

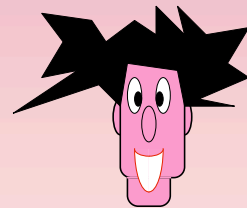
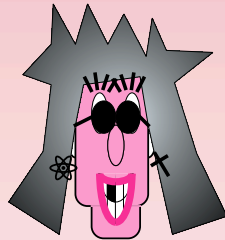
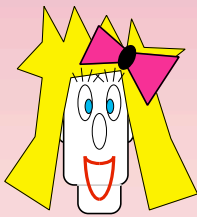
### (3.1.2) One-time pad



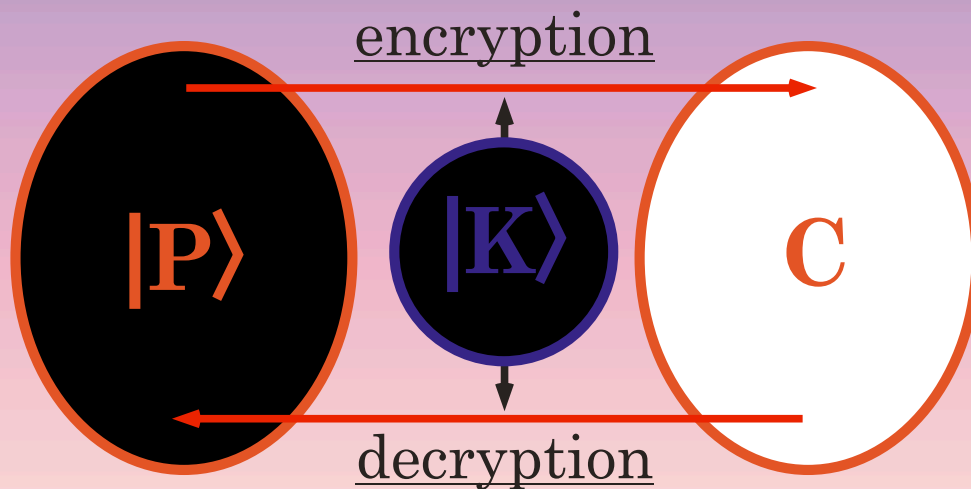
Classical key : Vernam  $\mathbb{Q}$ -cipher (various sources)  
Quantum Ciphertext



Quantum key : one-time  $\mathbb{Q}$ -pad ( $\mathbb{Q}$ -teleportation)  
Classical Ciphertext (BBCJPW)



### symmetric encryption of Quantum messages

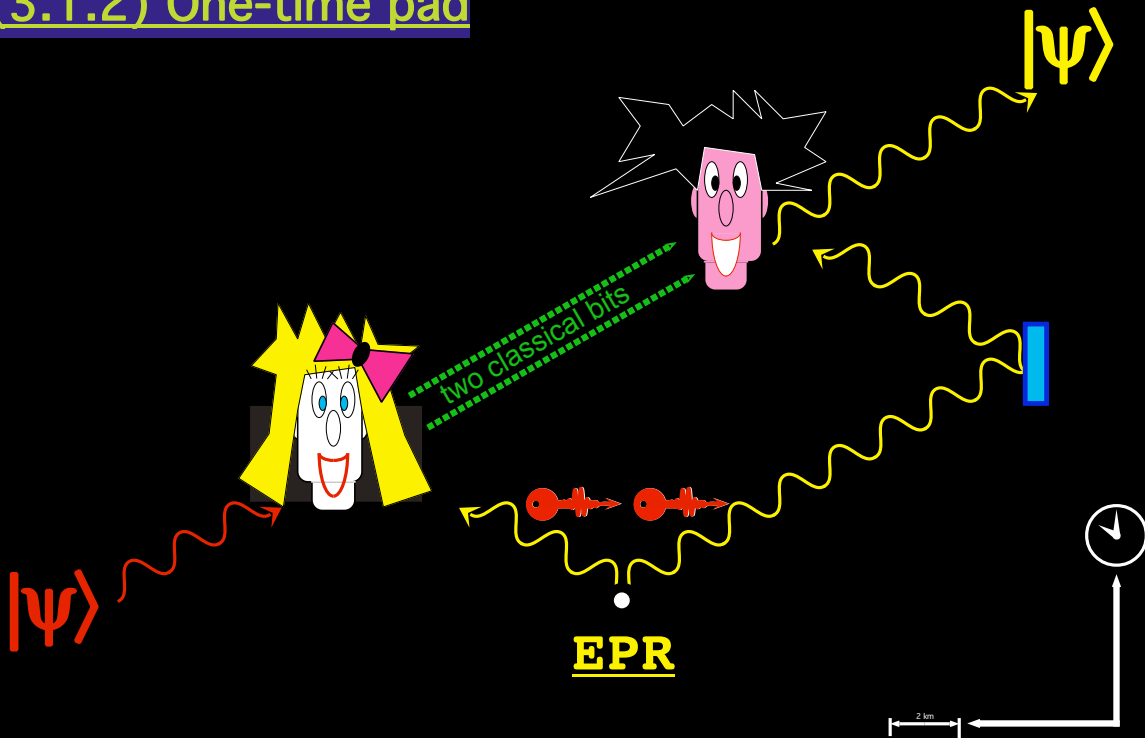


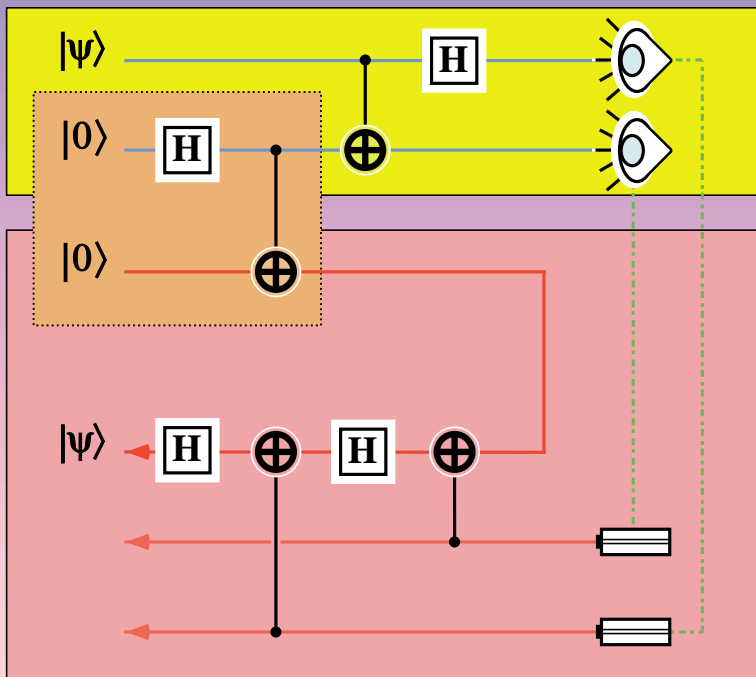
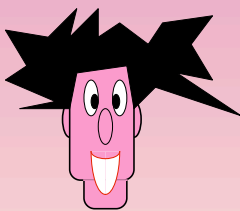
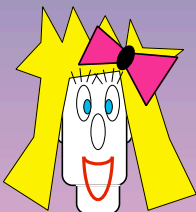
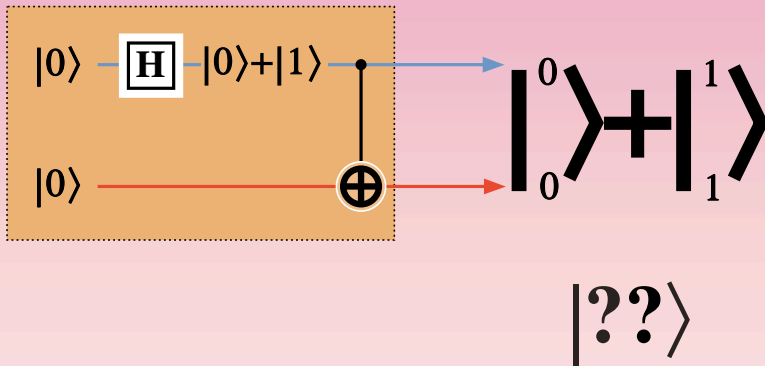
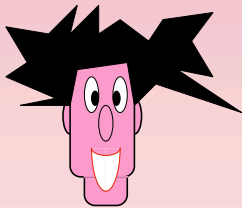
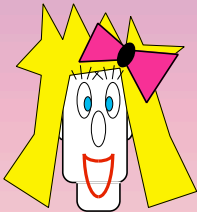
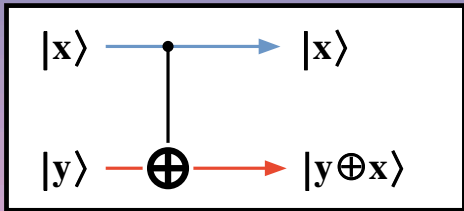
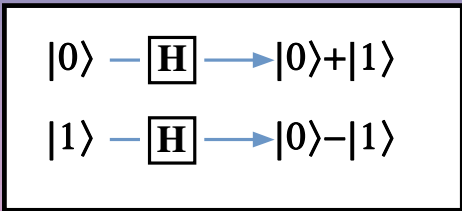
Information Theoretical Security

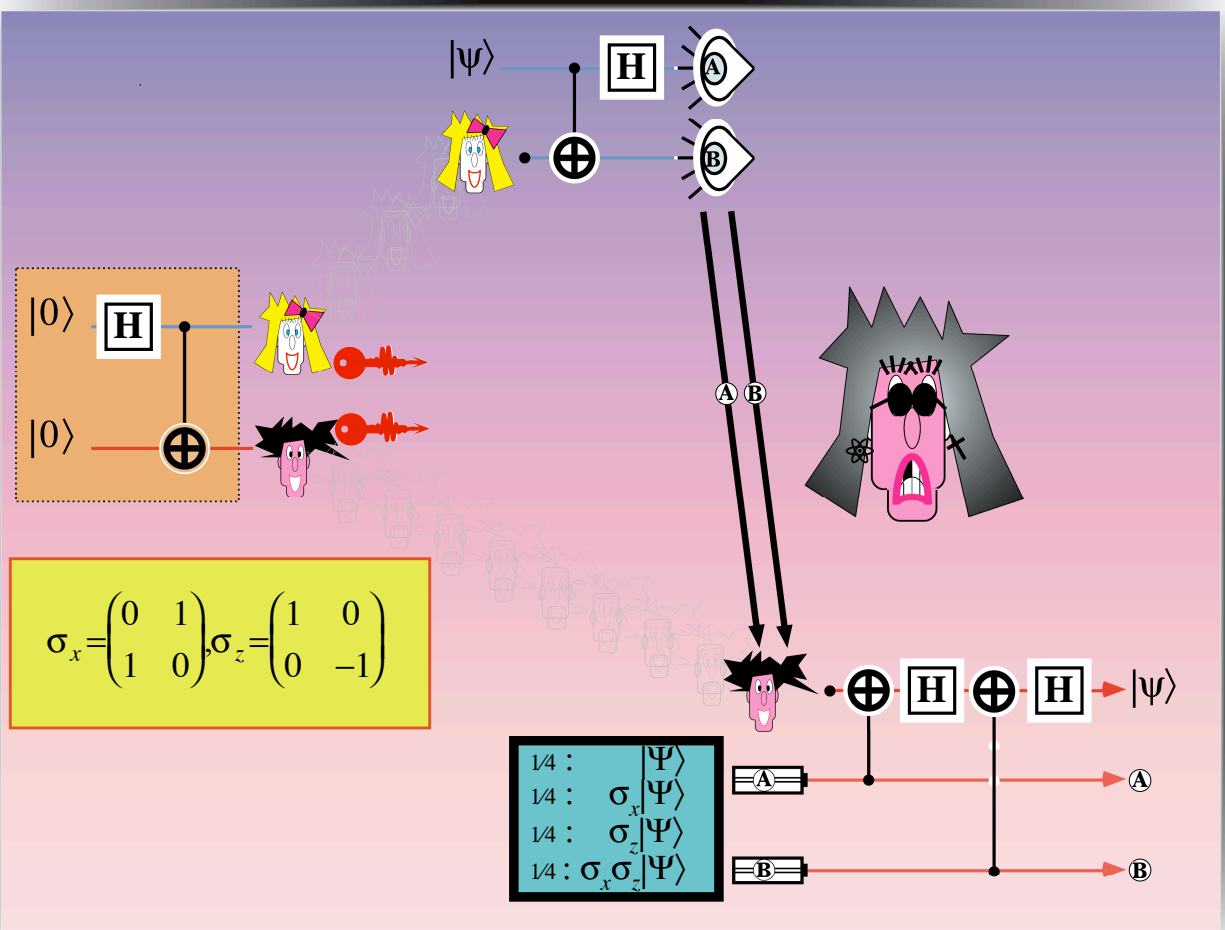
# One-time Q-pad



# (3.1.2) One-time pad







### (3.1.2b) one-time pad



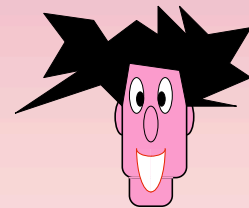
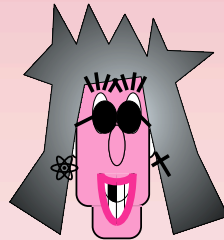
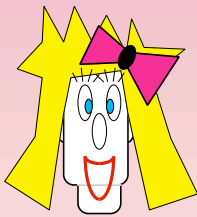
### (3.1.2a) One-time pad



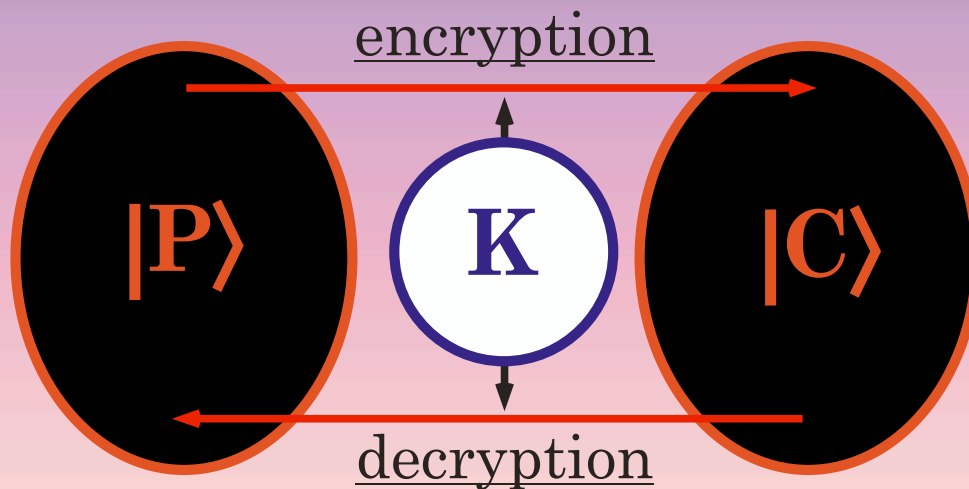
Classical key : Vernam Q-cipher (various sources)  
Quantum Ciphertext



Quantum key : one-time Q-pad (BBCJPW)  
Classical Ciphertext



### symmetric encryption of Quantum messages

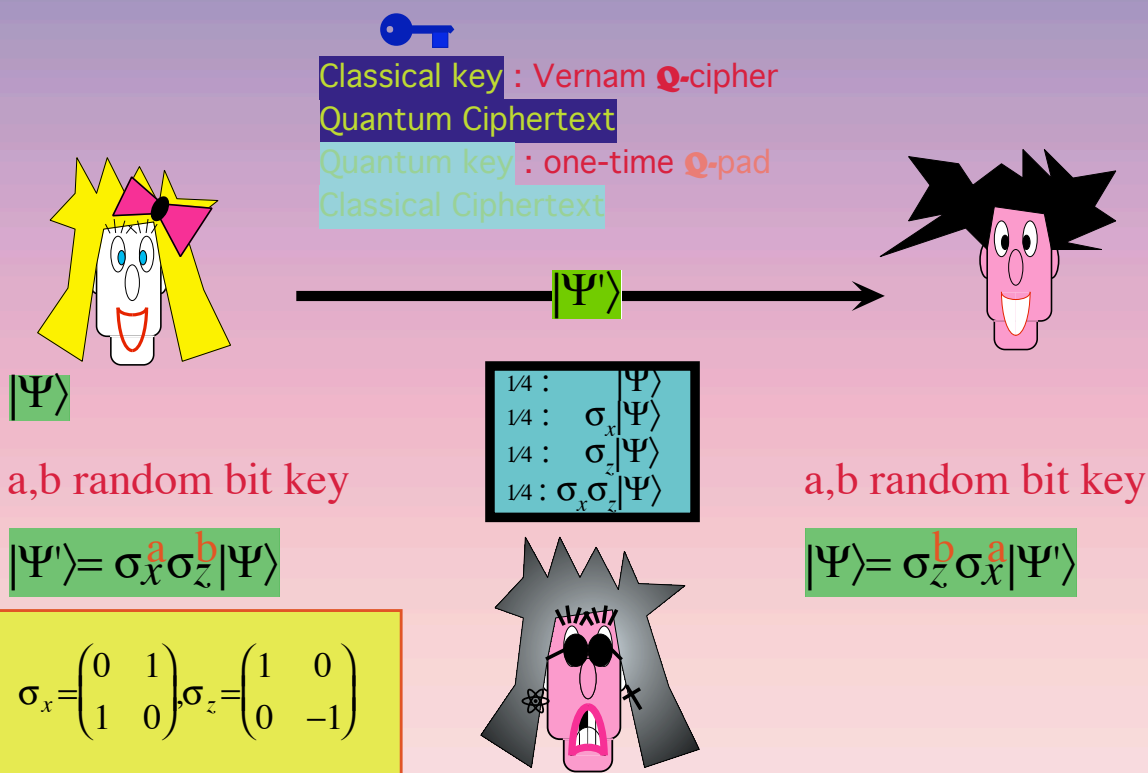


Information Theoretical Security

# Vernam Q-cipher

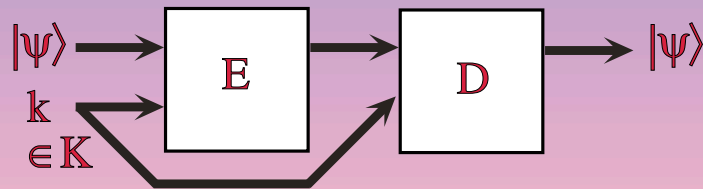


## (3.1.2a) one-time pad

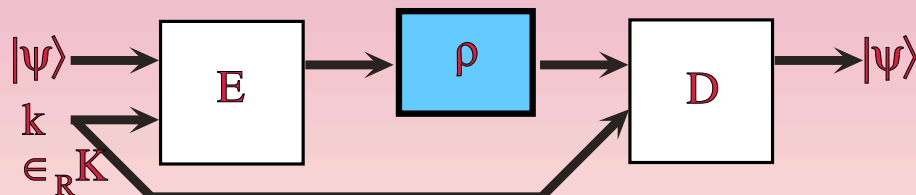


## One-time Q-encryption with error $\varepsilon$

Completeness:



