

# Classical and $|\text{Quantum}\rangle$ strategies for two-prover bit commitments

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**School of Computer Science  
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**Joint work with J-R Simard and A Tapp**

# Bit Commitment Strikes Back

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# EPISODE 11

## Classical and Quantum Strategies against Two-Prover Bit Commitments

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**Abstract.** First we show that the assumption behind the Two-Prover Zero-knowledge Interactive proof of BenOr, Goldwasser, Kilian and Wigderson is too weak and need be upgraded to preserve soundness of their construction. Secondly, we introduce a Two-Prover Zero-knowledge Interactive proof similar to theirs and demonstrate that classically it is equally secure as the original. However, we later show that if the provers are allowed to share quantum entanglement, they are able to successfully prove false statements to the verifier with probability one. Then we demonstrate that a small variation on the BGKW Two-Prover Zero-knowledge Interactive proof is classically secure with probability nearly one but obviously quantum insecure with probability nearly one. We finally show that another variation of the original scheme of BGKW is quantumly secure.

### 1 Introduction

The notion of Multi-Prover Interactive proofs was introduced by BenOr, Goldwasser, Kilian and Wigderson [?] together with the Zero-knowledge property of such proofs. In the Two-prover scenario, we have two provers, Peggy and Paula, that are allowed to share arbitrary information before the proof, but they become physically separated and isolated during the execution of the proof in order to prevent them from communicating. The Two-prover Interactive proofs of BGKW rely on their construction of a

## EPISODE I

A Quantum Bit Commitment Scheme  
Provably Unbreakable by both PartiesGilles Brassard\*  
Université de Montréal †Claude Crépeau ‡  
École Normale Supérieure §Richard Jozsa  
Université de Montréal †Denis Langlois †  
Université Paris-Sud ¶

## Abstract

Assume that a party, *Alice*, has a bit  $x$  in mind, to which she would like to be committed toward another party, *Bob*. That is, *Alice* wishes, through a procedure  $commit(x)$ , to provide *Bob* with a piece of evidence that she has a bit  $x$  in mind and that she cannot change it. Meanwhile, *Bob* should not be able to tell from that evidence what  $x$  is. At a later time, *Alice* can reveal, through a procedure  $unveil(x)$ , the value of  $x$  and prove to *Bob* that the piece of evidence sent earlier really corresponded to that bit. Classical bit commitment schemes (by which *Alice*'s piece of evidence is classical information such as a bit string) cannot be secure against unlimited computing power and none have been proven secure against algorithmic sophistication. Previous quantum bit commitment schemes (by which *Alice*'s piece of evidence is quantum information such as a stream of polarized photons) were known to be invulnerable to unlimited computing power and algorithmic sophistication, but not to *arbitrary* measurements allowed by quantum physics: perhaps more sophisticated use of quantum physics could have defeated them.

We present a new quantum bit commitment scheme. The major contribution of this work is to provide the first *complete* proof that, according to the laws of quantum physics, neither participant in the protocol can cheat, except with arbitrarily small probability. In addition, the new protocol can be implemented with current technology.

\*Supported in part by NSERC's E. W. R. Steacie Memorial Fellowship and Québec's FCAR.

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‡ Part of this work was performed while visiting †.

§ Laboratoire d'Informatique de l'École Normale Supérieure, (CNRS URA1327), 45 rue d'Ulm, 75230 Paris CEDEX 05, France. e-mail: crepeau@dmi.ens.fr.

¶ Labo. de Recherche en Informatique, Université Paris-Sud, Bâtiment 490, 91405 Orsay, France. e-mail: langlois@lri.lri.fr.

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Bit commitment schemes have several applications in the field of cryptographic protocols. In particular one can implement *zero-knowledge proofs* of a variety of statements using bit commitment schemes [GMR89, GMW91, BCC88]. The first implementations of bit commitment schemes were given in a computational complexity scenario [Blu82]. Unfortunately, proofs of their (computational) security have always required an unproved assumption since otherwise they would imply very strong results such as  $\mathcal{P} \neq \mathcal{NP}$ .

Over the last two decades a number of researchers have investigated the connection between cryptography and quantum physics, starting with the work of Wiesner in the late 1960's (though published much later [Wie83]), and continuing with the work of Bennett and Brassard [BBW83, BB84, BB85, BBR88, BB89, BBBSS92] and later of Crépeau [CK88, Cré90, BC91, BBCS92, Cré93]. The security of these protocols would not be compromised if a cheater had unlimited computing power, but in essentially all cases it has not yet been ruled out that still more sophisticated use of quantum physics might defeat them.

The first quantum bit commitment scheme ever proposed is due to Bennett and Brassard [BB84] (actually, the protocol they describe is only claimed to implement coin tossing, but implicitly it implements bit commitment). Their scheme had two major flaws: it was impossible to use in practice because faint pulses

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Yoda said:  
"Happens to every guy sometimes  
. this does"

