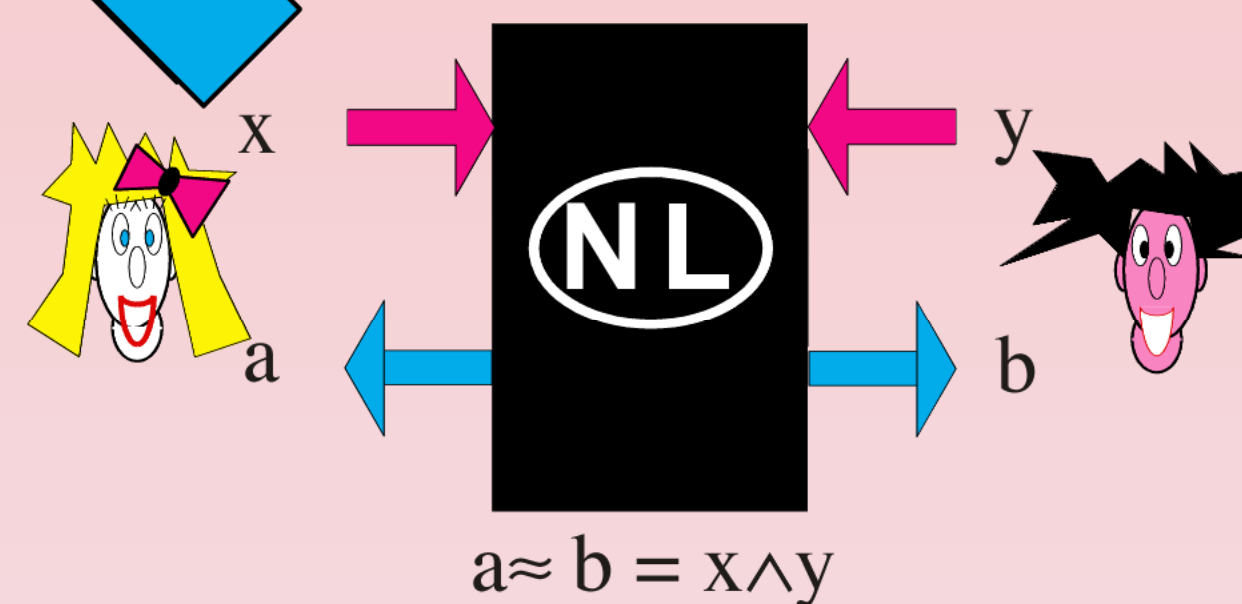


$$C: 3/4 \quad Q: \cos^2(\pi/8) \approx 85\%$$

# Quantumly



Non-Locality Box



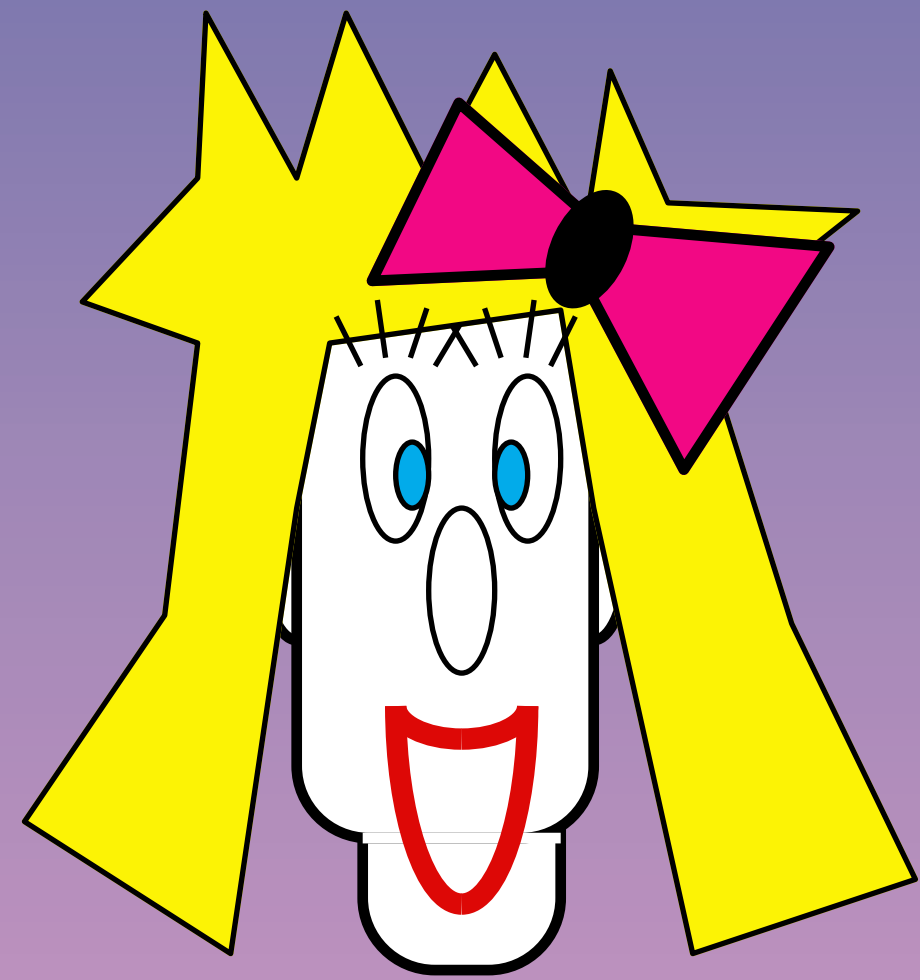
- 1) Wolf, Wullschleger ?
- 2) Short, Gisin, Popescu
- 3) Buhrman, Christandl, Unger, Wehner, Winter

**(5)**

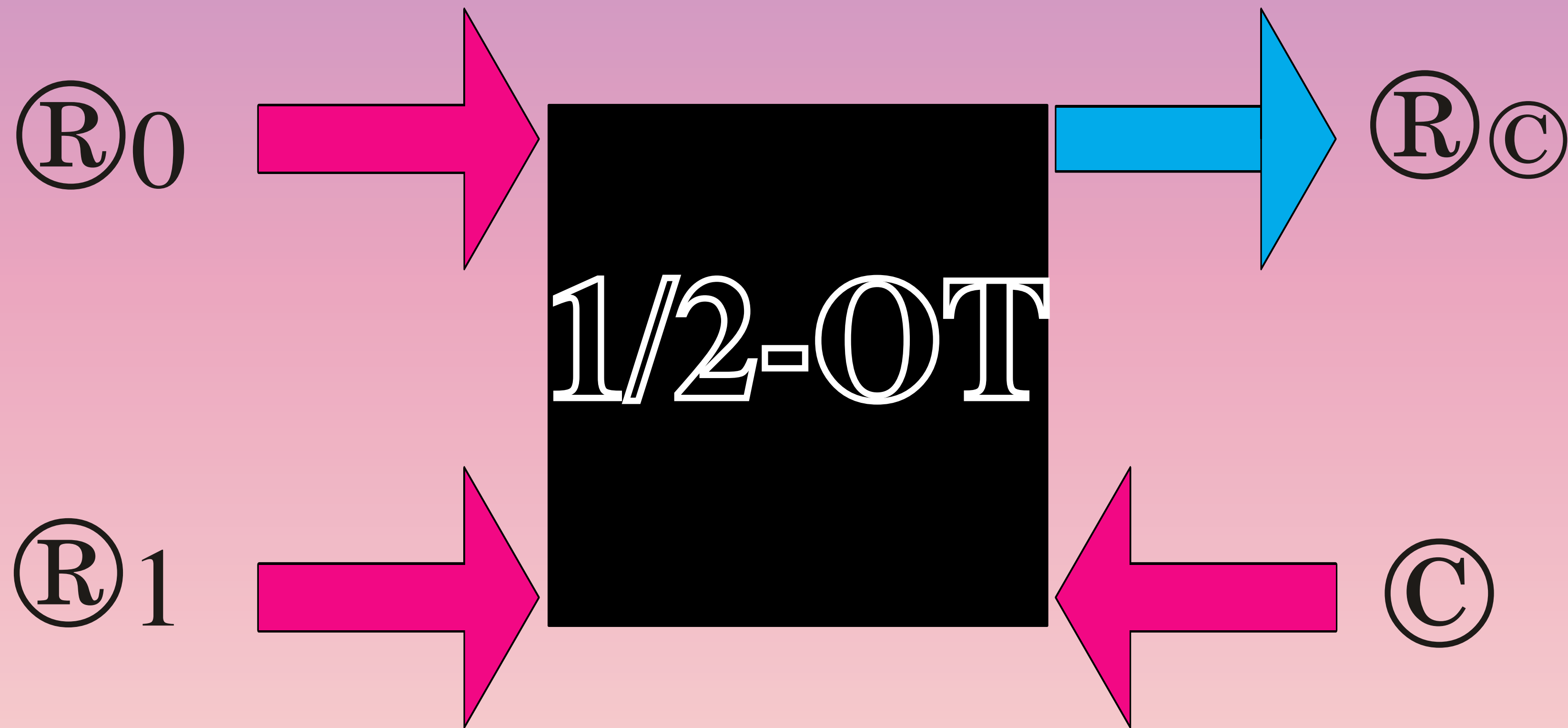
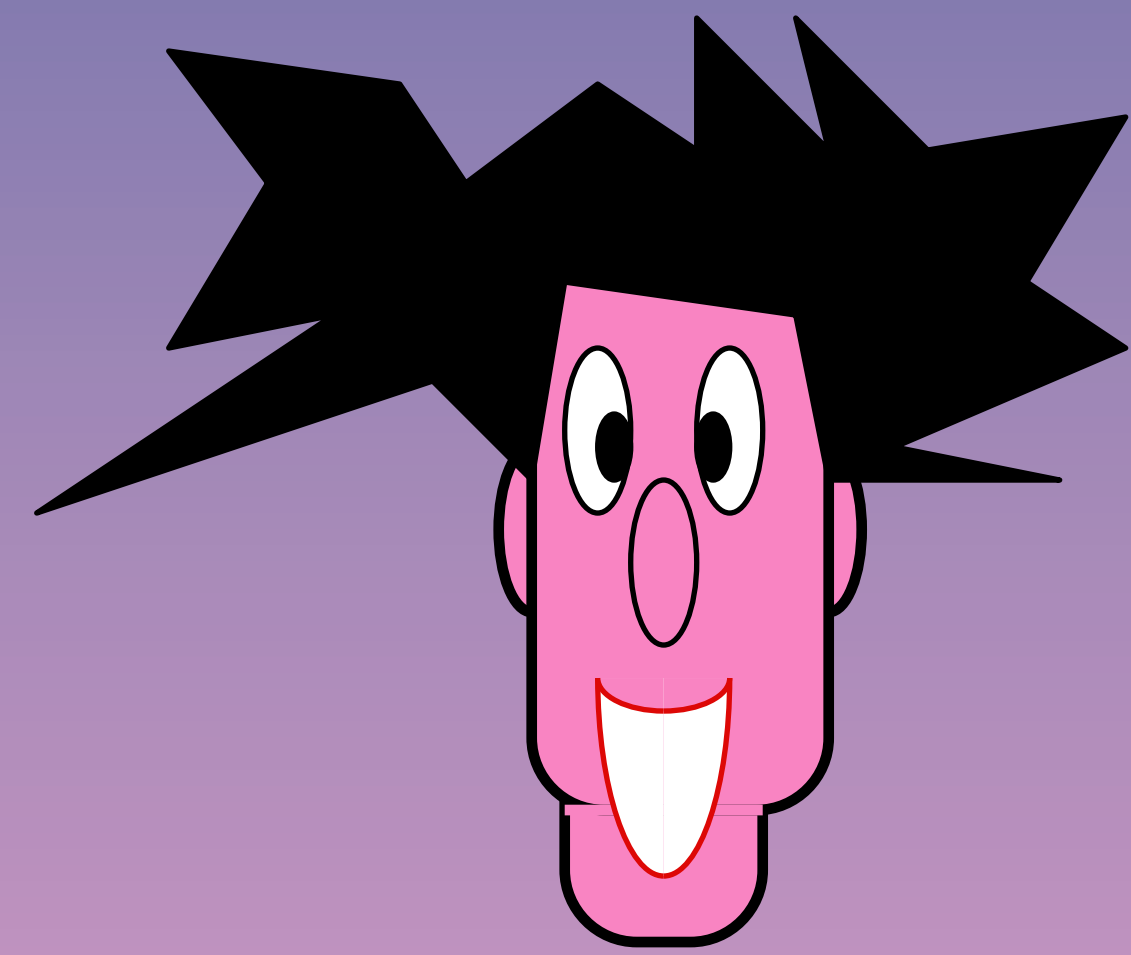
**Quantum**

**Oblivious**

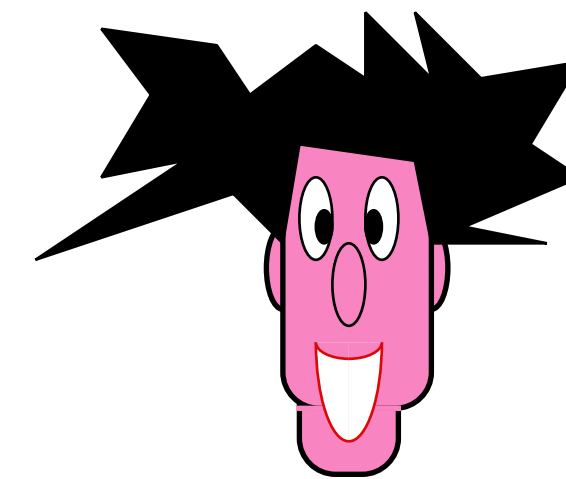
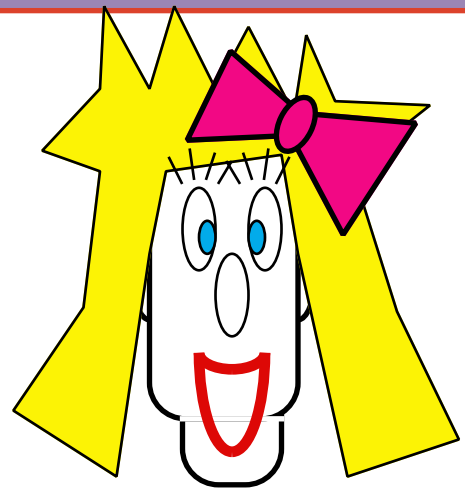
**Transfer**



# Randomized Oblivious Transfer



# Q-ROT



A: 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0  
 × + × + + × × × × + + + + × × × + × + + + × +

B: × × + + × + + + × + + × × × + × × × + + × + × +  
 0 0 1 0 0 1 0 0 1 0 0 0 0 1 1 1 0 0 0 1 1 0 0 0

A: × + × + + + × × × × + + + + × × × + × + + + × +

B: 0 🐸 🐸 0 🐸 1 🐸 🐸 1 🐸 0 🐸 🐸 🐸 🐸 1 0 🐸 🐸 1 🐸 0 0 0

B: 0 0 1 1 0 1 0 1 0 0 0 🐸 🐸 🐸 🐸 🐸 🐸 🐸 🐸 🐸 🐸 🐸 🐸

A: 0 0 1 1 0 1 0 1 0 0 0 1 1 0 0 0 1 1 0 0 0 1 0 1

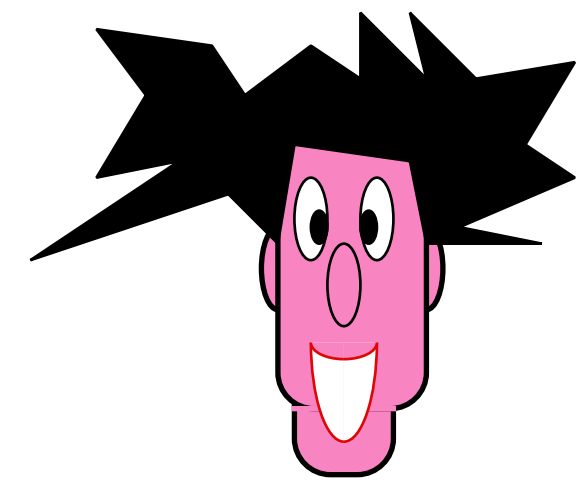
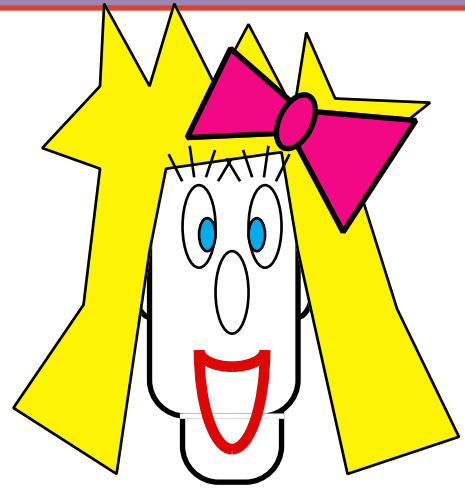
B: 0 0 1 1 0 1 0 1 = 0 🐸 = 🐸 🐸 🐸 🐸 🐸 🐸 🐸 🐸

A: 0 0 1 1 0 1 0 1 = 0 =  $\mathbb{R}_0$      $\mathbb{R}_1 = 0 = 1 1 0 0 0 1 0 1$

**Crépeau-Kilian**

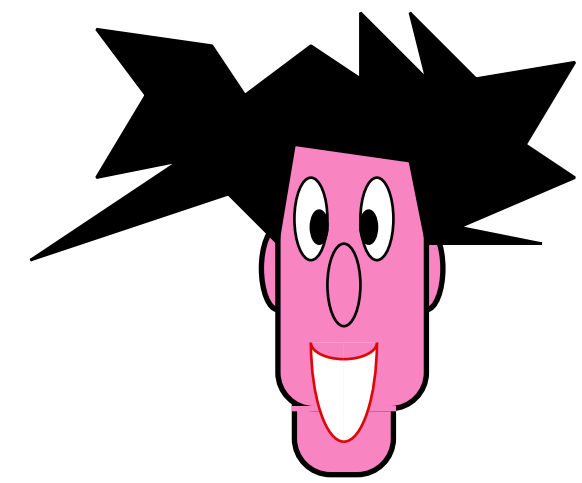
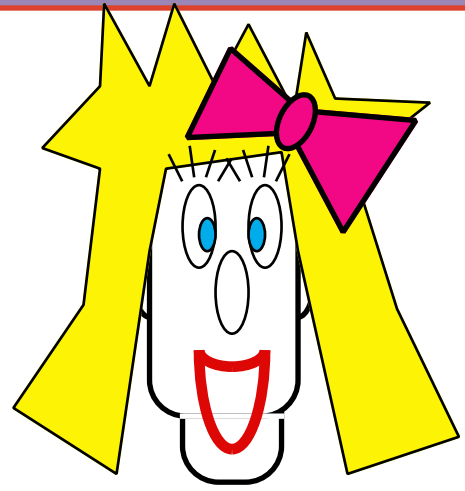


# Q-OT



A: 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0  
× + × + + + × × × × + + + + × × × + × + + + × +

# Q-OT



**A:** 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0

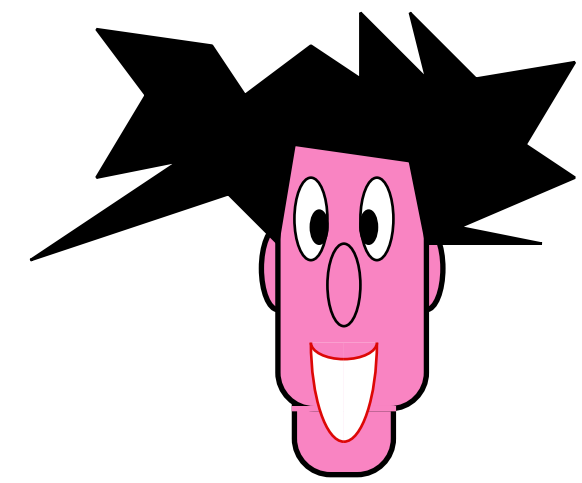
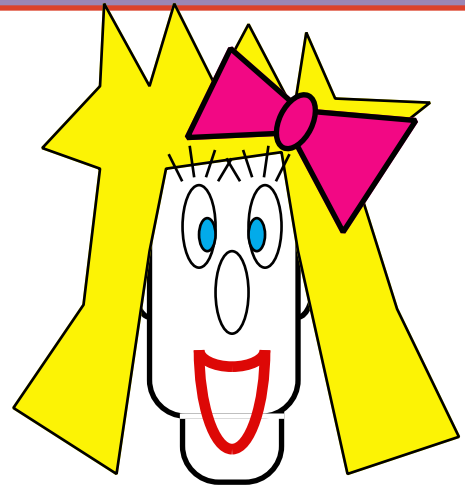
× + × + + + × × × × + + + + × × × + × + + + × +

**B:** × × + + × + + + × + + × × × + × × × + + × + × +

0 0 1 0 0 1 0 0 1 0 0 0 0 1 1 1 0 0 0 1 1 0 0 0

---

# Q-OT



**A:** 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0

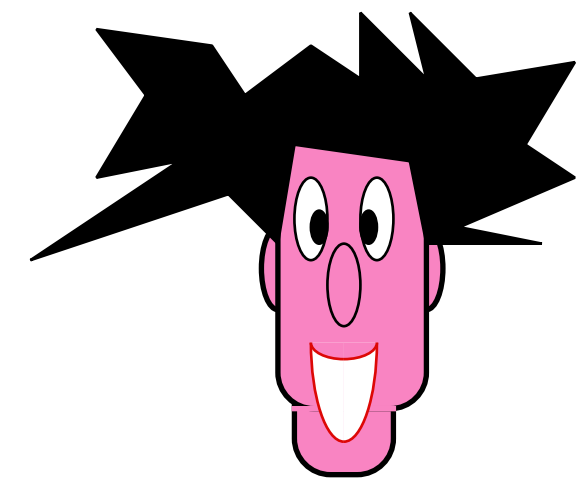
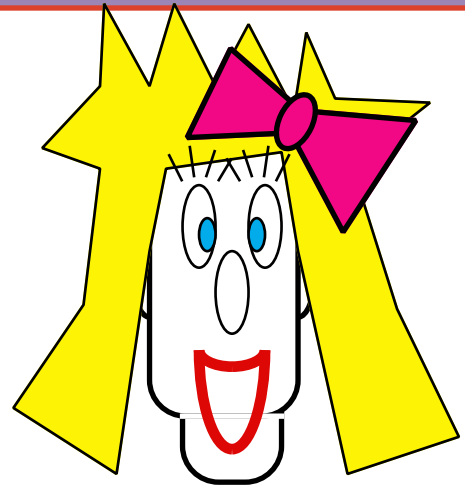
× + × + + + × × × × + + + + × × × + × + + + × +

**B:** × × + + × + + + × + + × × × + × × × + + × + × +

0 0 1 0 0 1 0 0 1 0 0 0 0 1 1 1 0 0 0 1 1 0 0 0

**A:** × + × + + + × × × × + + + + × × × + × + + + × +

# Q-OT

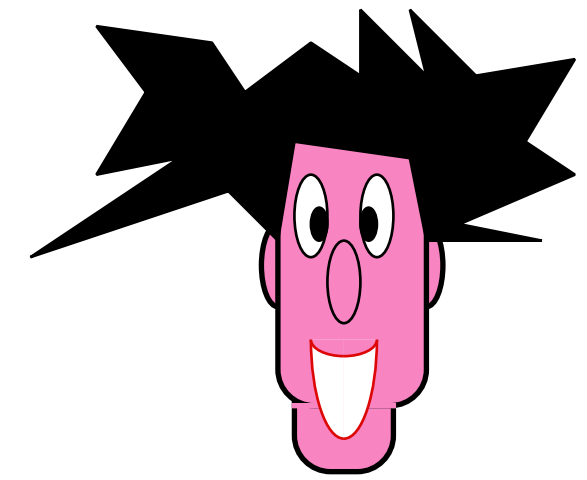
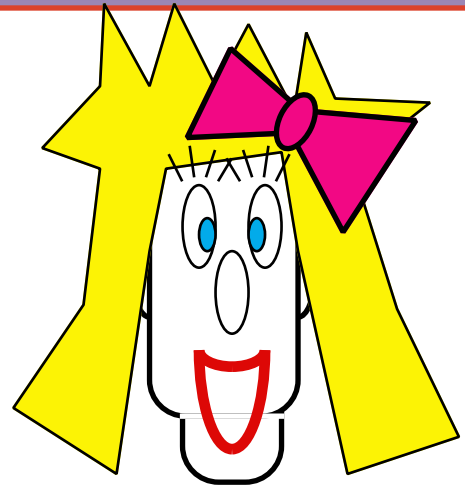