Secrecy:



$$\forall |\psi_0\rangle, |\psi_1\rangle \quad D(\rho_0, \rho_1) = \text{Tr}(|\rho_0 - \rho_1|) < \varepsilon$$

## One-time Q-encryption with error $\varepsilon > 0$

Lower bounds:

**[MTW00]**

Arbitrary quantum state = 2 bits / qubit

**[HLSW03]**

Arbitrary quantum state but not entangled with eavesdropper ~ 1 bit / qubit

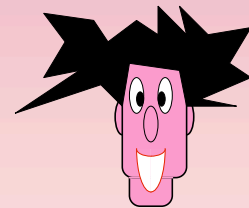
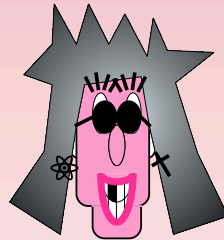
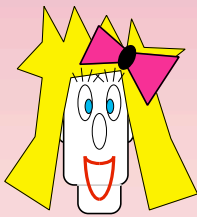
### (3.1.3) One-time Q-Authentication



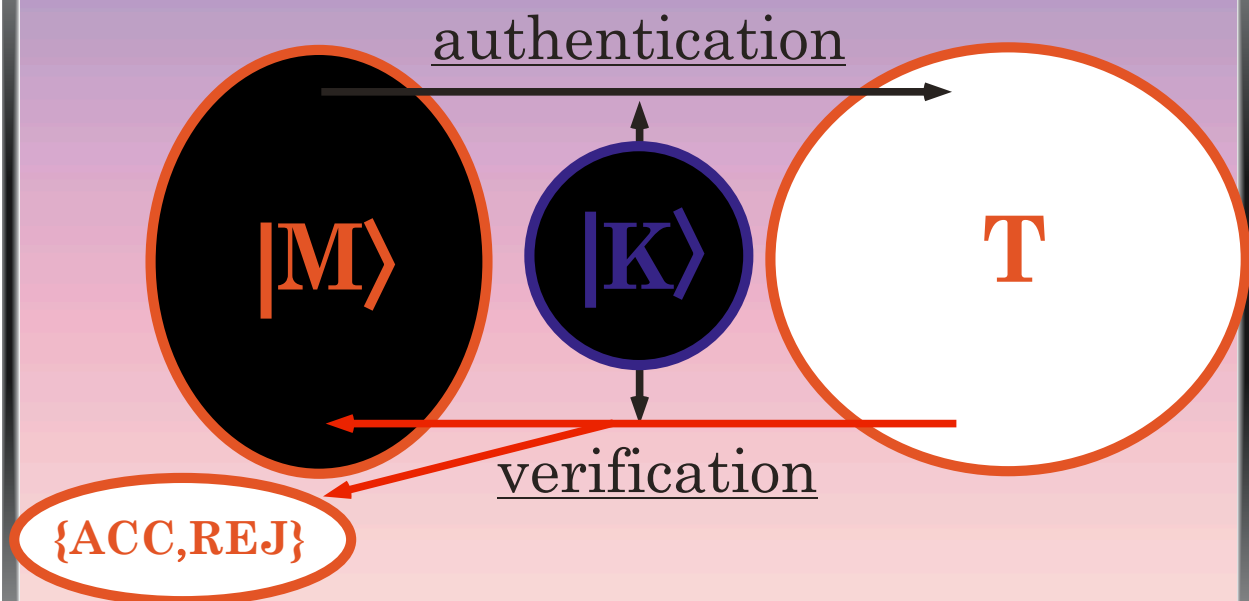
Classical key : Q-Authentication (BCGST)  
Quantum message+tag



Quantum key : Authenticated Q-teleportation  
Classical message+tag (BBCJPW)

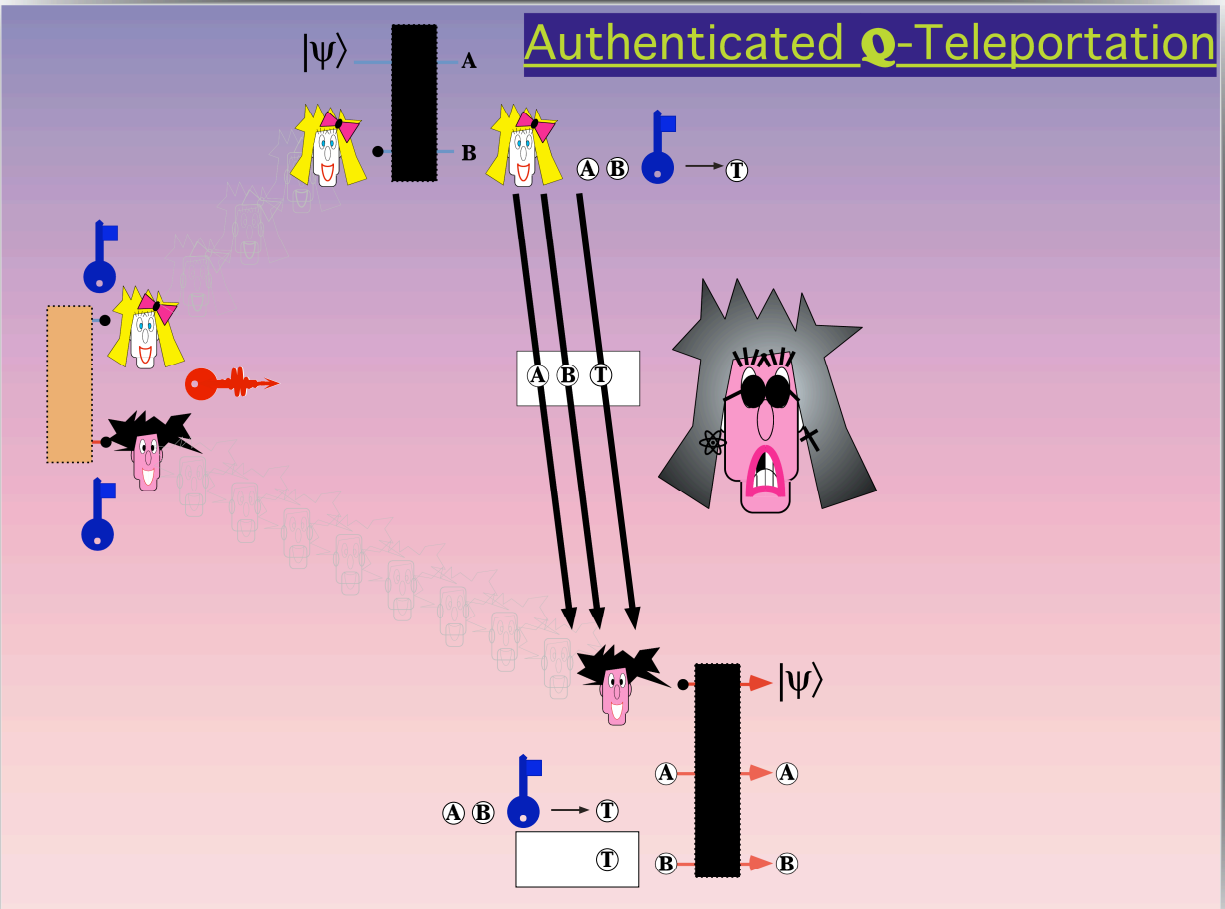
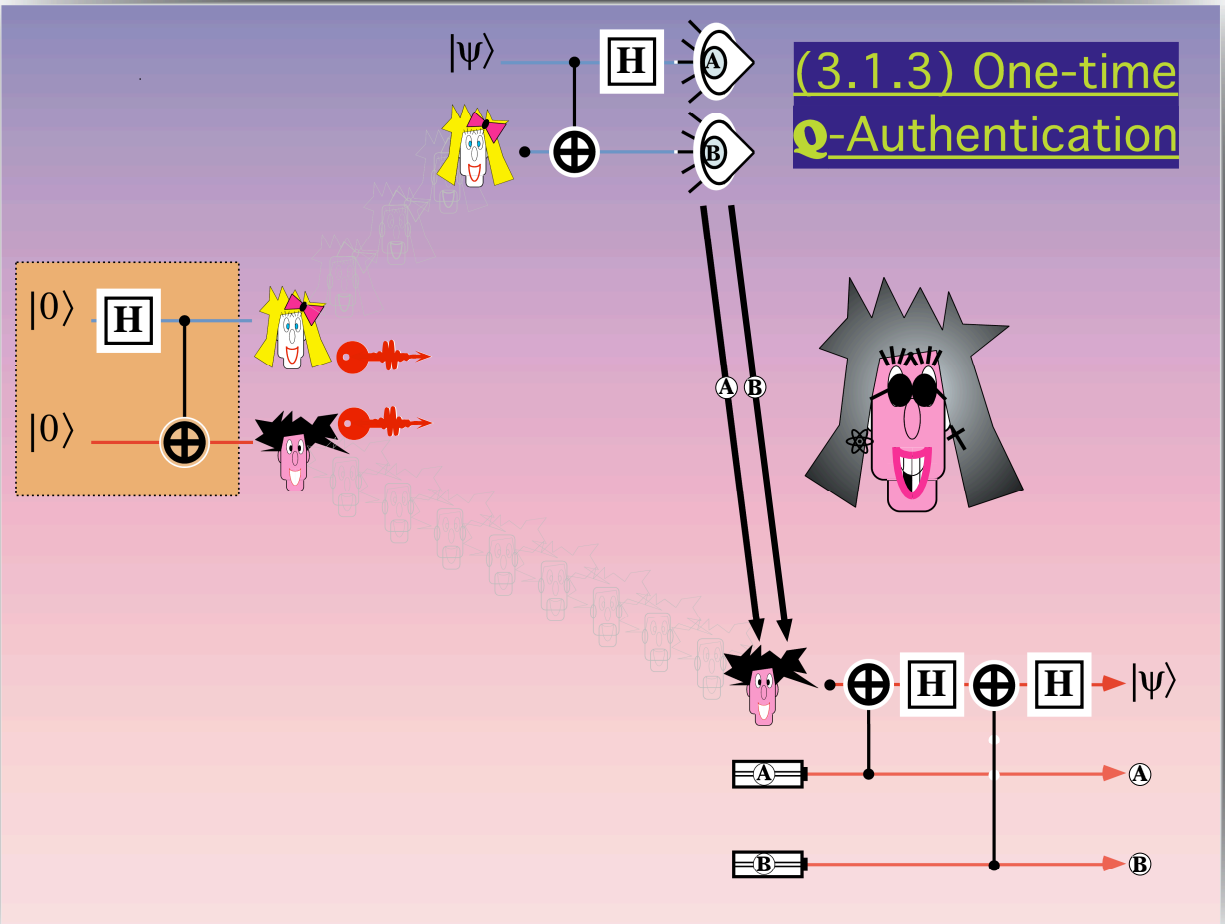


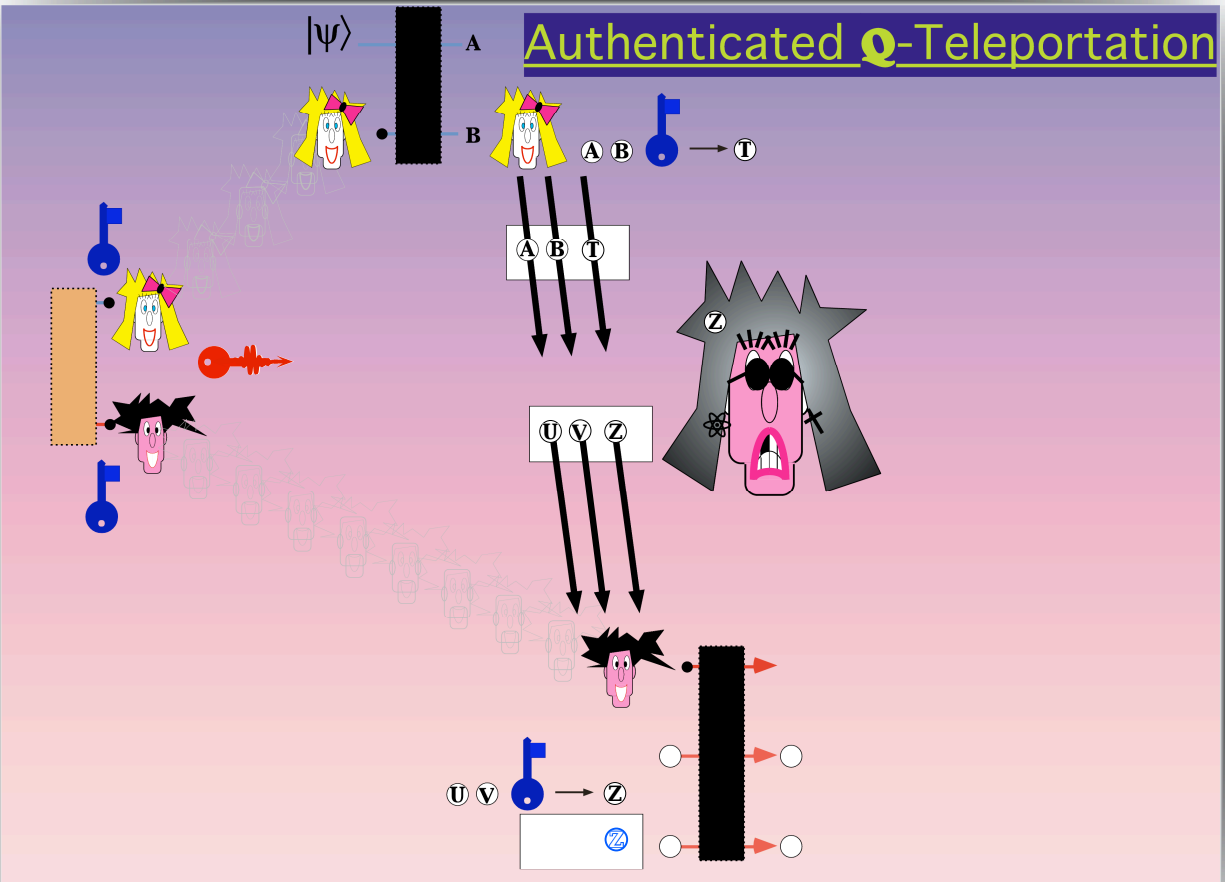
### symmetric authentication



Information Theoretical Security







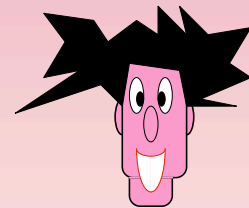
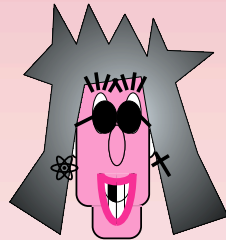
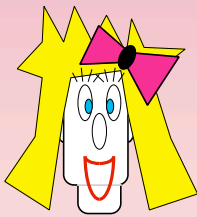
### (3.1.3a) One-time Q-Authentication



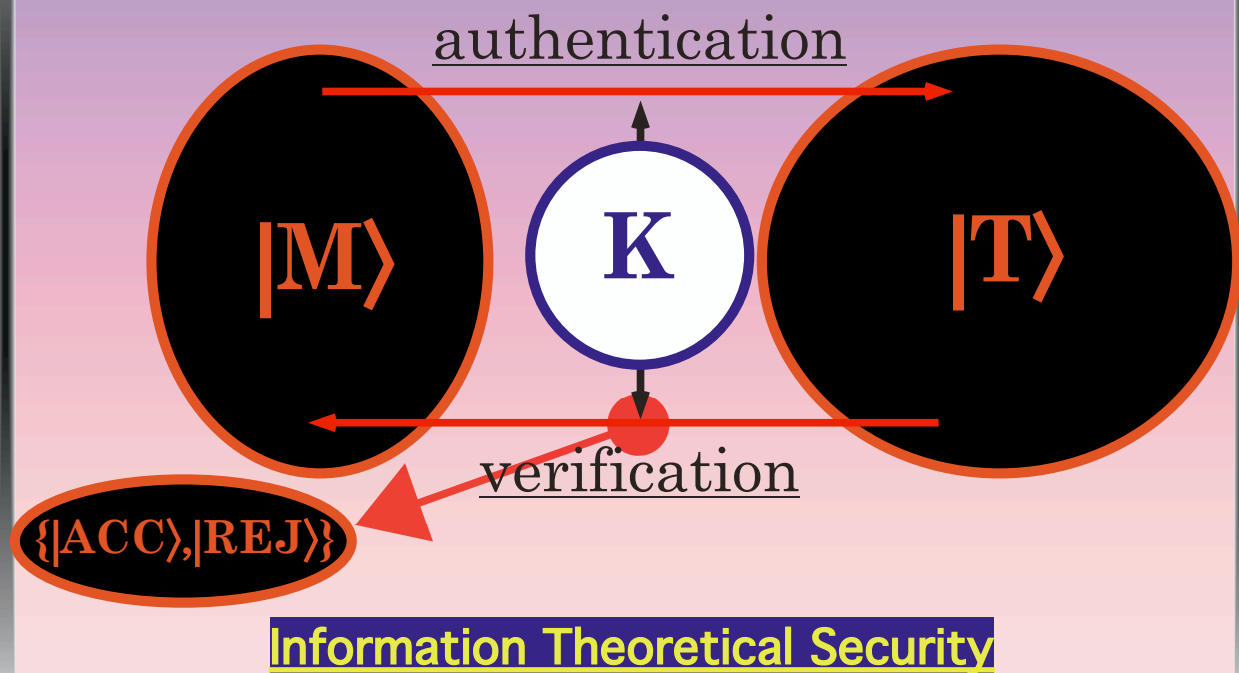
Classical key : Q-Authentication (BCGST)  
Quantum message+tag



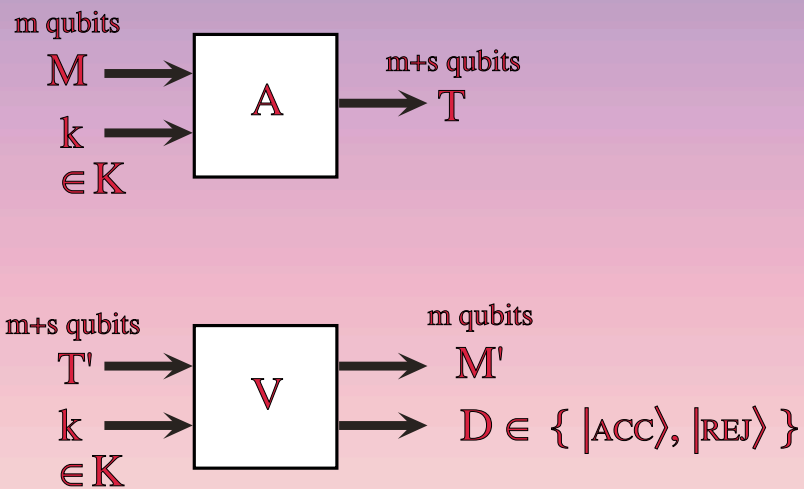
Quantum key : Authenticated Q-teleportation (BBCJPW)  
Classical message+tag