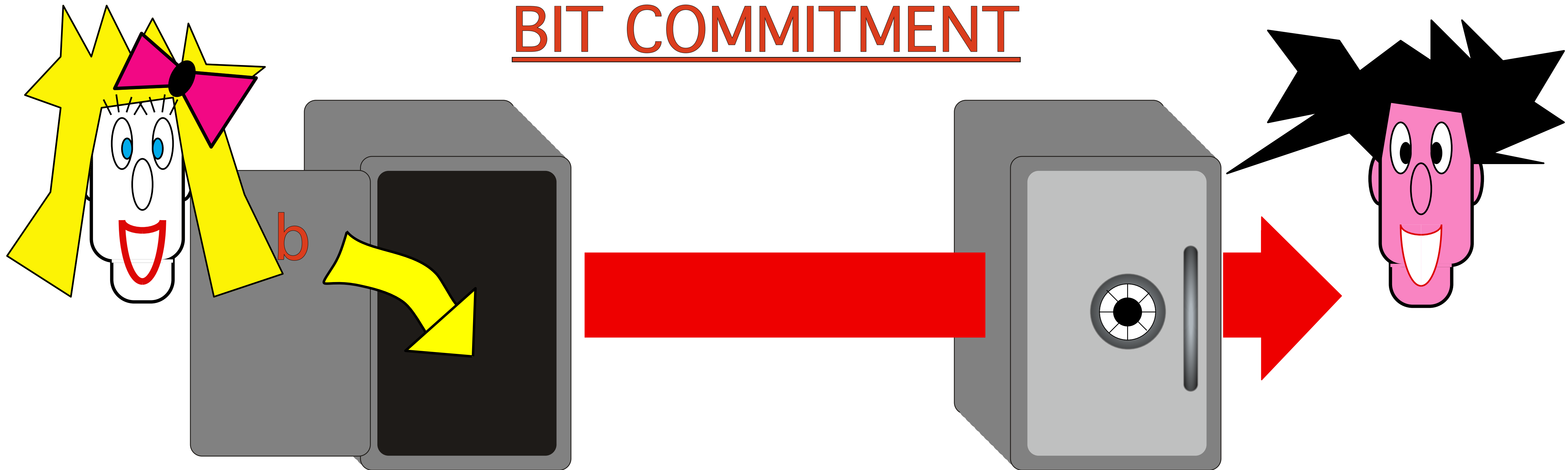
**(1)**

**two-party**

**Cryptographic Protocols**

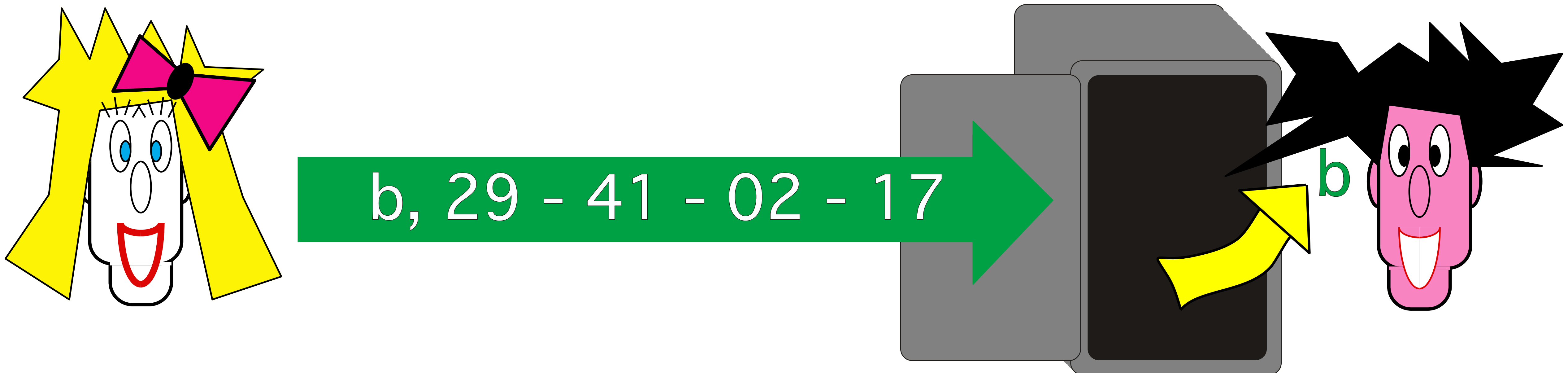


# BIT COMMITMENT

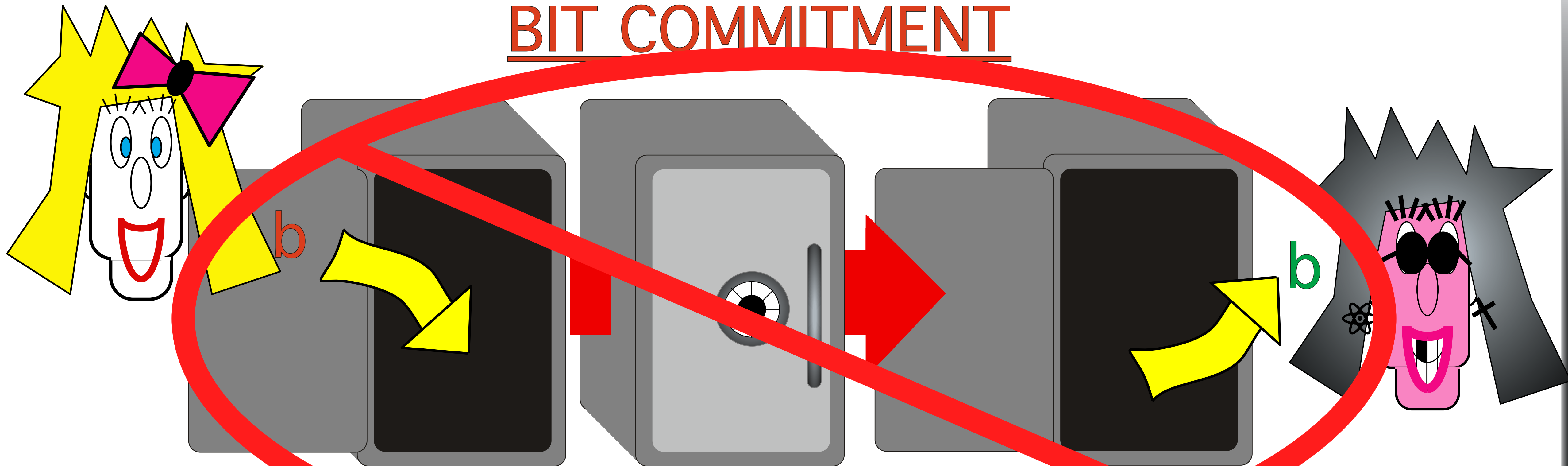


COMMIT

UNVEIL

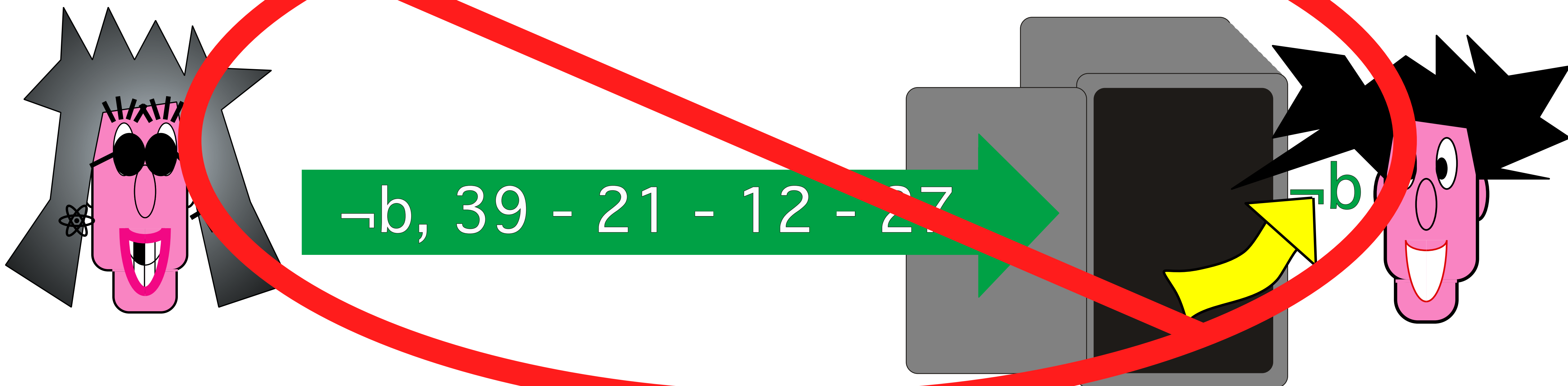


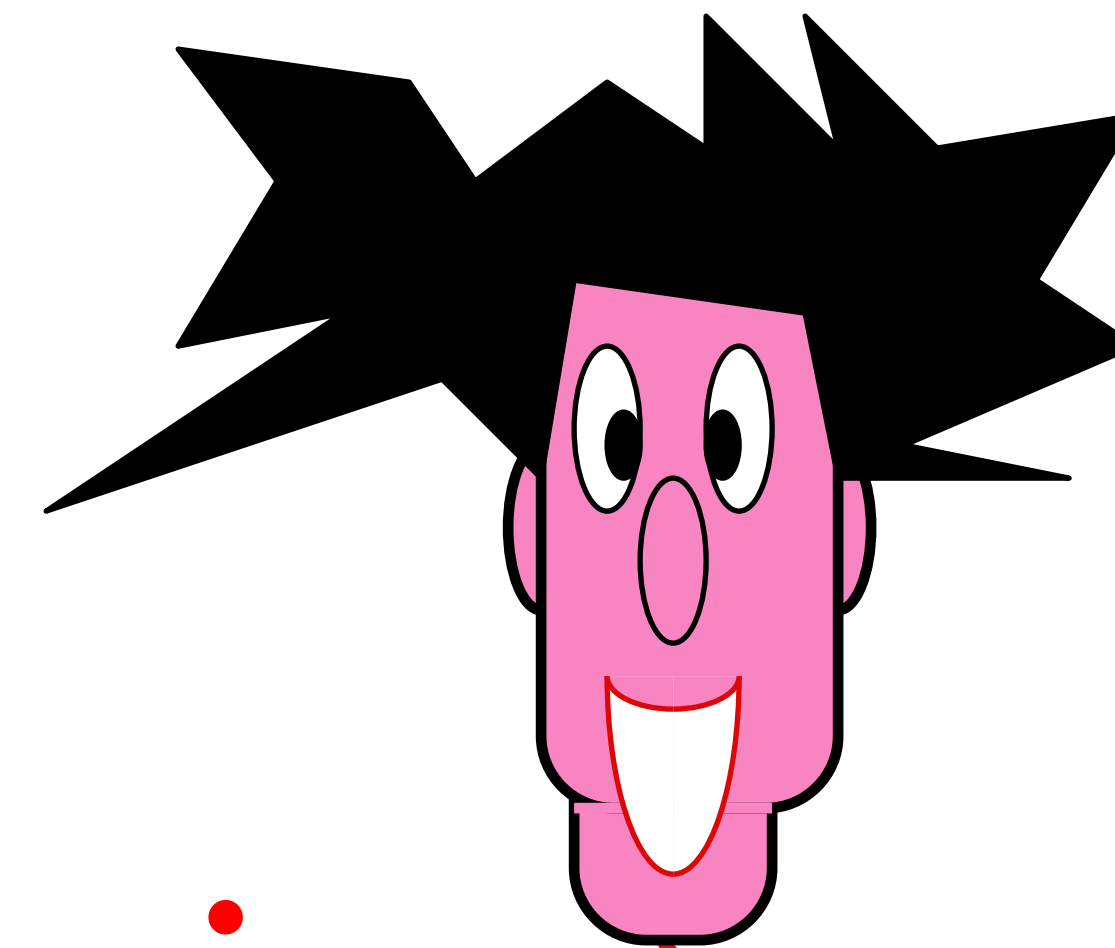
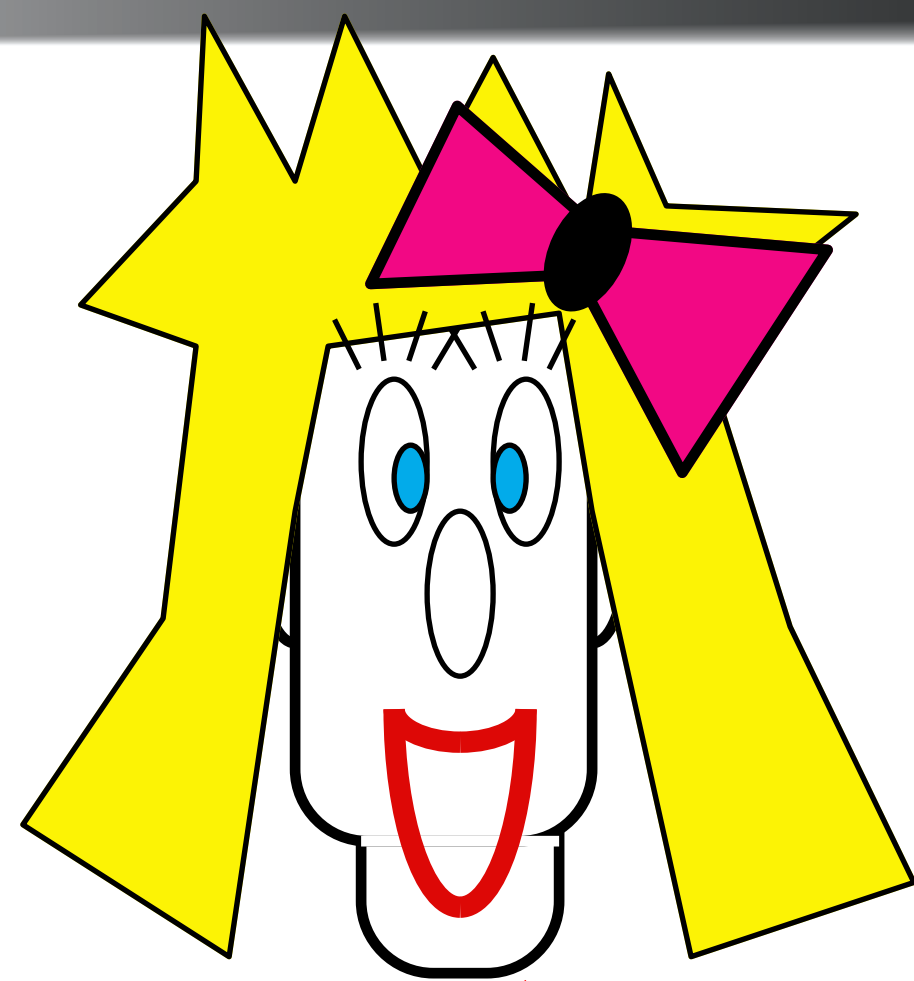
# BIT COMMITMENT



CONCEALING

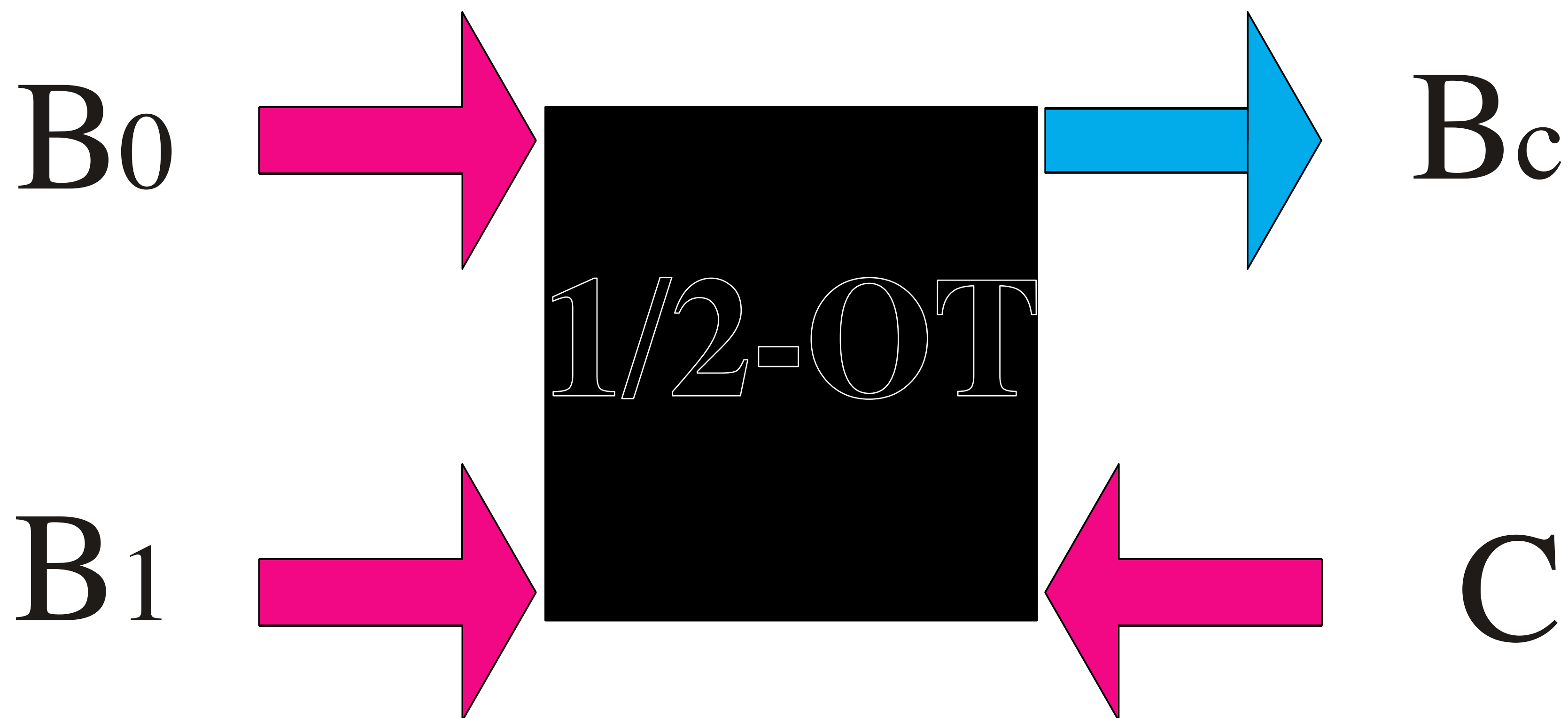
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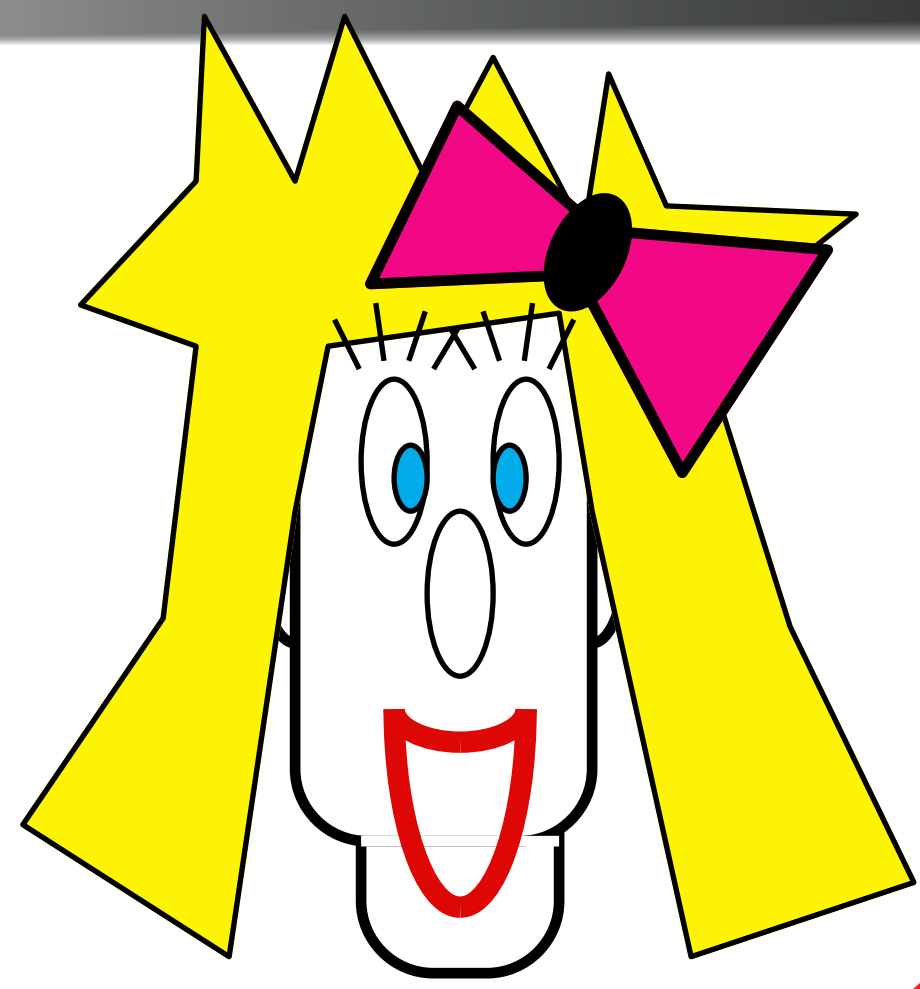




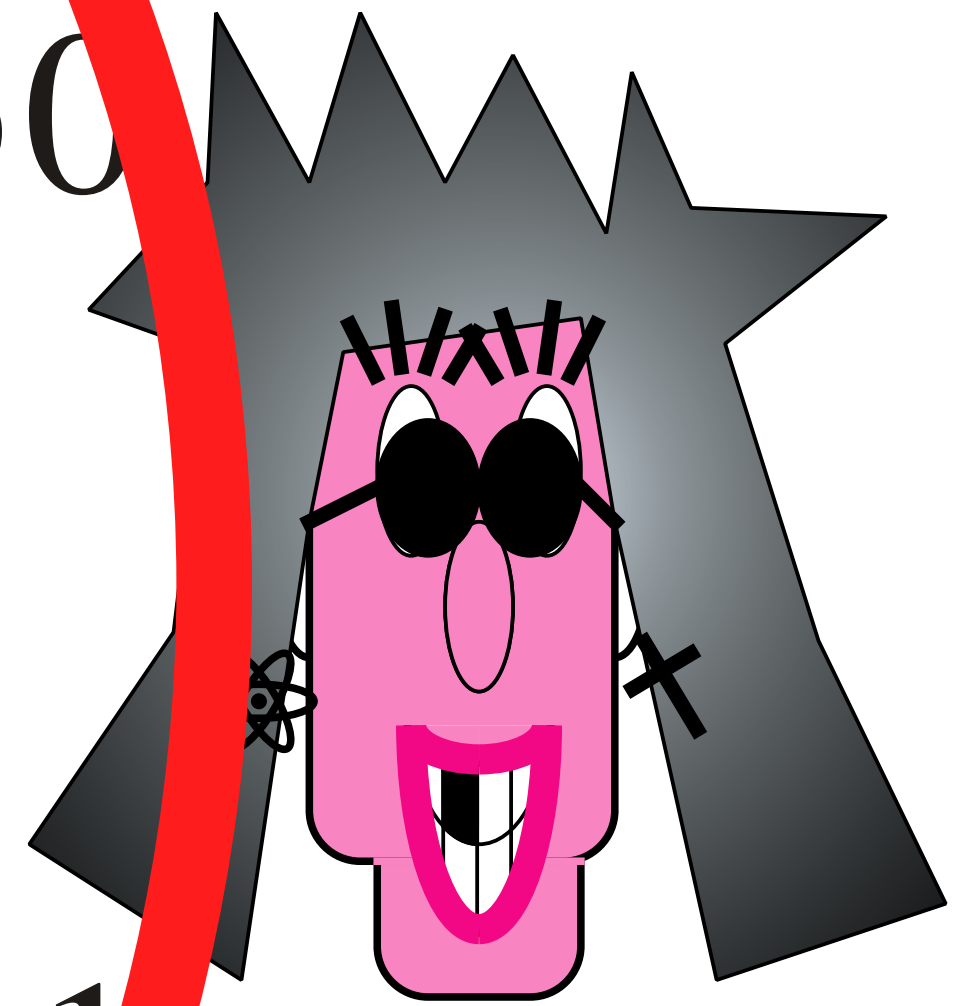
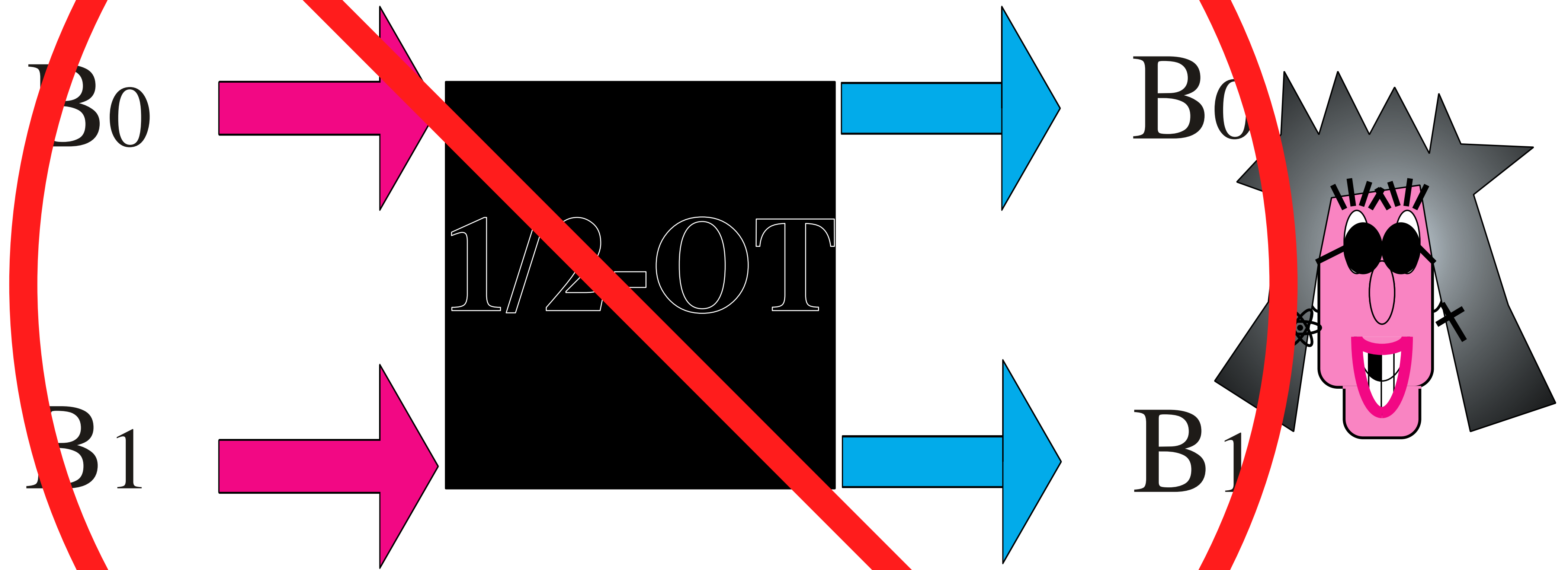
# Oblivious Transfer