**B:**    × × + + × + + + × + + × × × + × × × + + × + × +  
         0 0 1 0 0 1 0 0 1 0 0 0 0 1 1 1 0 0 0 1 1 0 0 0

**A:**    × + × + + + × × × × + + + + × × × + × + + + × +

**B:**    0   0  1   1  0     1 0   1  0 0 0

# Q-OT



A: 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0

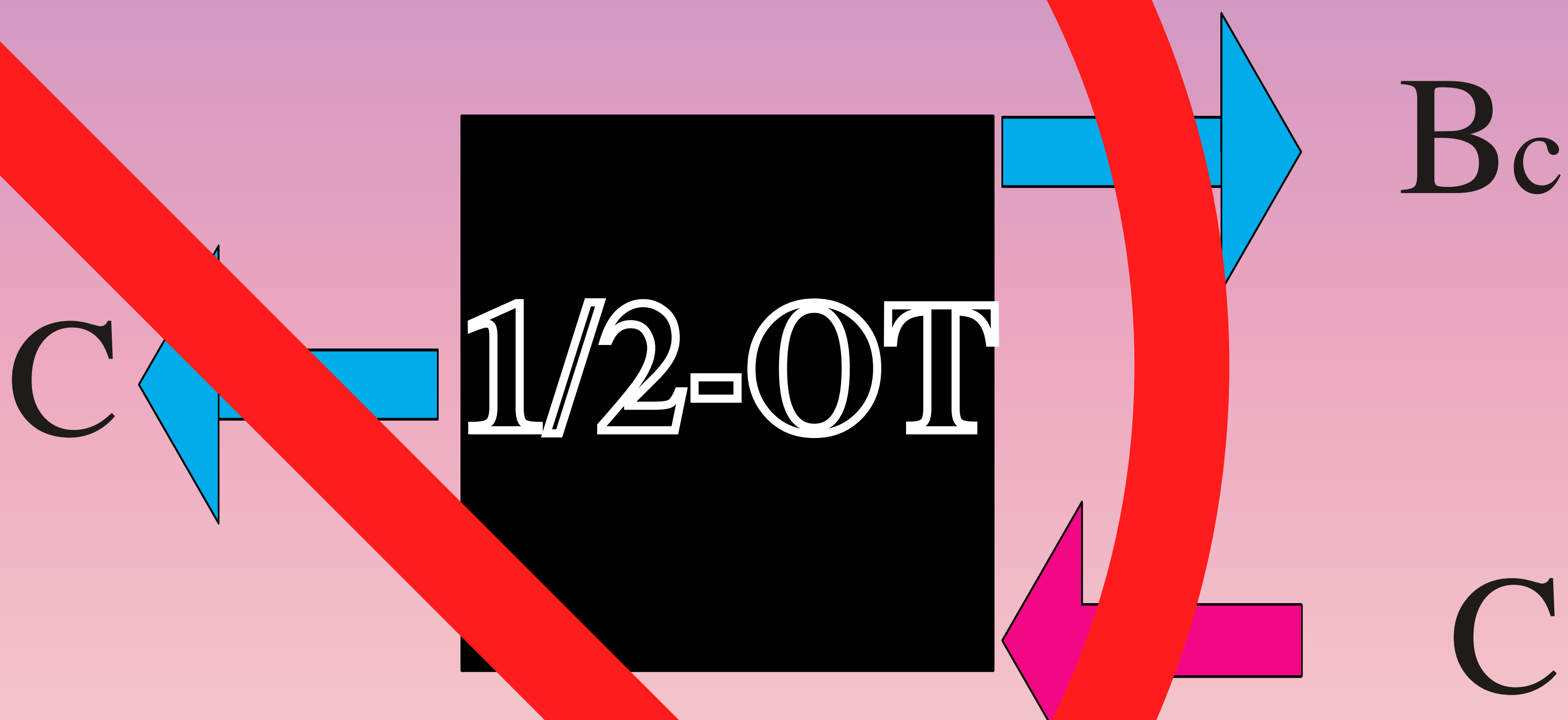
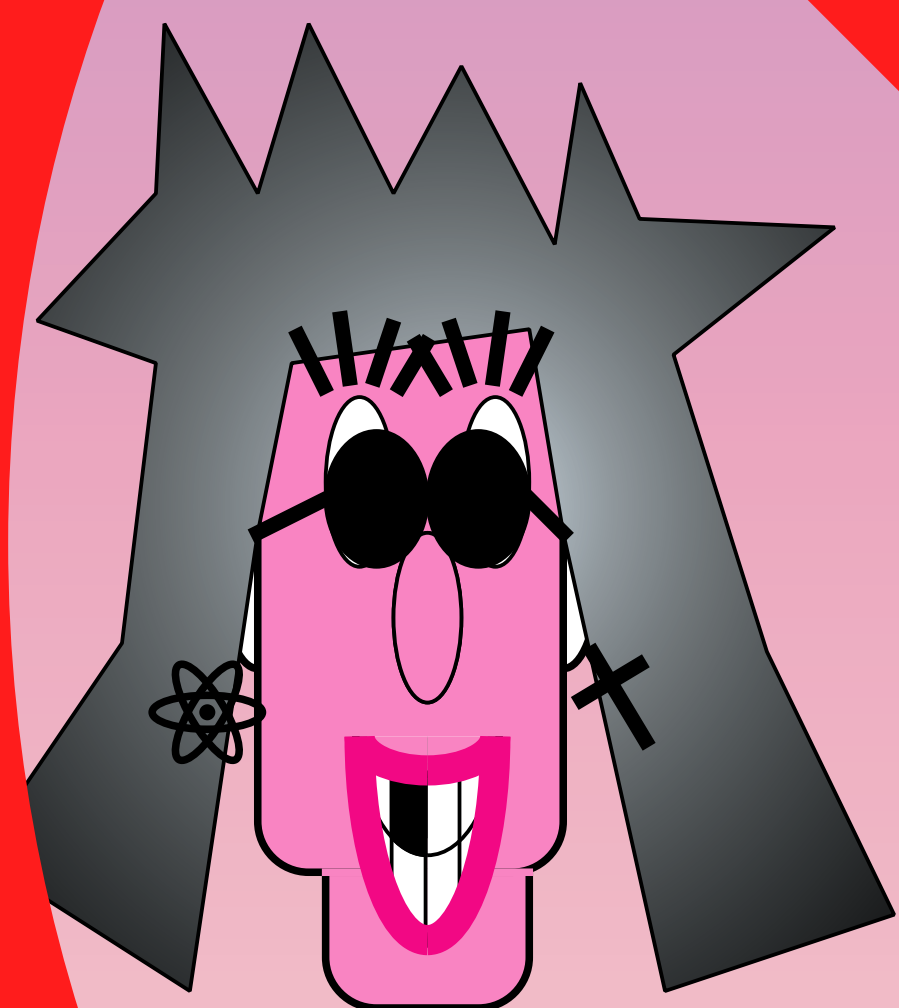
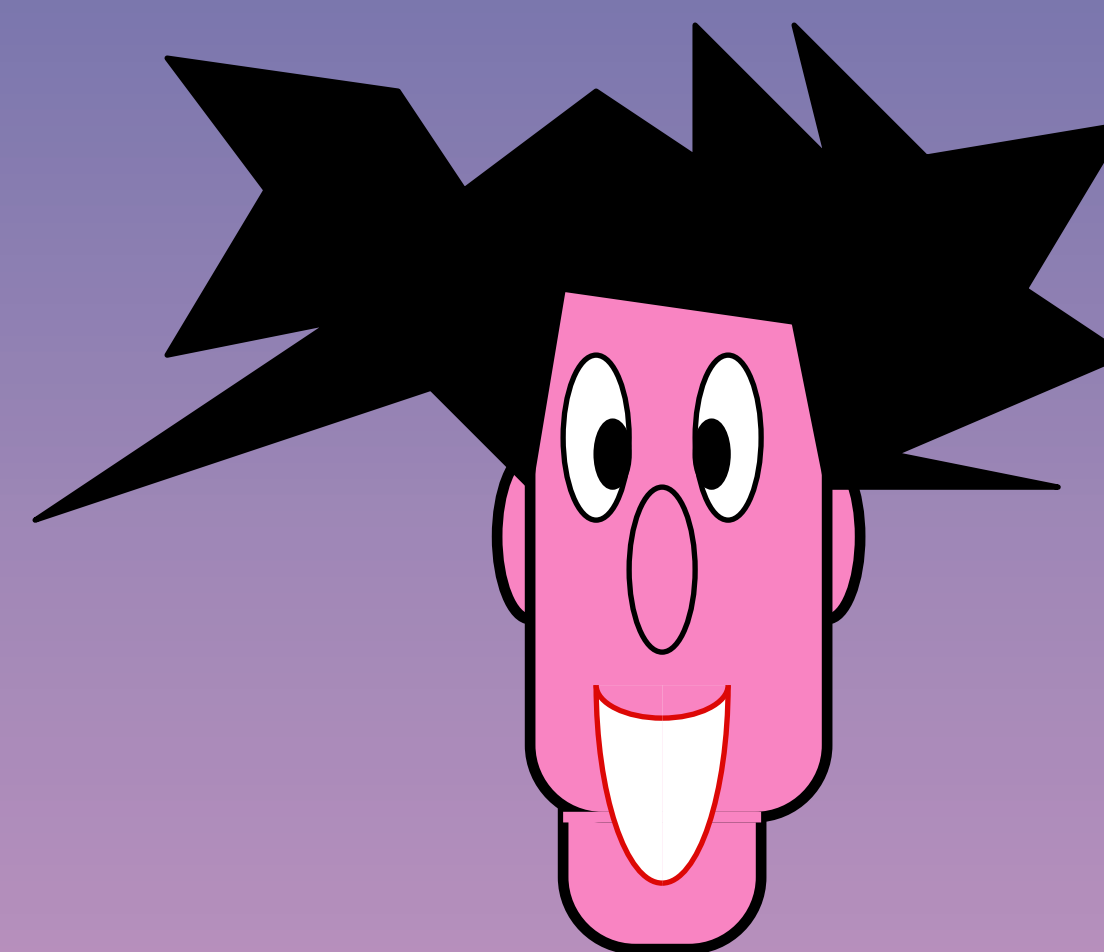
B: 0   0  1   1  0      1 0   1  0 0 0

B: 0 0 1 1 0 1 0 1                     

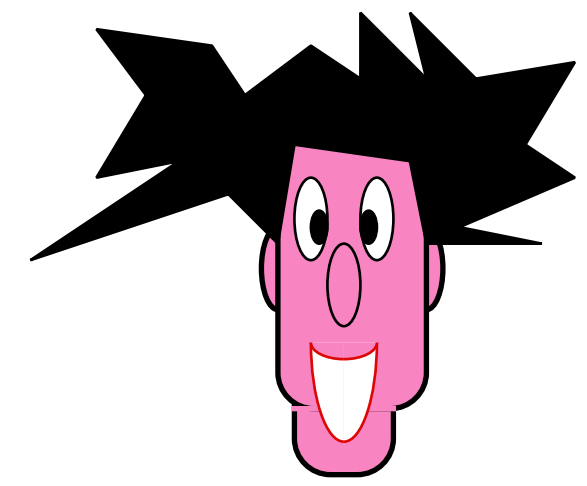
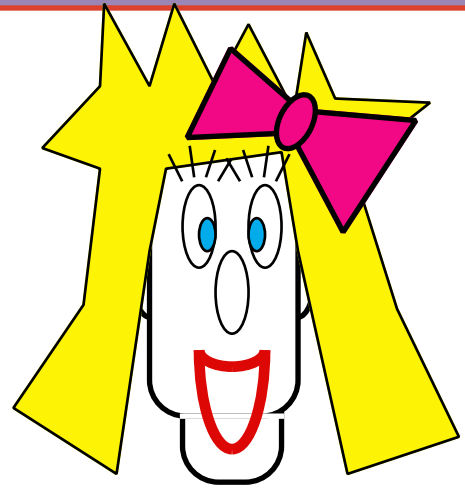
A: 0 0 1 1 0 1 0 1 0 0 0 1 1 0 0 0 1 1 0 0 0 1 0 1



# Oblivious Transfer

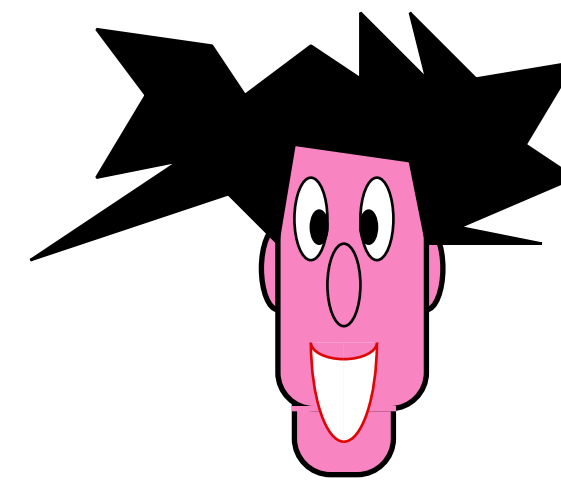
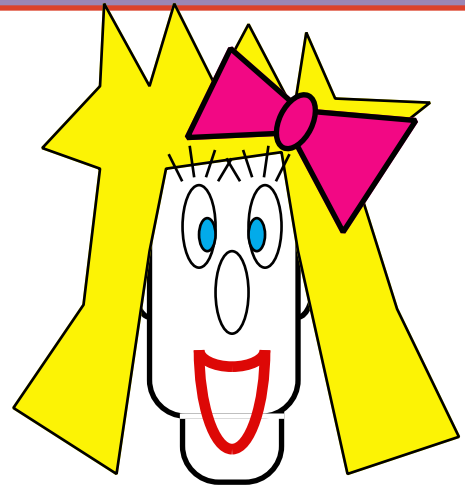


# Q-OT



B:	0 0 1 1 0 1 0 1	0 0 0	☹ ☹ ☹ ☹ ☹	☹ ☹ ☹ ☹ ☹ ☹ ☹ ☹
A:	0 0 1 1 0 1 0 1	0 0 0 1 1 0 0 0	1 1 0 0 0 1 0 1	
B:	0 0 1 1 0 1 0 1	= 0	☹ =	☹ ☹ ☹ ☹ ☹ ☹ ☹ ☹
A:	0 0 1 1 0 1 0 1	= 0 = $\mathbb{R}_0$	$\mathbb{R}_1 = 0 =$	1 1 0 0 0 1 0 1

# Q-OT

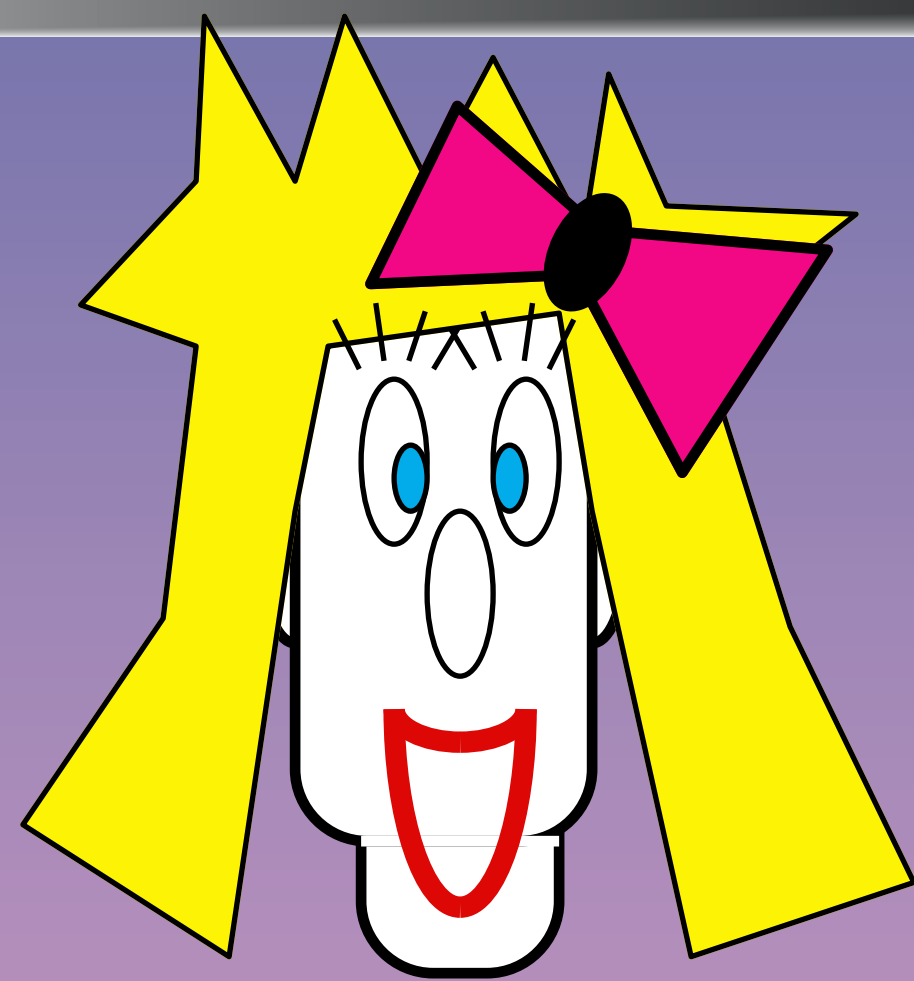


B: 0 0 1 1 0 1 0 1 = 0

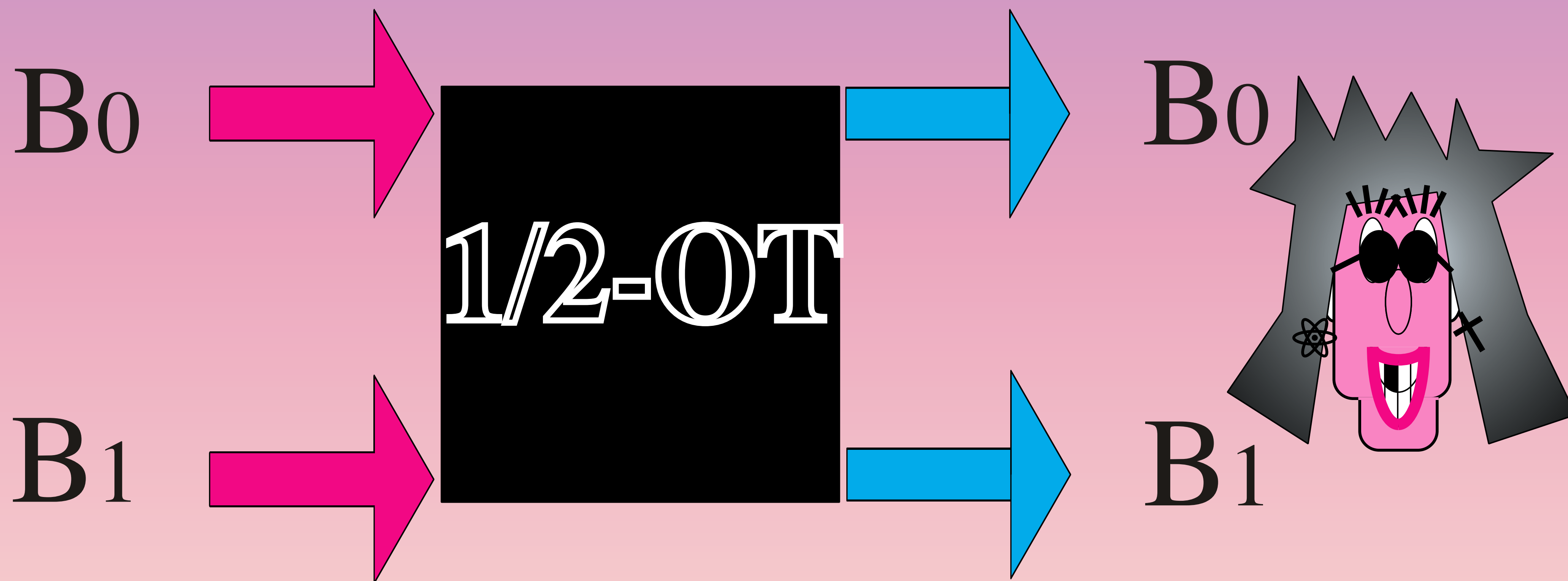
☞ = ☞ ☞ ☞ ☞ ☞ ☞ ☞ ☞

A: 0 0 1 1 0 1 0 1 = 0 =  $\mathbb{R}_0$

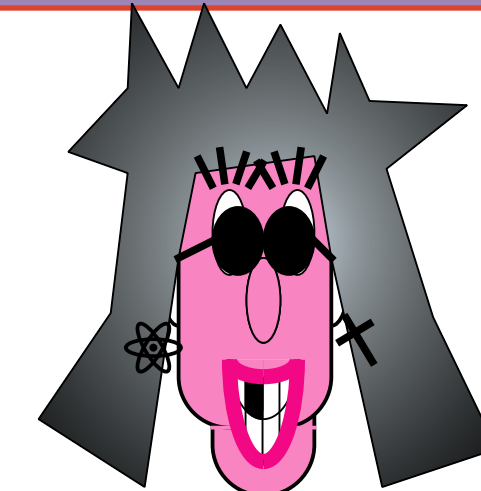
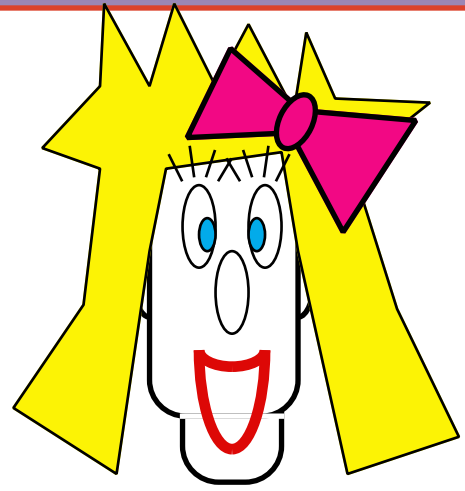
$\mathbb{R}_1 = 0 = 1 1 0 0 0 1 0 1$



# Oblivious Transfer



# Q-OT



A: 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0  
 × + × + + + × × × × + + + + × × × + × + + + × +

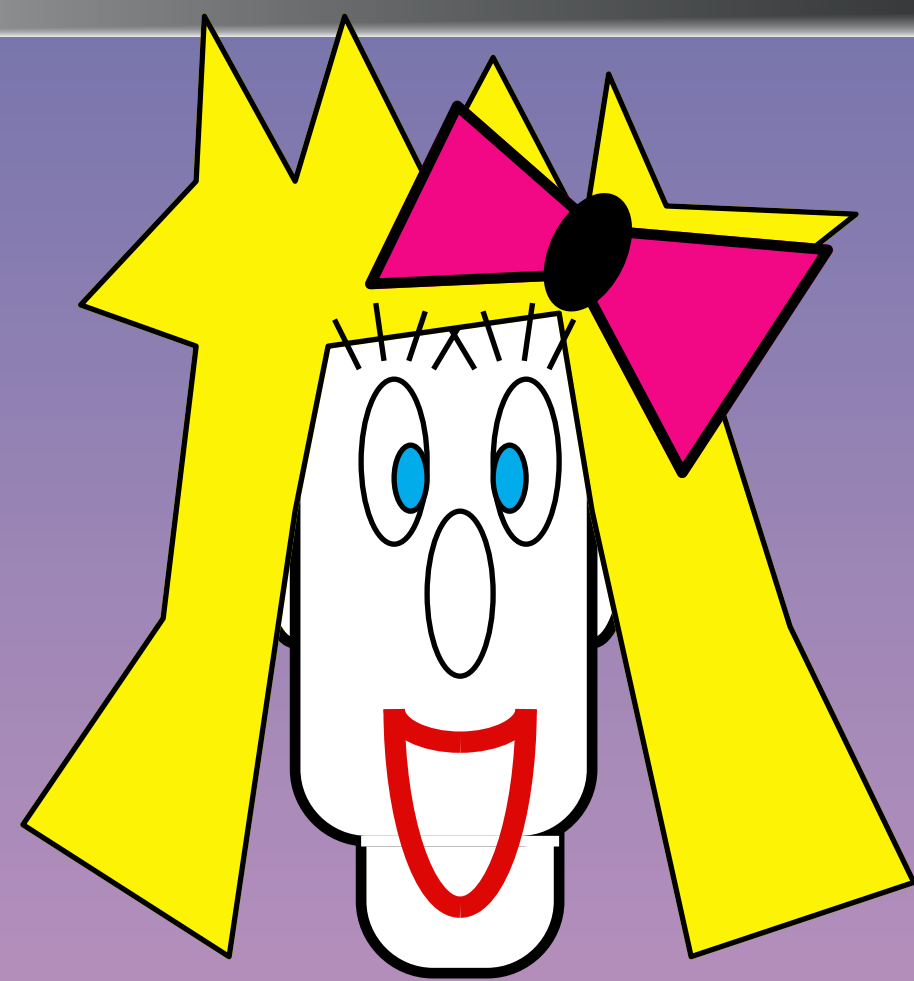
A: × + × + + + × × × × + + + + × × × + × + + + × +