### symmetric authentication of Quantum Messages



## One-time Q-Authentication



## One-time Q-Authentication

For any pure state  $|\psi\rangle$  consider the measurement on  $(M',D)$  such that

- output Right if  $M' = |\psi\rangle$  or if  $D = |REJ\rangle$
- output Wrong otherwise



The corresponding projectors are

$$R_{|\psi\rangle} = |\psi\rangle\langle\psi| \otimes I_D + I_{M'} \otimes |REJ\rangle\langle REJ| - |\psi\rangle\langle\psi| \otimes |REJ\rangle\langle REJ|$$

$$W_{|\psi\rangle} = (I_{M'} - |\psi\rangle\langle\psi|) \otimes |ACC\rangle\langle ACC|$$



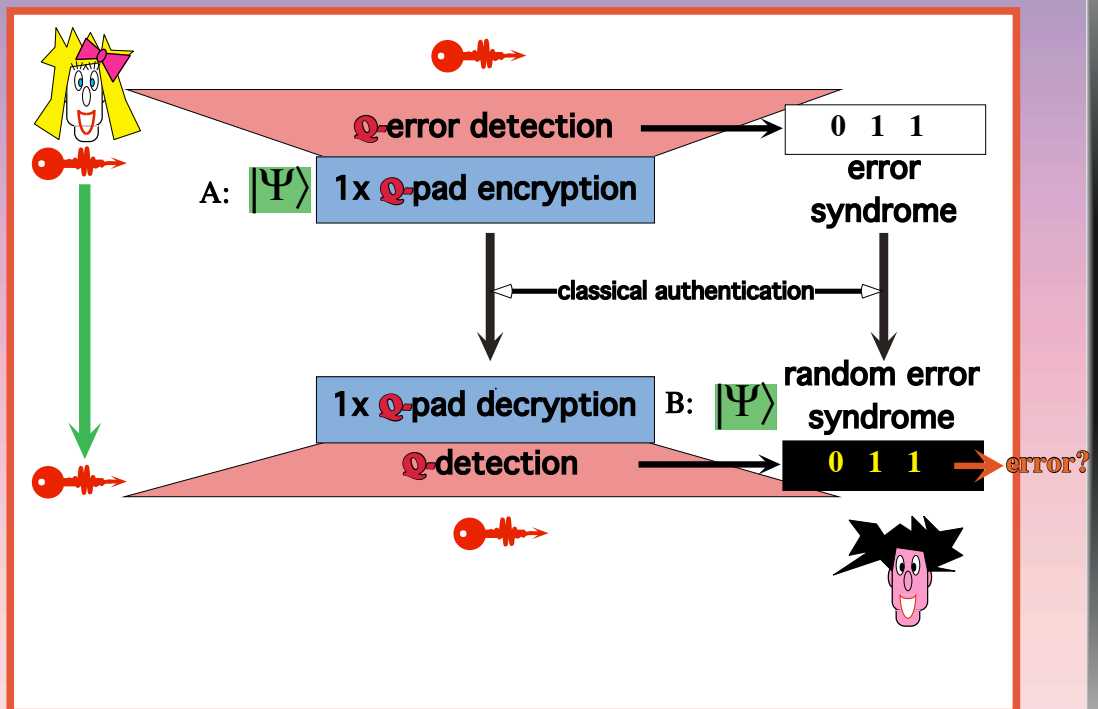
### (3.3C) One-time interactive Q-Authentication

.....

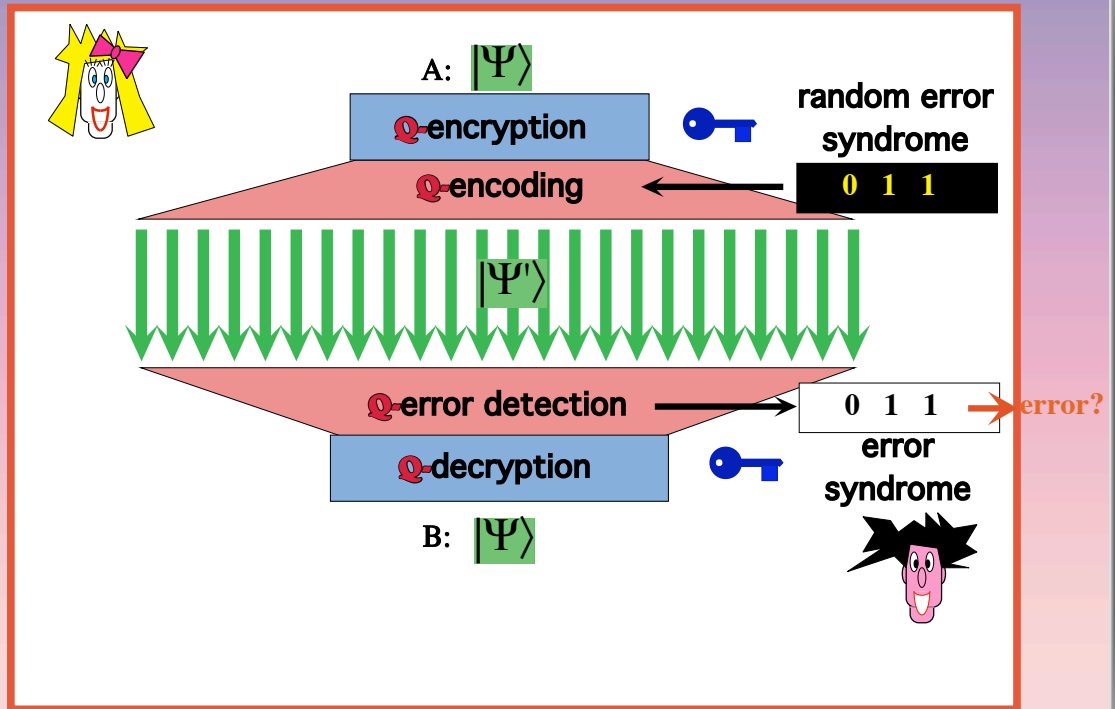
- Transmit quantum key (EPR states)
- Quantum error-correction is used to purify (or test purity of) EPR states to form a smaller pure set
- one-time Authenticated Quantum pad is used to send message

.....

### (3.3C) One-time interactive Q-Authentication

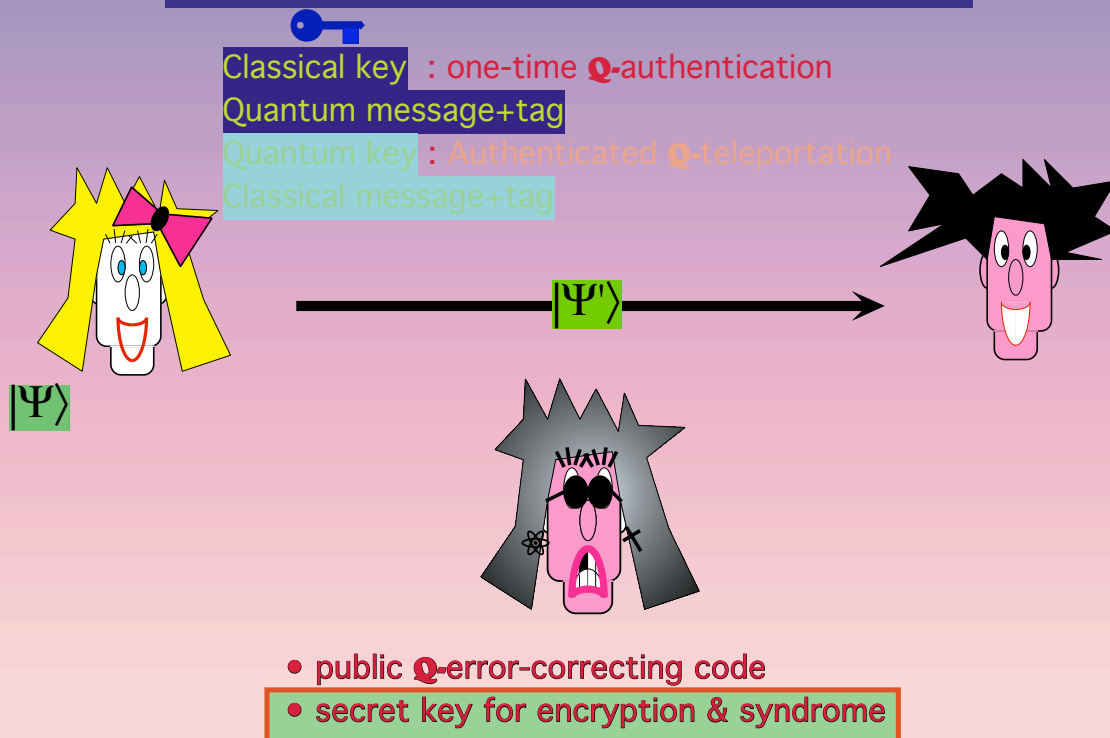


### (3.1.3a) One-time Q-Authentication



### Barnum-Crépeau-Gottesman-Smith-Tapp

### (3.1.3a) One-time Q-Authentication





one-time **Q**-authentication



Vernam **Q**-cipher

(authenticated quantum messages must be encrypted  
which is false for classical messages! )

Main Lower Bound

A Quantum Authentication Scheme  
with error probability  $\epsilon$

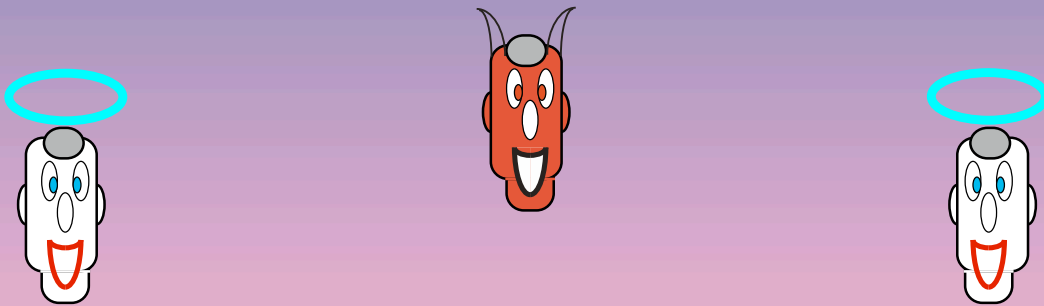
is

A Quantum Encryption Scheme  
with error probability  $4\epsilon^{1/6}$ .



# (3.2) Complexity Theoretical Quantum Cryptography

## (3.2) Complexity Theoretical Cryptography



(3.2.1) Public key cryptosystem : public-key  $\mathbb{Q}$ -cryptosystem

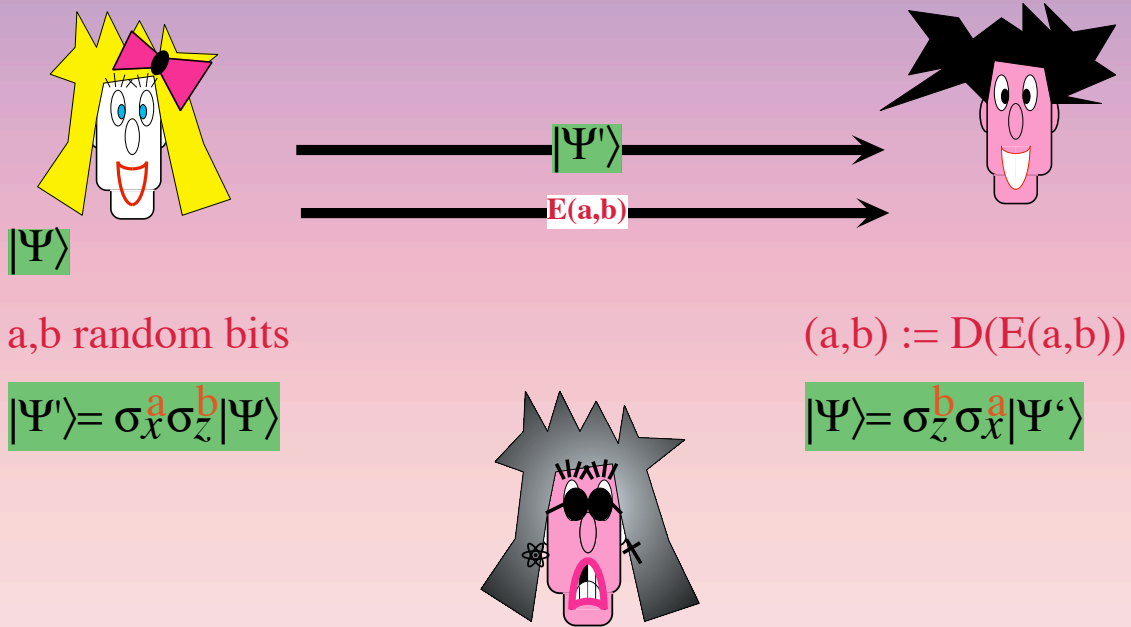
(3.2.2) Digital signature scheme : public-key  $\mathbb{Q}$ -Authentication  
 $\mathbb{Q}$ -digital signature scheme

(3.2.3) (trapdoor) one-way functions :  $\mathbb{Q}$ -cryptanalysis  
(trapdoor)  $\mathbb{Q}$ -one-way functions



### (3.2.1) Public-Key Q-Cryptosystem

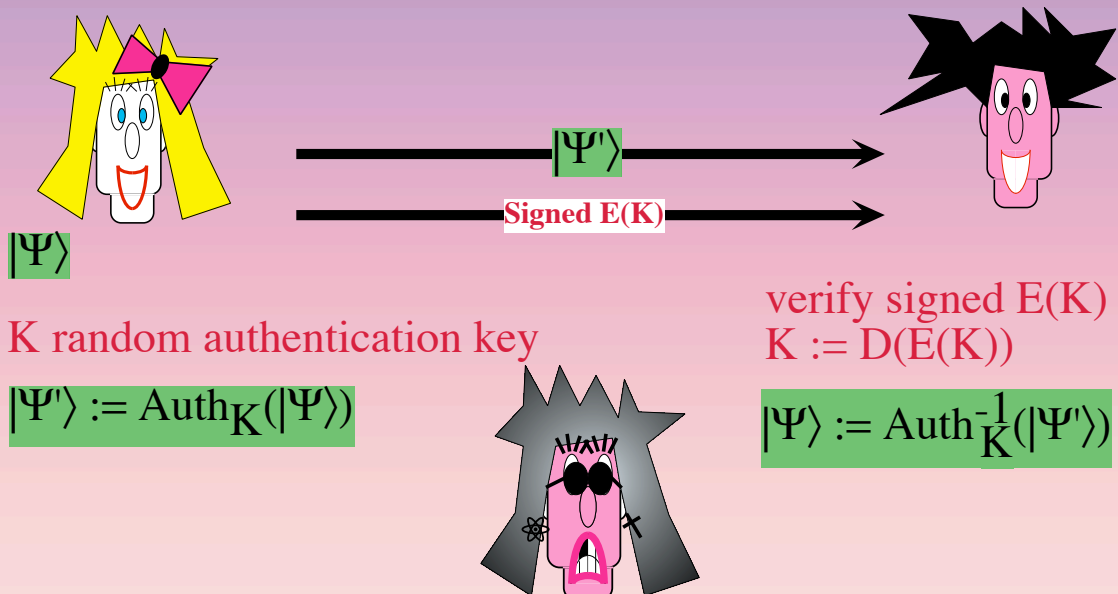
Assuming Classical Public Key Cryptography



### (3.2.2a) Public-Key Q-Authentication

Assuming Classical Public Key Cryptography

Assuming Classical Digital Signature

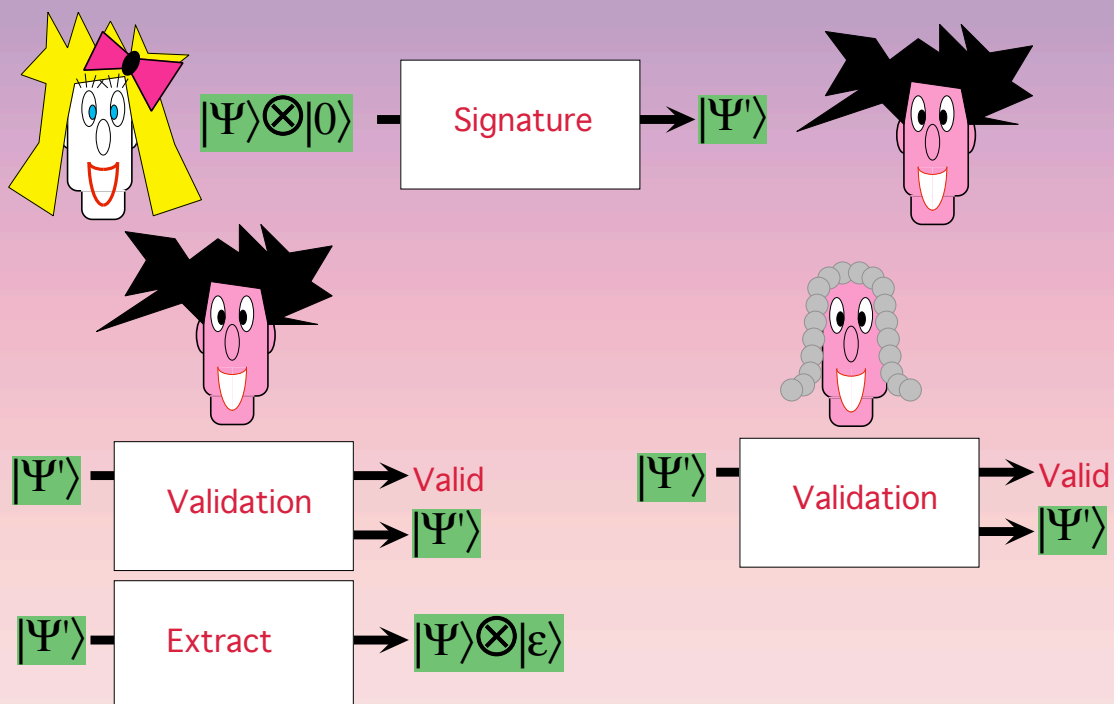


## (3.2.2b) Q-Digital Signature Scheme

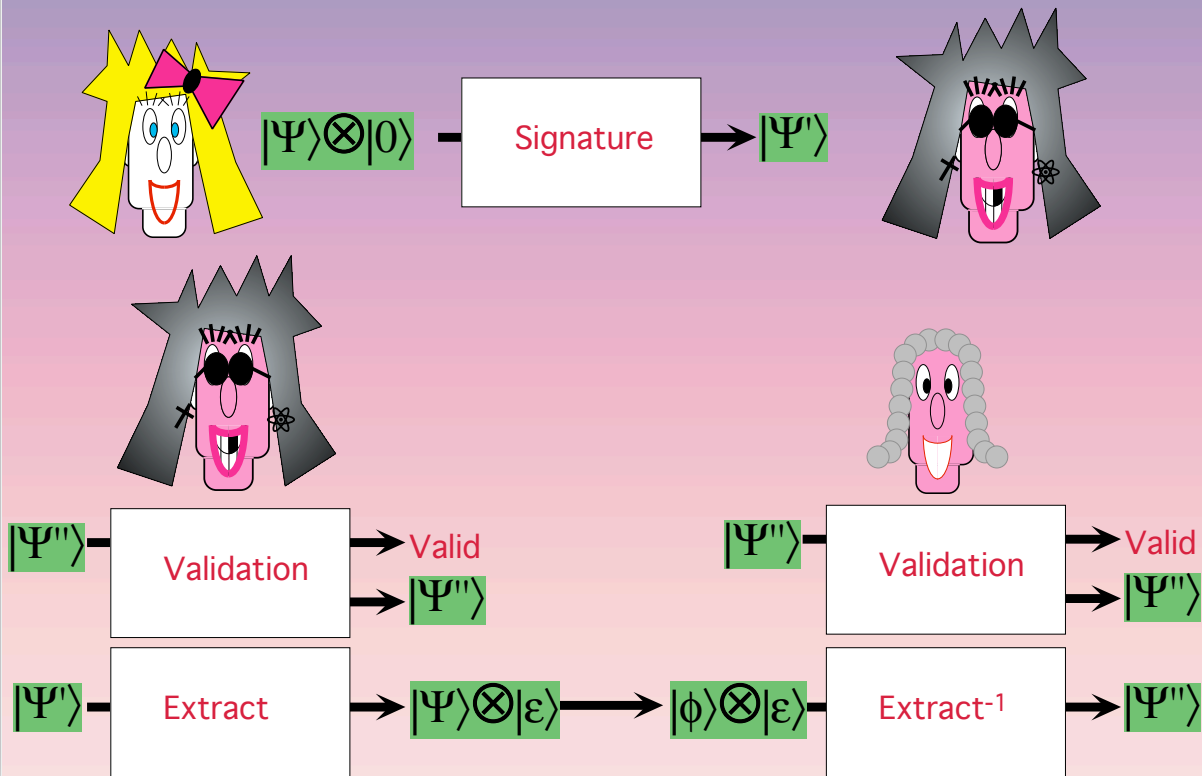
Assuming Classical Public Key Cryptography  
Assuming Classical Digital Signature