(message multiplexing)

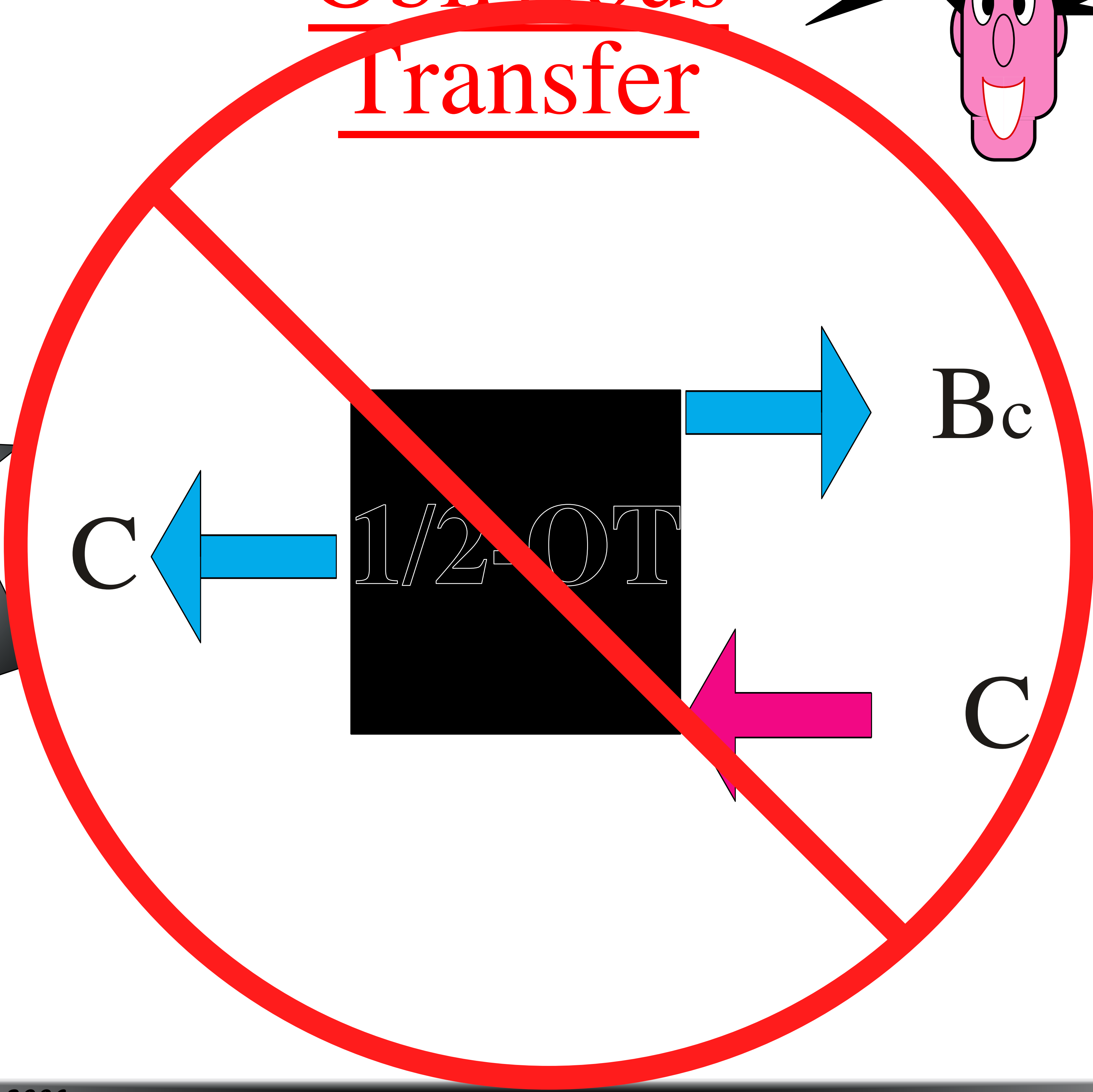
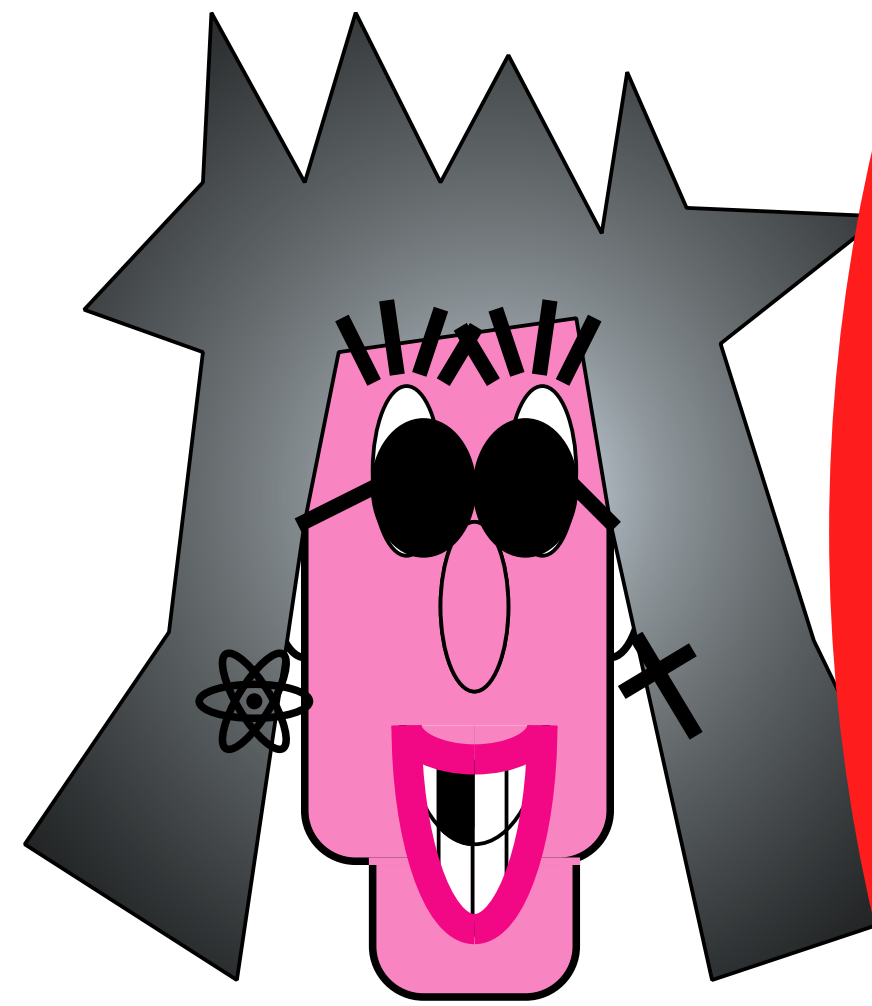
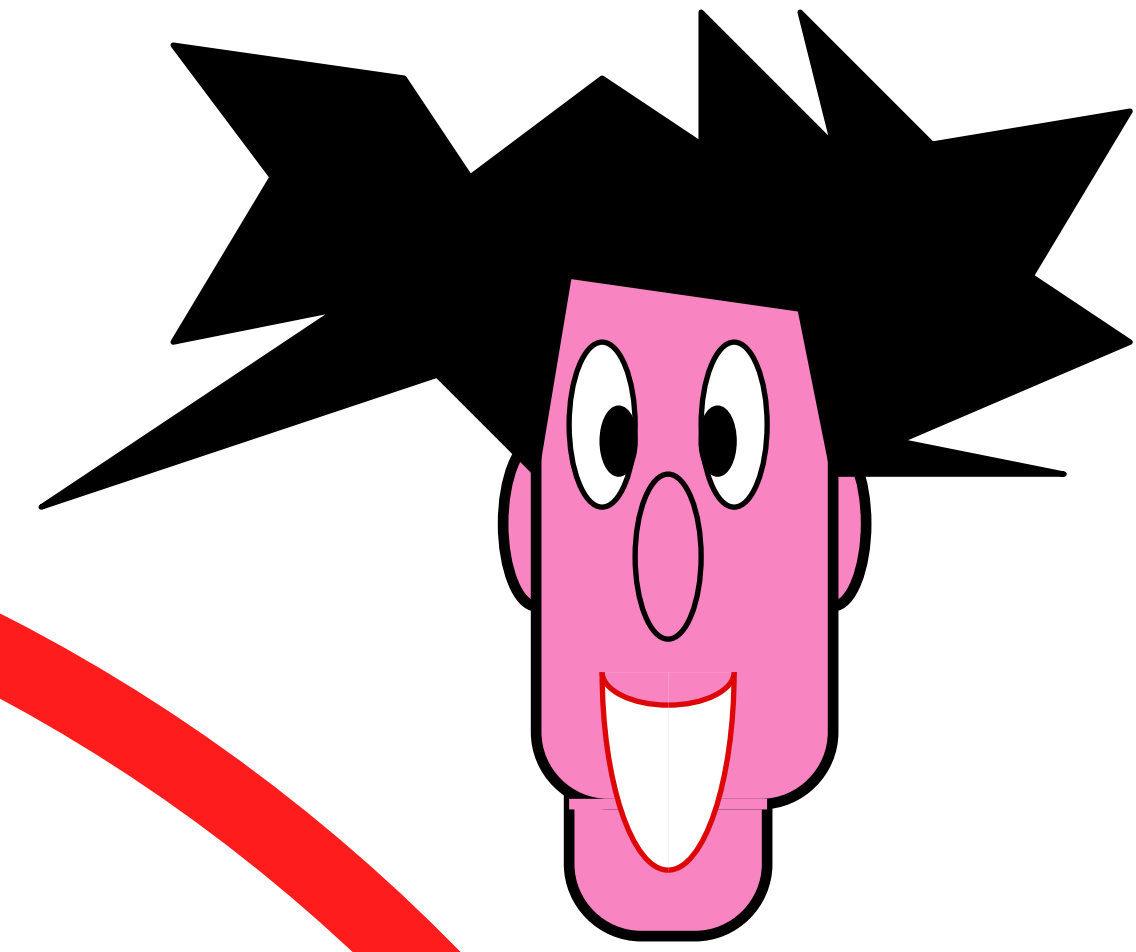