B: × + × + + + × × × × + + + + × × × + × + × + × +  
 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0

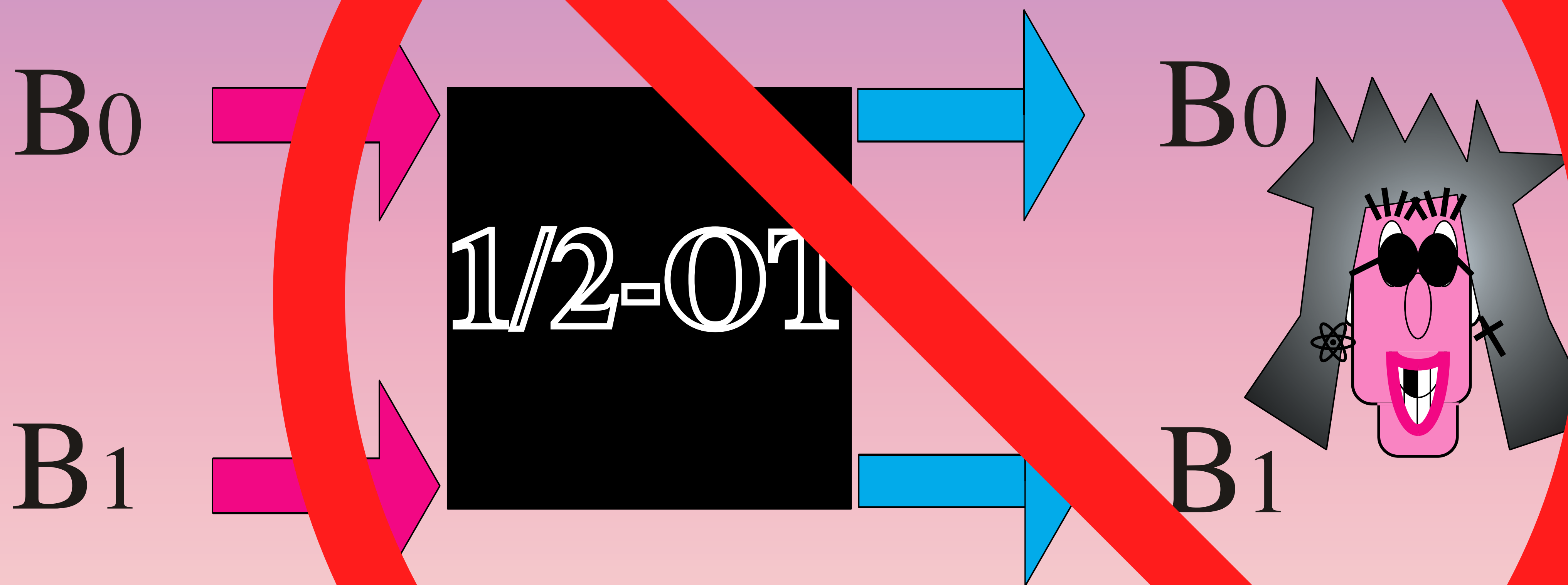
B: 0 0 1 1 0 1 0 1 = 0                      0 = 1 1 0 0 0 1 0 1

A: 0 0 1 1 0 1 0 1 = 0 =  $\mathbb{R}_0$                        $\mathbb{R}_1 = 0 = 1 1 0 0 0 1 0 1$



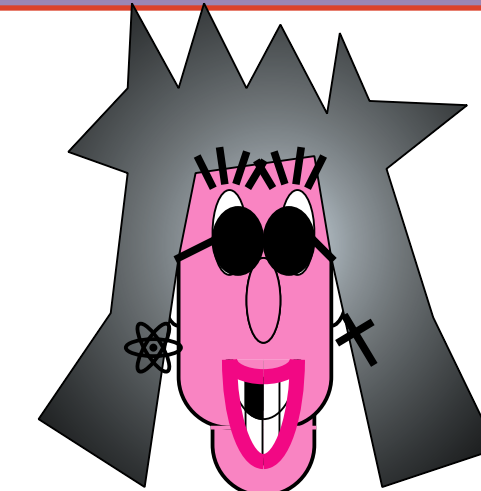
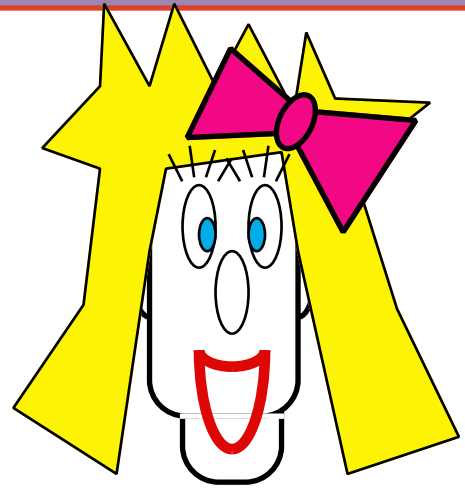


# Oblivious Transfer



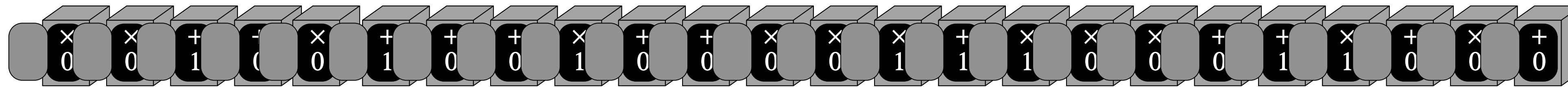
# Q-OT

## from Q-BC

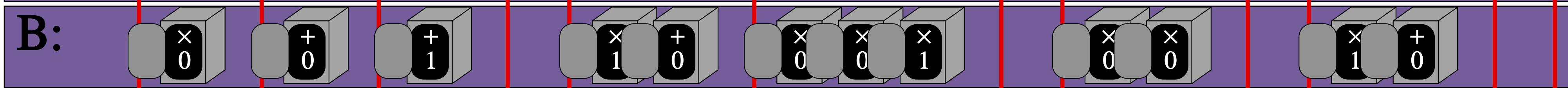


A: 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0  
 × + × + + + × × × × + + + + × × × + × + + + × +

B: × × + + × + + + × + + × × × + × × × + + × + × +  
 0 0 1 0 0 1 0 0 1 0 0 0 0 1 1 1 0 0 0 1 1 0 0 0



A: [24 empty grey blocks]

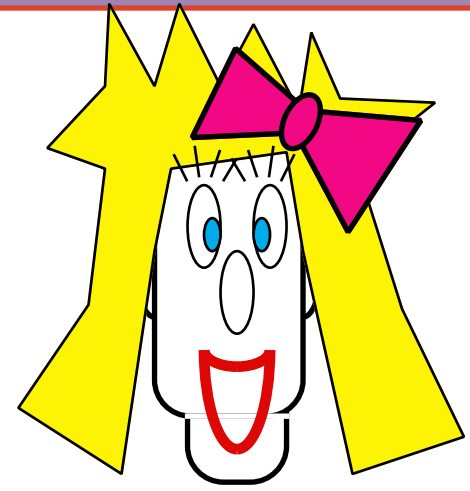


A: 1 0 1 1 1 0 0 1 1 1 0  
 + + + × × + + + × + + +

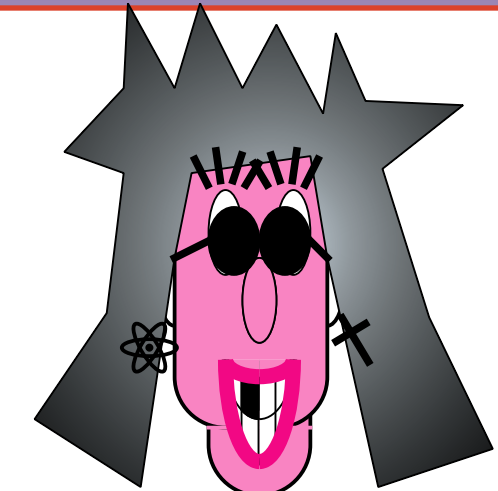
A: × × + × × + × × × + × + × +

B: × + × + + + 0 + × + + + × +  
 0 0 0 0 0 1 1 1 1 0 0

# Q-OT



## from Q-BC

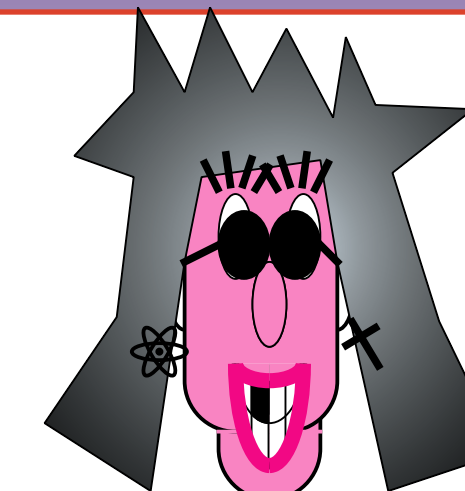
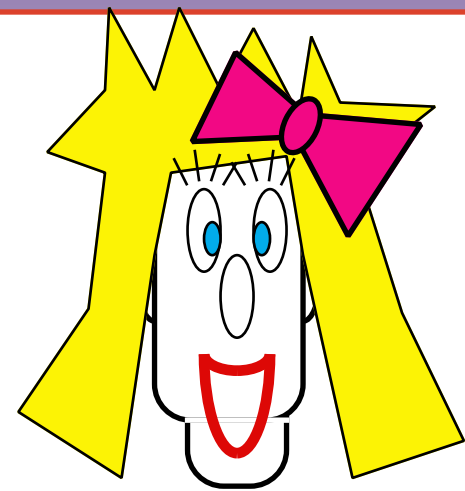


**A:** 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0  
× + × + + + × × × × + + + + × × × + × + + + × +

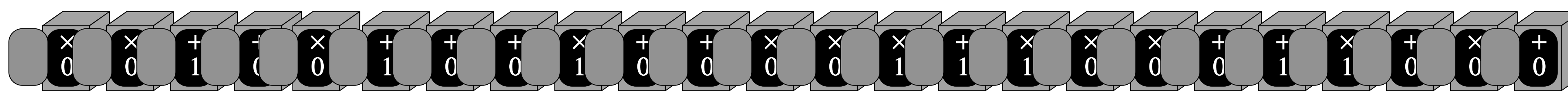
**B:** × × + + × + + + × + + × × × + × × × + + × + × +  
0 0 1 0 0 1 0 0 1 0 0 0 0 1 1 1 0 0 0 1 1 0 0 0

# Q-OT

## from Q-BC



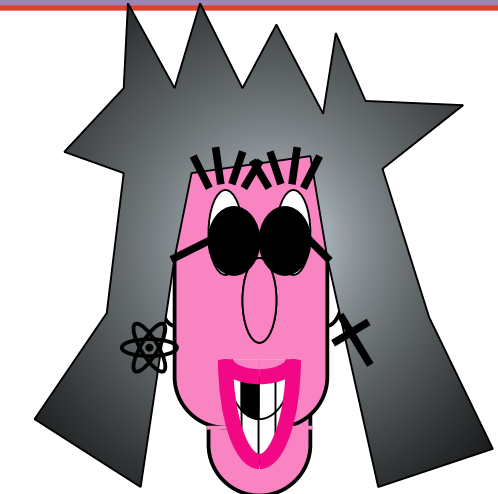
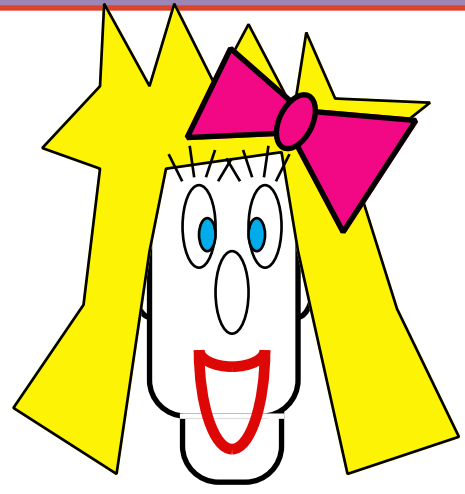
**B:**    × × + + × + + + × + + × × × + × × × + + × + × +  
         0 0 1 0 0 1 0 0 1 0 0 0 0 1 1 1 0 0 0 1 1 0 0 0



**A:**    [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ] [ ]

# Q-OT

## from Q-BC



**A:** [20 empty gray boxes]

**A:** [20 orange arrows pointing right]

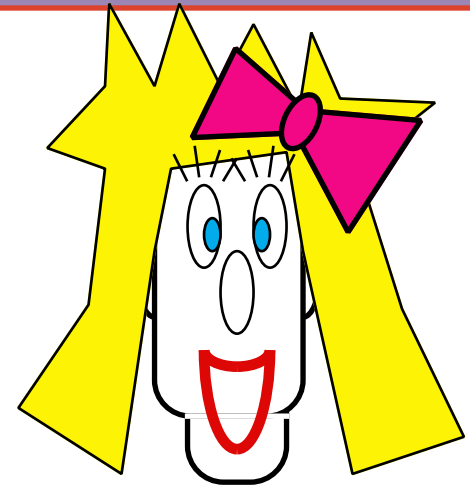
**B:** [3 boxes: ×0, +0, +1] [2 boxes: ×1, +0] [3 boxes: ×0, ×0, ×1] [2 boxes: ×0, ×0] [2 boxes: ×1, +0]

**A:** 1 0 1 1 1 1 0 0 1 1 1 0

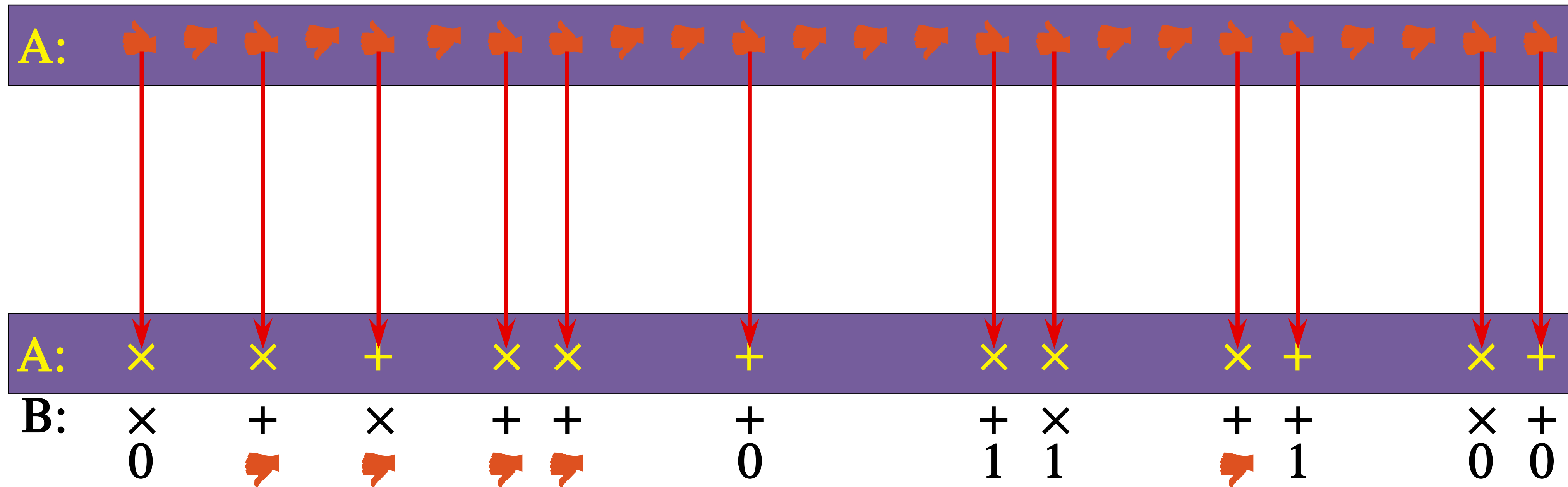
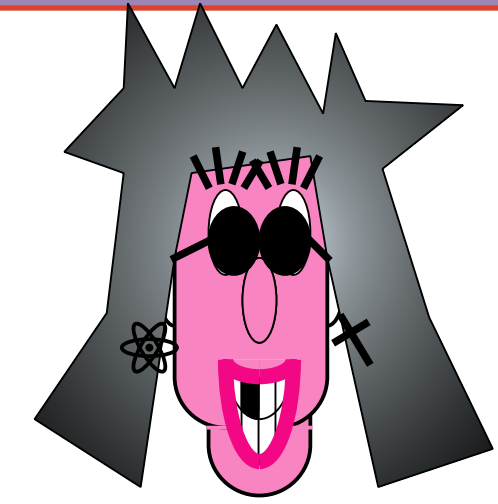
          +   +   +   × ×   + + +   × +   + +



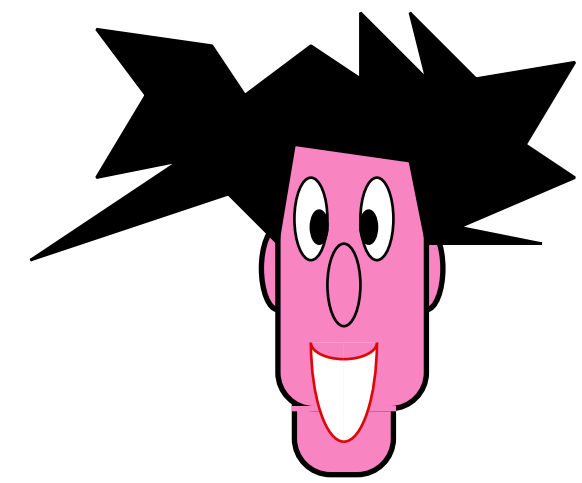
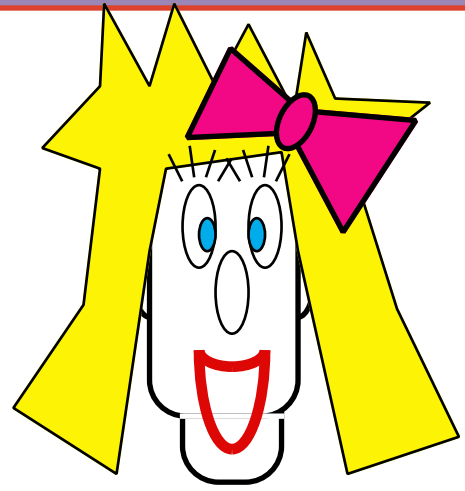
# Q-OT