## (3.2.2b) Q-Digital Signature Scheme



### (3.2.2b) $\mathcal{Q}$ -Digital Signature Scheme



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### (3.2.3) (Trapdoor) $\mathcal{Q}$ -One-way functions

- generate a function  $f$  (and trapdoor) s.t.
- computing  $f(x)$  is easy
- finding  $x$  s.t.  $f(x)=y$  is hard
- finding  $x$  s.t.  $f(x)=y$  is easy with trapdoor

$\mathcal{Q}$ -cryptanalysis : Shor

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## Q-One-way function

Fischer-Stern  
one-way function  
(error correction code based)

generation : classical easy  
computing  $f$  : classical easy  
inverting  $f$  : classical / quantum hard ???

## Trapdoor Q-One-way function

Okamoto-Tanaka-Uchiyama  
trapdoor one-way permutation  
(subset products problem based)

generation : quantum easy  
computing  $f$  : classical easy  
inverting  $f$  : classical / quantum hard ???  
trapdooring  $f$  : classical easy

## Trapdoor Q-One-way function

### McEliece

trapdoor one-way permutation  
(error correction code based)

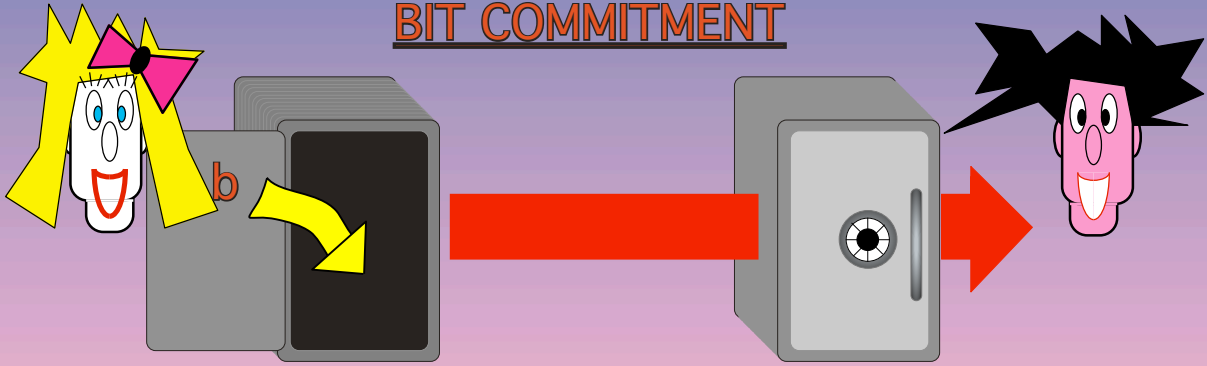
generation : classical easy  
computing  $f$  : classical easy  
inverting  $f$  : classical / quantum hard ???  
trapdooring  $f$  : classical easy

(4)

two-party

Cryptographic Protocols

# BIT COMMITMENT



COMMIT

UNVEIL

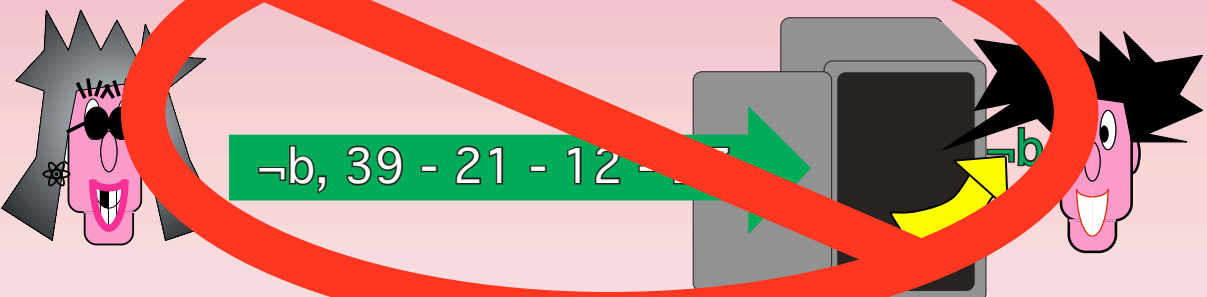


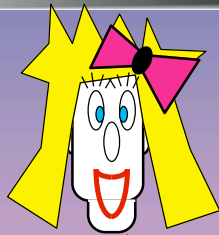
# BIT COMMITMENT



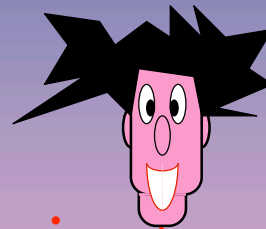
CONCEALING

BINDING

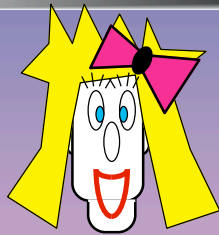
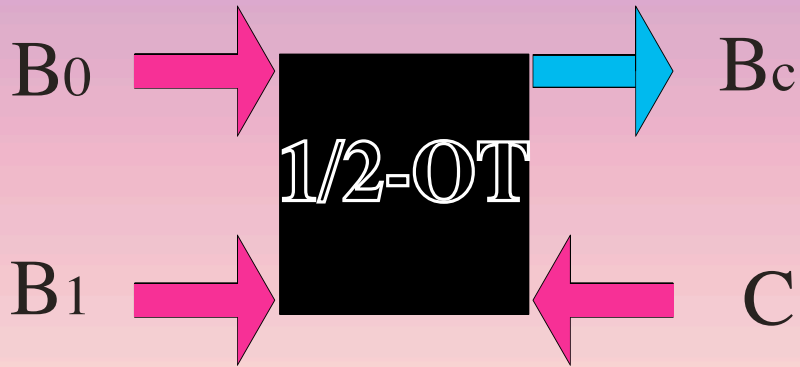




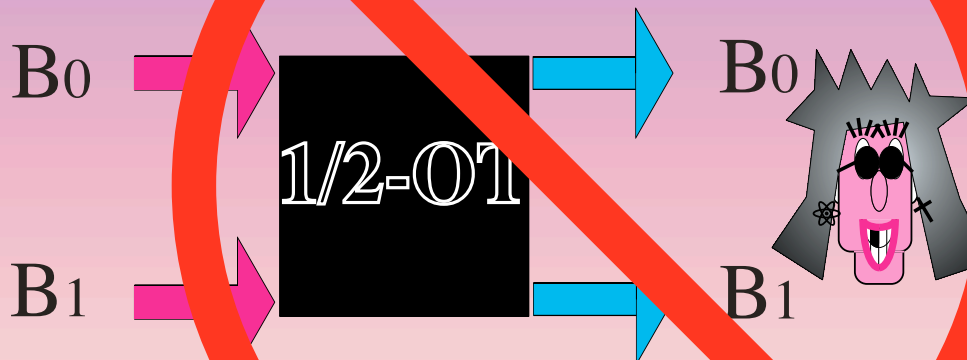
# Oblivious Transfer



(message multiplexing)

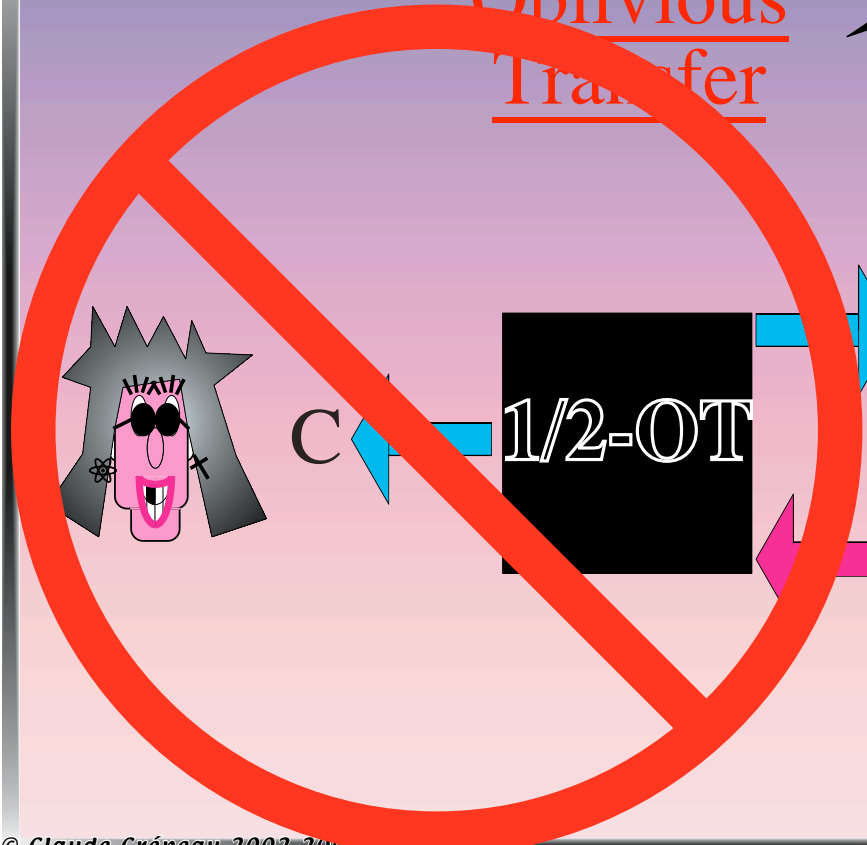
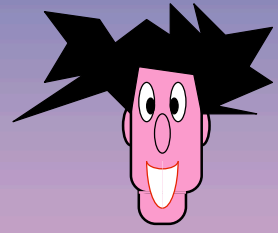


# Oblivious Transfer

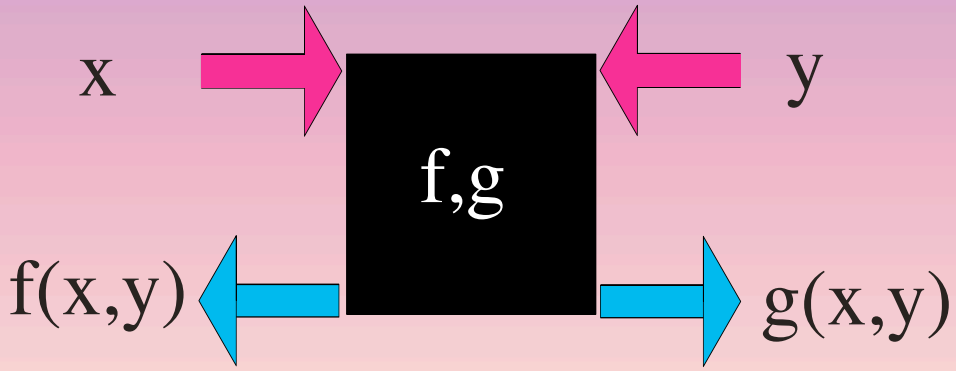
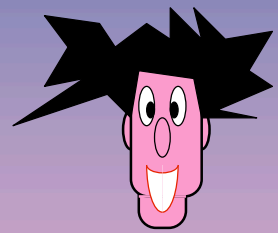
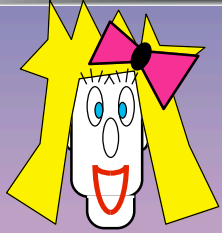




# Oblivious Transfer

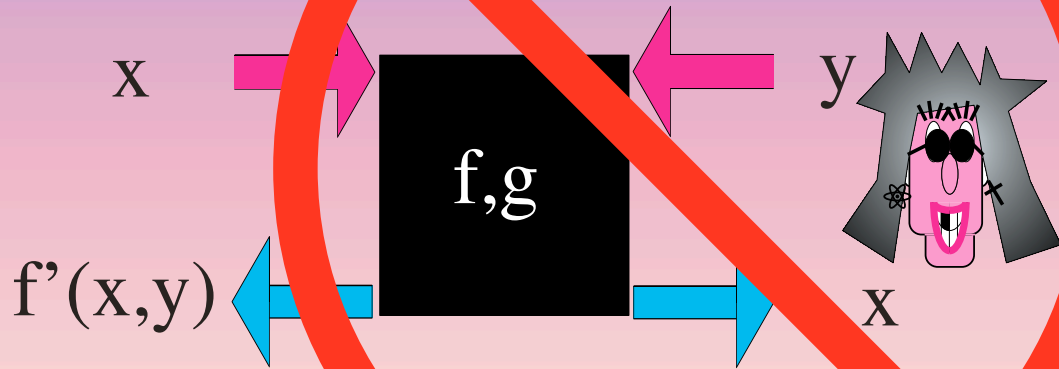


# Oblivious Function Evaluation

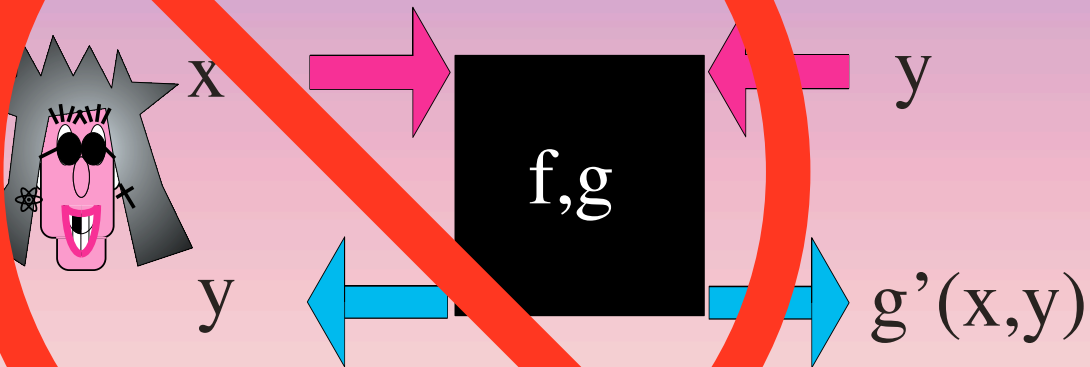
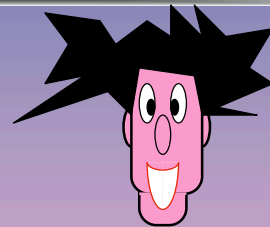


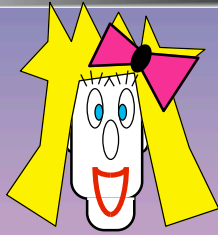


# Oblivious Function Evaluation

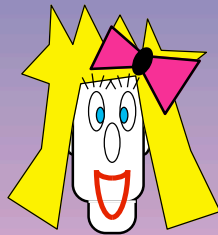
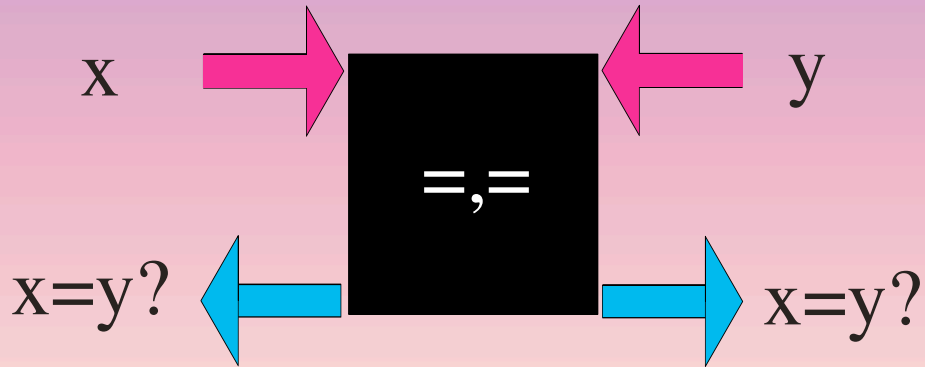
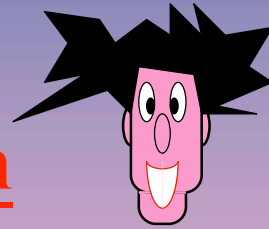


# Oblivious Function Evaluation

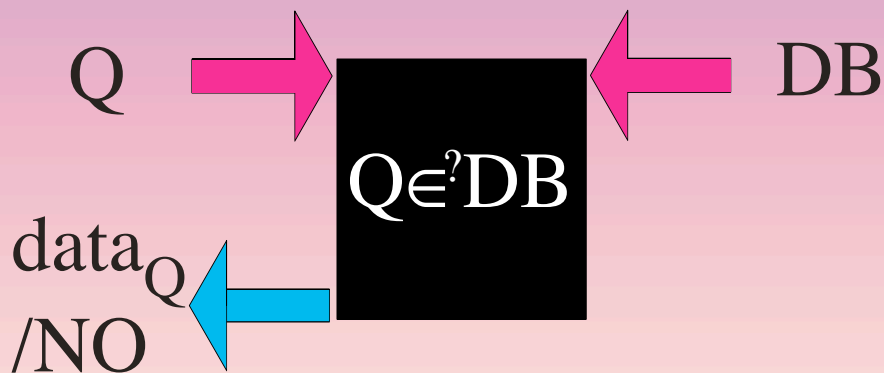
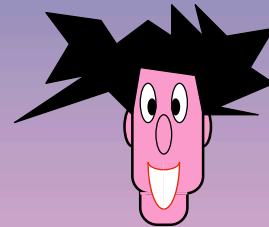