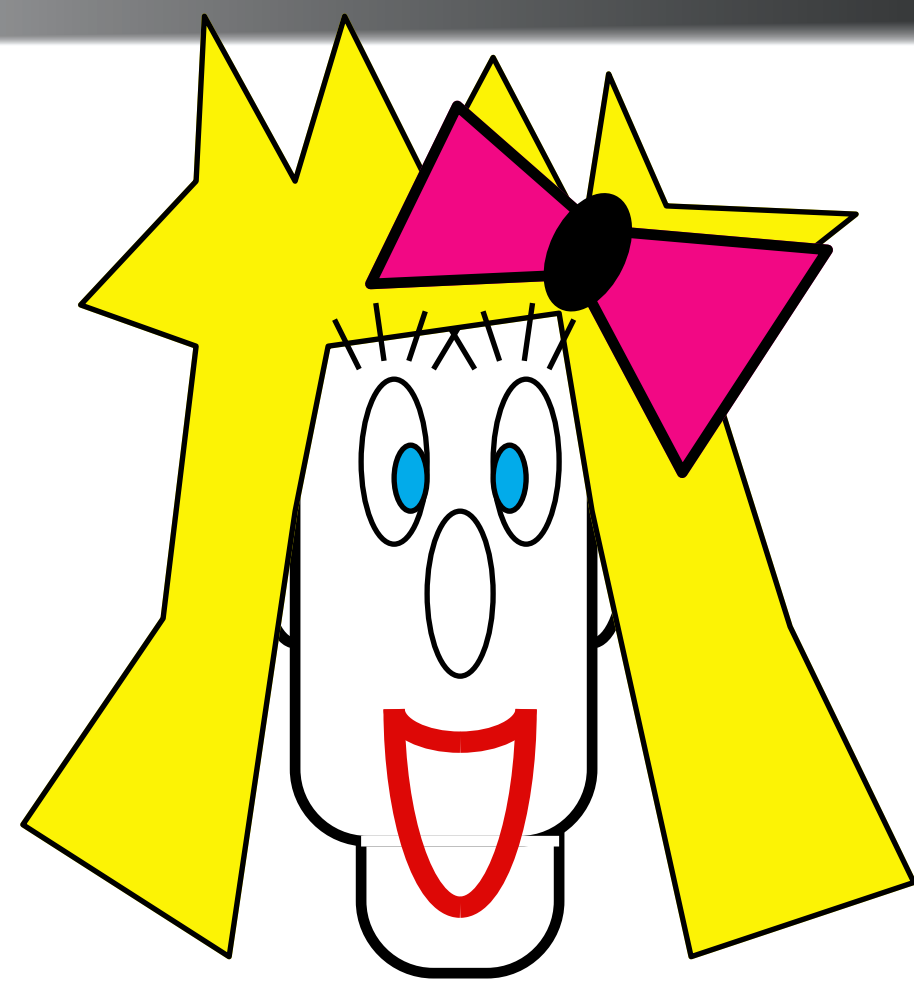


# Oblivious Transfer

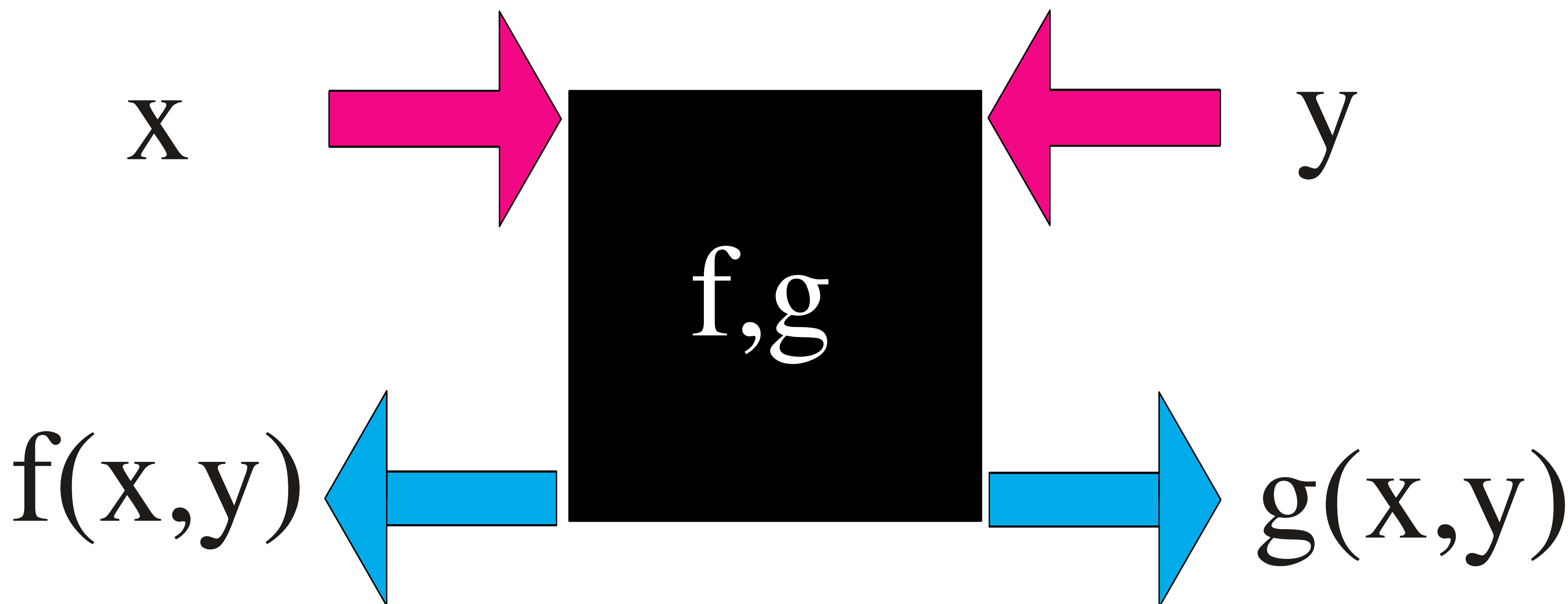
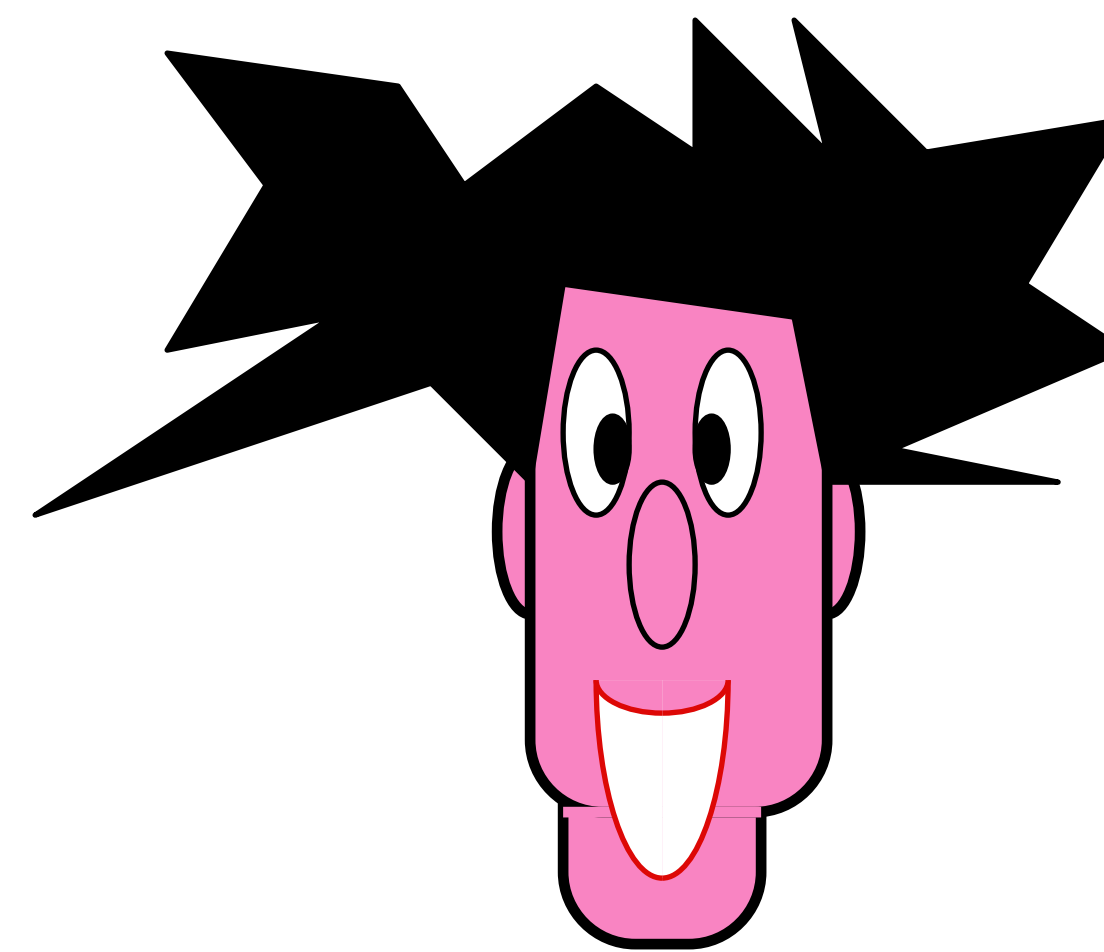


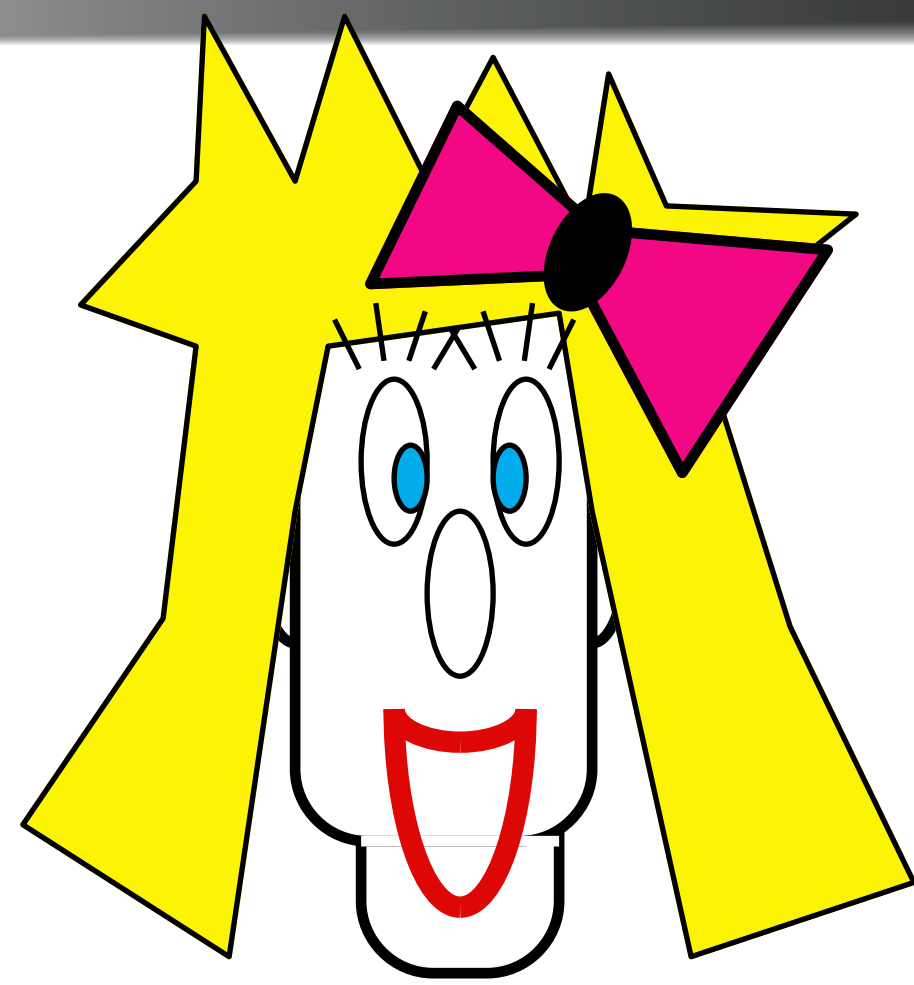
# Oblivious Transfer



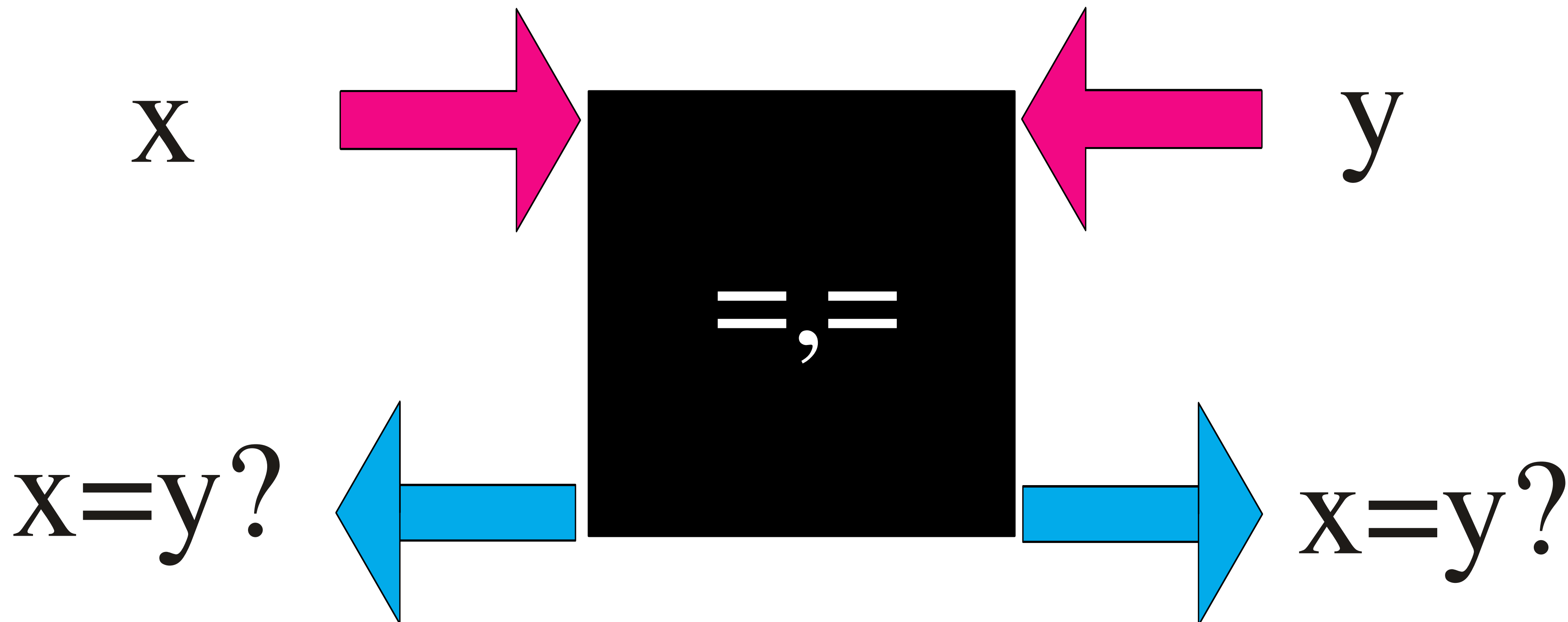
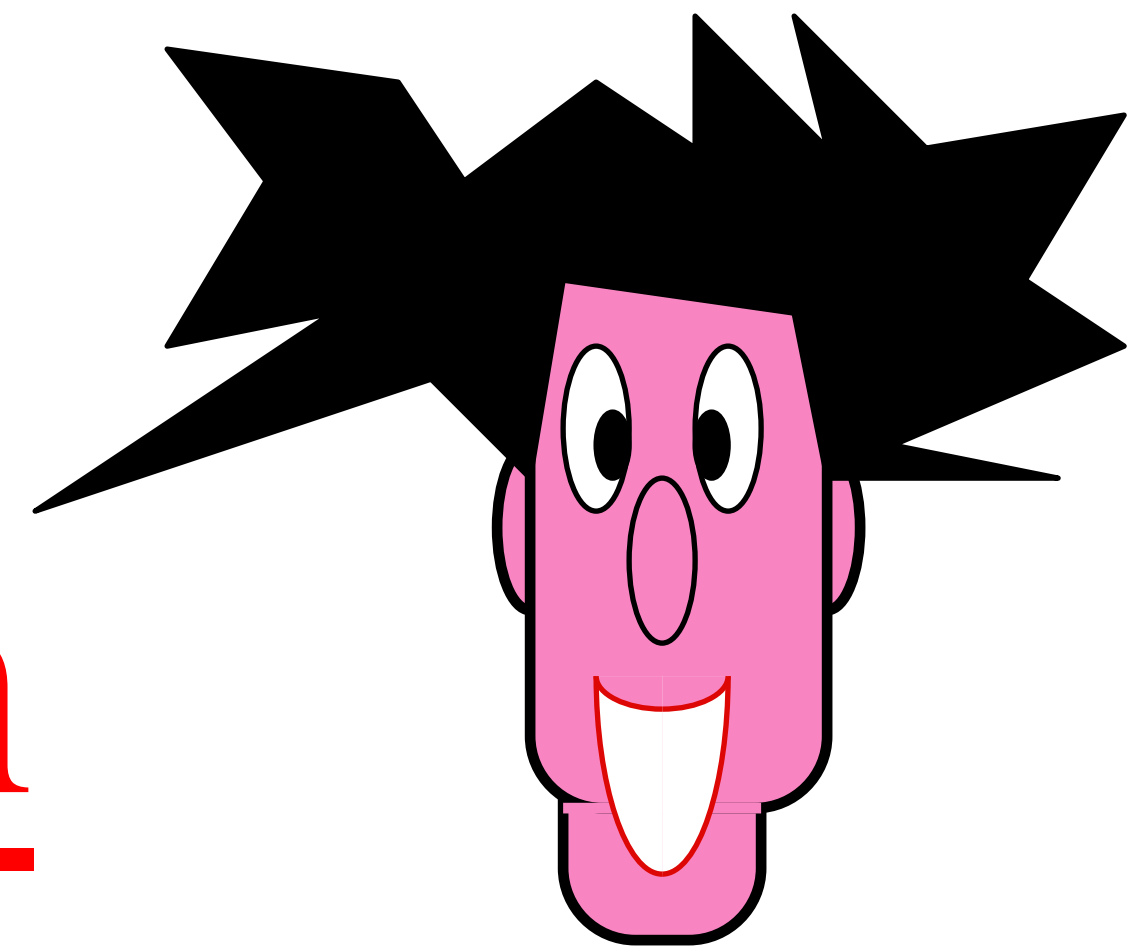