## from Q-BC



# Q-OT



**A:** 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0

× + × + + + × × × × + + + + × × × + × + + + × +

**B:** × × + + × + + + × + + × × × + × × × + + × + × +

0 0 1 0 0 1 0 0 1 0 0 0 0 1 1 1 0 0 0 1 1 0 0 0

**A:** × + × + + + × × × × + + + + × × × + × + + + × +

**B:** 0   0  1   1  0     1 0   1  0 0 0

**(6)**

**two provers**

**Cryptographic Protocols**

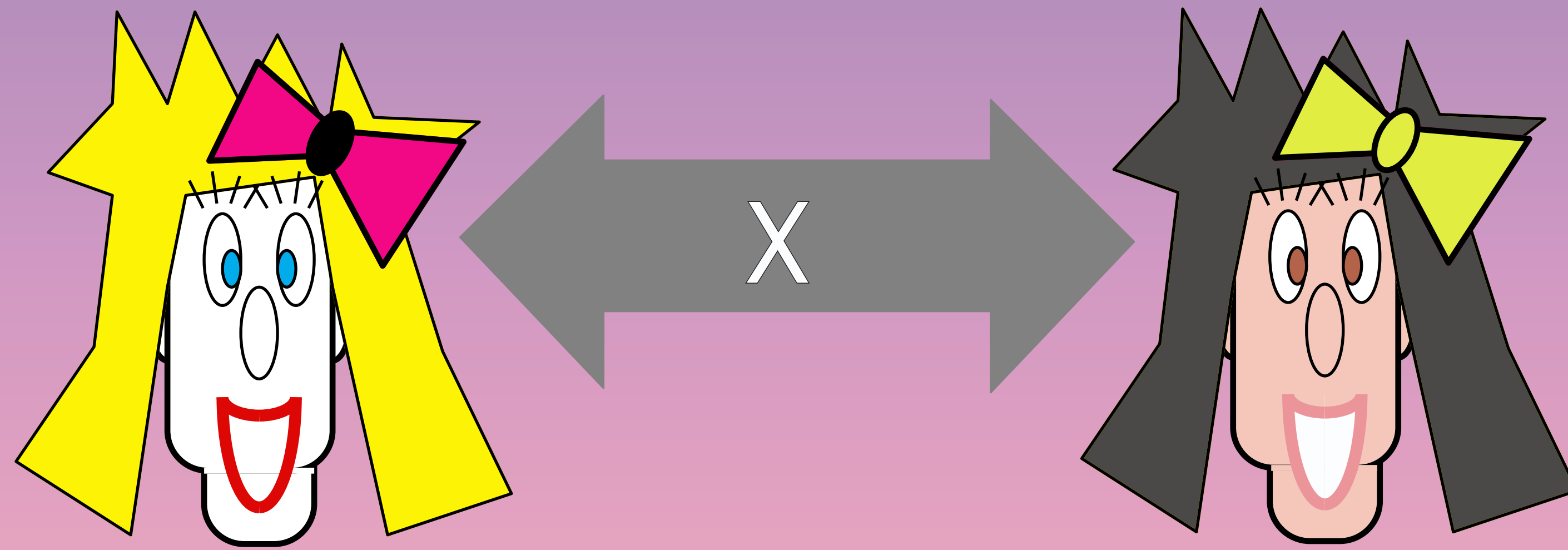
Classically

BIT COMMITMENT



BGKW88

# Classically

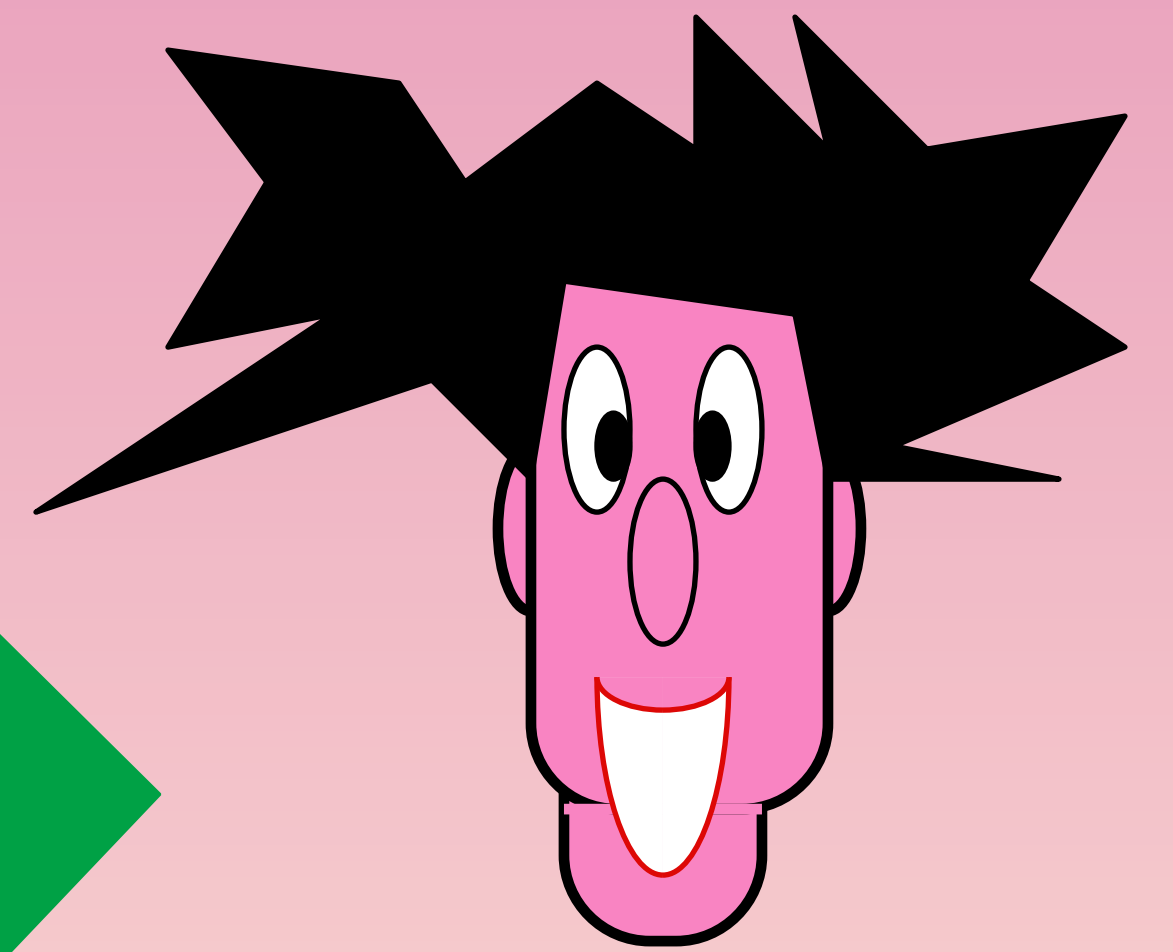
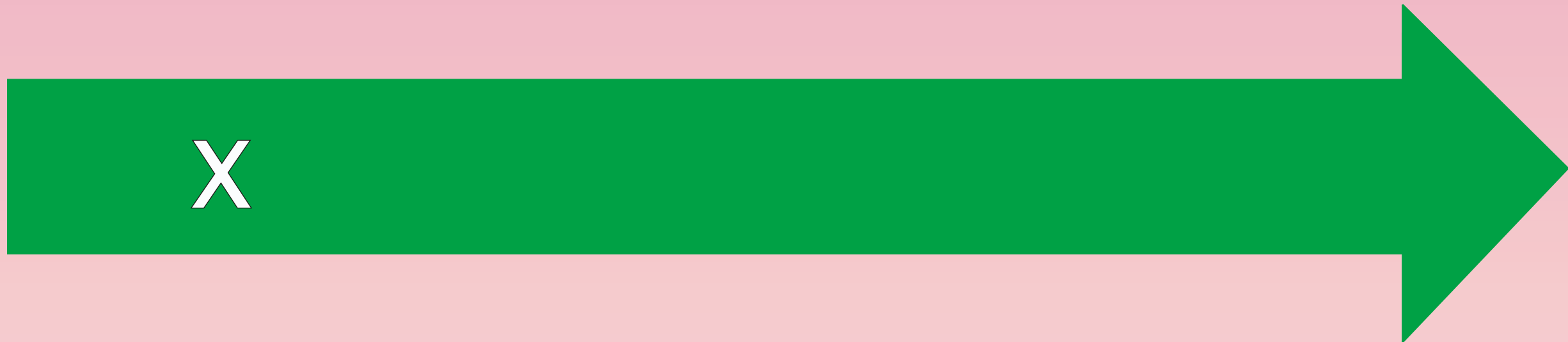
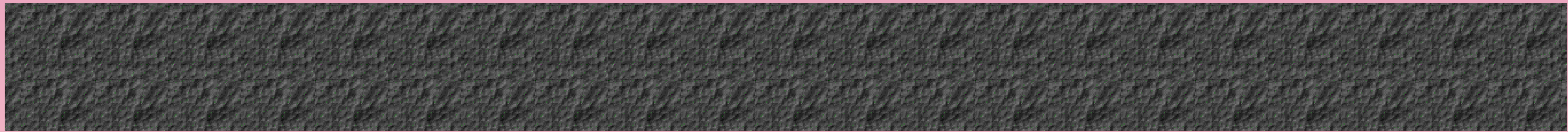
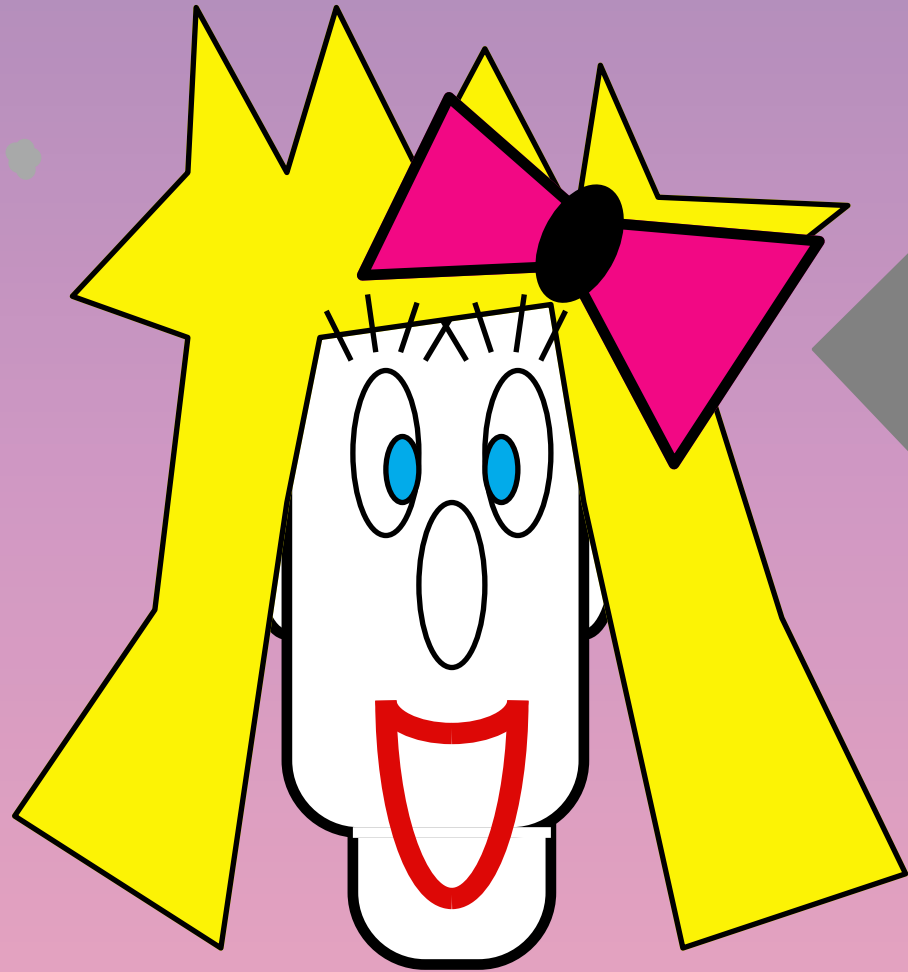


Ben-Or, Goldwasser, Kilian, Wigderson



b

$$z = x \quad \text{if } b = 0$$
$$z = x \oplus y \quad \text{if } b = 1$$



$$x \oplus z = b \cdot y?$$

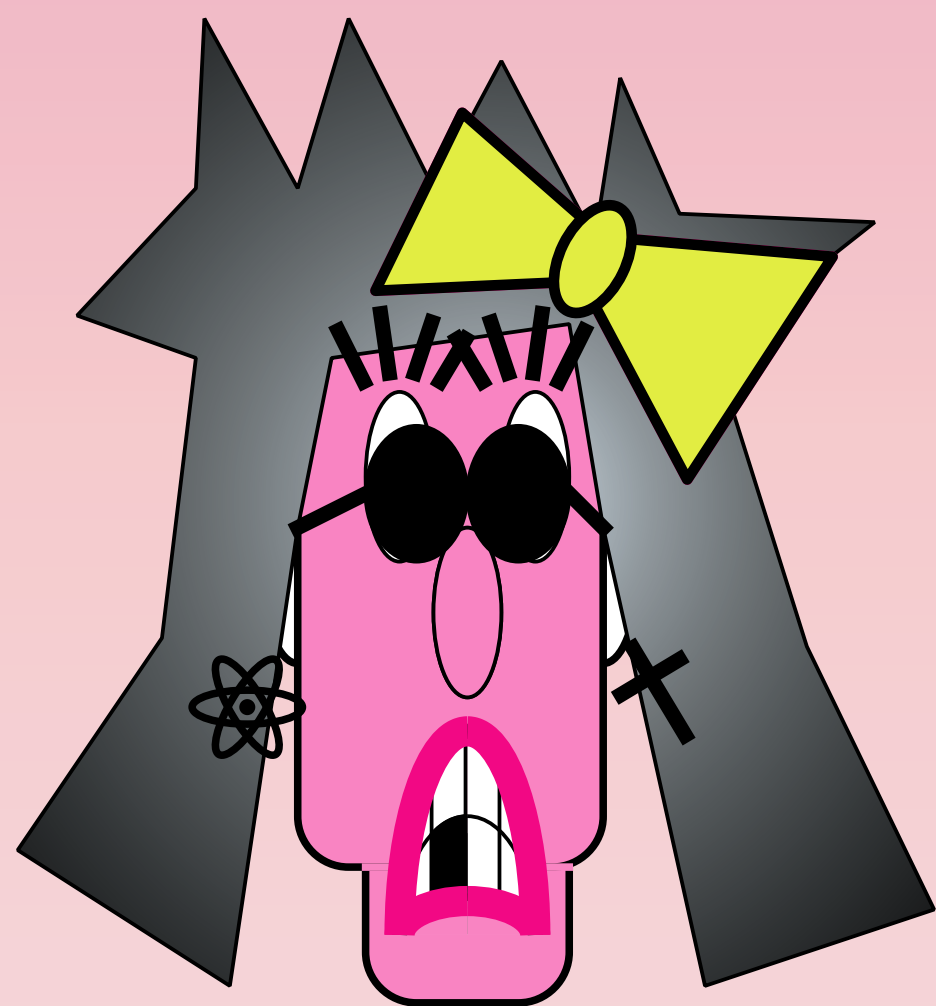
Ben-Or, Goldwasser, Kilian, Wigderson

$$x_0 \oplus z = 0 \cdot y = 0$$

$$x_1 \oplus z = 1 \cdot y = y$$

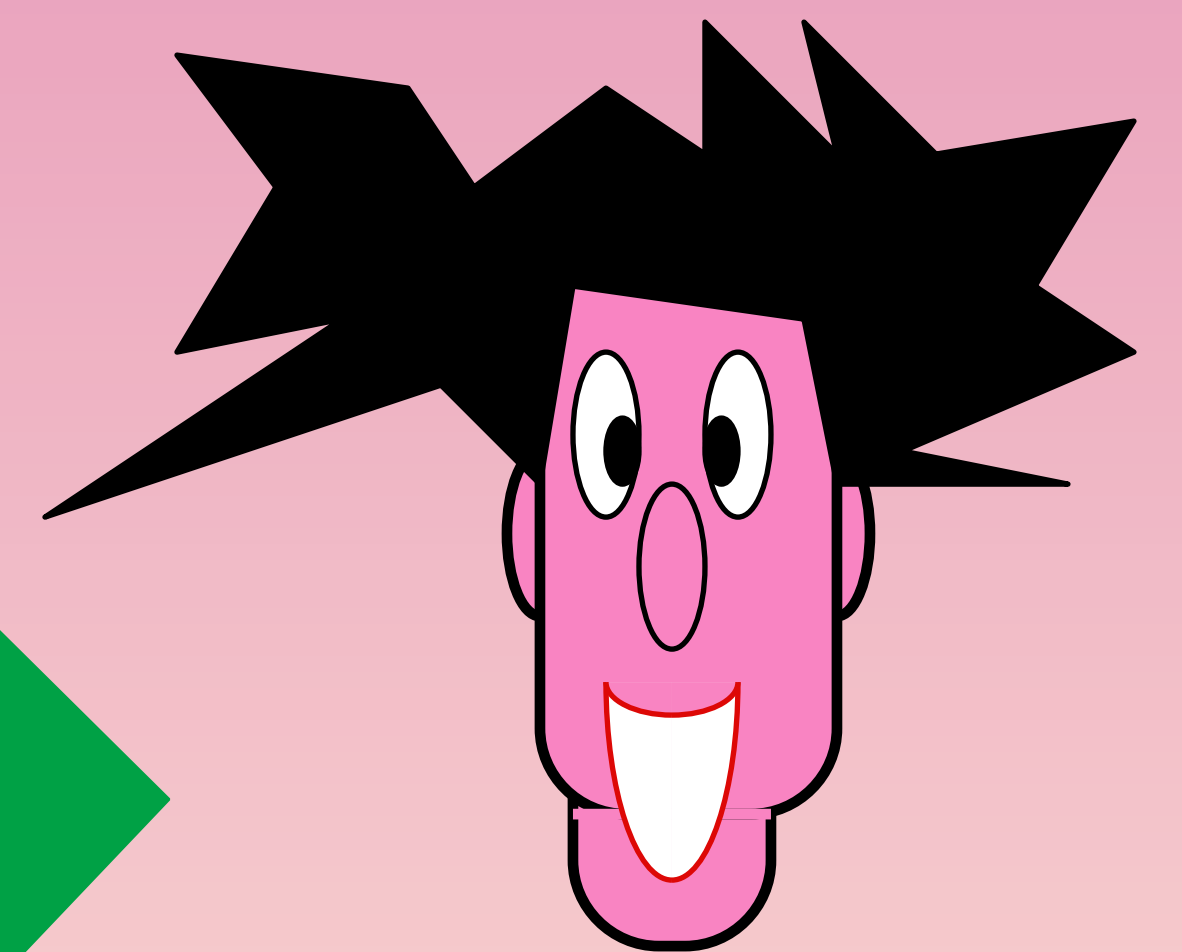
$$x_0 \oplus x_1 = (x_0 \oplus z) \oplus (x_1 \oplus z) = y$$

possible with prob. at most  $2^{-n}$



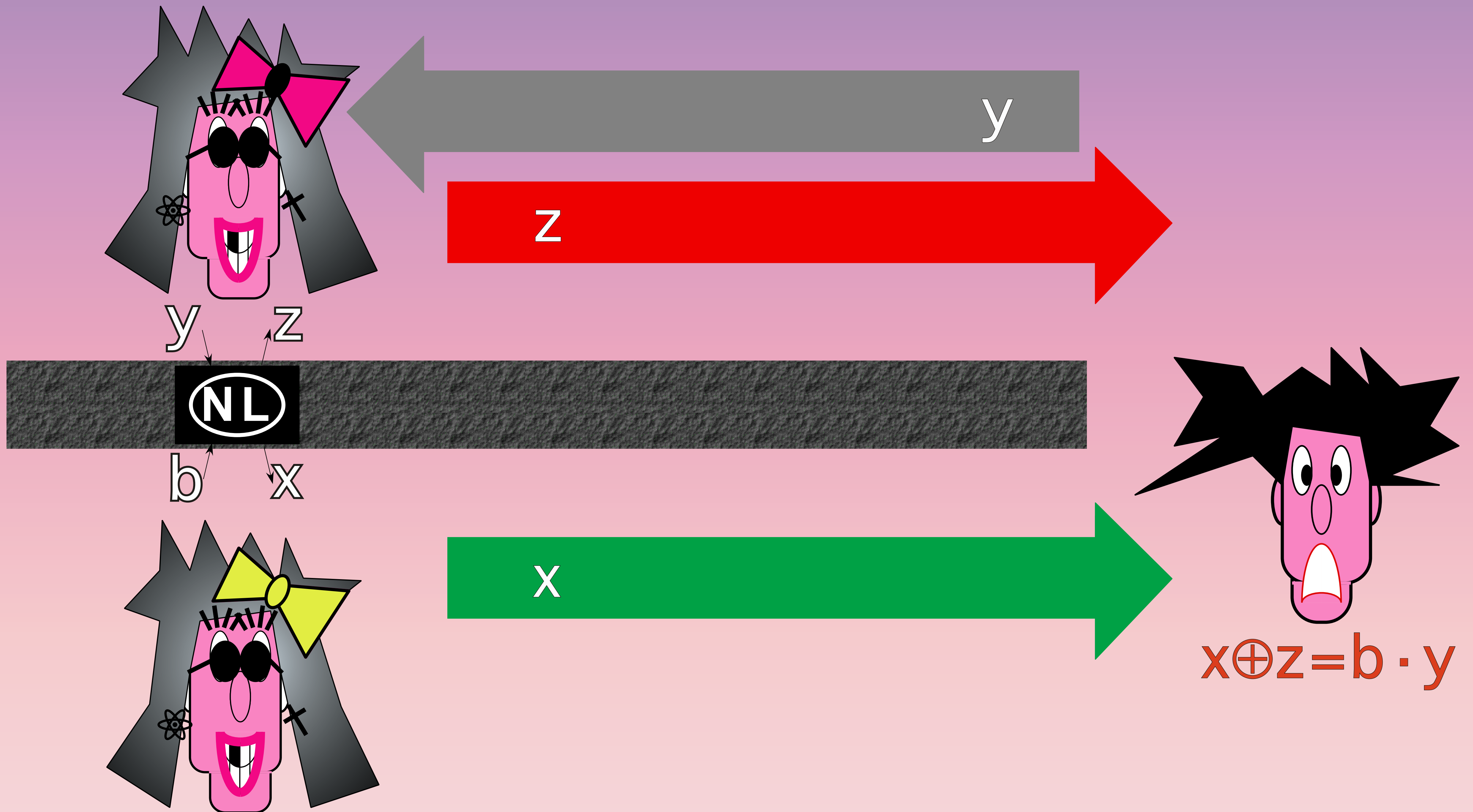
$x_0$

$x_1$



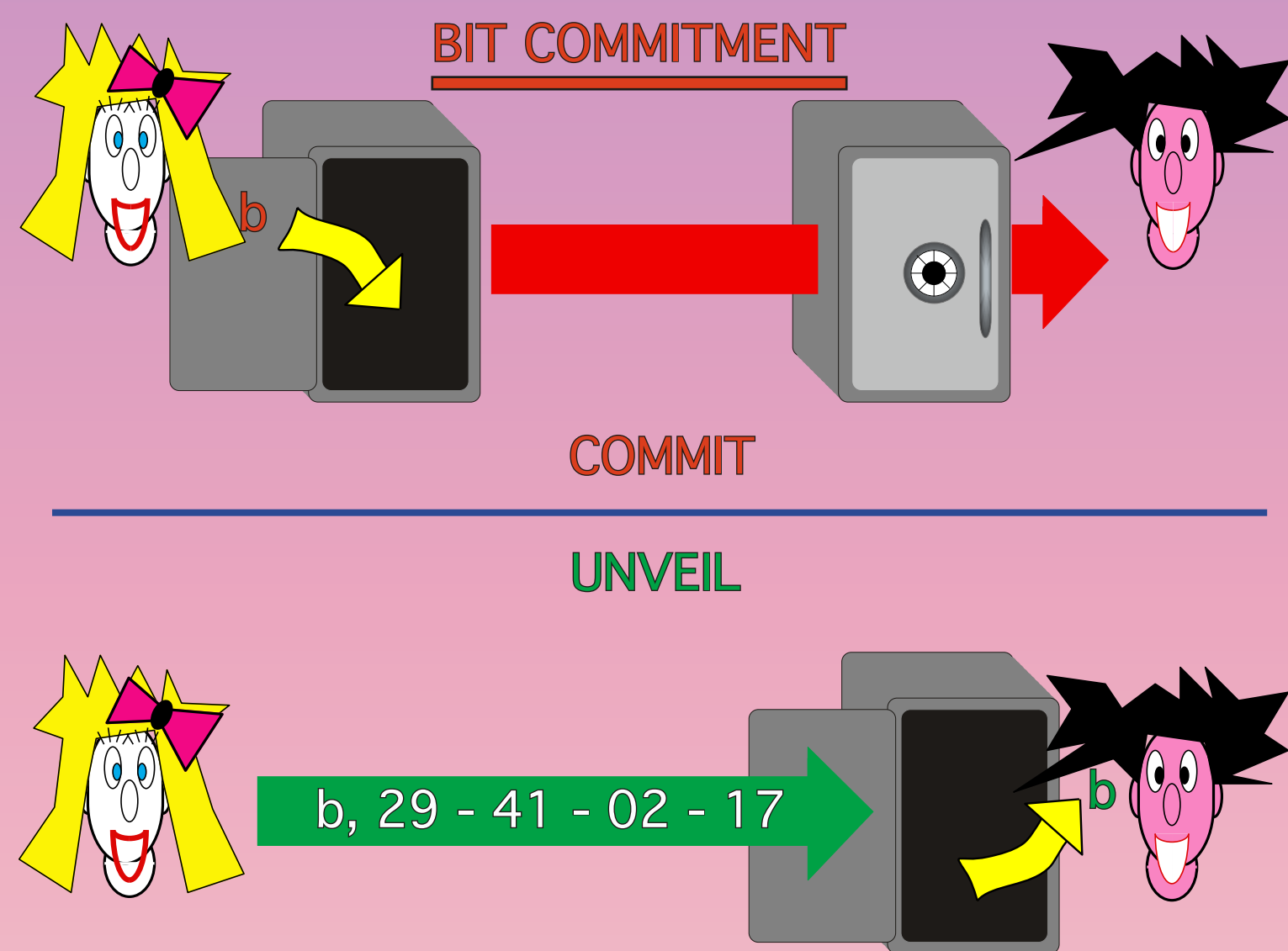
Ben-Or, Goldwasser, Kilian, Wigderson

# Classically

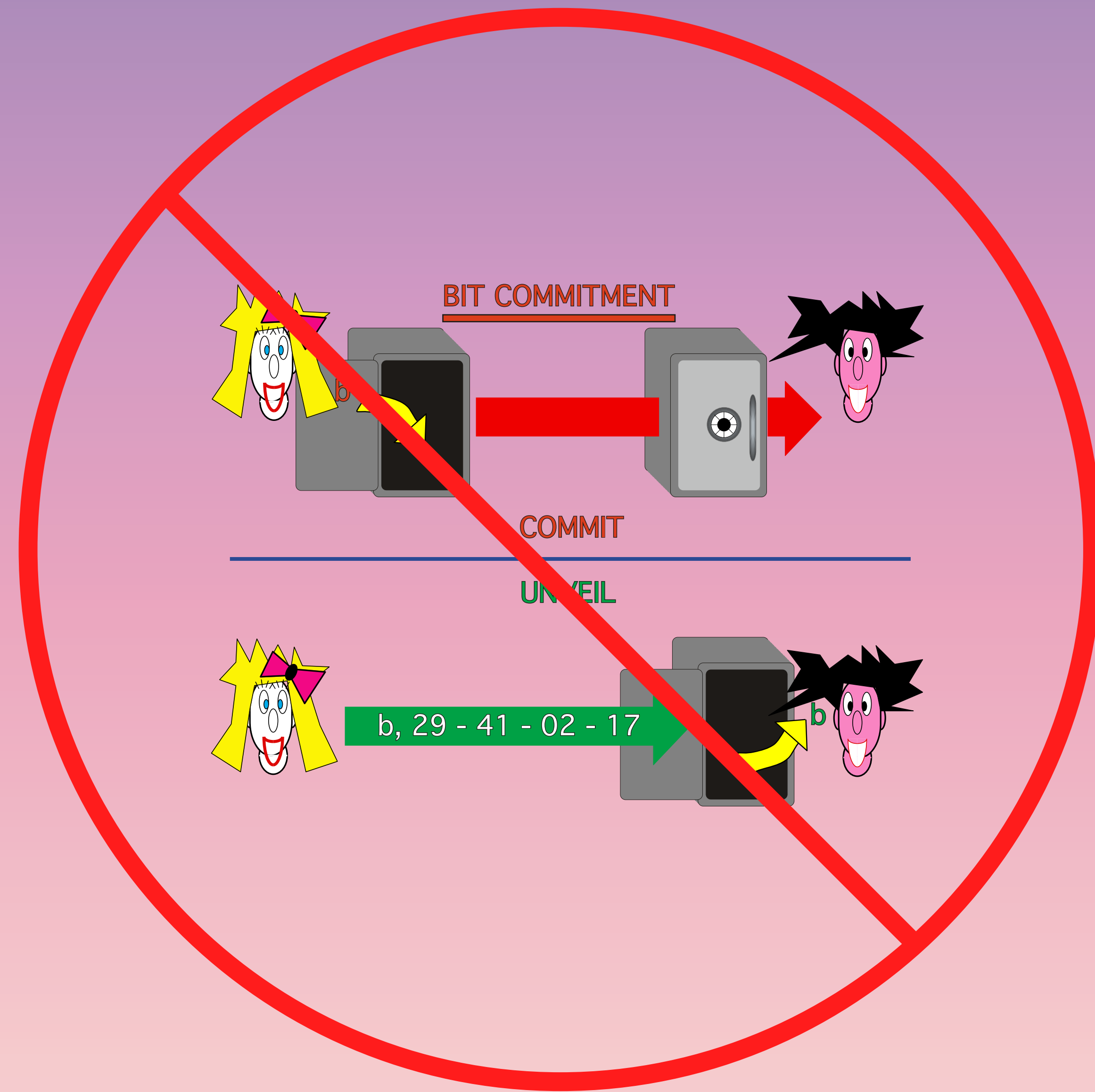


~~Ben-Or, Goldwasser, Kilian, Wigderson~~

# Quantumly



or



???