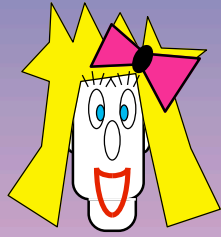


## Mutual Identification

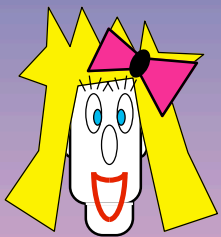
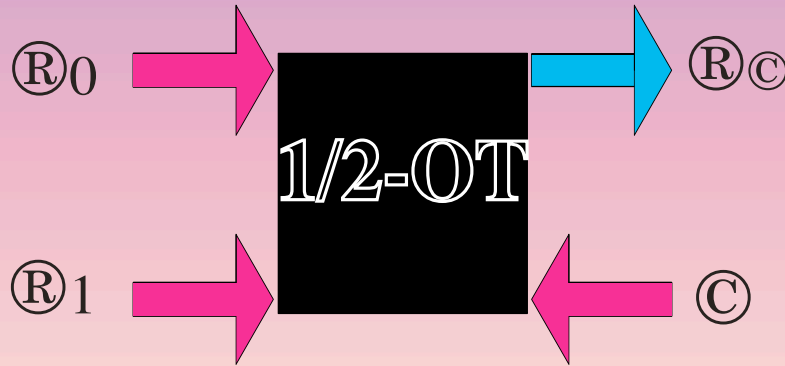
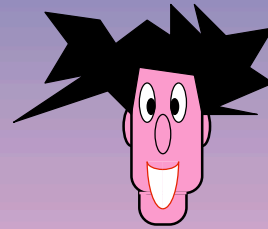


## Oblivious DB query

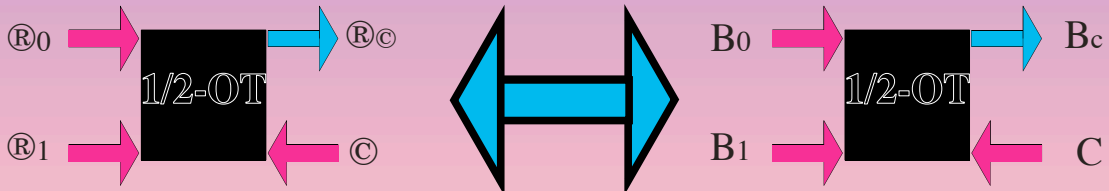
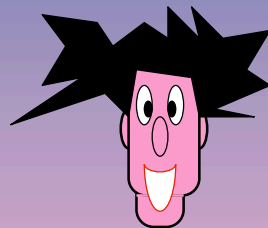


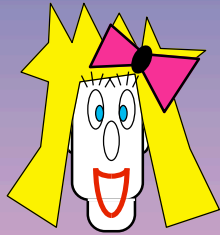


# Randomized Oblivious Transfer

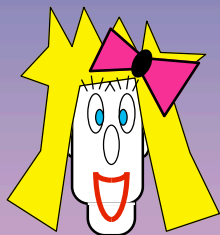
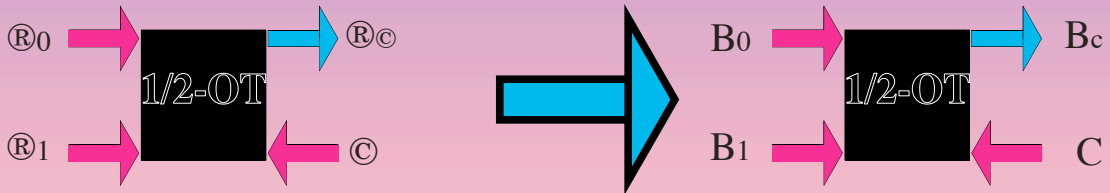
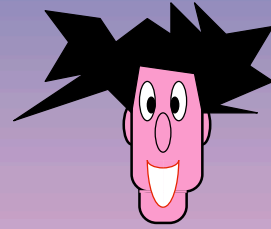


# Randomized Oblivious Transfer

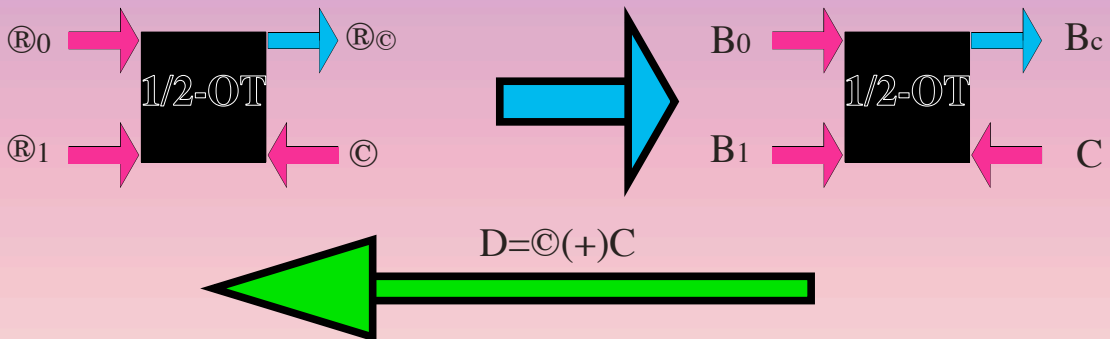
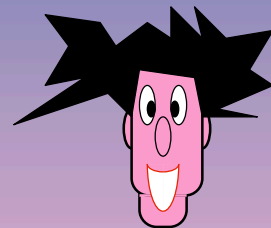


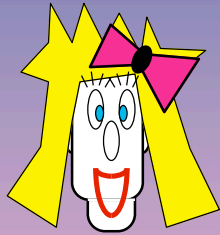


# Randomized Oblivious Transfer

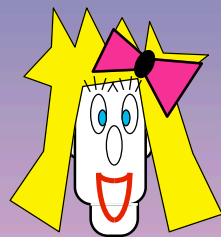
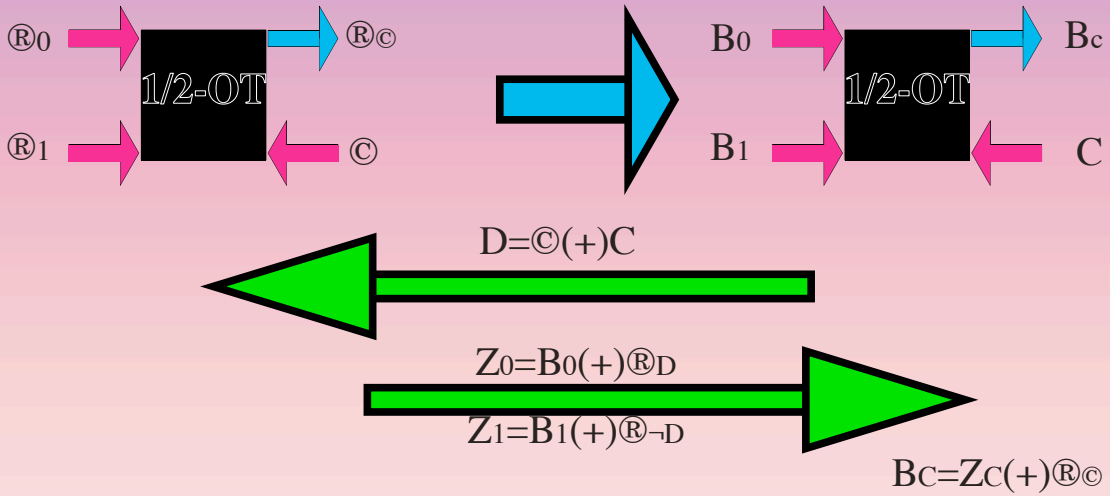
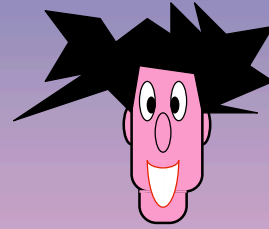


# Randomized Oblivious Transfer

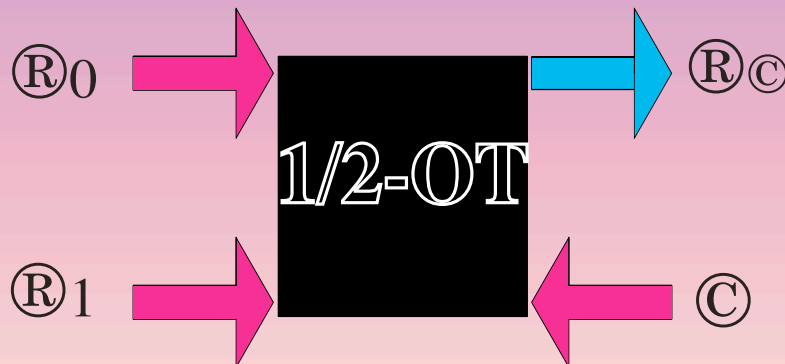
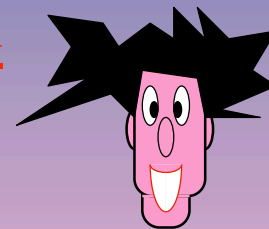




# Randomized Oblivious Transfer

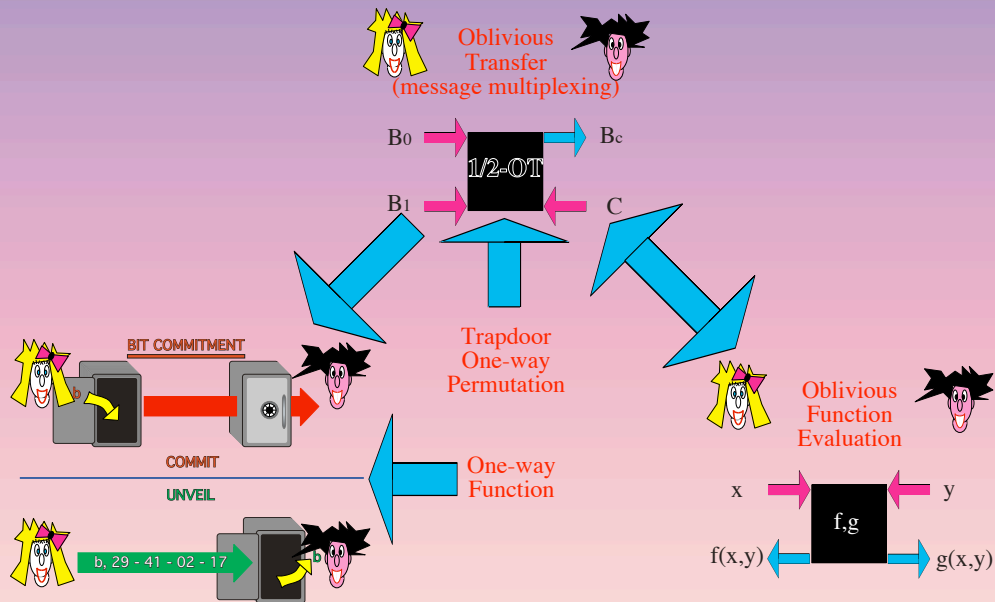


# Randomized Oblivious Transfer

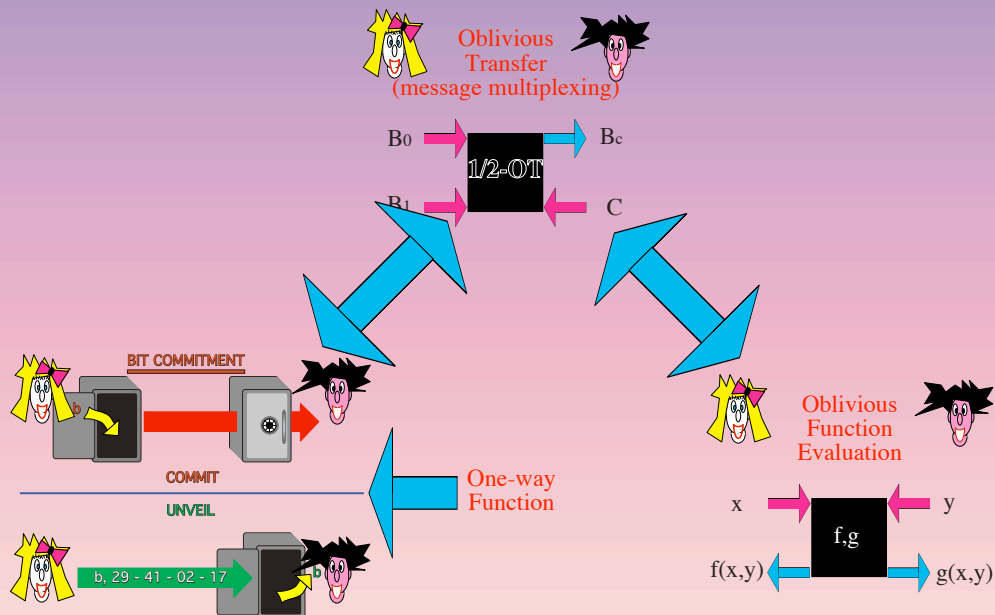


***IS AN INVESTMENT  
IN THE FUTURE***

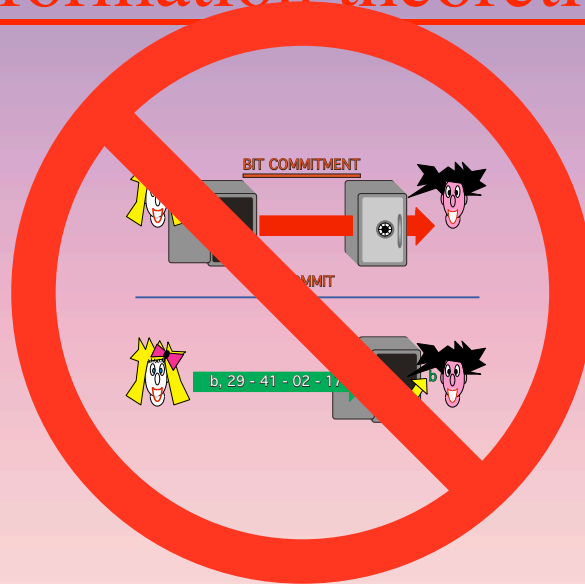
# Classically



# Quantumly

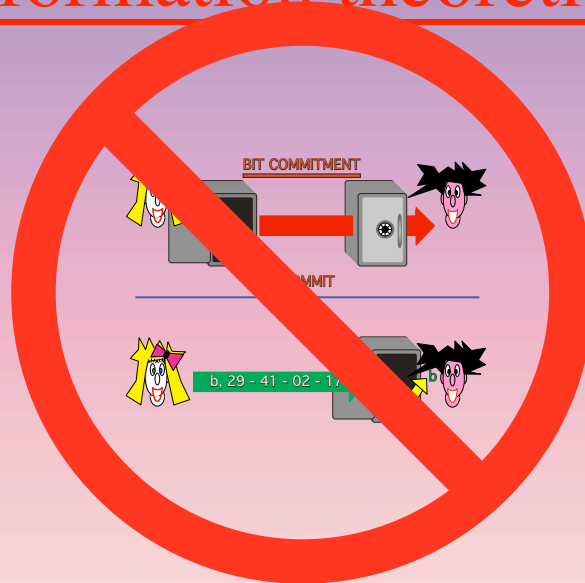


# Classically (information theoretical)



# Folklore

# Quantumly (information theoretical)

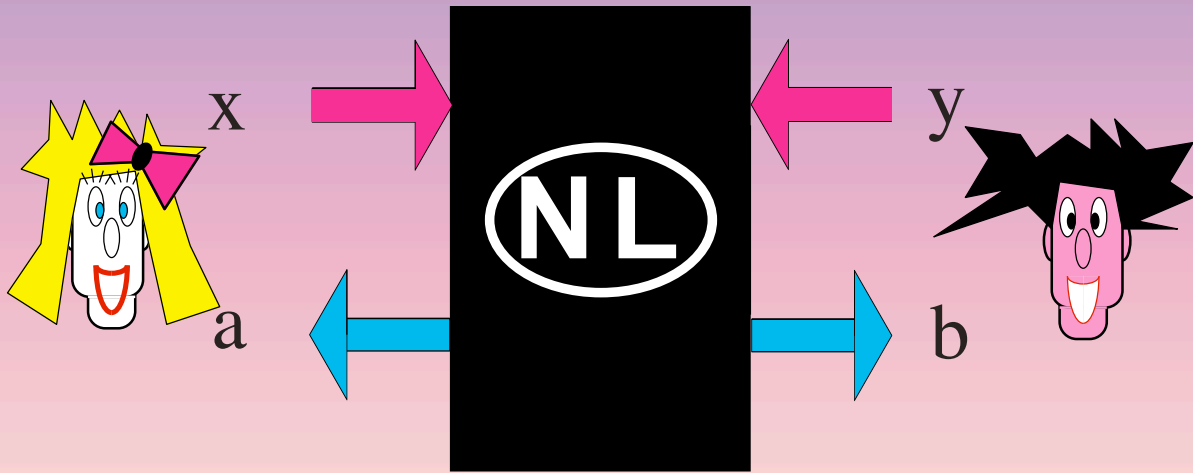


# Mayers, Lo-Chau



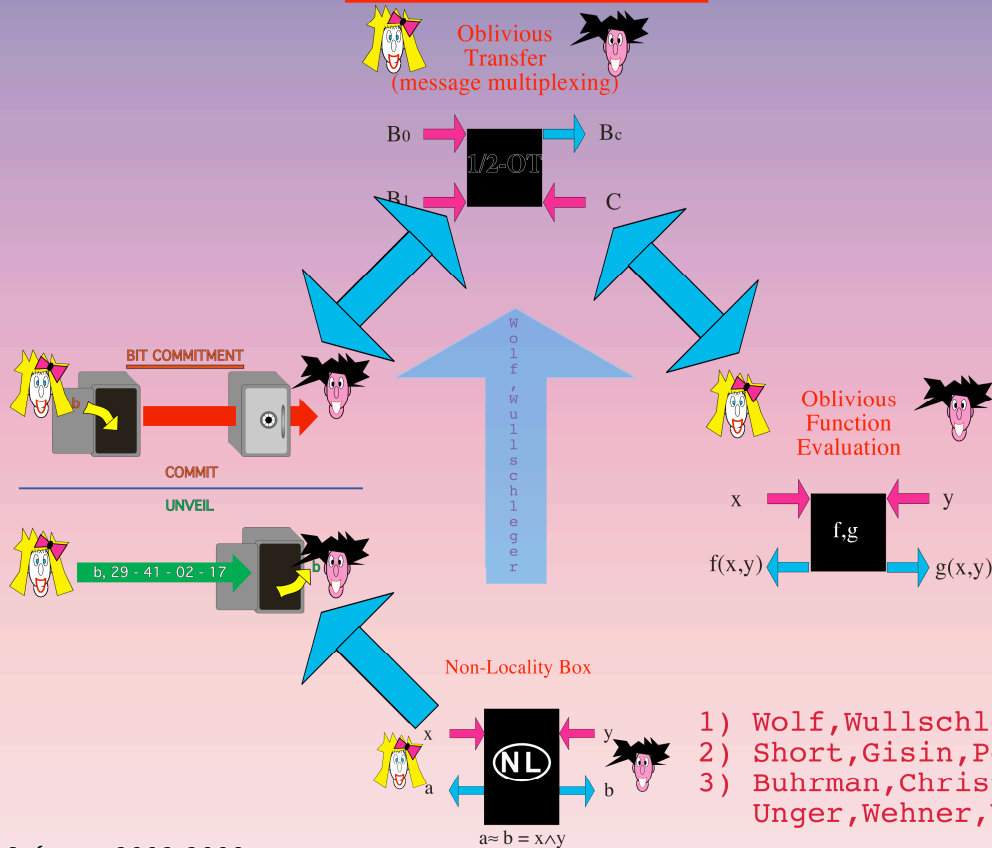
# Non-Locality Box

$$a \oplus b = x \otimes y$$



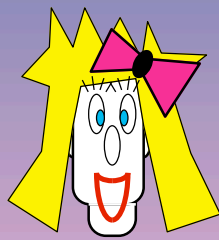
$$C: 3/4 \quad Q: \cos^2(\pi/8) \approx 85\%$$