# Oblivious Function Evaluation



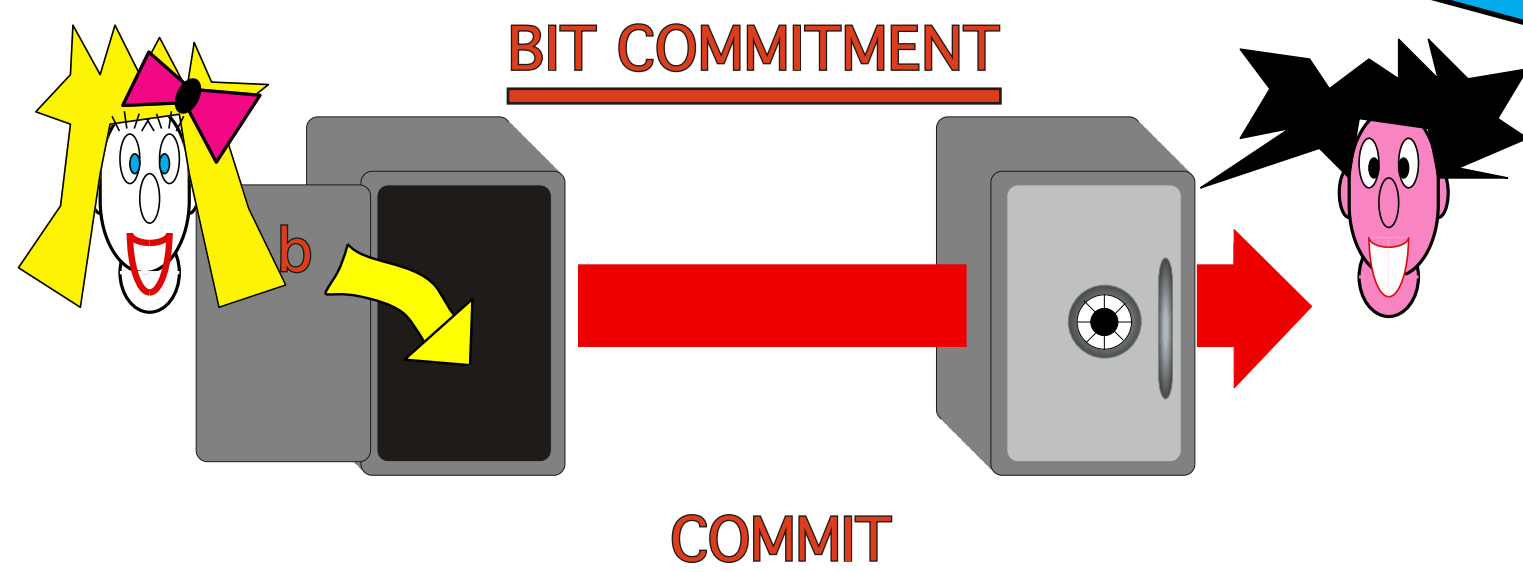
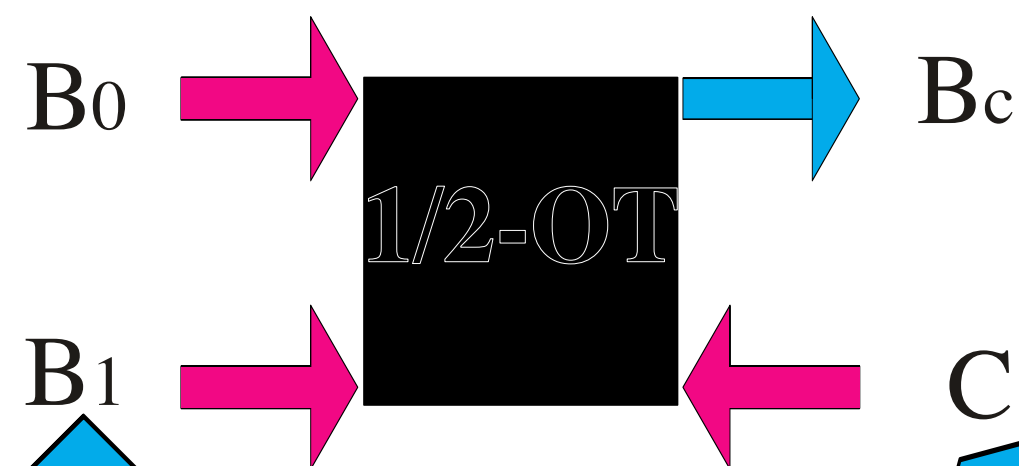


# Mutual Identification

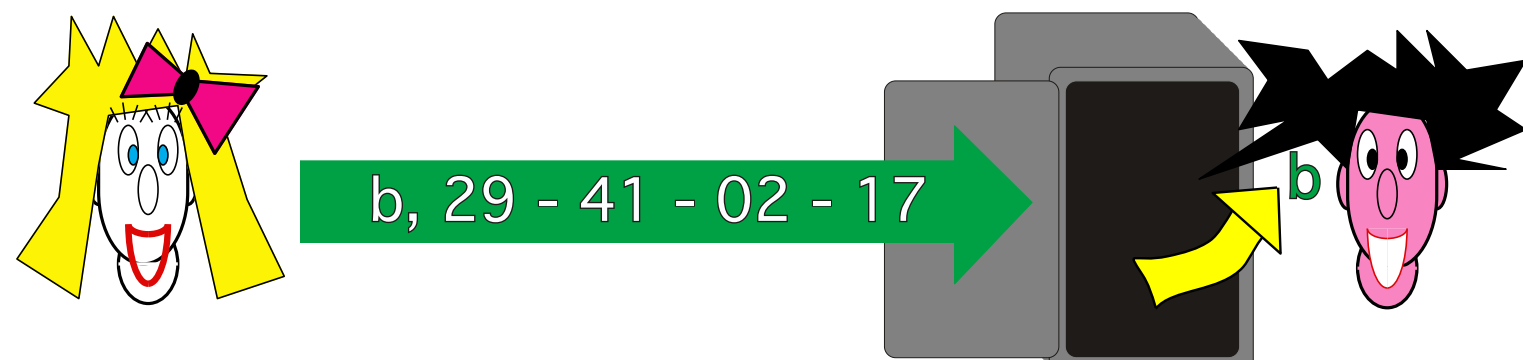


# Classically

Oblivious Transfer  
(message multiplexing)