**(7)**

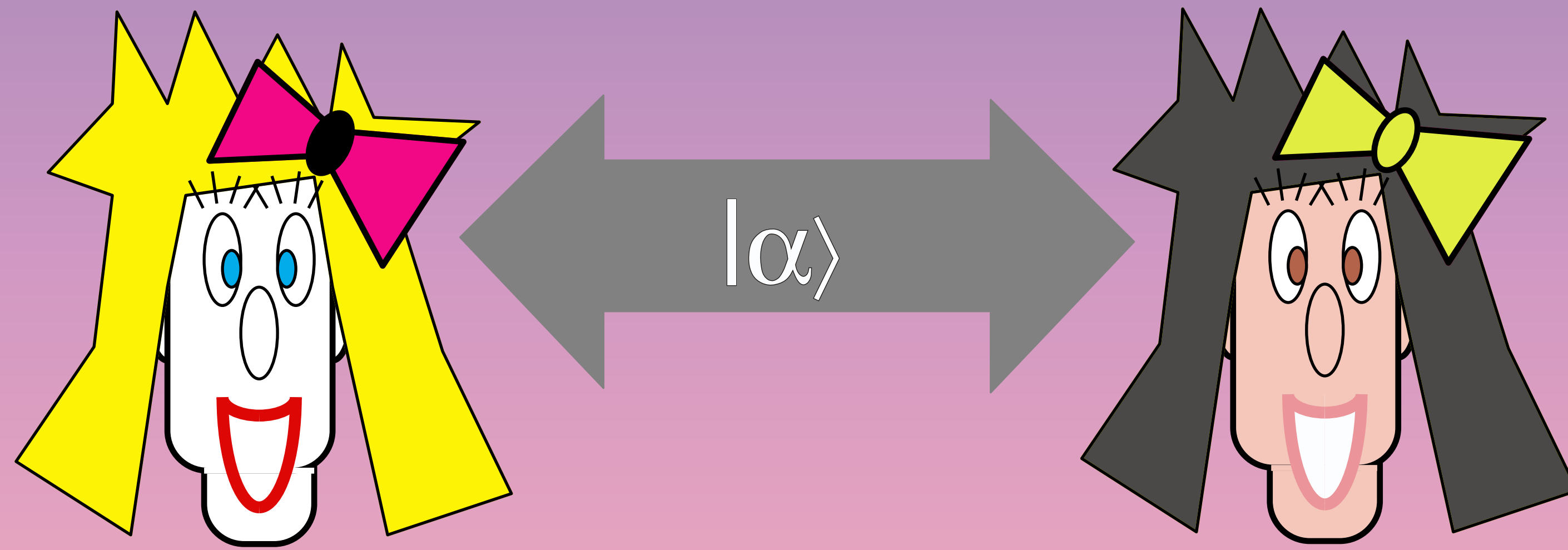
**two provers BC**

**Classically Secure**

**Quantumly Insecure**



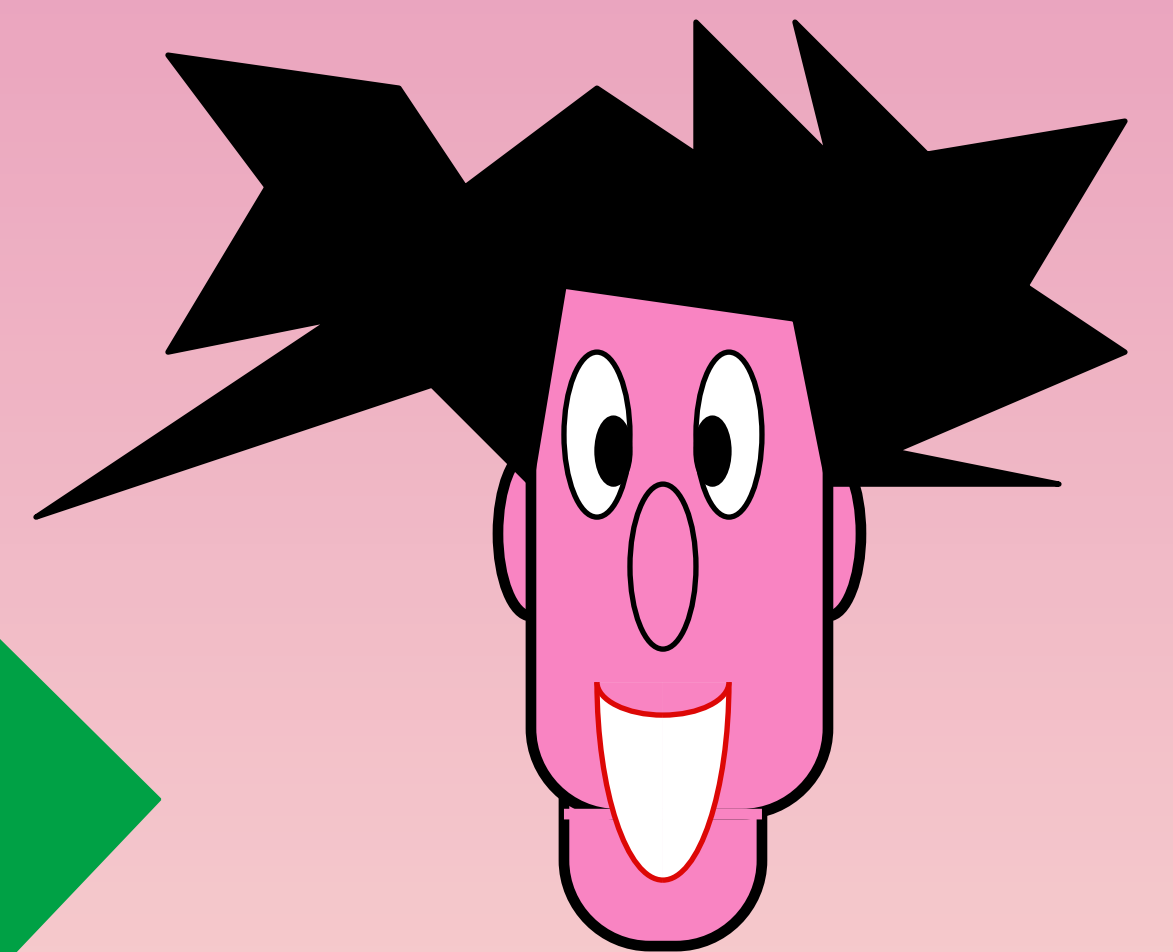
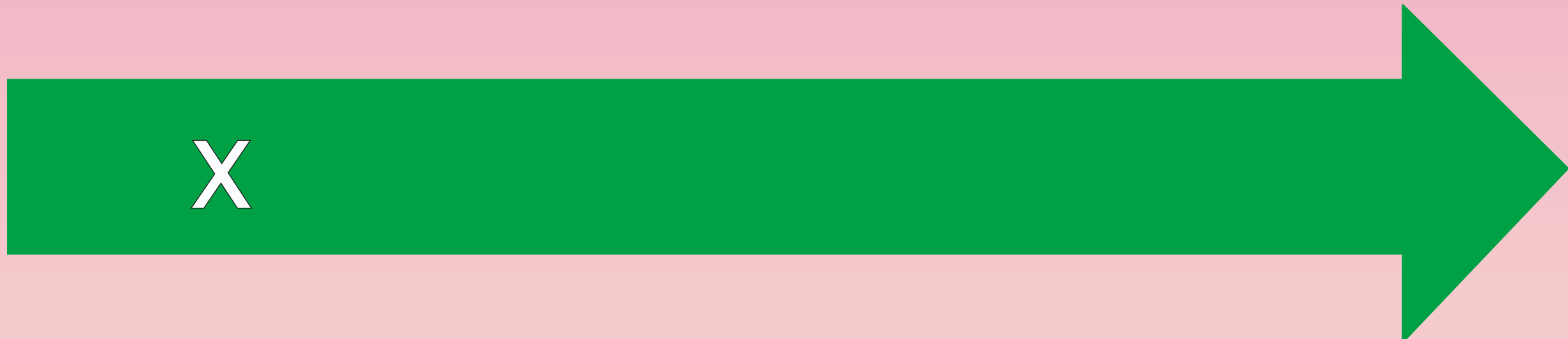
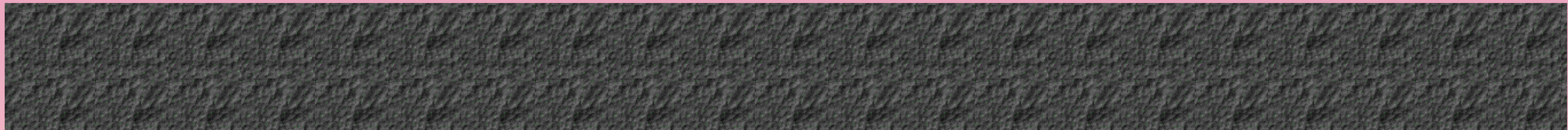
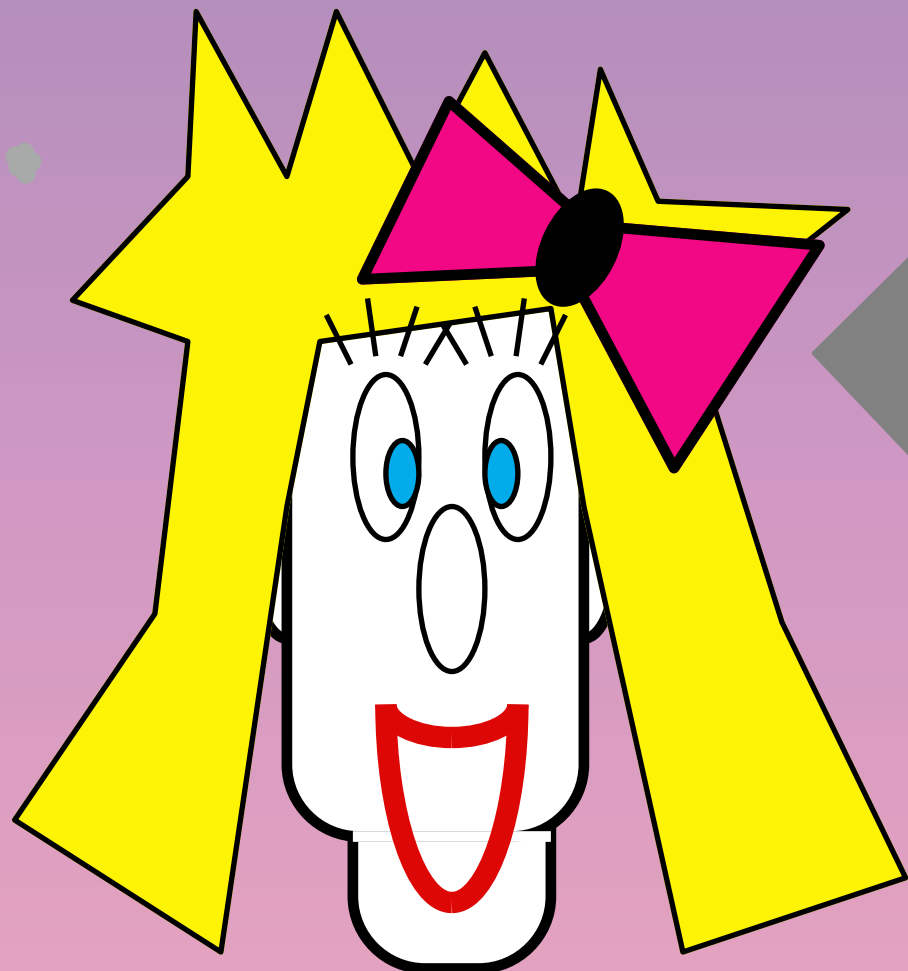
# Quantumly



Ben-Or, Goldwasser, Kilian, Wigderson

b

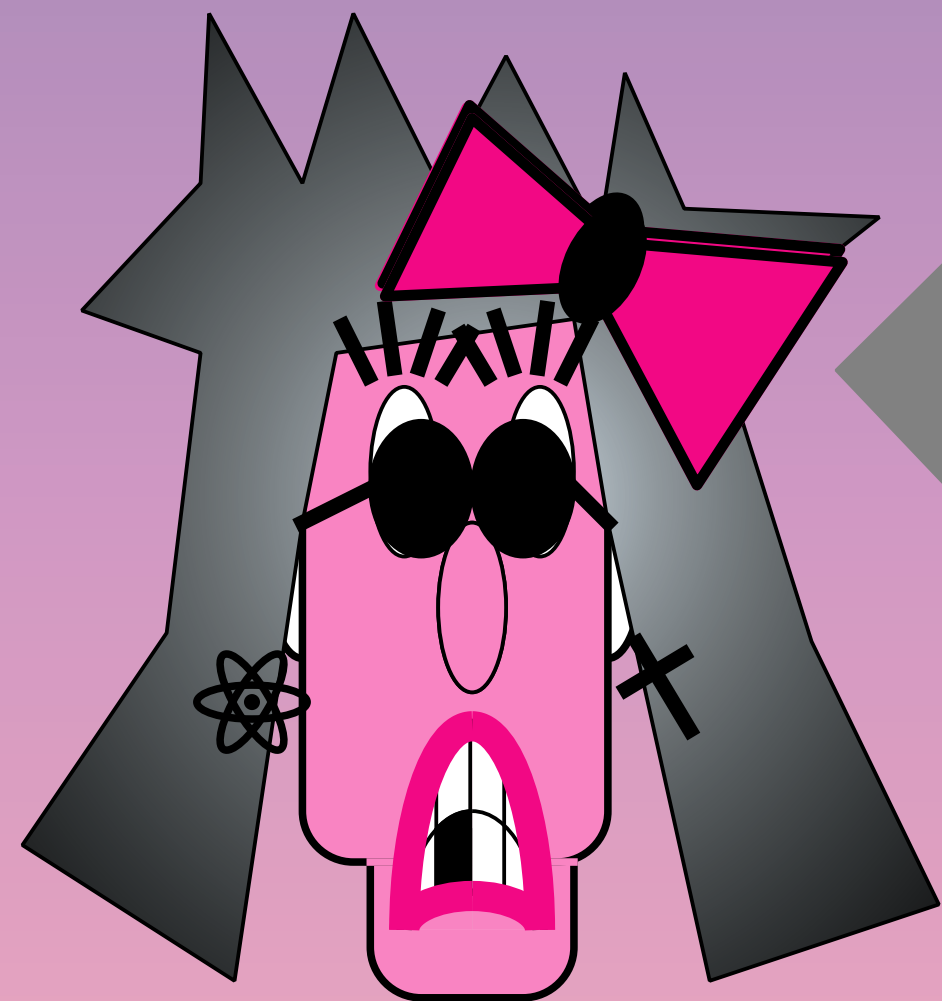
$$\begin{aligned} z &= x && \text{if } b = 0 \\ z &= x \oplus y && \text{if } b = 1 \end{aligned}$$



$$D_H(x \oplus z, b \cdot y) < n/5?$$

Ben-Or, Goldwasser, Kilian, Wigderson

# Classically

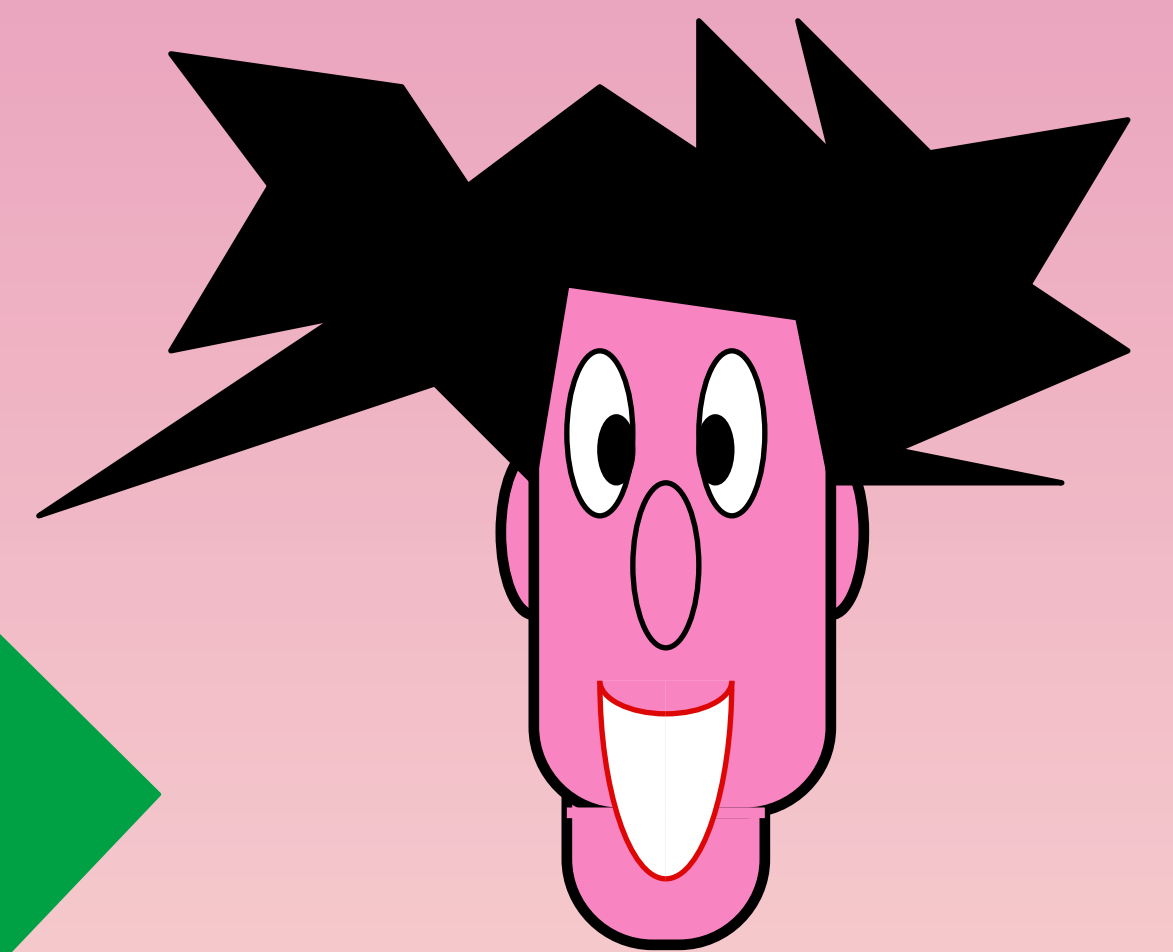
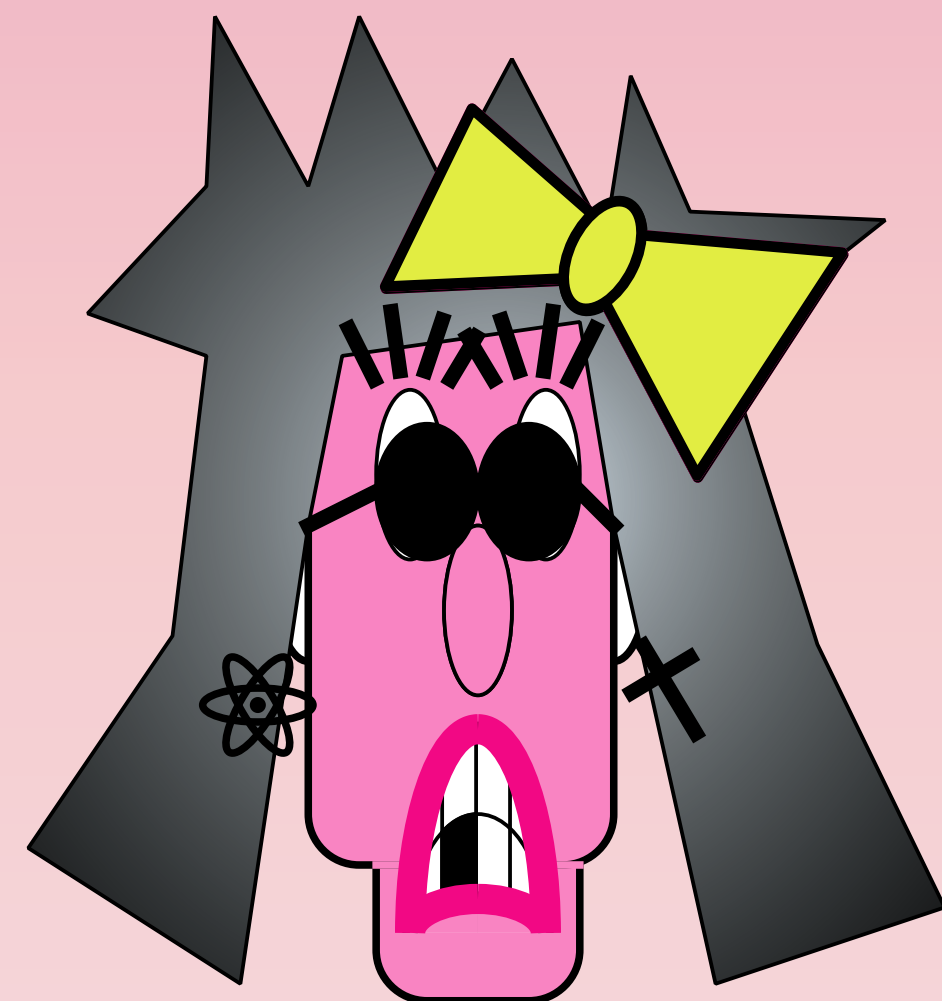