# Quantumly

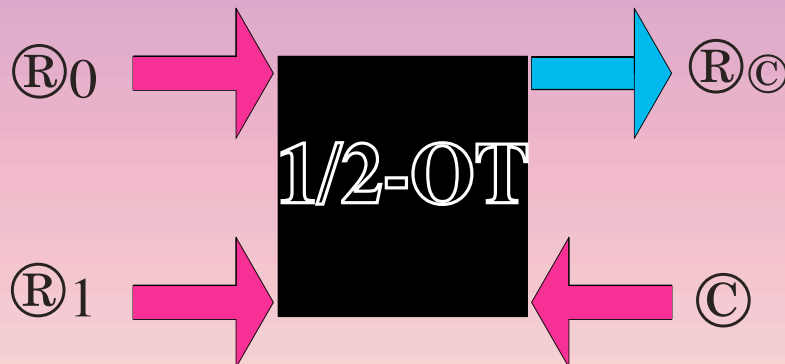
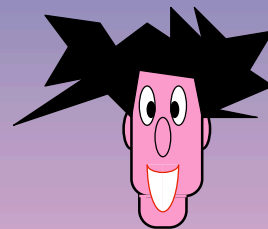


- 1) Wolf, Wullschlegler ?
- 2) Short, Gisin, Popescu
- 3) Buhrman, Christandl, Unger, Wehner, Winter

(5)  
Quantum  
Oblivious  
Transfer



Randomized  
Oblivious  
Transfer



# Q-ROT



A: 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0

x + x + + + x x x x + + + + x x x + x + + + x +

B: x x + + x + + + x + + x x x + x x x + + x + x +

0 0 1 0 0 1 0 0 1 0 0 0 0 1 1 1 0 0 0 1 1 0 0 0

A: x + x + + + x x x x + + + + x x x + x + + + x +

B: 0 ♡ ♡ 0 ♡ 1 ♡ ♡ 1 ♡ 0 ♡ ♡ ♡ ♡ 1 0 ♡ ♡ 1 ♡ 0 0 0

B: 0 0 1 1 0 1 0 1 0 0 0 ♡ ♡ ♡ ♡ ♡ ♡ ♡ ♡ ♡ ♡ ♡ ♡

A: 0 0 1 1 0 1 0 1 0 0 0 1 1 0 0 0 1 1 0 0 0 1 0 1

B: 0 0 1 1 0 1 0 1 = 0 ♡ = ♡ ♡ ♡ ♡ ♡ ♡ ♡ ♡

A: 0 0 1 1 0 1 0 1 = 0<sup>Ⓡ</sup><sub>0</sub> Ⓡ<sub>1</sub>=0 = 1 1 0 0 0 1 0 1

## Crépeau-Kilian

# Q-OT



A: 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0

x + x + + + x x x x + + + + x x x + x + + + x +

# Q-OT



A: 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0

x + x + + + x x x x + + + + x x x + x + + + x +

B: x x + + x + + + x + + x x x + x x x + + x + x +

0 0 1 0 0 1 0 0 1 0 0 0 0 1 1 1 0 0 0 1 1 0 0 0

# Q-OT



A: 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0

x + x + + + x x x x + + + + x x x + x + + + x +

B: x x + + x + + + x + + x x x + x x x + + x + x +

0 0 1 0 0 1 0 0 1 0 0 0 0 1 1 1 0 0 0 1 1 0 0 0

A: x + x + + + x x x x + + + + x x x + x + + + x +

# Q-OT



B: × × + + × + + + × + + × × × + × × × + + × + × +  
 0 0 1 0 0 1 0 0 1 0 0 0 0 1 1 1 0 0 0 1 1 0 0 0

A: × + × + + + × × × × + + + + × × × + × + + + × +

B: 0 0 1 1 0 1 0 1 0 0 0

# Q-OT



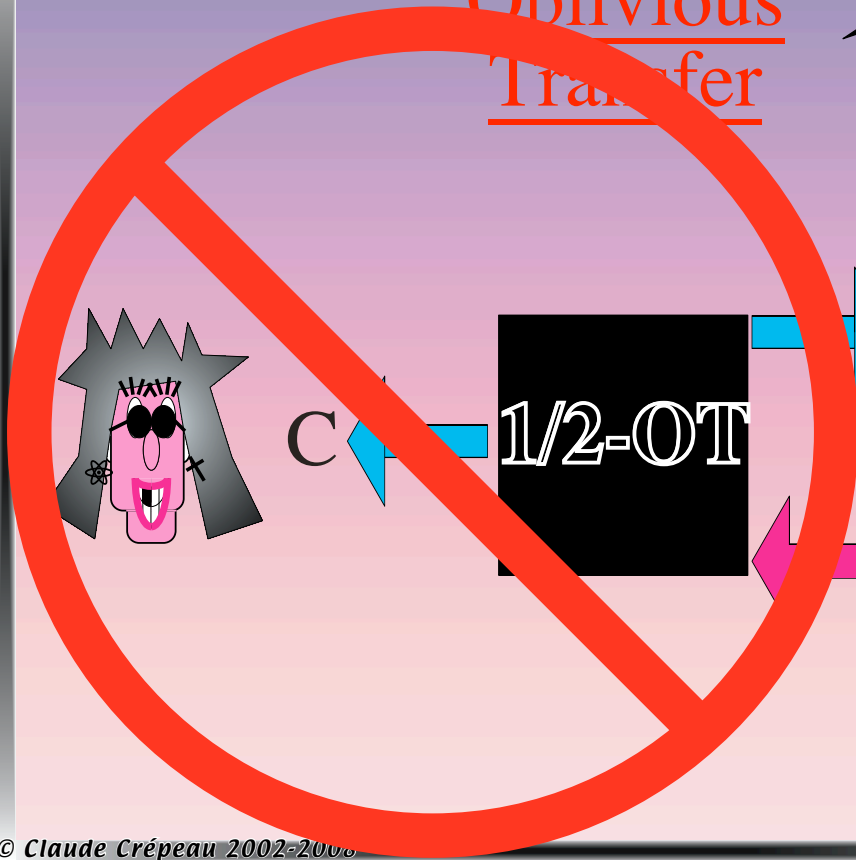
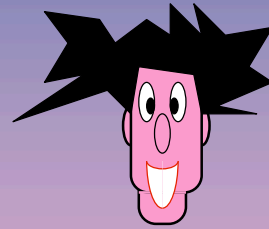
A: 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0

B: 0 0 1 1 0 1 0 1 0 0 0

B: 0 0 1 1 0 1 0 1 0 0 0

A: 0 0 1 1 0 1 0 1 0 0 0 1 1 0 0 0 1 1 0 0 0 1 0 1

# Oblivious Transfer



# Q-OT

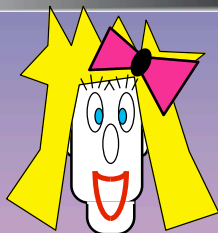


B:	0 0 1 1 0 1 0 1	0 0 0	⚡ ⚡ ⚡ ⚡ ⚡ ⚡ ⚡ ⚡	⚡ ⚡ ⚡ ⚡ ⚡ ⚡ ⚡ ⚡
A:	0 0 1 1 0 1 0 1	0 0 0 1 1 0 0 0	1 1 0 0 0 1 0 1	
B:	0 0 1 1 0 1 0 1	= 0	⚡ =	⚡ ⚡ ⚡ ⚡ ⚡ ⚡ ⚡ ⚡
A:	0 0 1 1 0 1 0 1	= 0 <sup>Ⓡ<sub>0</sub></sup>	Ⓡ <sub>1</sub> =0 =	1 1 0 0 0 1 0 1

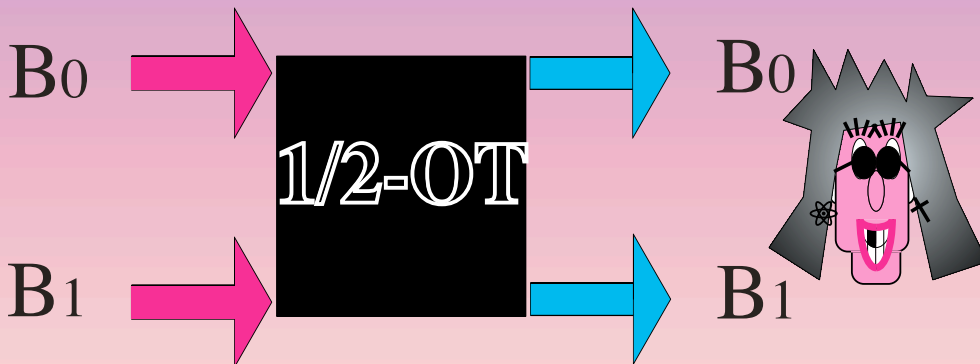
# Q-OT



B: 0 0 1 1 0 1 0 1 = 0       $\heartsuit = \heartsuit \heartsuit \heartsuit \heartsuit \heartsuit \heartsuit \heartsuit \heartsuit$   
A: 0 0 1 1 0 1 0 1 =  $0 = \mathbb{R}_0$        $\mathbb{R}_1 = 0 = 1 1 0 0 0 1 0 1$



# Oblivious Transfer



# Q-OT



A: 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0  
 × + × + + × × × × + + + + × × × + × + + + × +

A: × + × + + × × × × + + + + × × × + × + + + × +

B: × + × + + × × × × + + + + × × × + × + × + × +

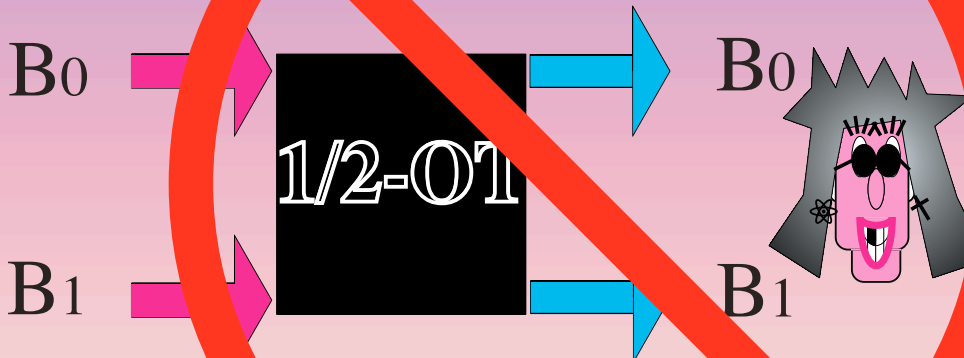
0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0

B: 0 0 1 1 0 1 0 1 = 0                      0 = 1 1 0 0 0 1 0 1

A: 0 0 1 1 0 1 0 1 = 0<sup>Ⓡ<sub>0</sub></sup>                      <sup>Ⓡ<sub>1</sub></sup>0 = 1 1 0 0 0 1 0 1



# Oblivious Transfer





# Q-OT

## from Q-BC



A: 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0  
 × + × + + + × × × × + + + + × × × + × + + + × +

B: × × + + × + + + × + + + × × × + × × × + + × + × +  
 0 0 1 0 0 1 0 0 1 0 0 0 0 1 1 1 0 0 0 1 1 0 0 0



A: [Empty boxes]



B: [Boxes containing 'x', '0', '1' and '+' signs]

A: 1 0 1 1 1 1 0 0 1 1 1 0  
 + + + × × + + + × + + +

A: × × + × × + × × × + × + × +

B: × + × + + + 0 1 1 + + × +

# Q-OT

## from Q-BC



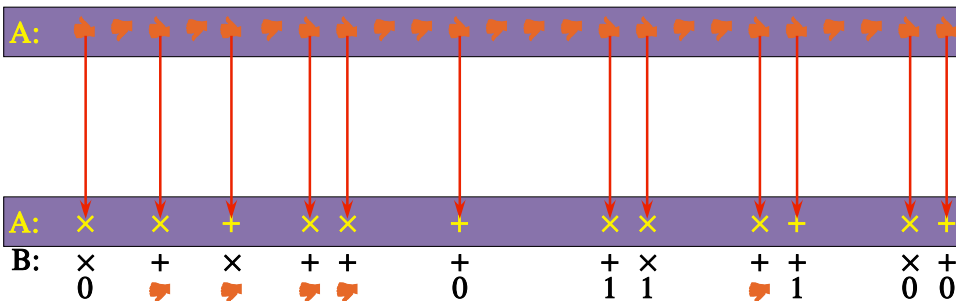
A: 0 1 1 0 0 1 0 0 1 1 0 1 0 0 0 1 1 1 0 1 1 0 0 0  
 × + × + + + × × × × + + + + × × × + × + + + × +

B: × × + + × + + + × + + + × × × + × × × + + × + × +  
 0 0 1 0 0 1 0 0 1 0 0 0 0 1 1 1 0 0 0 1 1 0 0 0

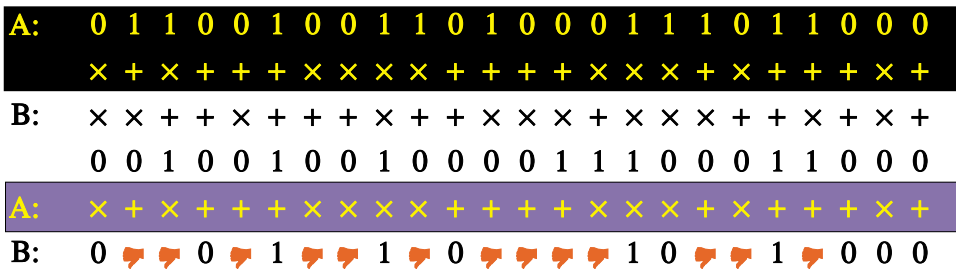


# Q-OT

## from Q-BC



# Q-OT



(6)

two provers

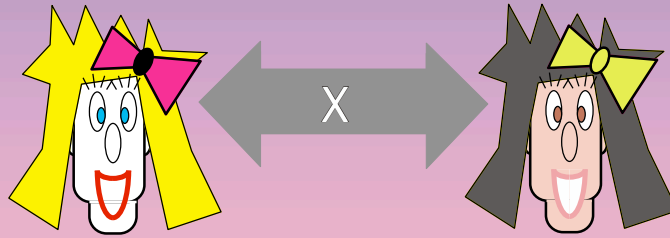
Cryptographic Protocols

Classically

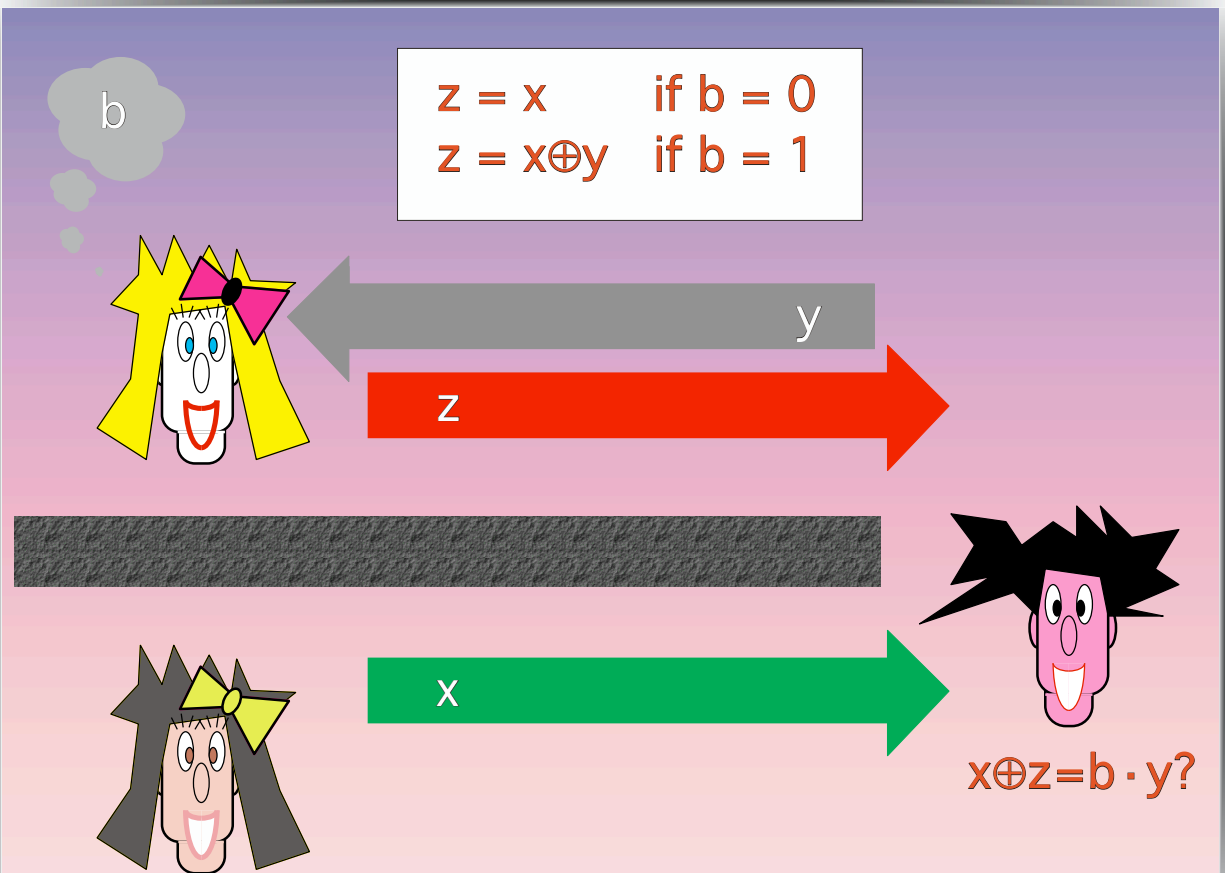
BIT COMMITMENT

BGKW88

# Classically



Ben-Or, Goldwasser, Kilian, Wigderson



Ben-Or, Goldwasser, Kilian, Wigderson

$$x_0 \oplus z = 0 \cdot y = 0$$

$$x_1 \oplus z = 1 \cdot y = y$$

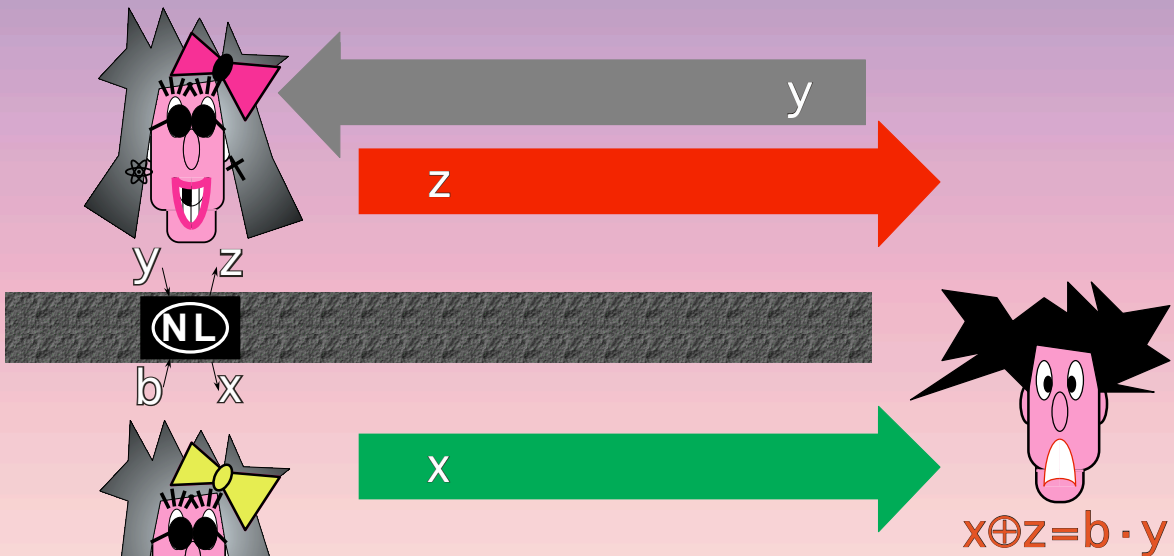
$$x_0 \oplus x_1 = (x_0 \oplus z) \oplus (x_1 \oplus z) = y$$