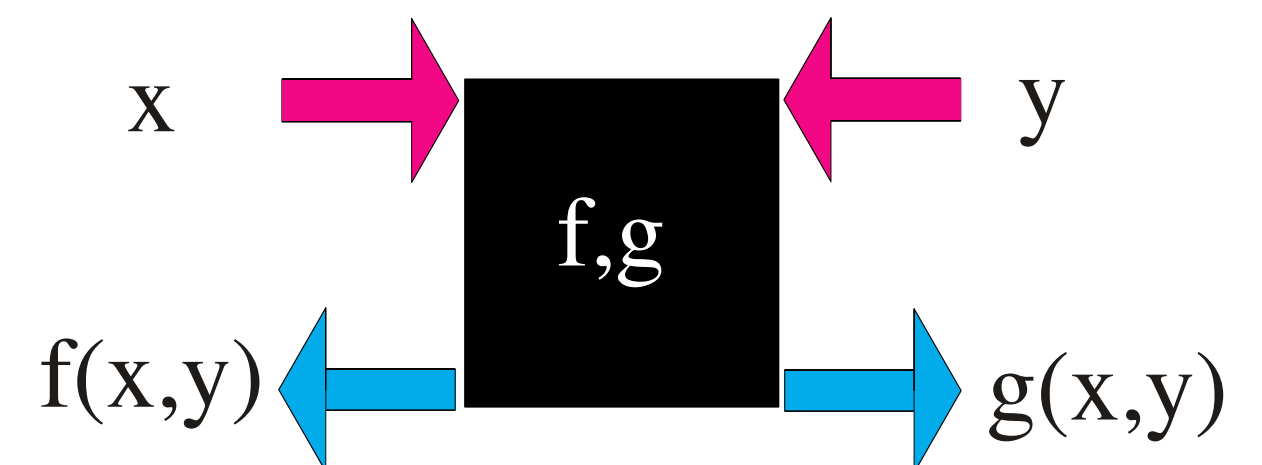


COMMIT



UNVEIL

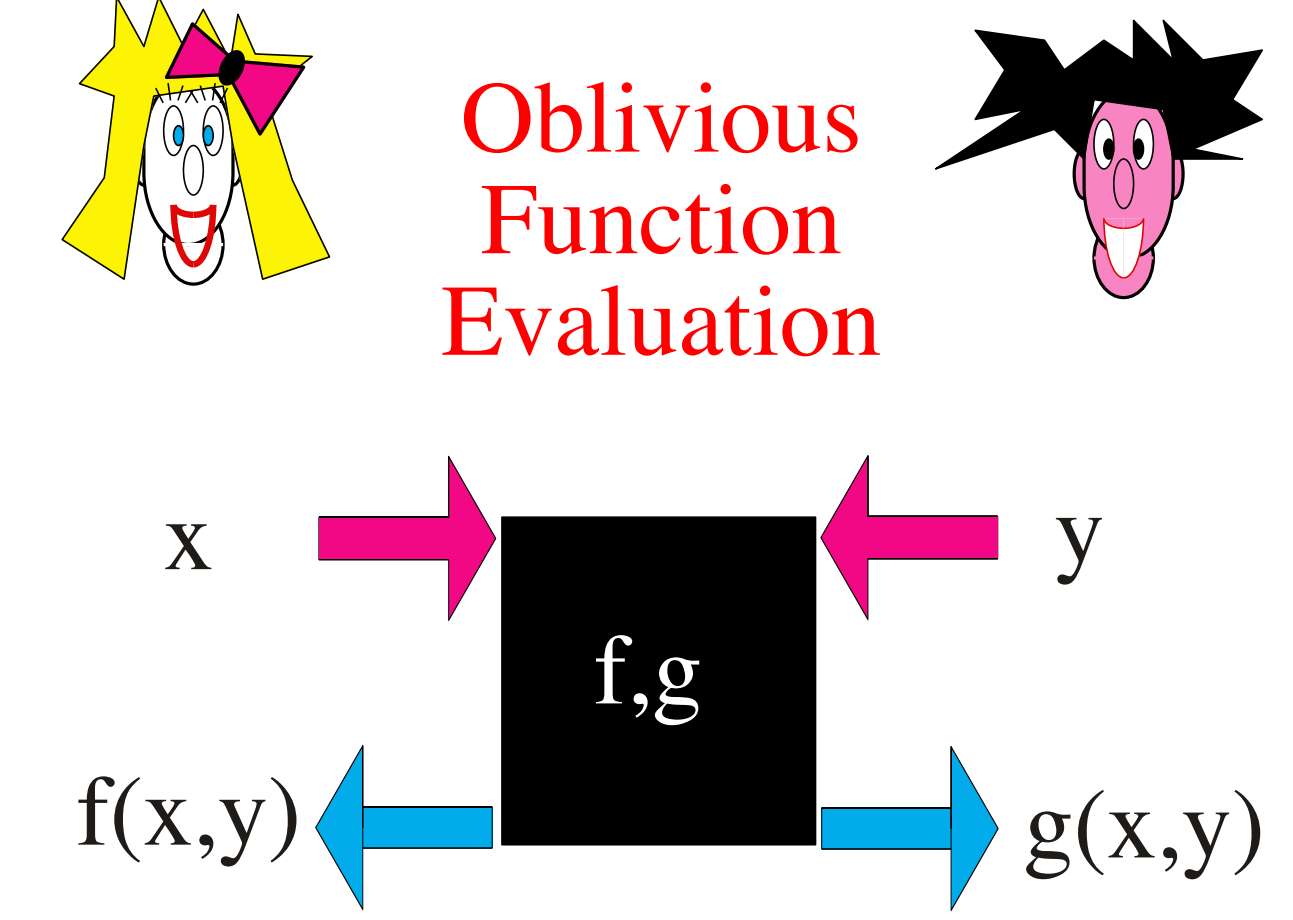
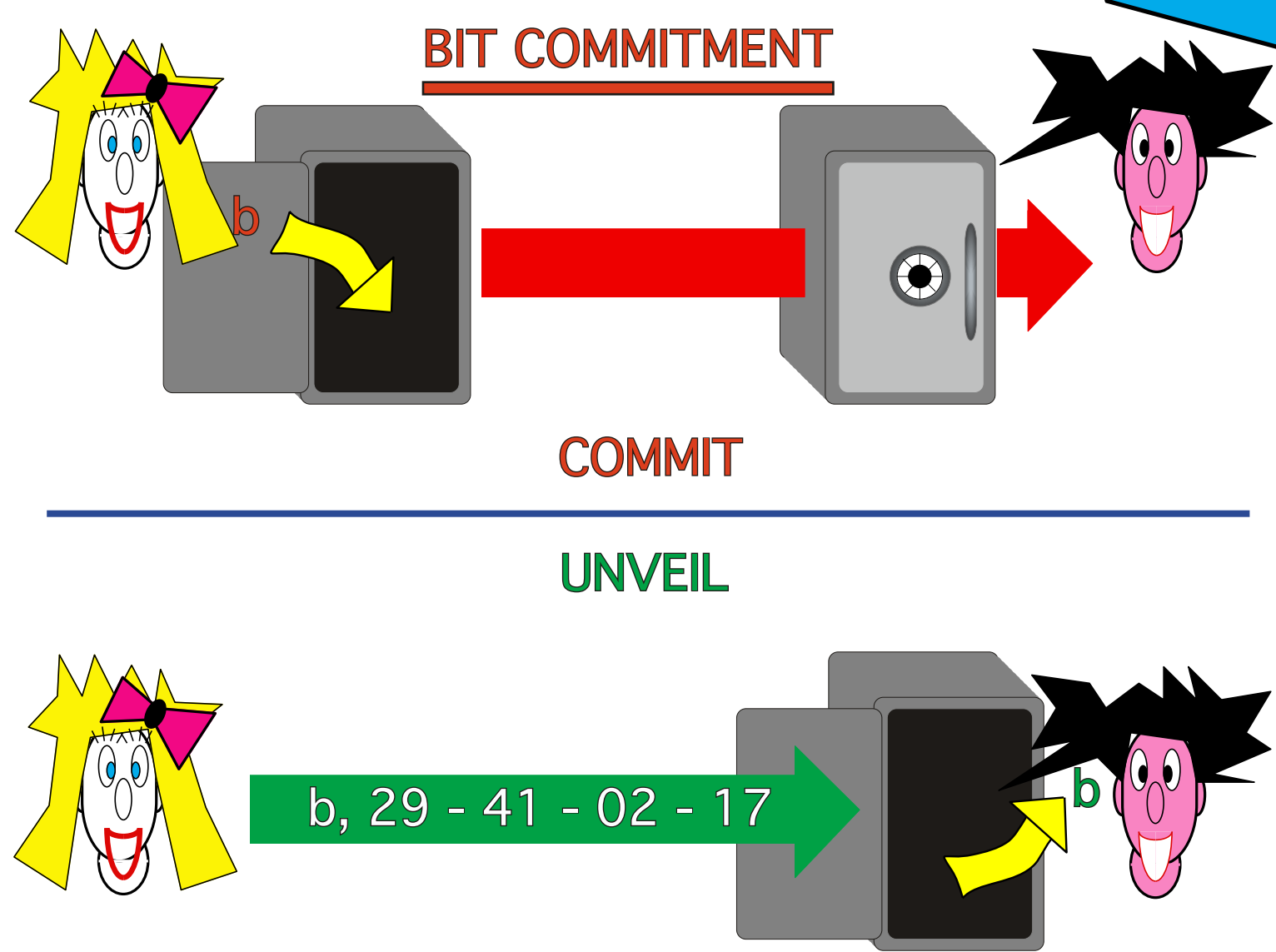
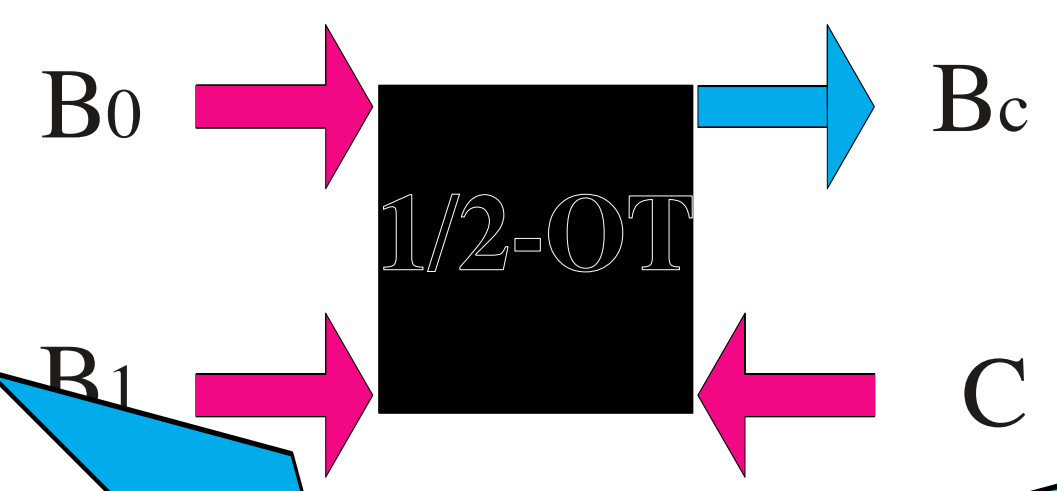
Oblivious Function Evaluation



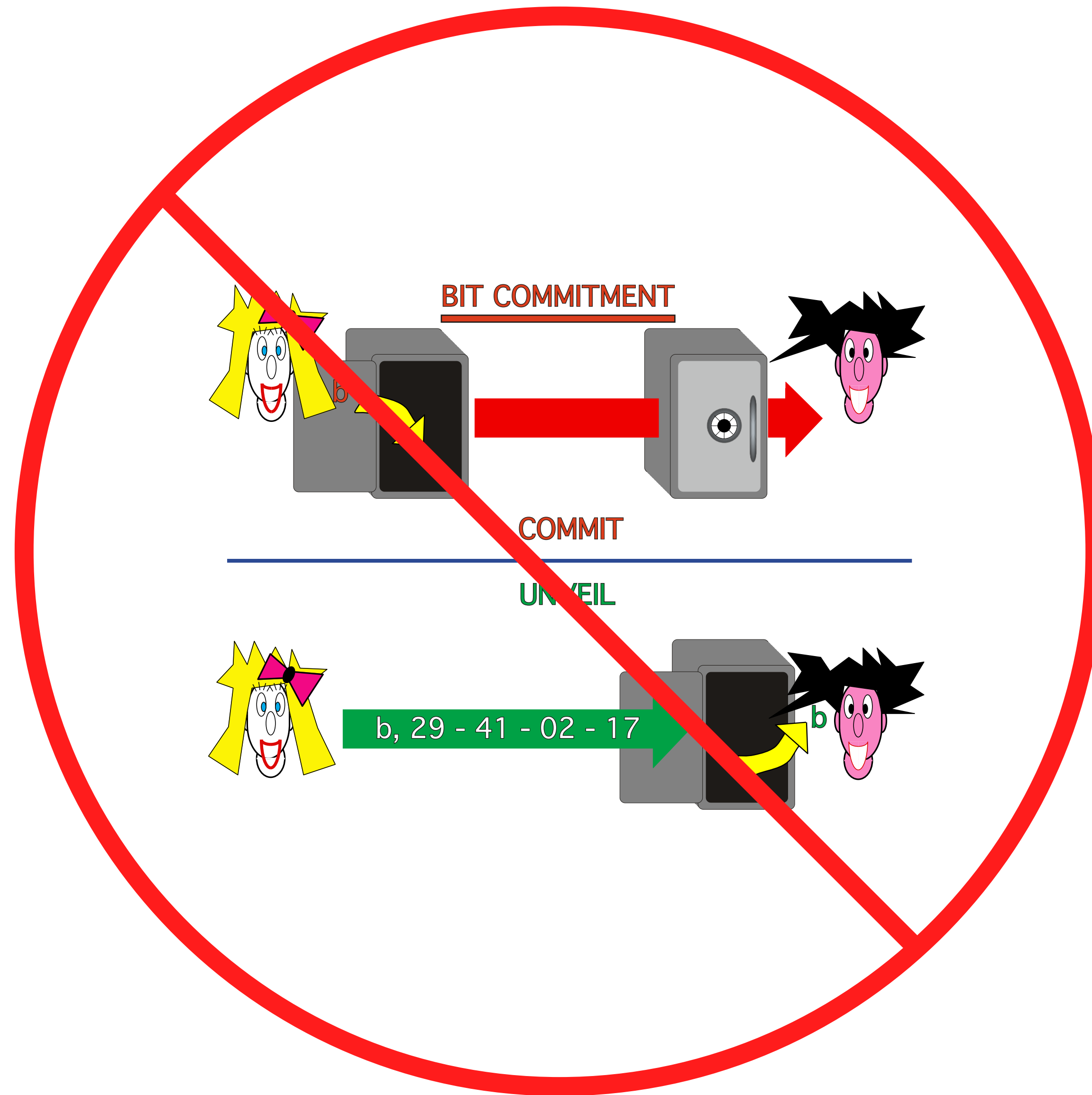


# Quantumly

Oblivious Transfer  
(message multiplexing)