y z



75% **NL**  $\rightarrow D_H(x \oplus z, b \cdot y) \approx 25\%n > n/5$

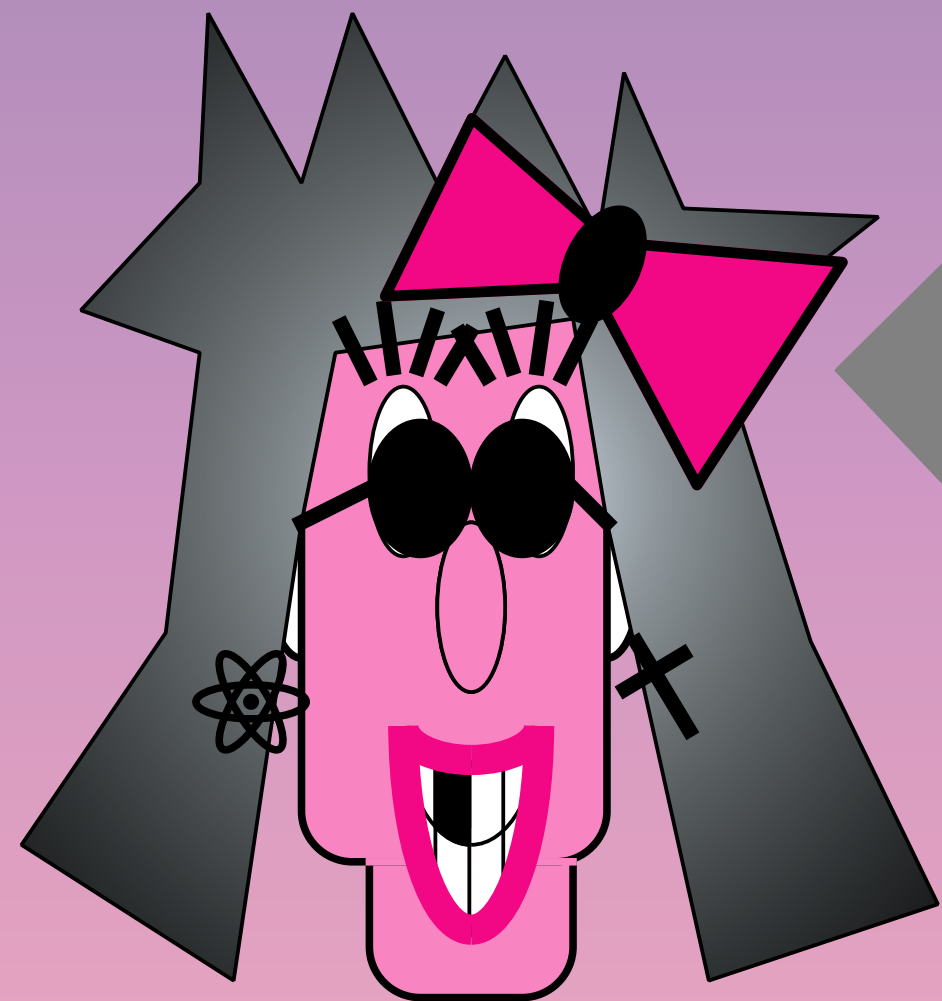
b x



$D_H(x \oplus z, b \cdot y) < n/5?$

Ben-Or, Goldwasser, Kilian, Wigderson

# Quantumly

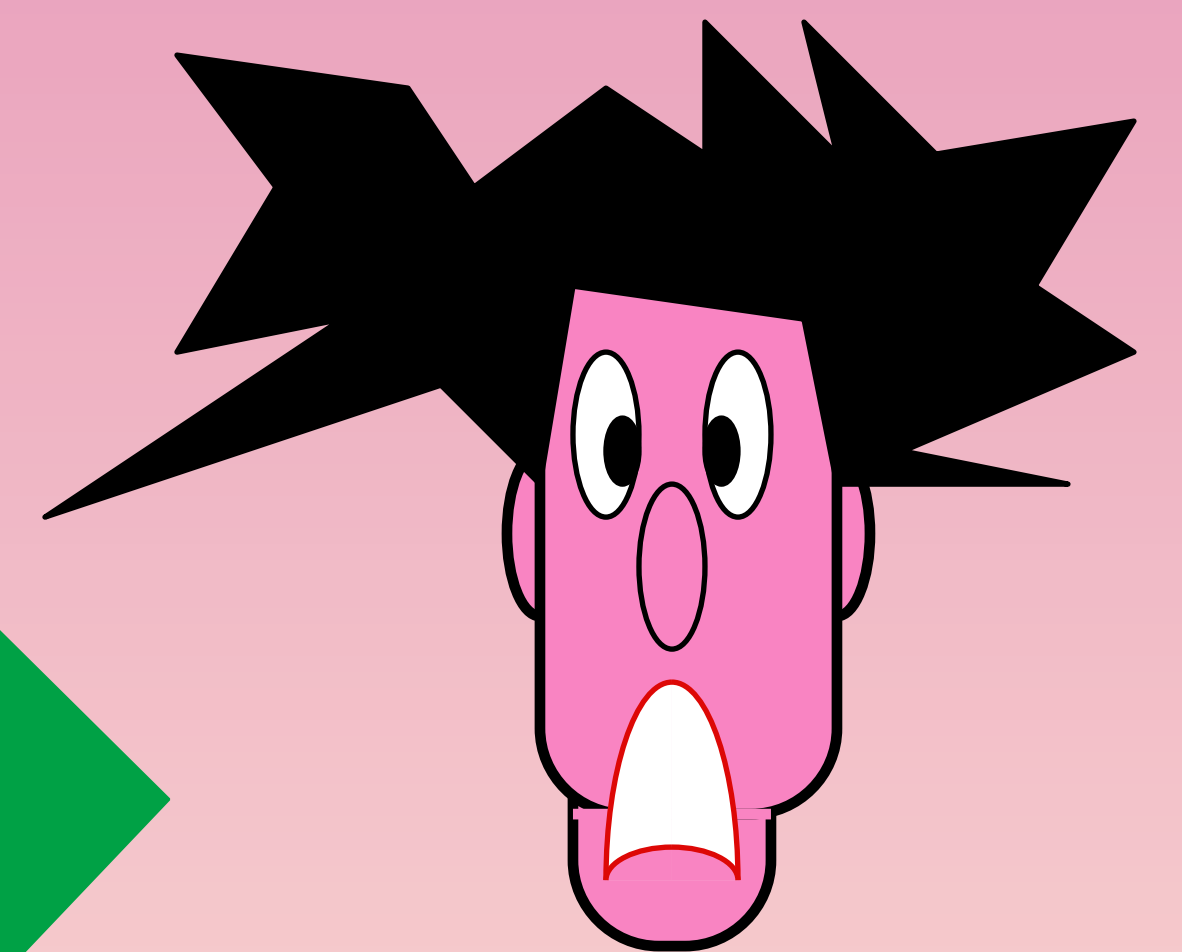
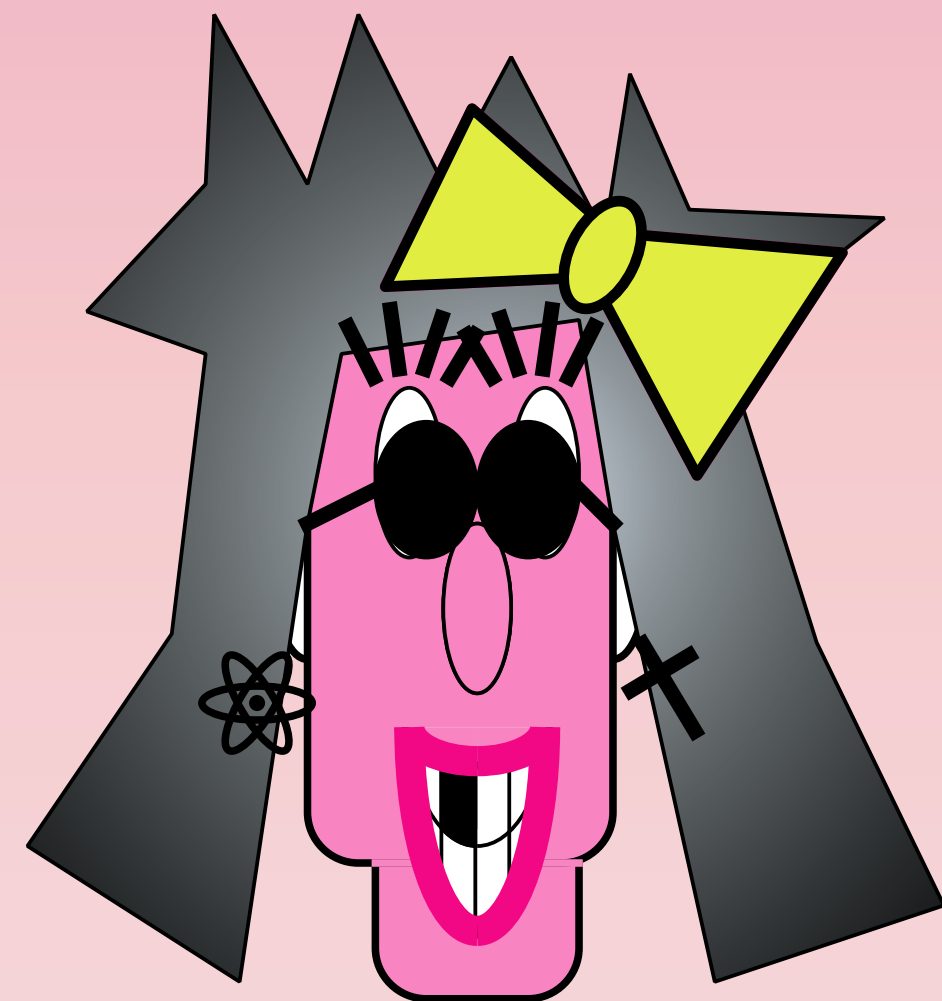


y z



$\approx 85\%$  **NL**  $\rightarrow D_H(x \oplus z, b \cdot y) \approx 15\% n < n/5$

b x



$D_H(x \oplus z, b \cdot y) < n/5?$

~~Ben-Or, Goldwasser, Kilian, Wigderson~~



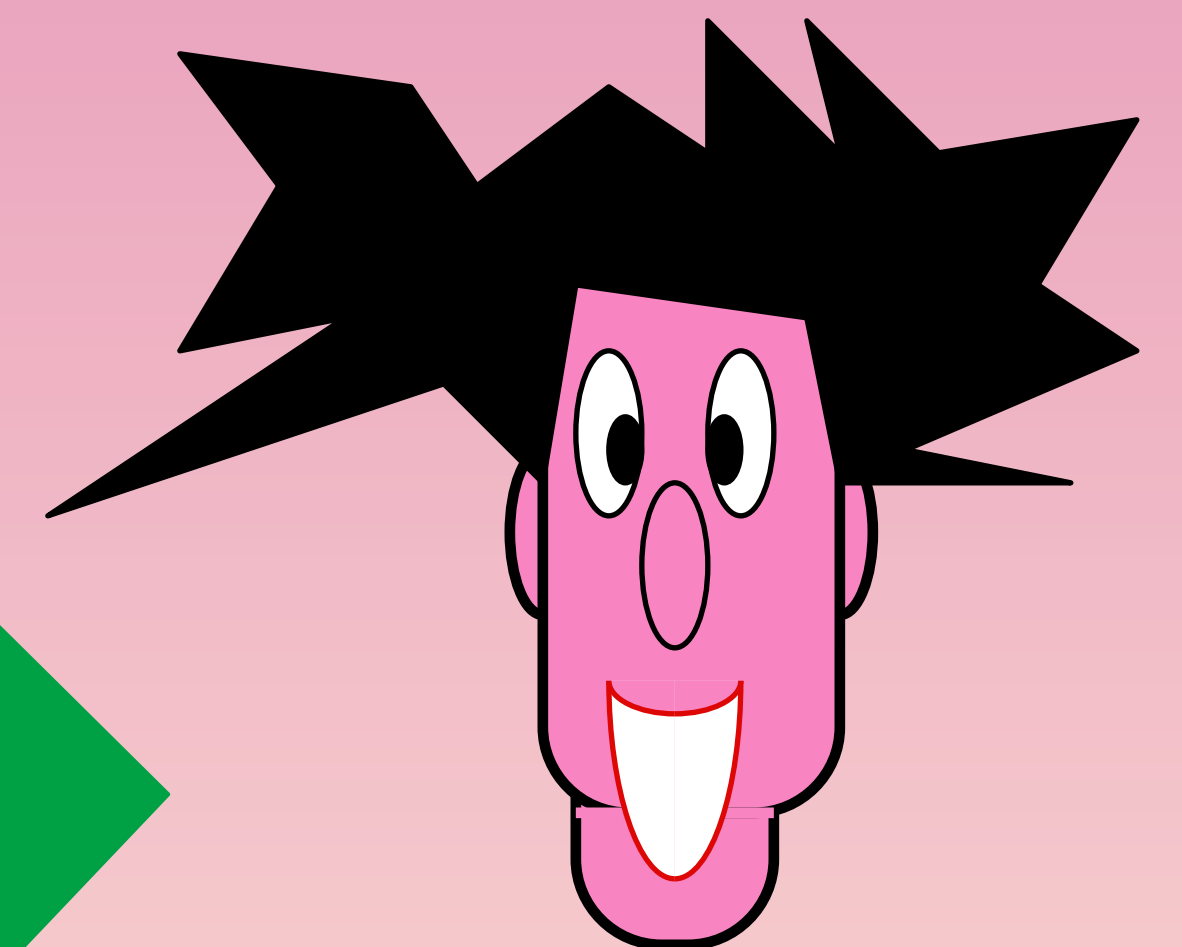
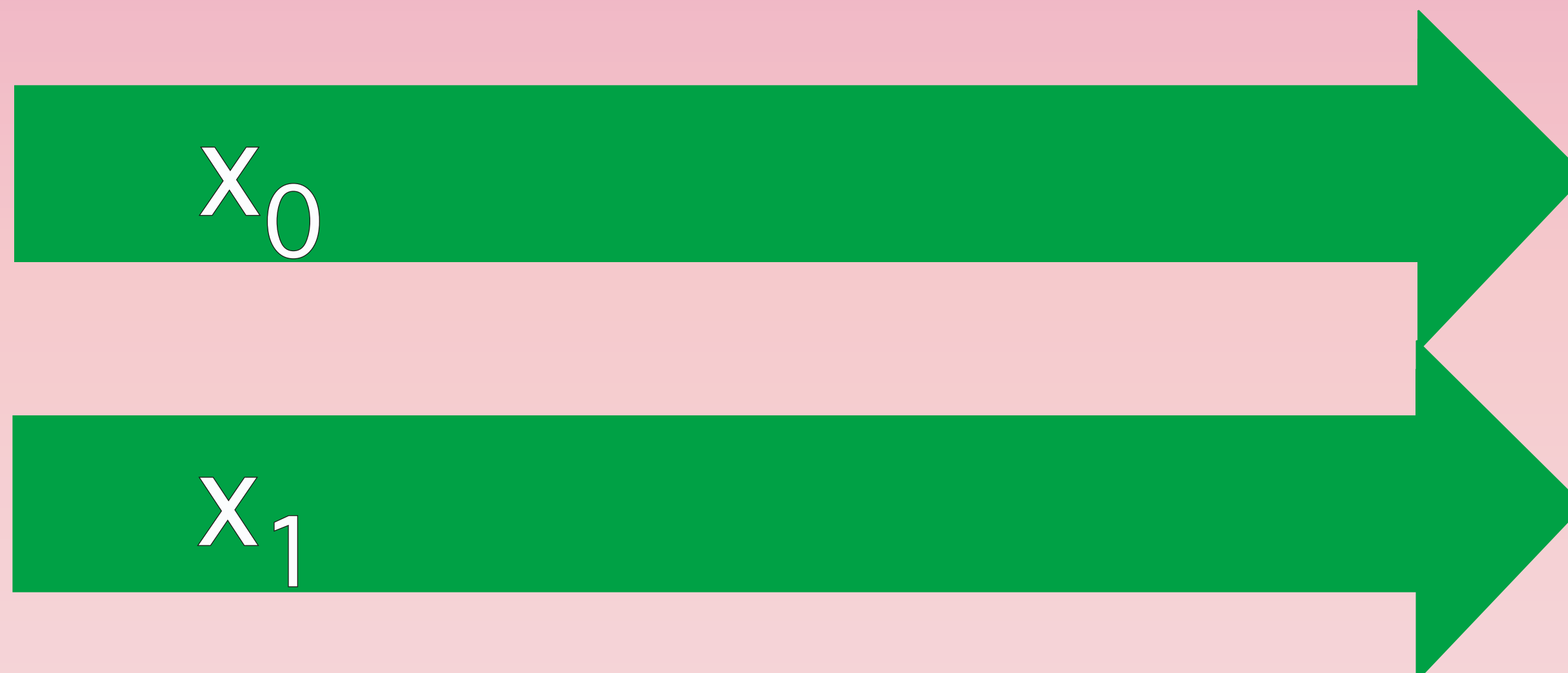
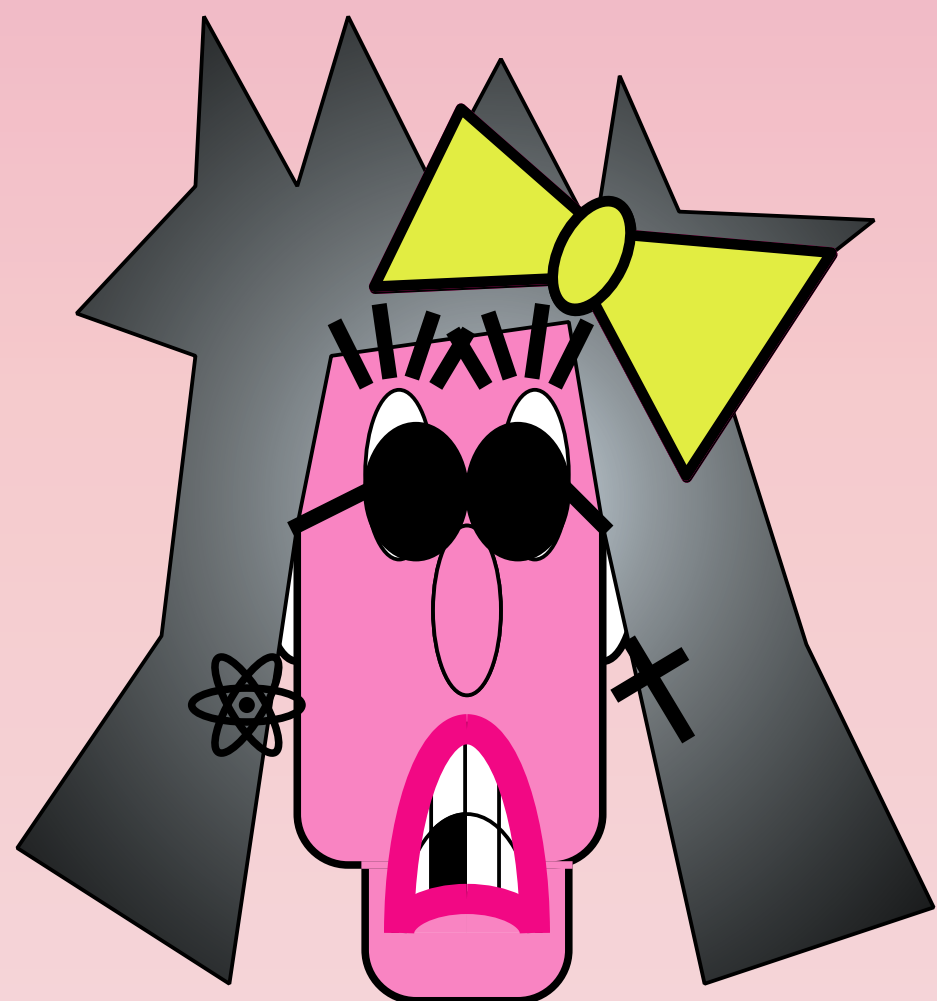
# Classically

$$D_H(x_0 \oplus z, 0 \cdot y) = D_H(x_0 \oplus z, 0) < n/5$$

$$D_H(x_1 \oplus z, 1 \cdot y) = D_H(x_1 \oplus z, y) < n/5$$

$$D_H(x_0 \oplus x_1, y) = D_H((x_0 \oplus z) \oplus (x_1 \oplus z), y) < 2n/5 < n/2$$

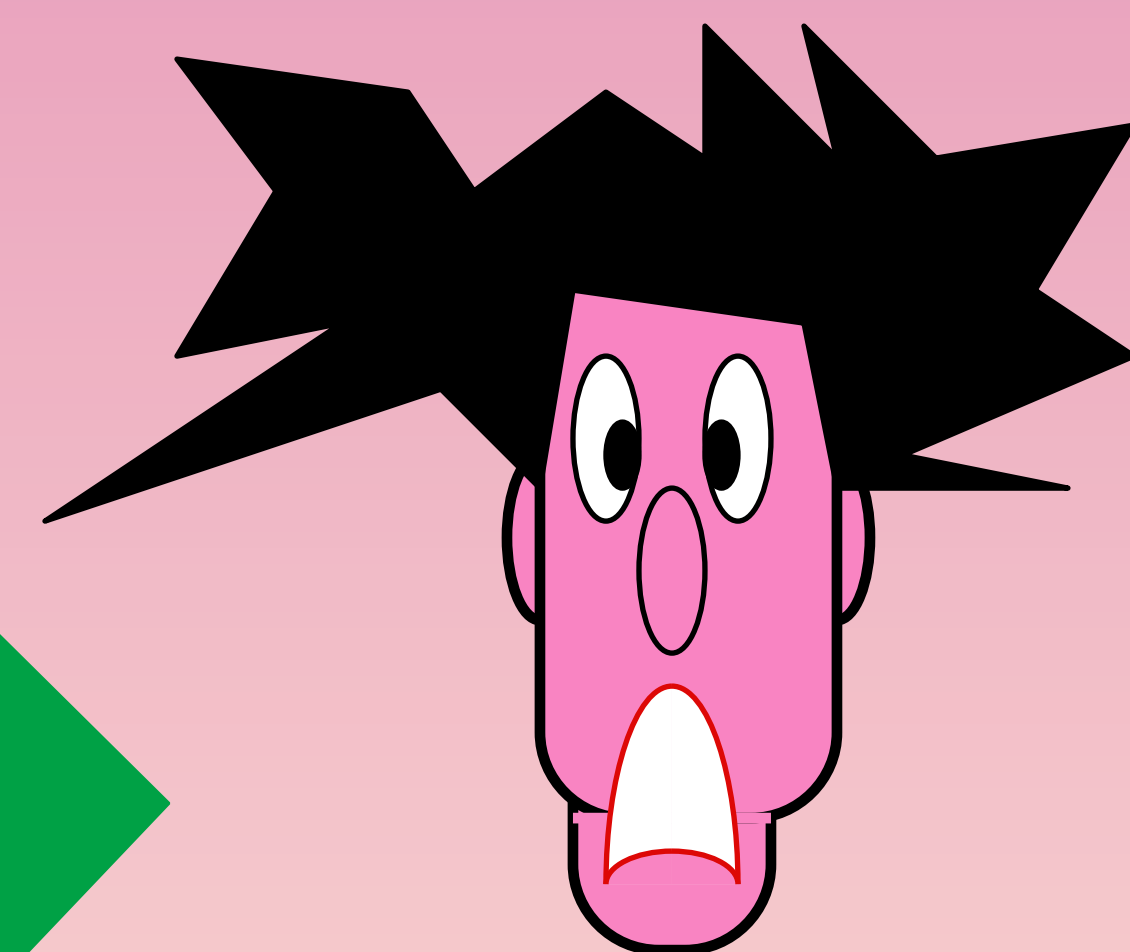
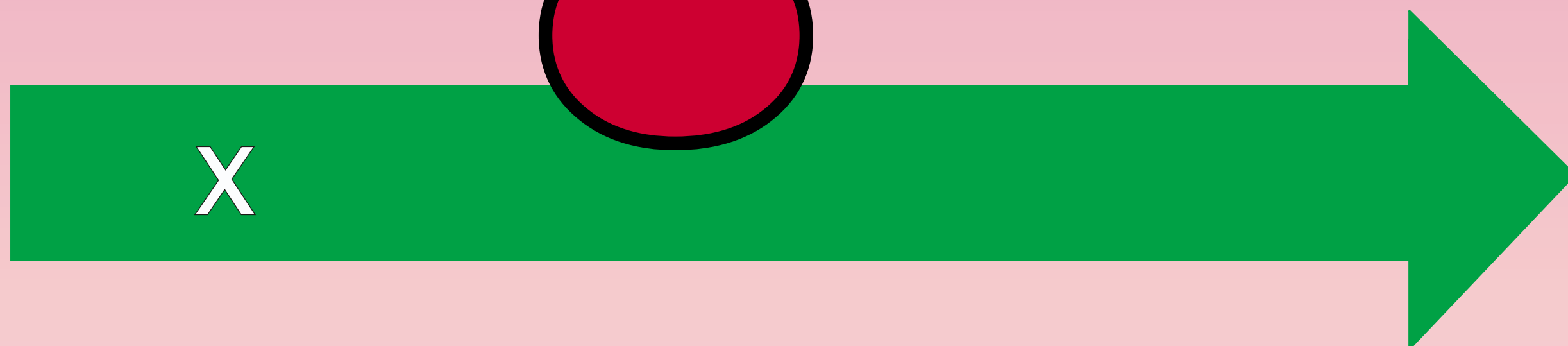
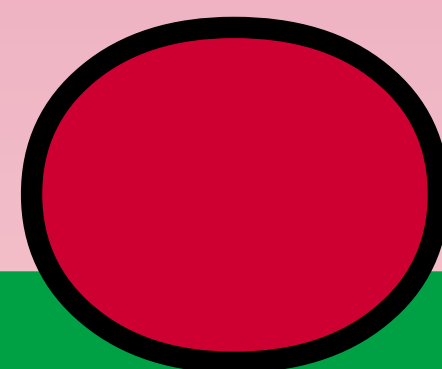
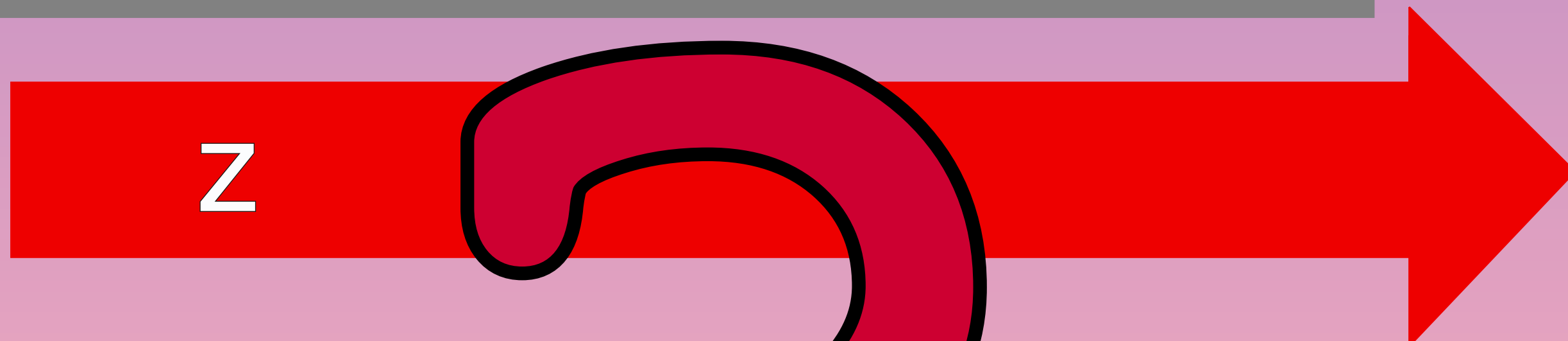
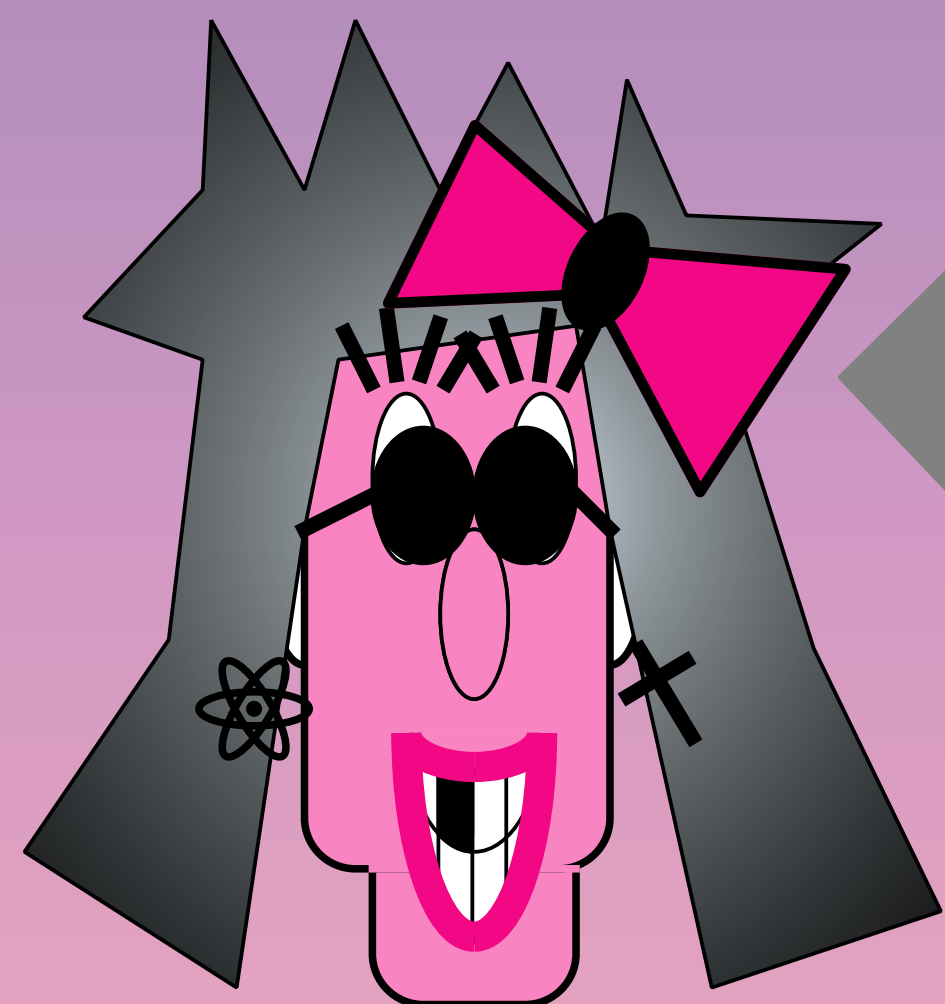
possible with prob. at most  $c^{-n}$