possible with prob. at most  $2^{-n}$



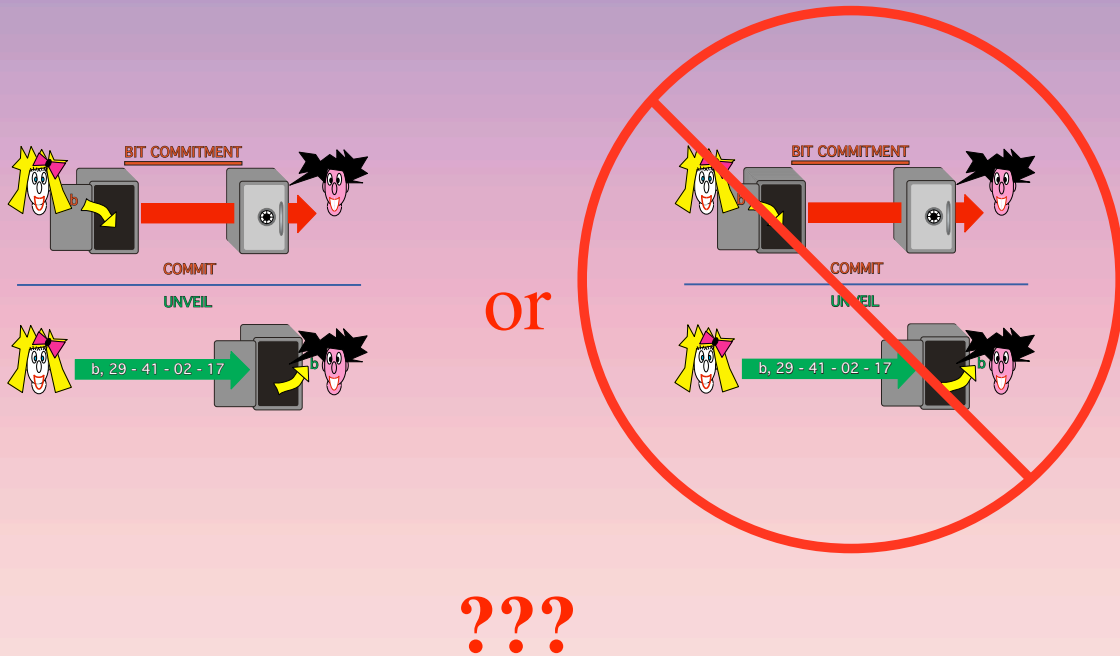
Ben-Or, Goldwasser, Kilian, Wigderson

## Classically



Ben-Or, Goldwasser, Kilian, Wigderson

# Quantumly



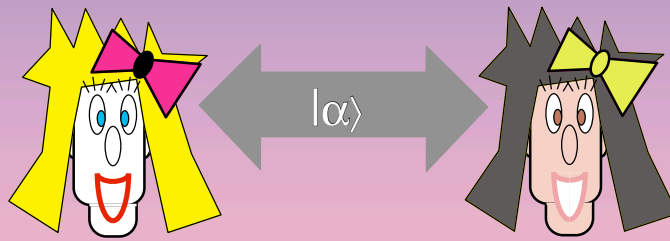
(7)

two provers BC

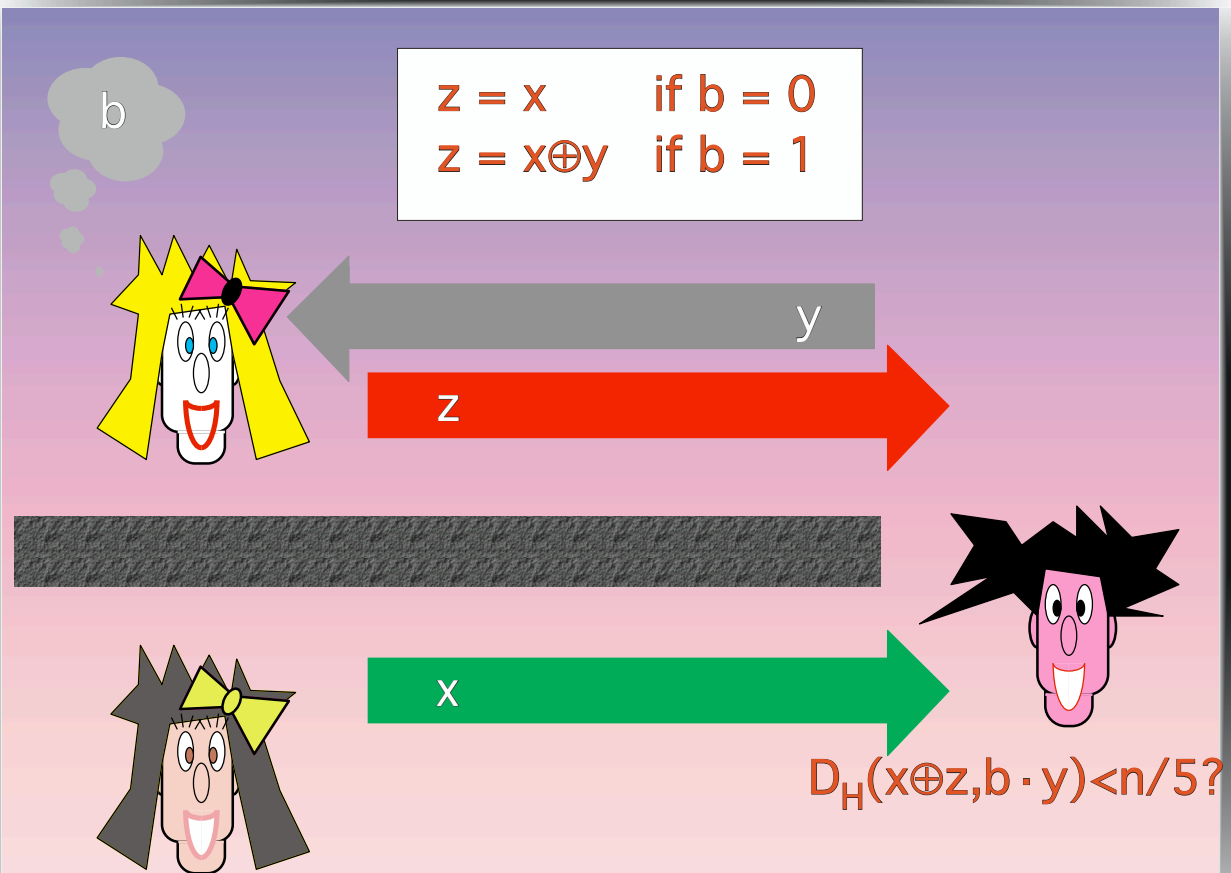
Classically Secure

Quantumly Insecure

# Quantumly



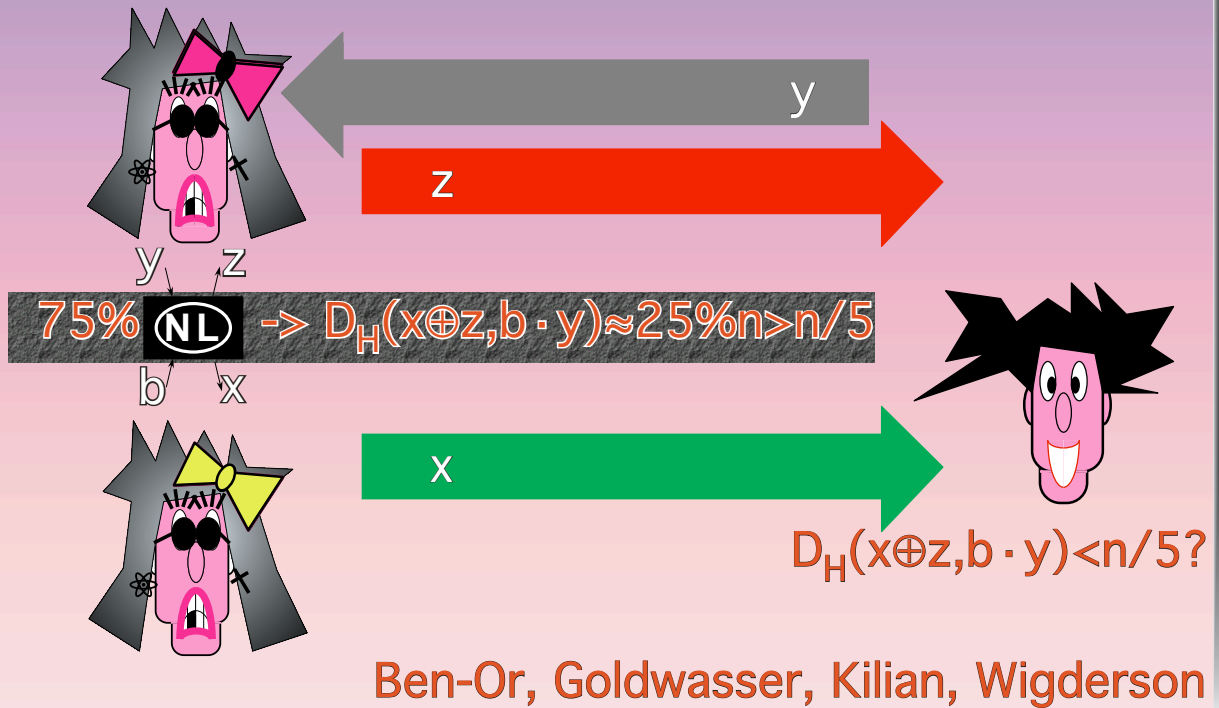
Ben-Or, Goldwasser, Kilian, Wigderson



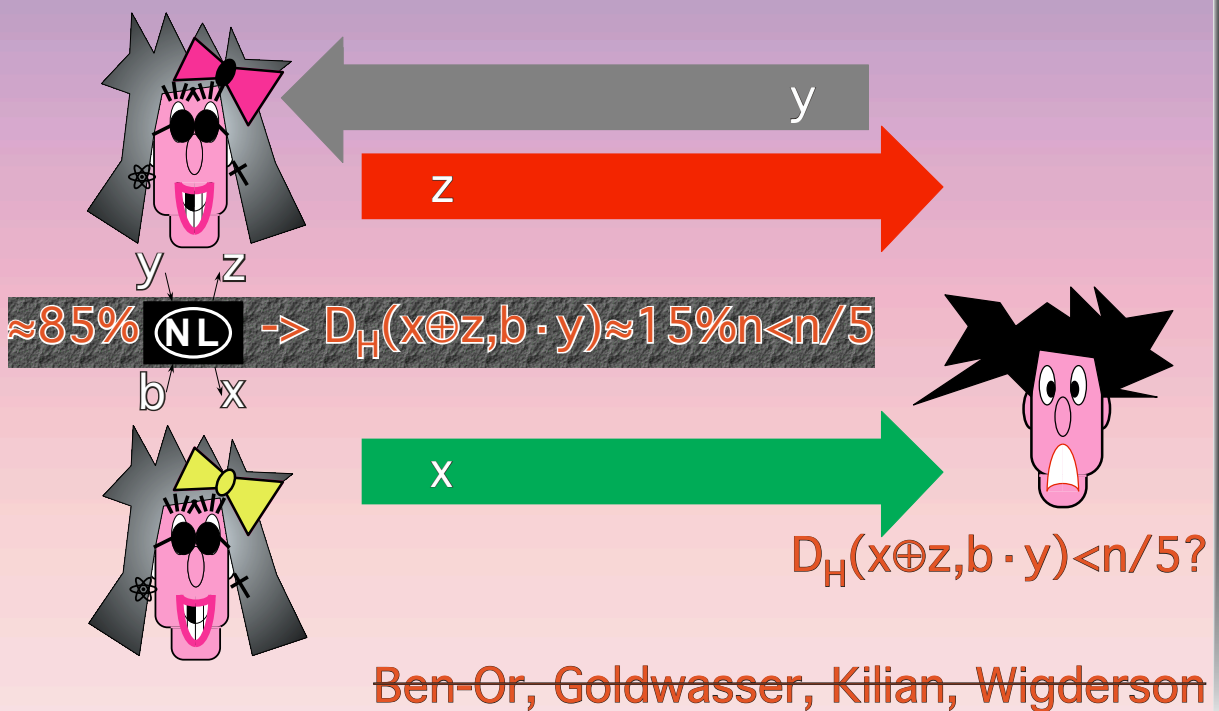
Ben-Or, Goldwasser, Kilian, Wigderson



# Classically



# Quantumly



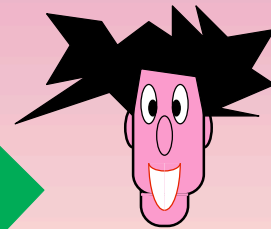
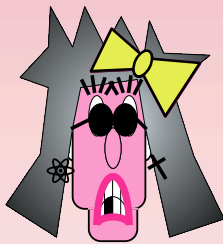
# Classically

$$D_H(x_0 \oplus z, 0 \cdot y) = D_H(x_0 \oplus z, 0) < n/5$$

$$D_H(x_1 \oplus z, 1 \cdot y) = D_H(x_1 \oplus z, y) < n/5$$

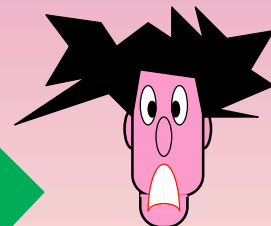
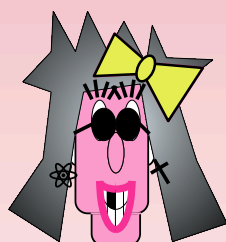
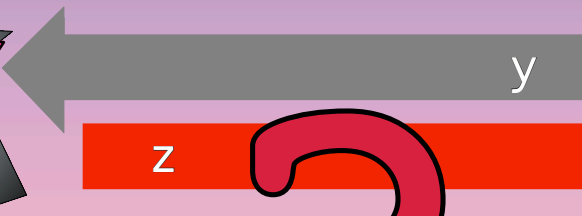
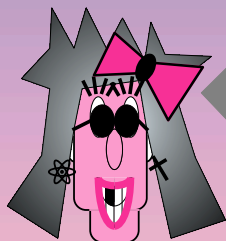
$$D_H(x_0 \oplus x_1, y) = D_H((x_0 \oplus z) \oplus (x_1 \oplus z), y) < 2n/5 < n/2$$