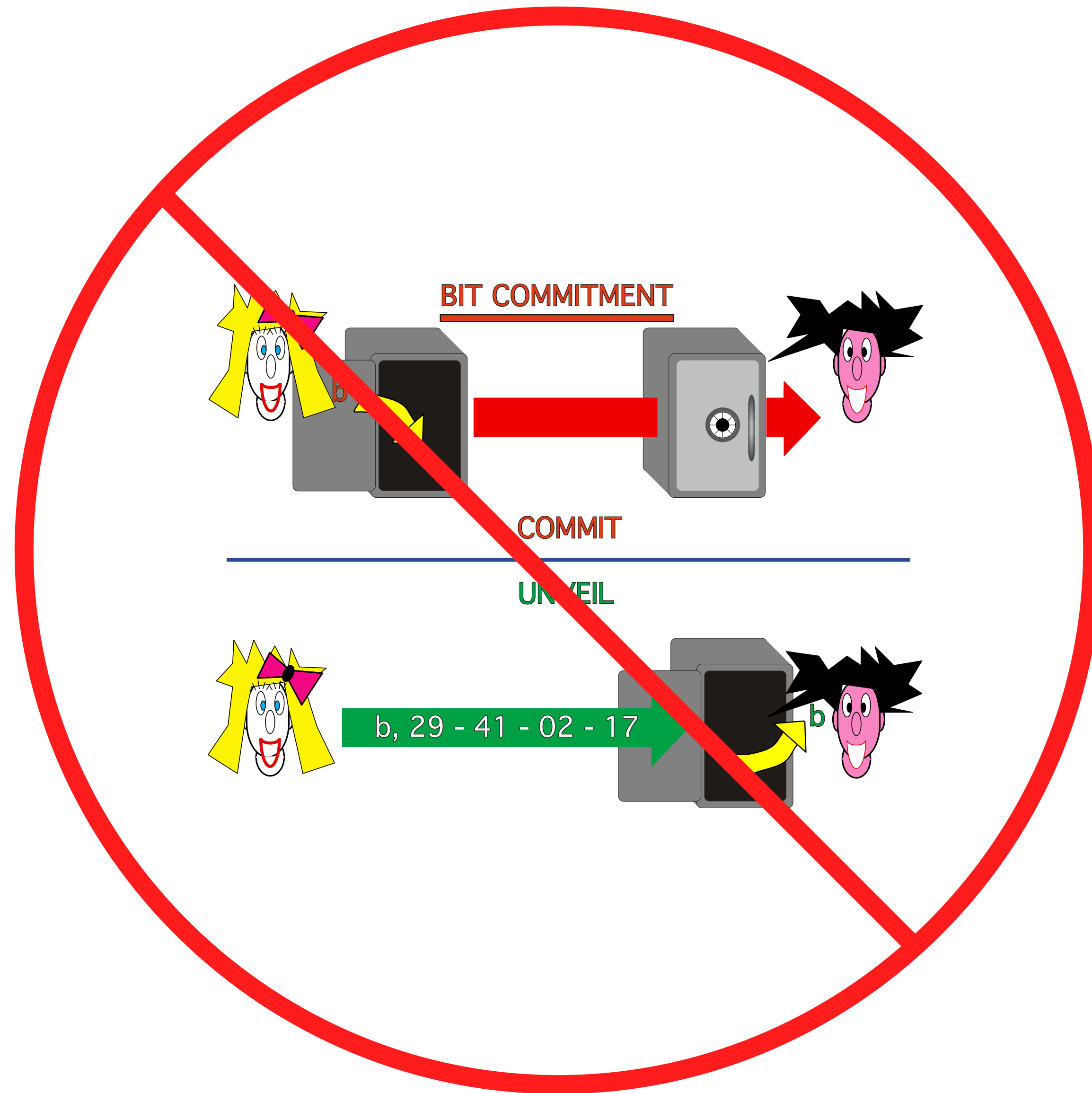
# Classically



# Folklore



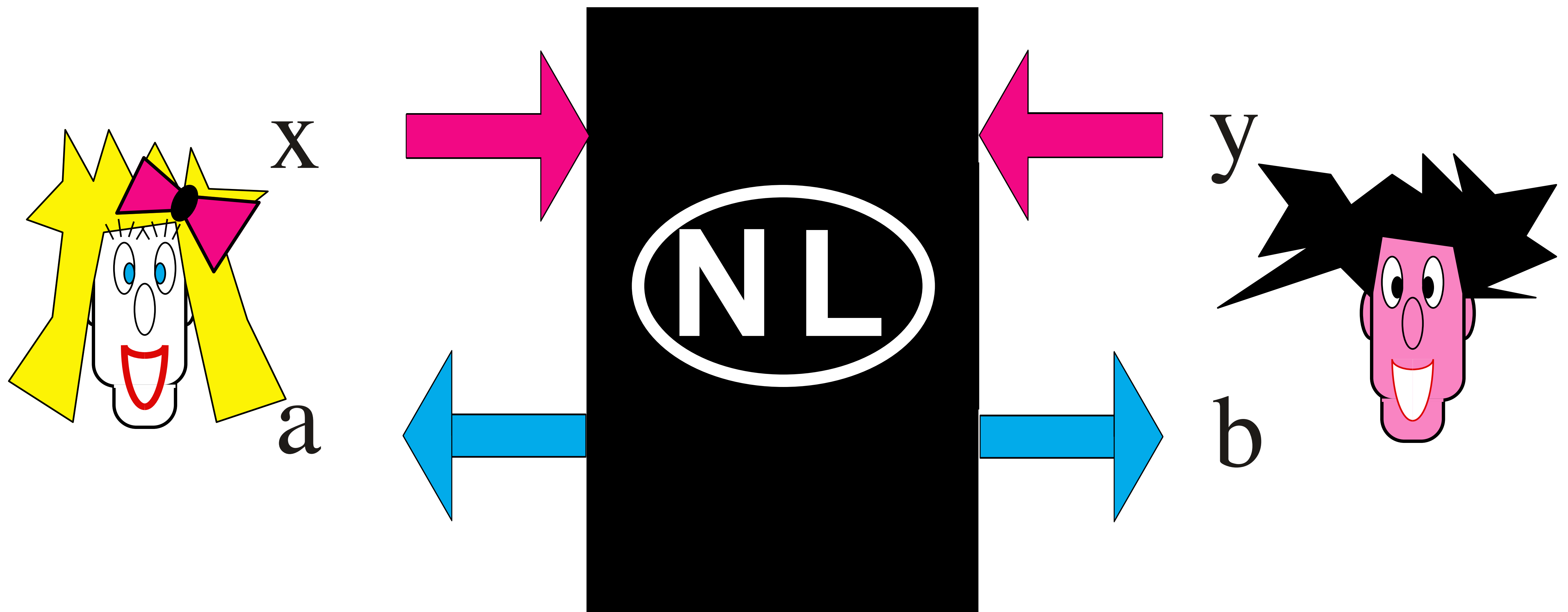
# Quantumly



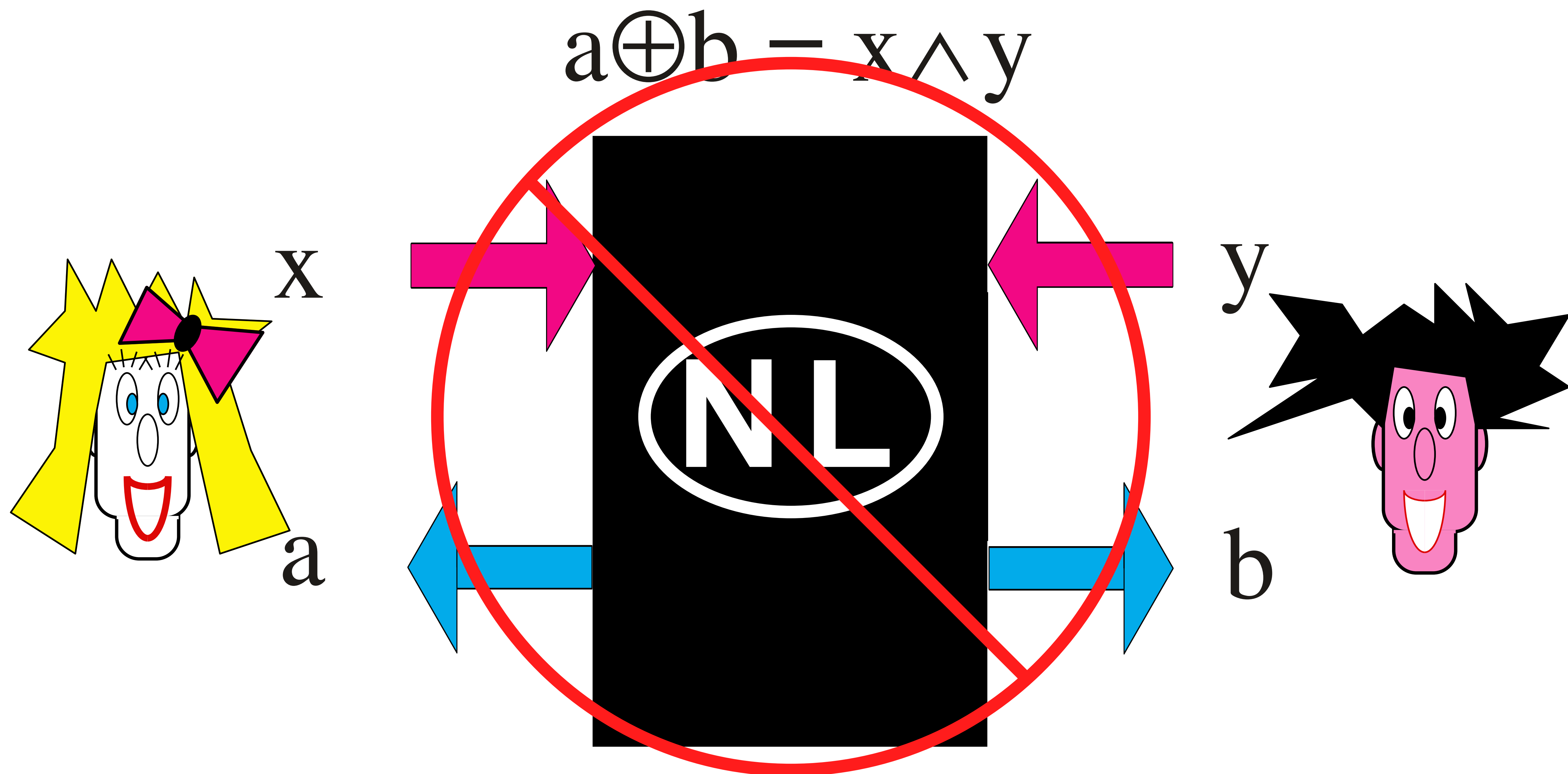
# Mayers, Lo-Chau

# Non-Locality Box

$$a \oplus b = x \wedge y$$

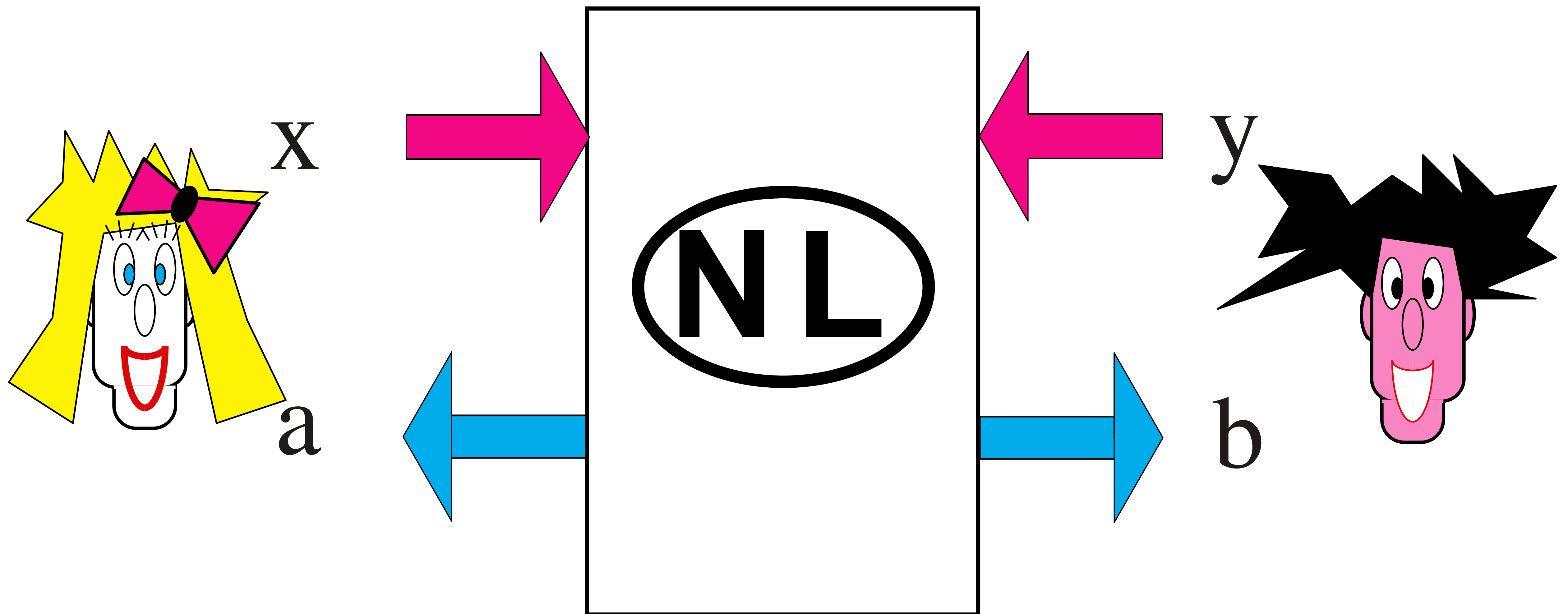


# Non-Locality Box



# Non-Locality Box

$$\Pr[a \oplus b = x \wedge y] < 1$$

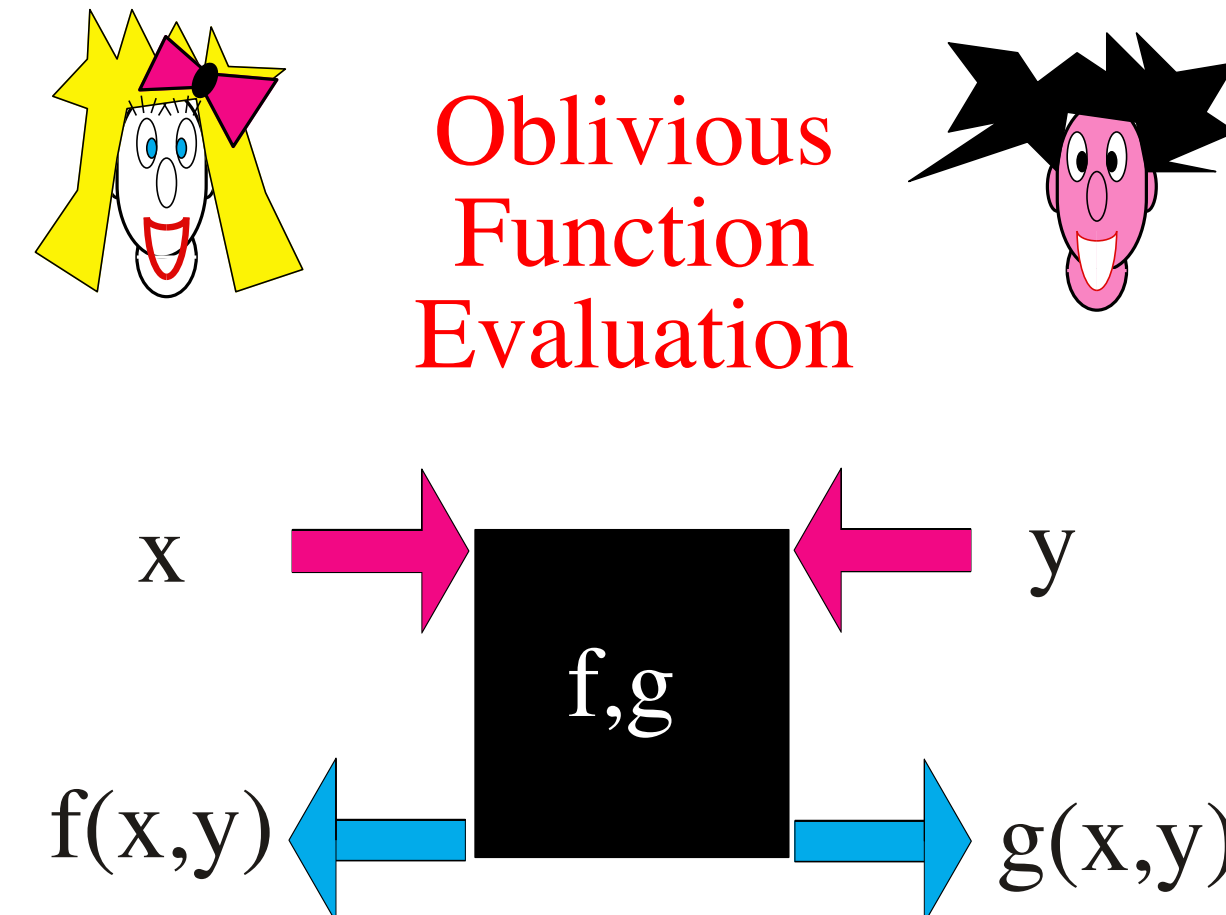
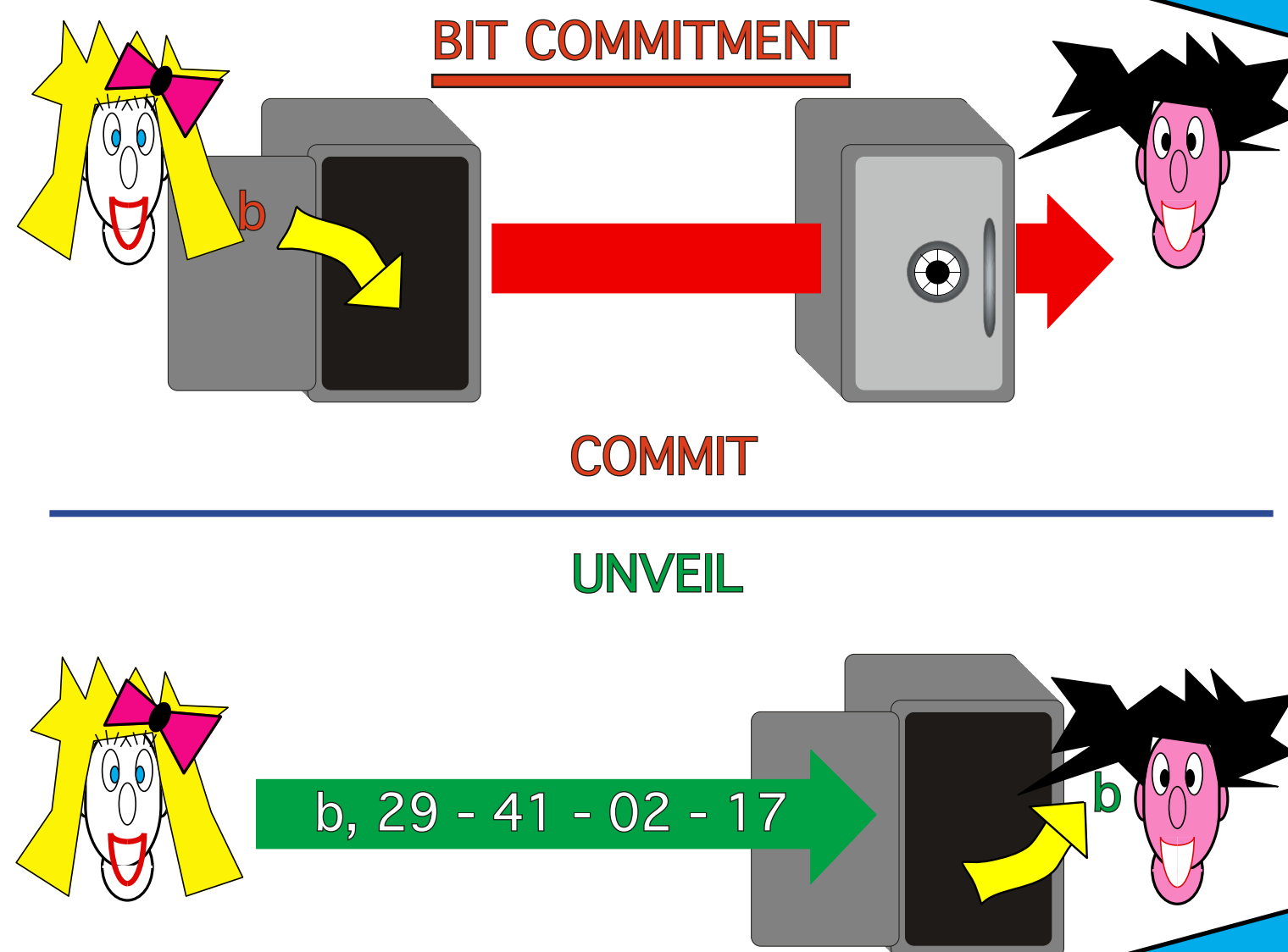
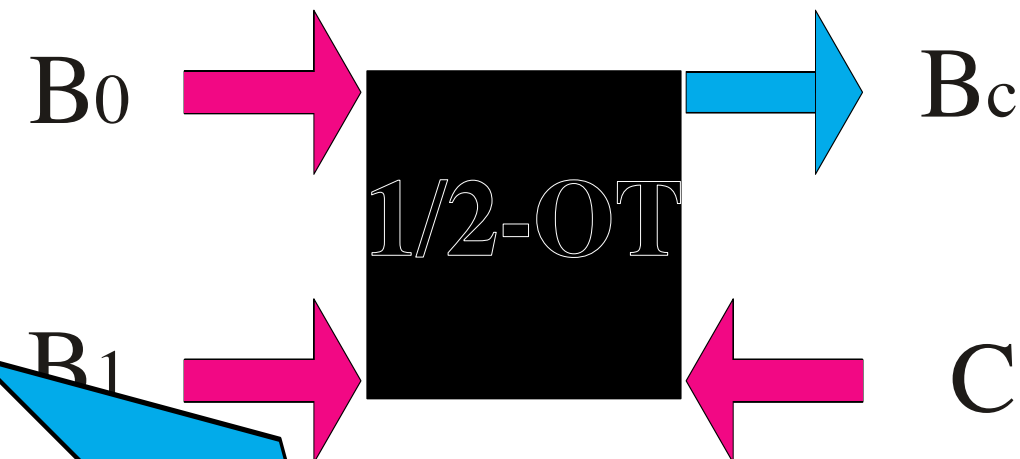