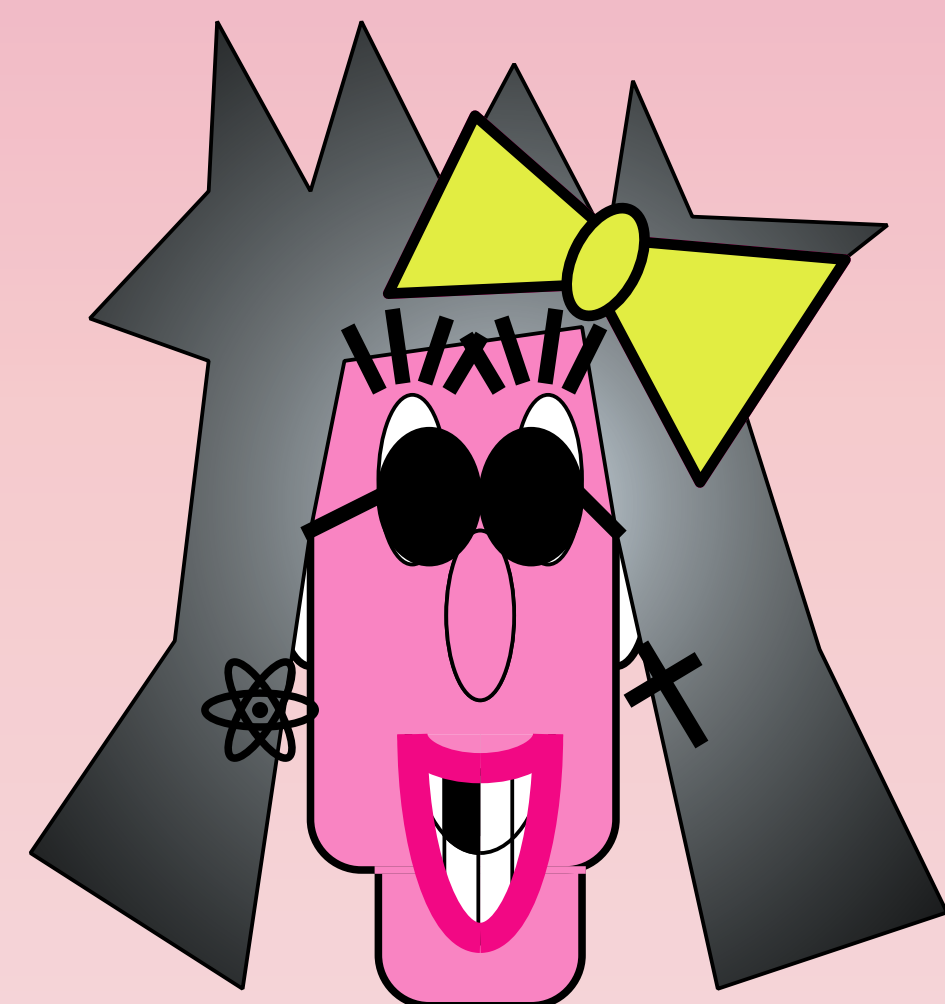
Ben-Or, Goldwasser, Kilian, Wigderson



# Quantumly



$$x \oplus z = b \cdot y$$



~~Ben-Or, Goldwasser, Kilian, Wigderson~~

**(8)**

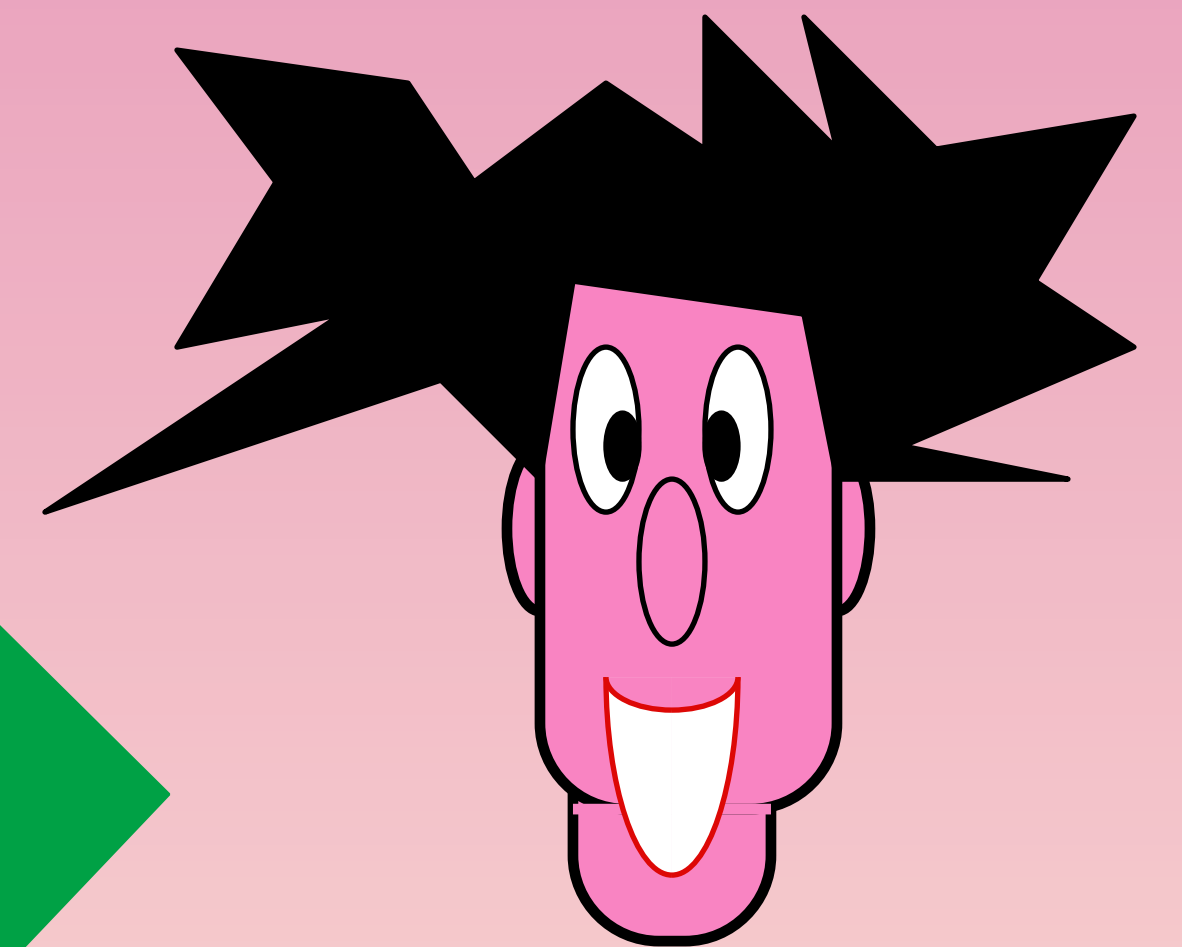
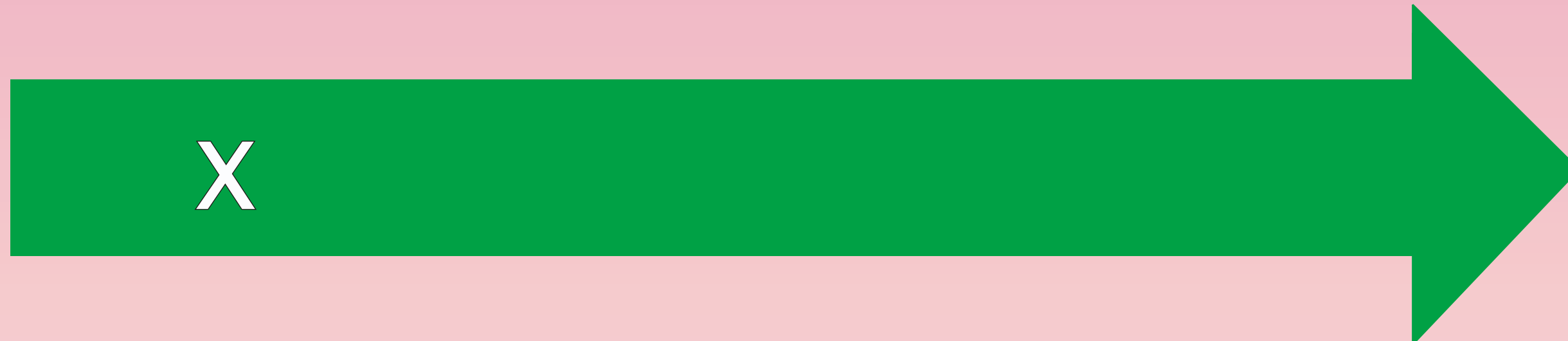
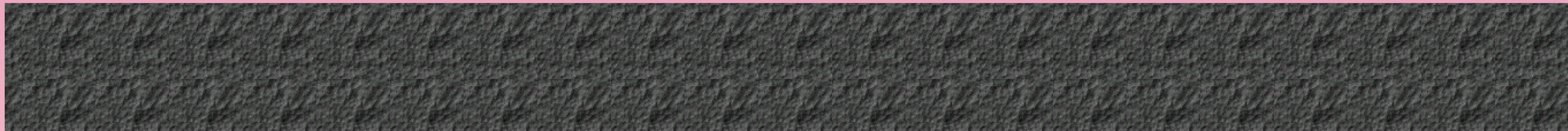
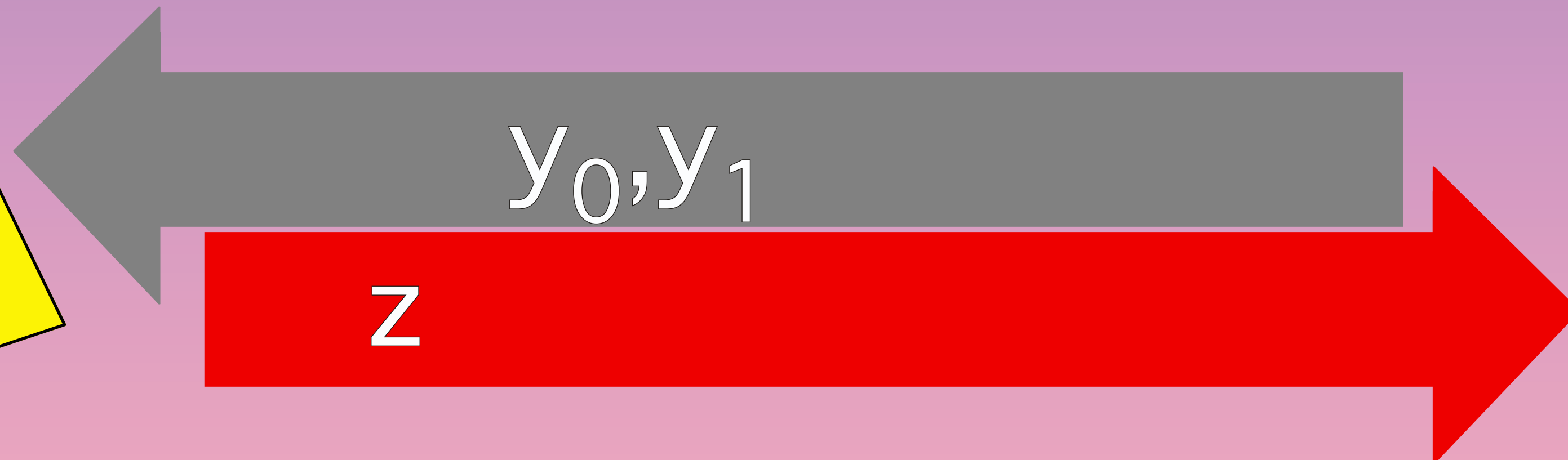
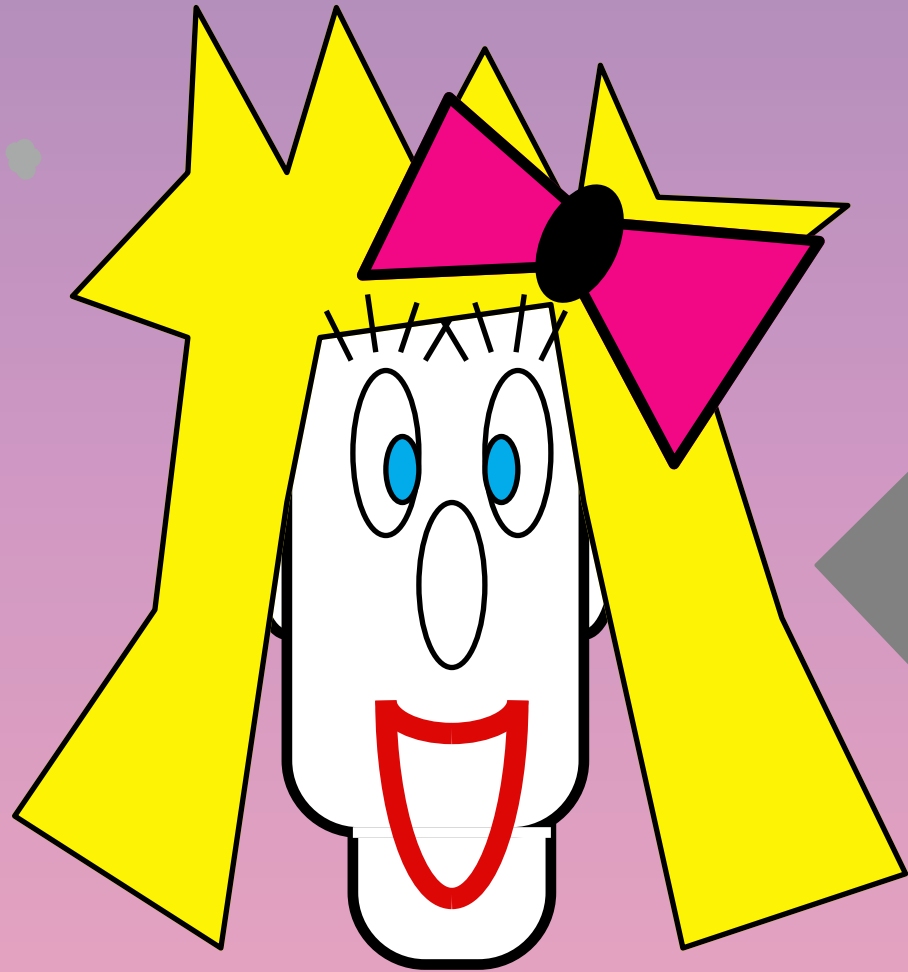
**two provers BC**

**Classically and**

**Quantumly Secure**

b

$$z = x \oplus y_b$$



$$x = z \oplus y_b ?$$

modified BGKW

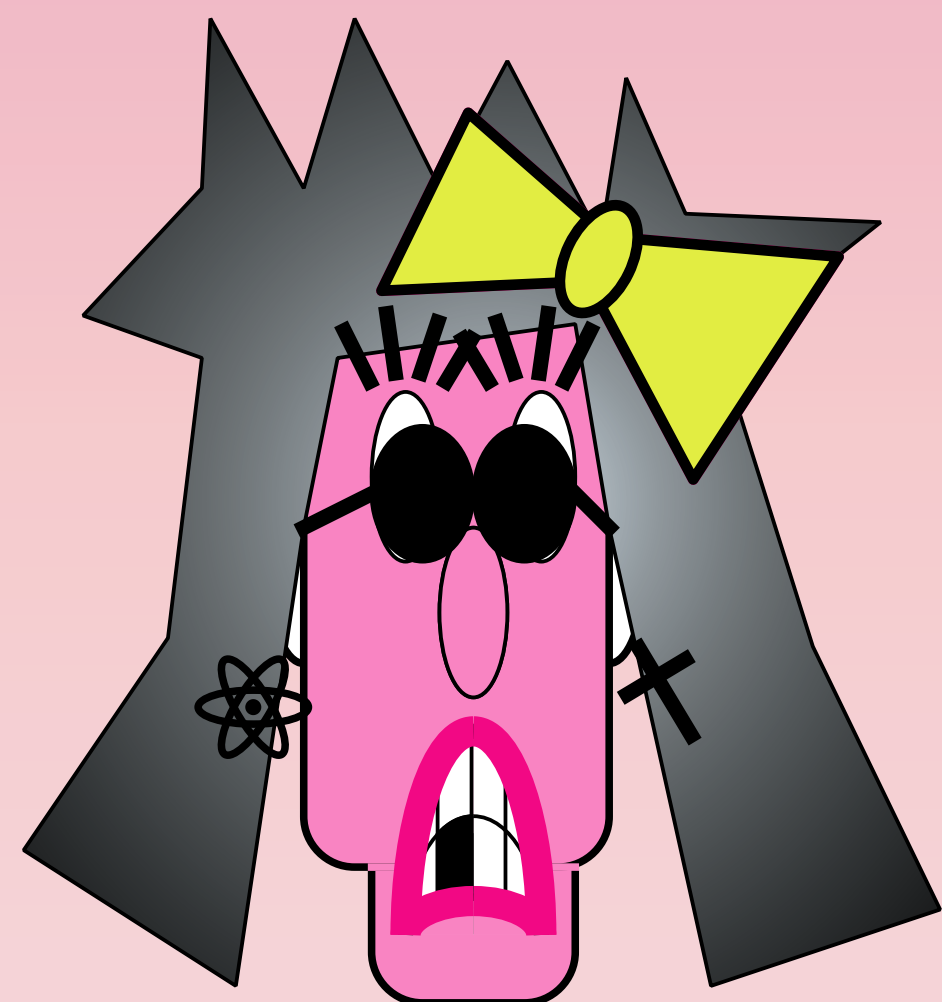
# Classically

$$x_0 \oplus z = y_0$$

$$x_1 \oplus z = y_1$$

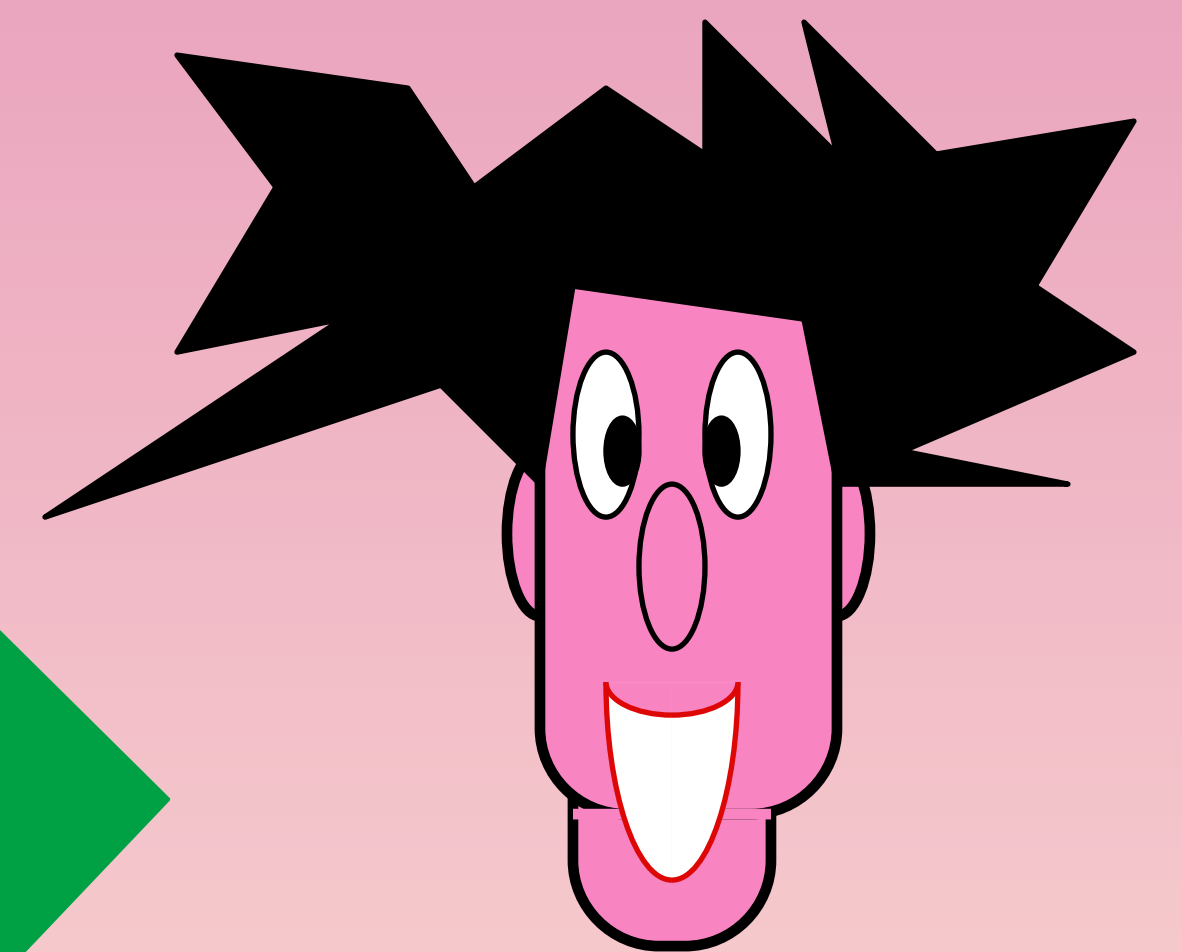
$$x_0 \oplus x_1 = (x_0 \oplus z) \oplus (x_1 \oplus z) = y_0 \oplus y_1$$

possible with prob. at most  $2^{-n}$



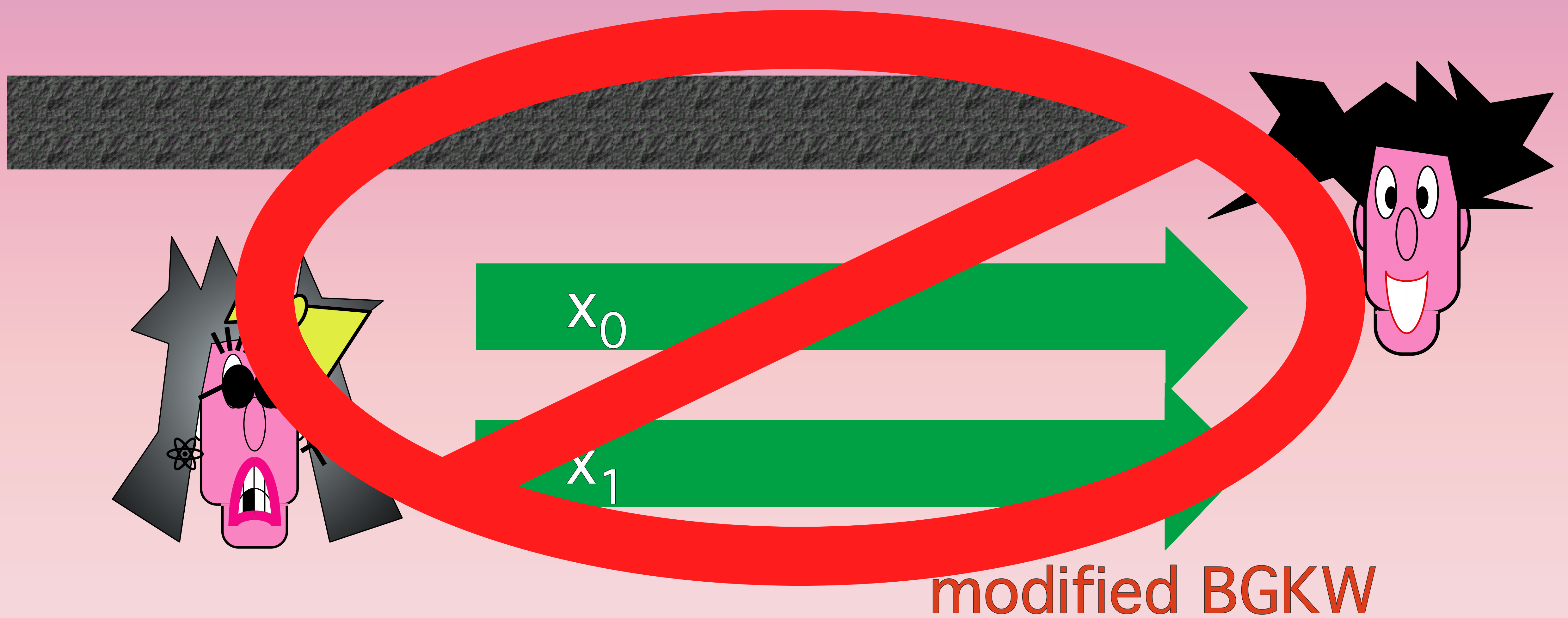
$x_0$

$x_1$



modified BGKW

# Quantumly





# Quantumly

## MAIN THEOREM

Let  $\mathbf{0}$  and  $\mathbf{1}$  be POVMs such that outputs  $x_0$  and  $x_1$  one could obtain by applying one of them to the state shared among the two provers.