possible with prob. at most  $c^{-n}$



Ben-Or, Goldwasser, Kilian, Wigderson

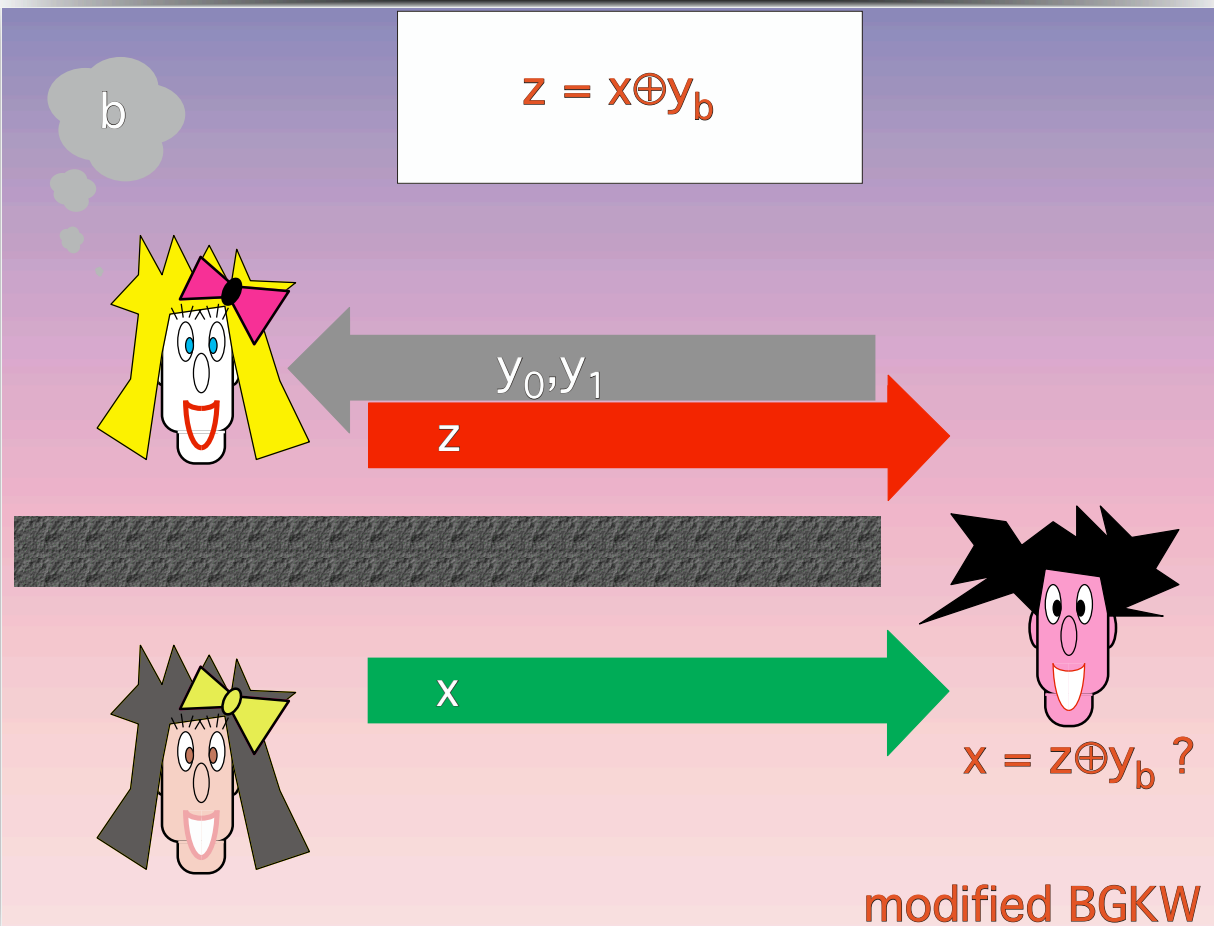
# Quantumly



$$x \oplus z = b \cdot y$$

Ben-Or, Goldwasser, Kilian, Wigderson

(8)  
two provers BC  
Classically and  
Quantumly Secure



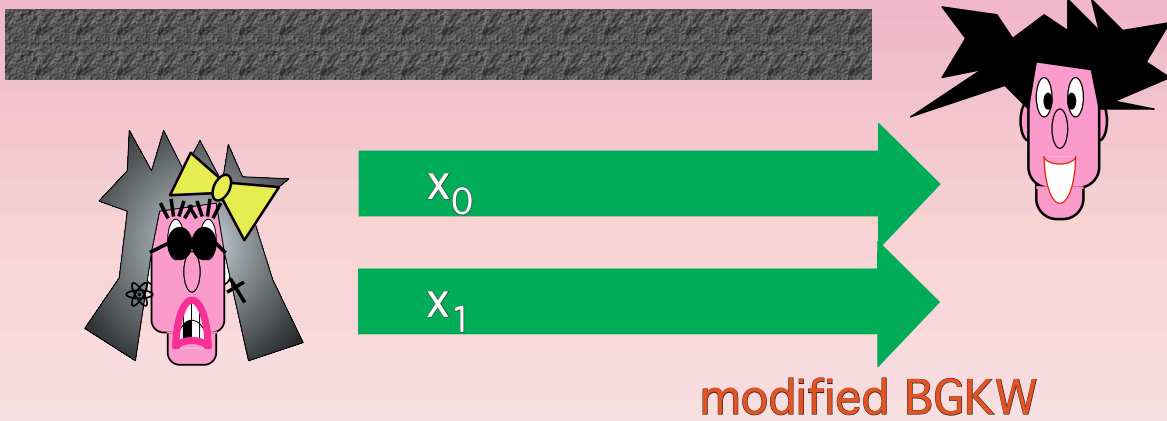
# Classically

$$x_0 \oplus z = y_0$$

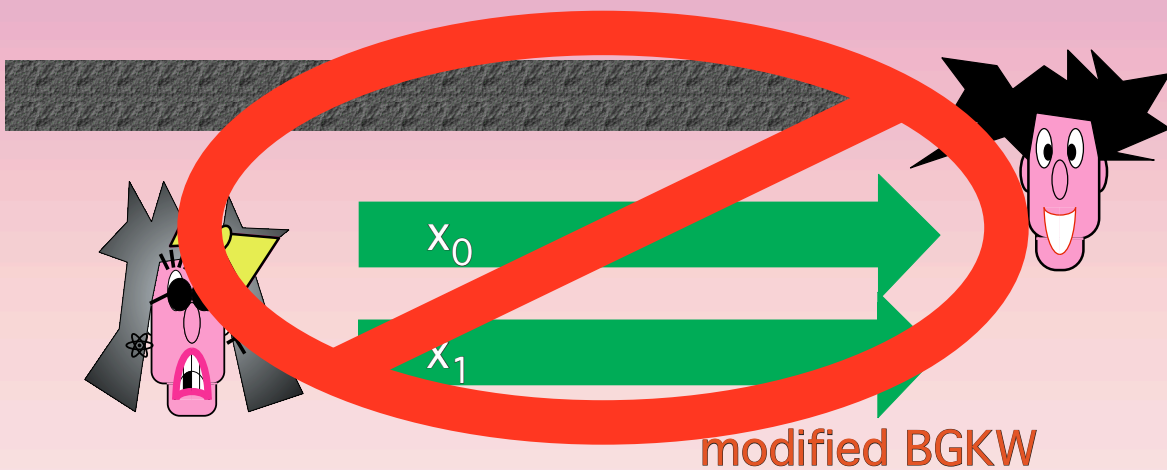
$$x_1 \oplus z = y_1$$

$$x_0 \oplus x_1 = (x_0 \oplus z) \oplus (x_1 \oplus z) = y_0 \oplus y_1$$

possible with prob. at most  $2^{-n}$



# Quantumly



# Quantumly

## MAIN THEOREM

Let  $0$  and  $1$  be POVMs such that outputs  $x_0$  and  $x_1$  one could obtain by applying one of them to the state shared among the two provers.

Suppose the success probability of unveiling is

$$p_0 + p_1 > 1 + \delta,$$

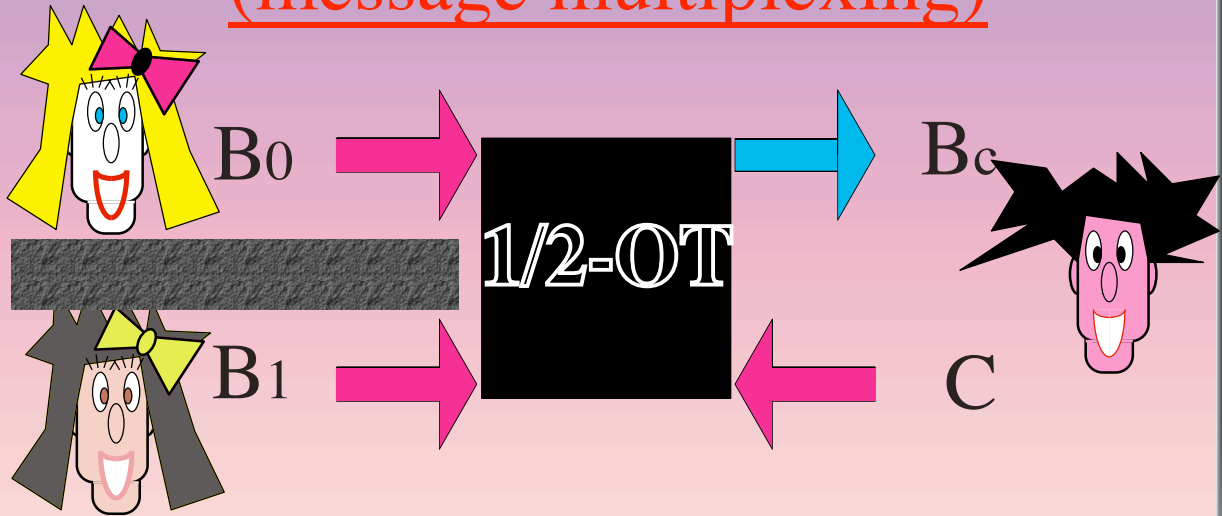
then the (prediction probability of  $y_0 \oplus y_1$ )  $> \delta$ .

This prediction probability is achieved by first applying  $0$  to the shared state followed by  $1$  on the leftover system or the other way around.

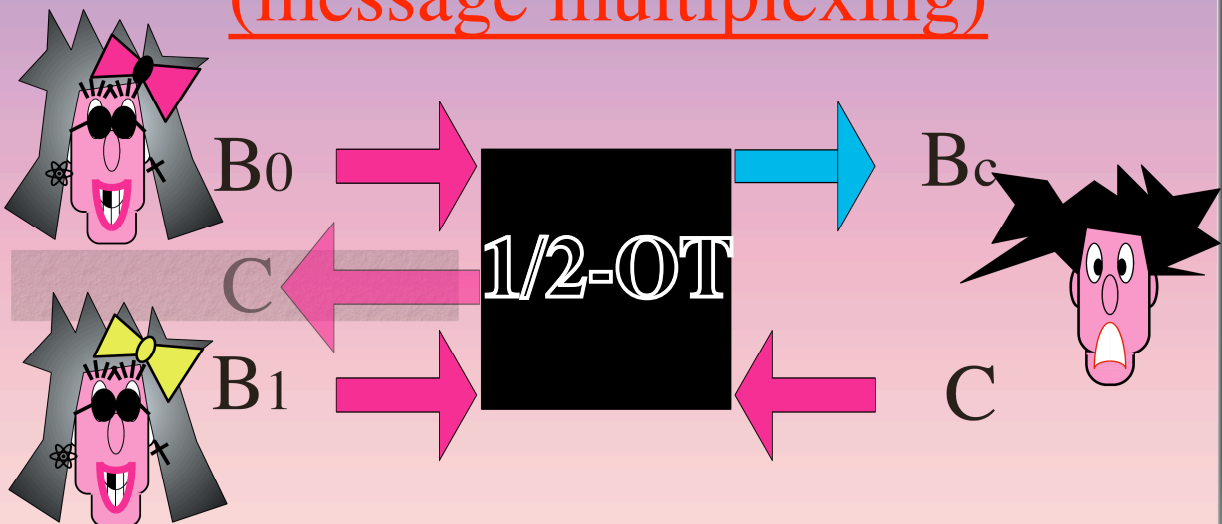
(9)

WARNING !

# Oblivious Transfer (message multiplexing)

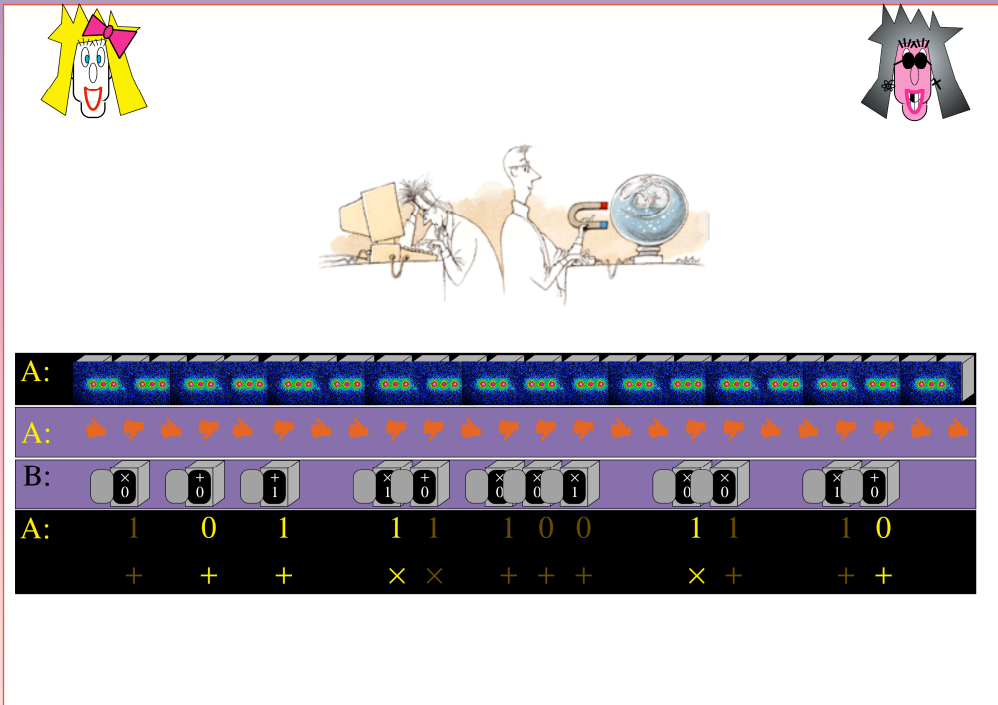


# Oblivious Transfer (message multiplexing)

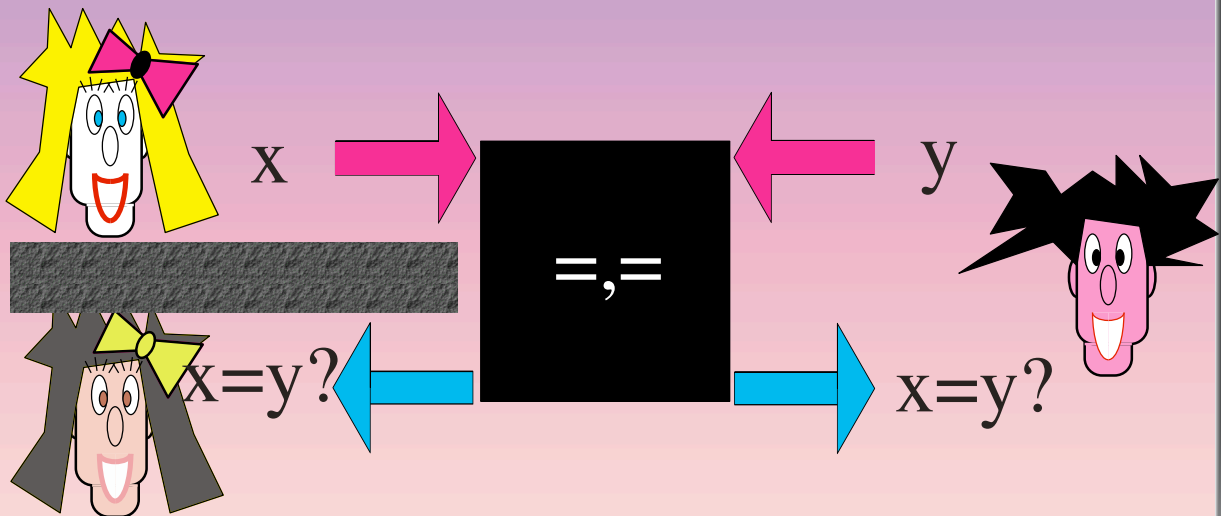


Brassard, Crépeau, Mayers, Salvail 97

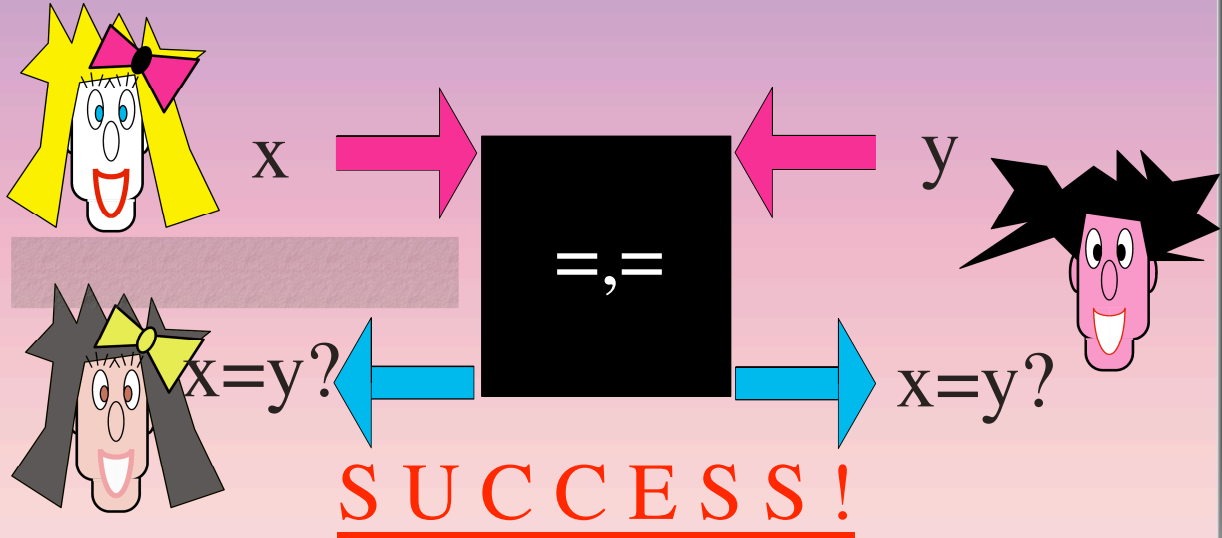
# BCMS' attack



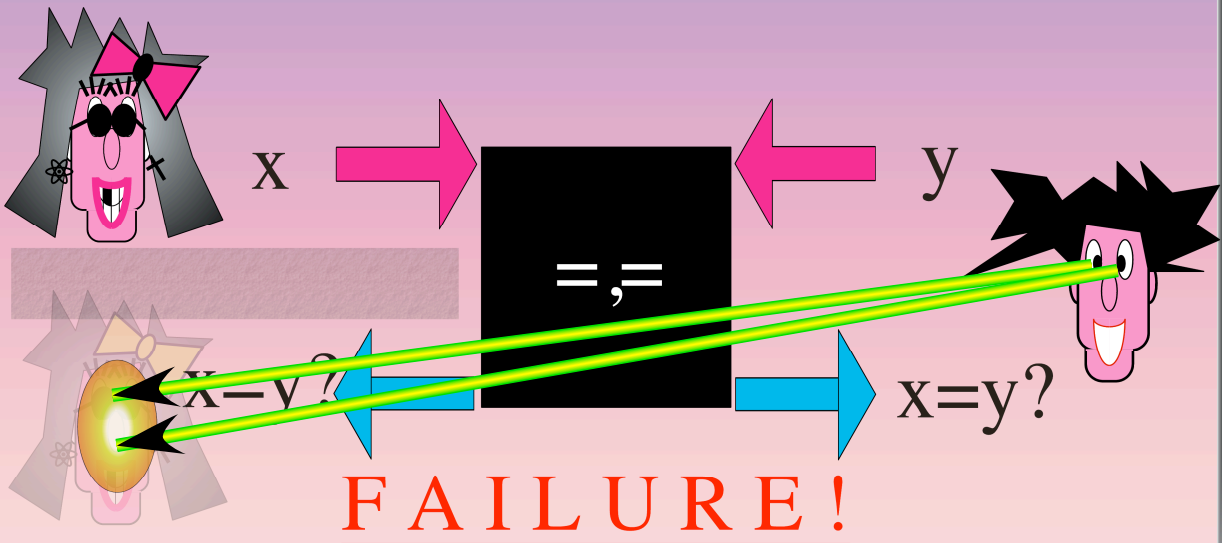
# Mutual Identification



# Mutual Identification



# Mutual Identification





an Introduction to  
theoretical quantum  
CRYPTOGRAPHY

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McGill University