Suppose the success probability of unveiling is

$$p_0 + p_1 > 1 + \delta,$$

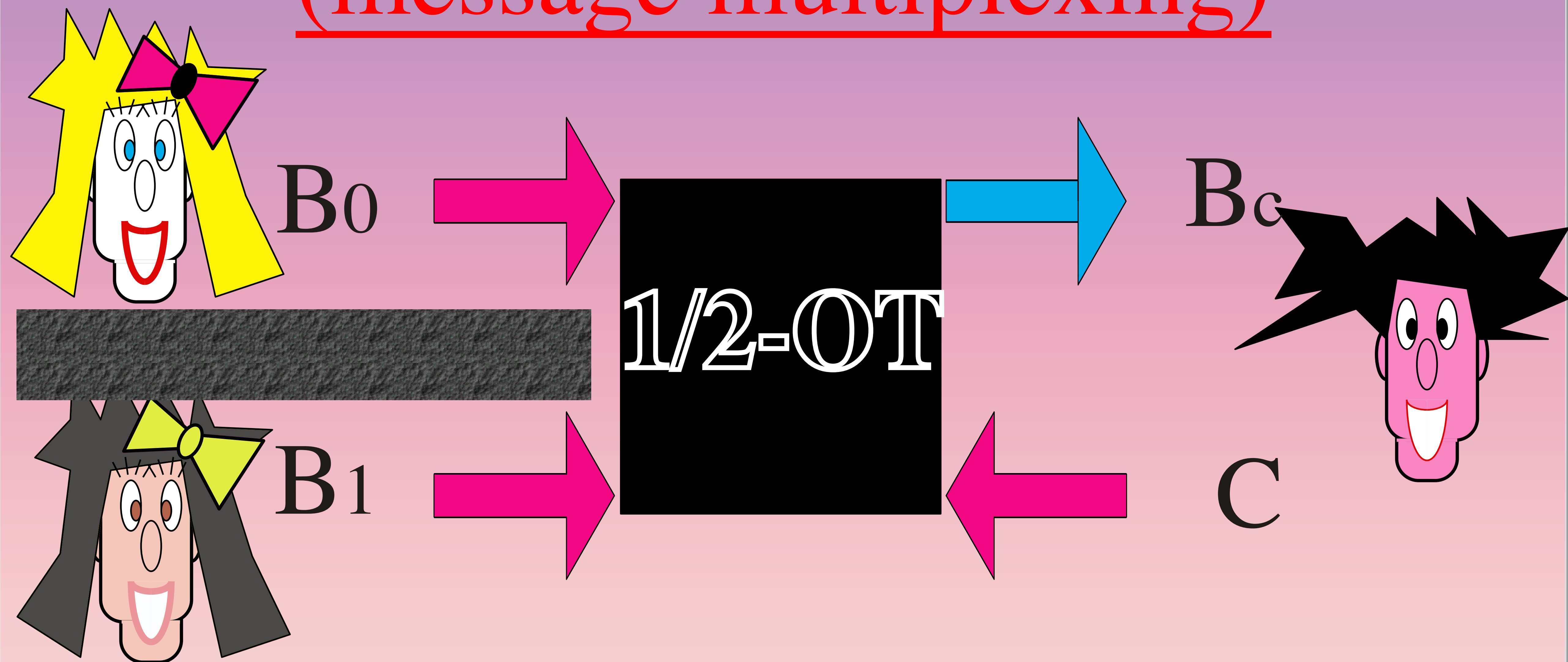
then the (prediction probability of  $y_0 \oplus y_1$ )  $> \delta$ .

This prediction probability is achieved by first applying  $\mathbf{0}$  to the shared state followed by  $\mathbf{1}$  on the leftover system or the other way around.

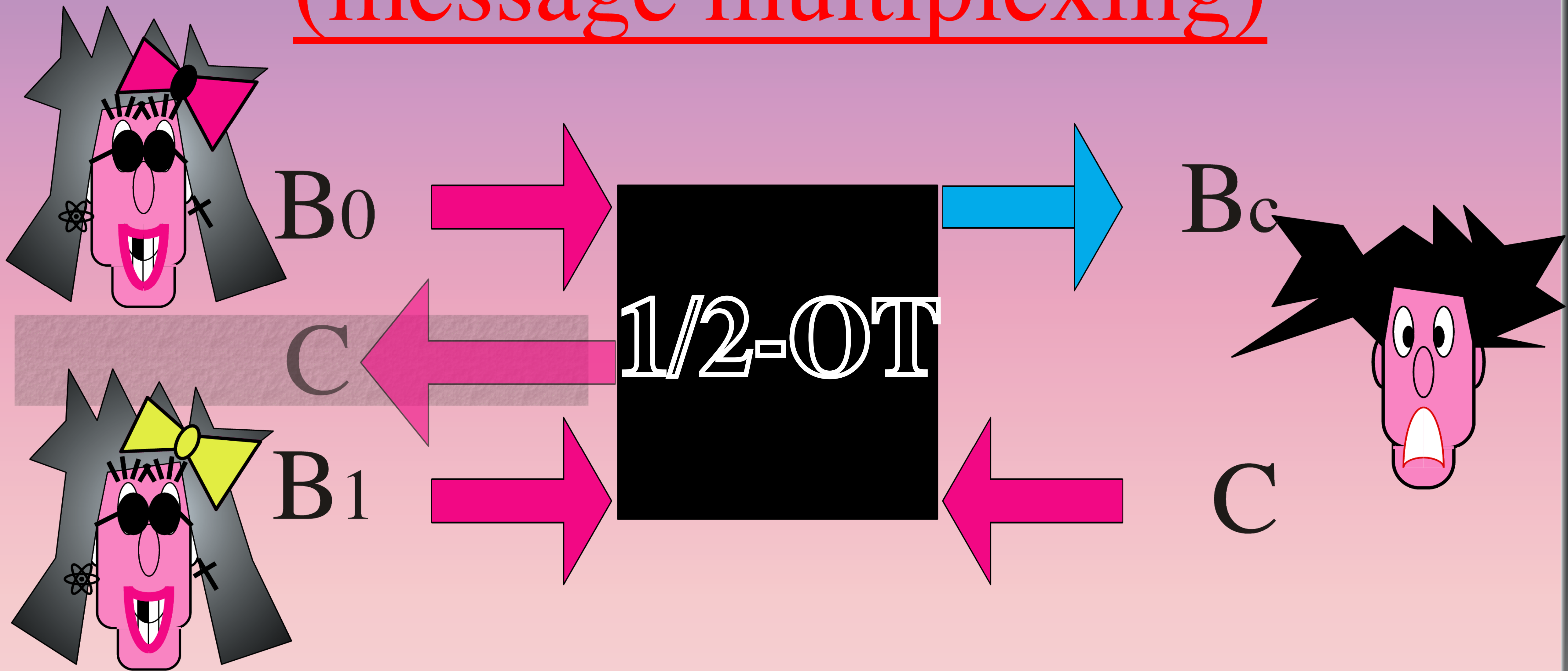
**(9)**

**WARNING !**

# Oblivious Transfer (message multiplexing)



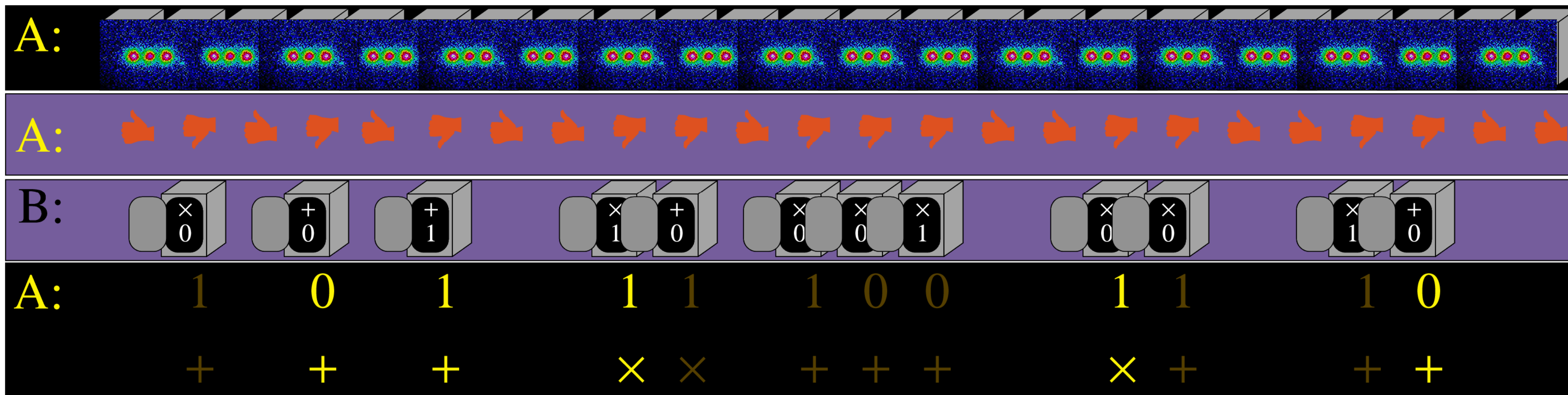
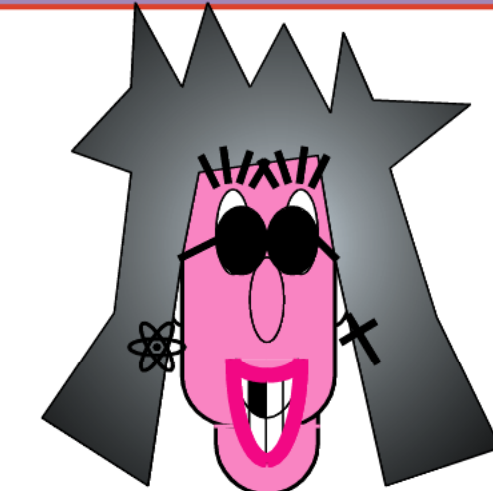
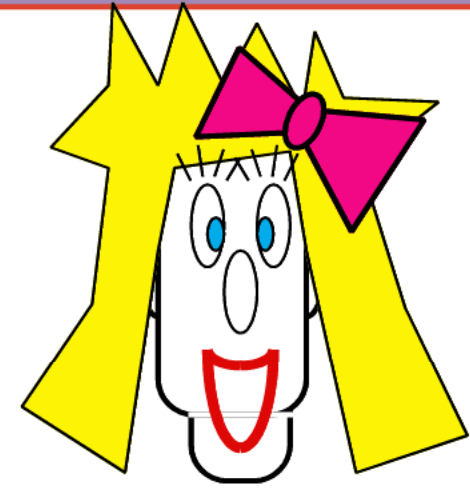
# Oblivious Transfer (message multiplexing)



Brassard, Crépeau, Mayers, Salvail 97

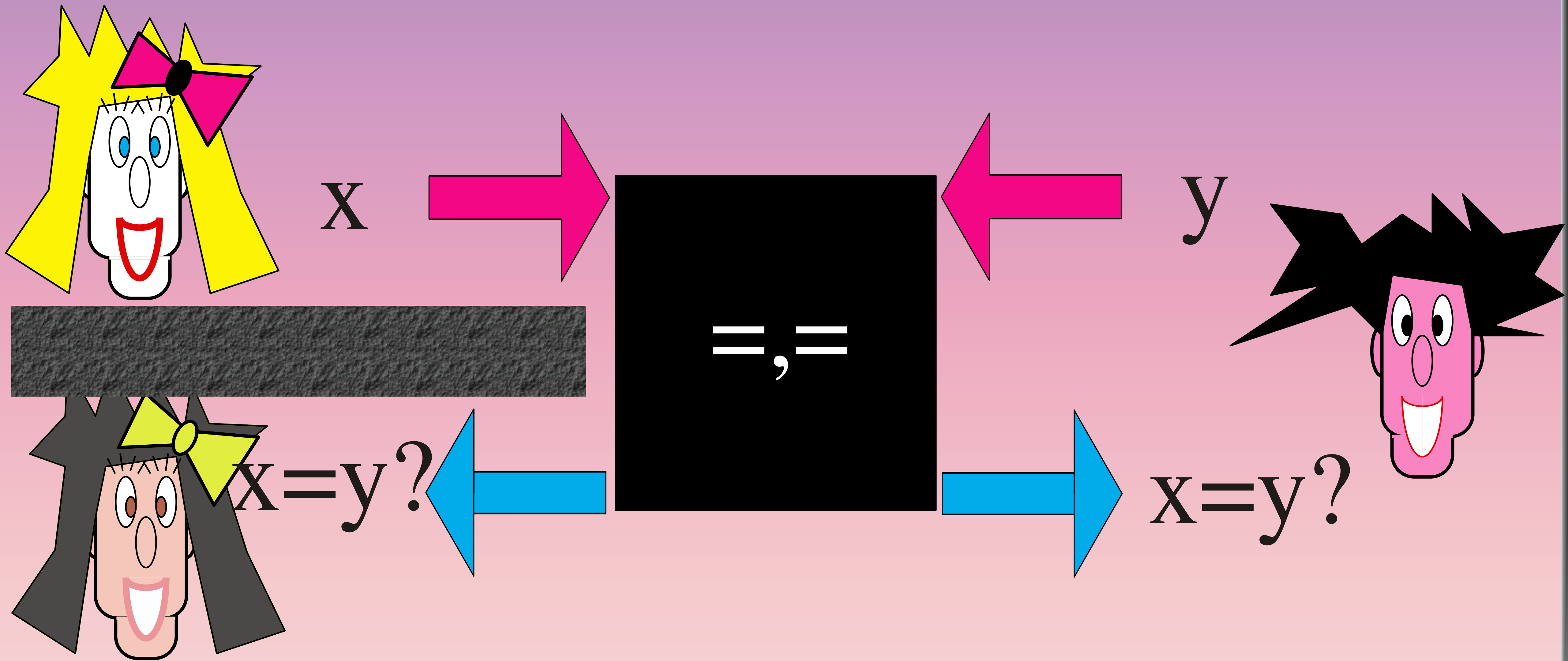


# BCMS' attack

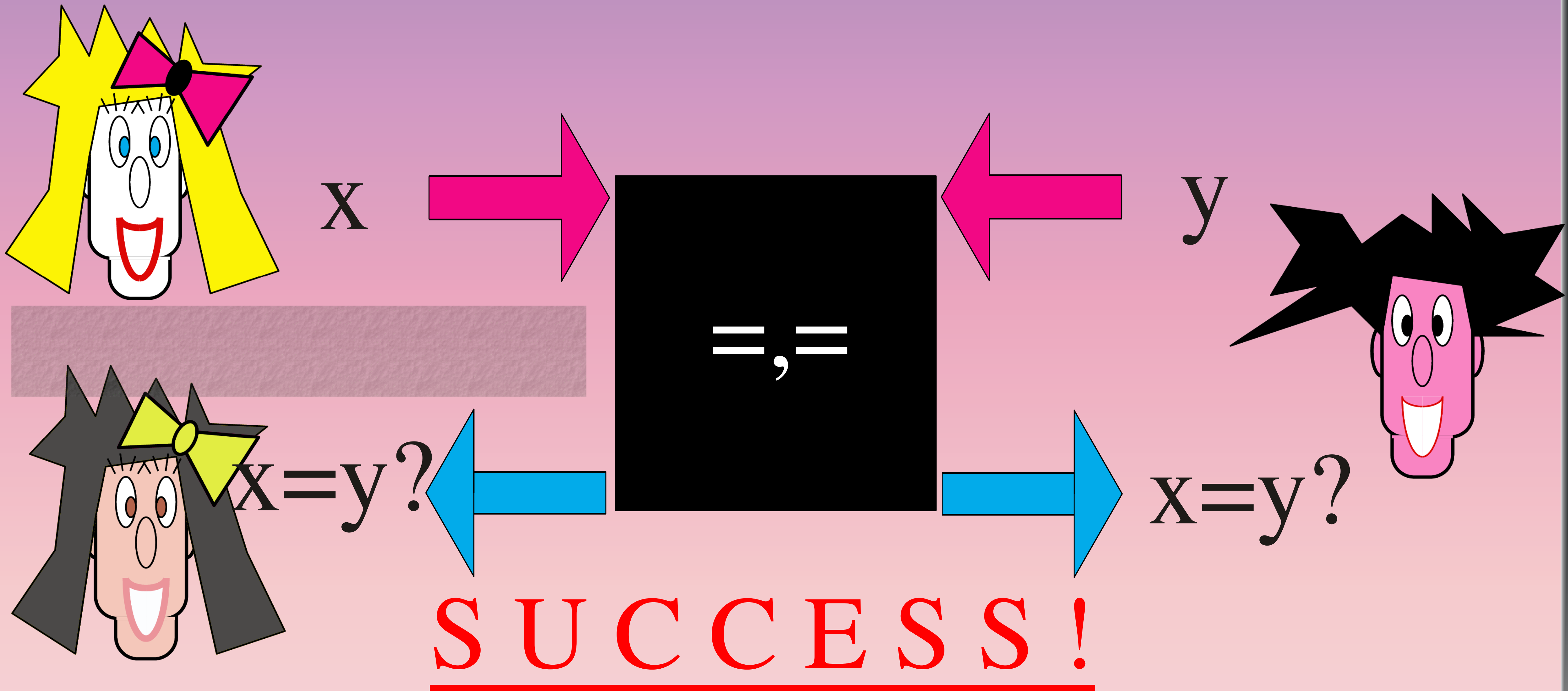




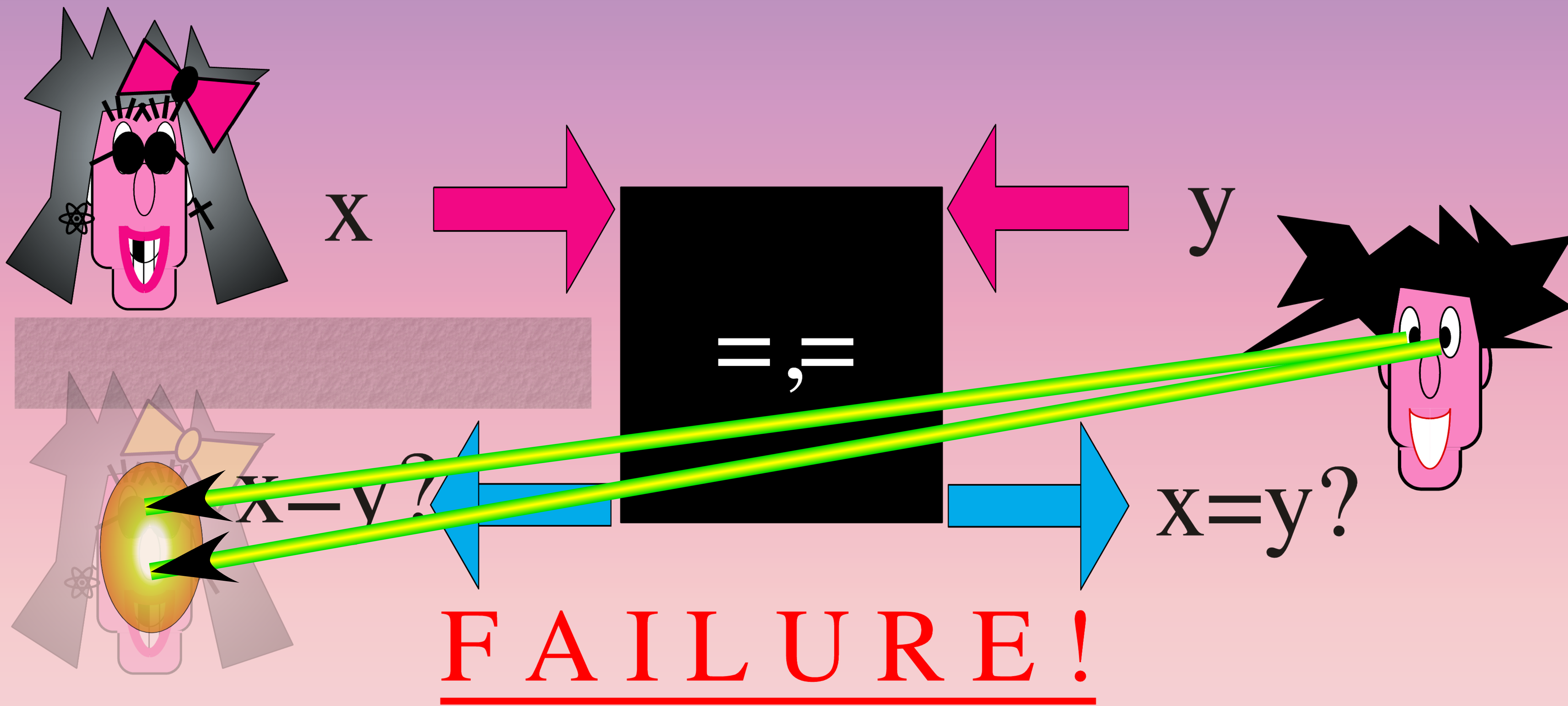
# Mutual Identification



# Mutual Identification



# Mutual Identification



an Introduction to  
theoretical quantum  
CRYPTOGRAPHY

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