Classical:  $\Pr[\dots] = 75\%$

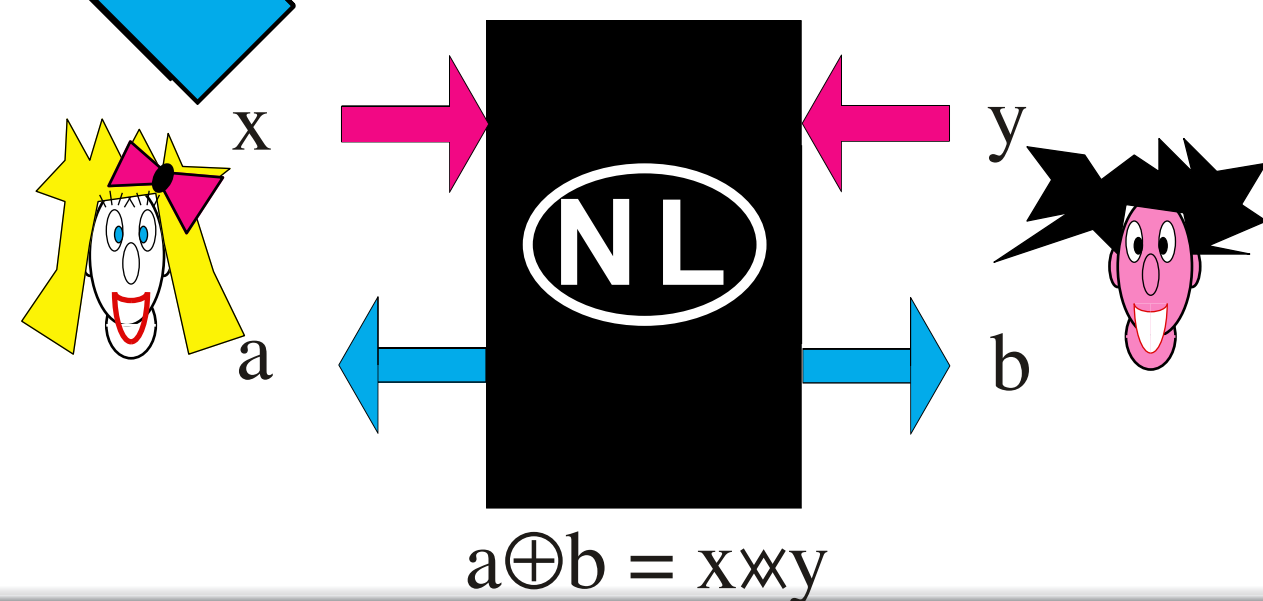
Quantum:  $\Pr[\dots] = \cos^2(\pi/8) \approx 85\%$

# Quantumly

Oblivious Transfer  
(message multiplexing)



**Non-Locality Box**



Buhrman, Christandl,  
Unger, Wehner, Winter  
(Wolf, Wullschlegler +  
Short, Gisin, Popescu)

Wolf, Wullschlegler

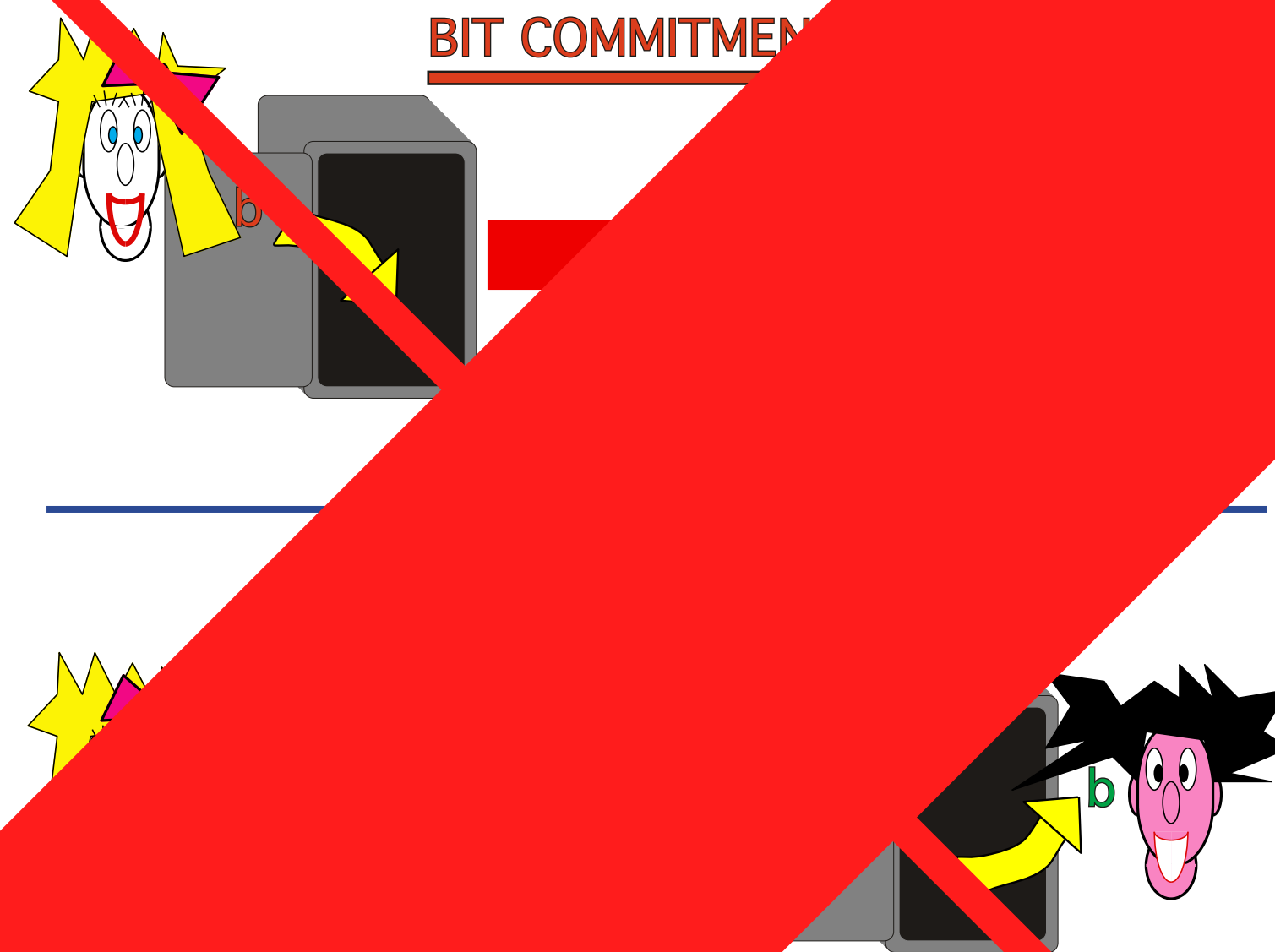
(2)

two provers

Cryptographic Protocols

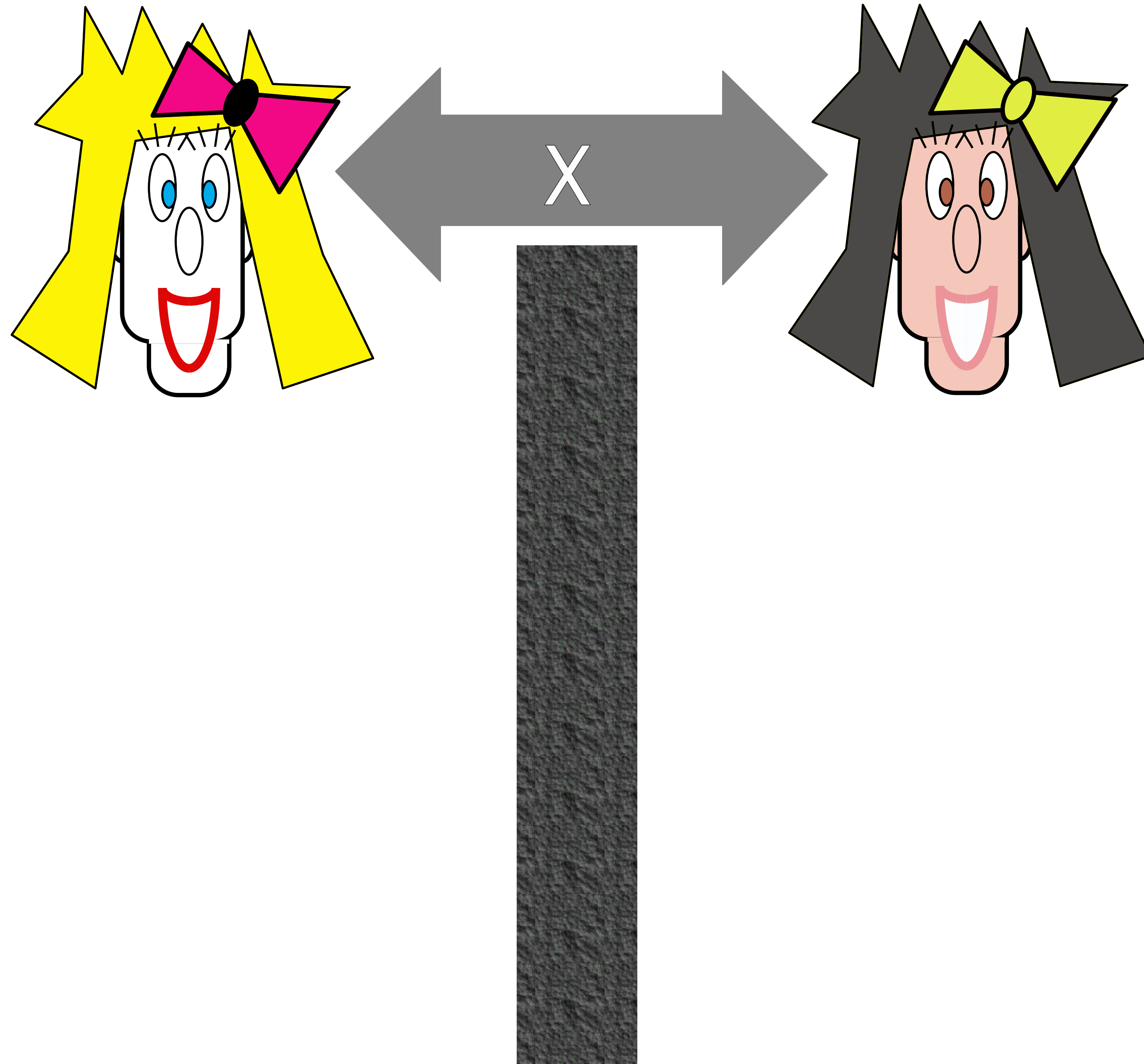


Classically

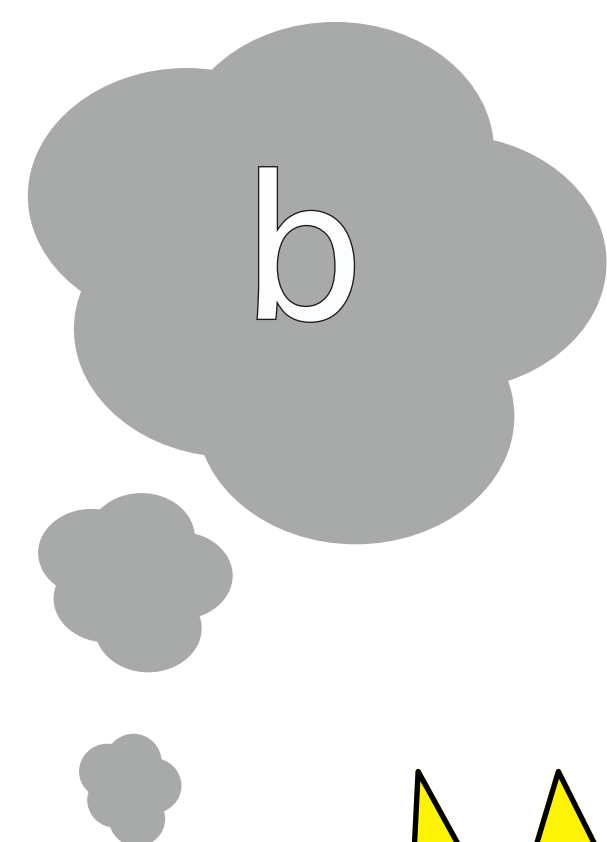


BGKW88

# Classically

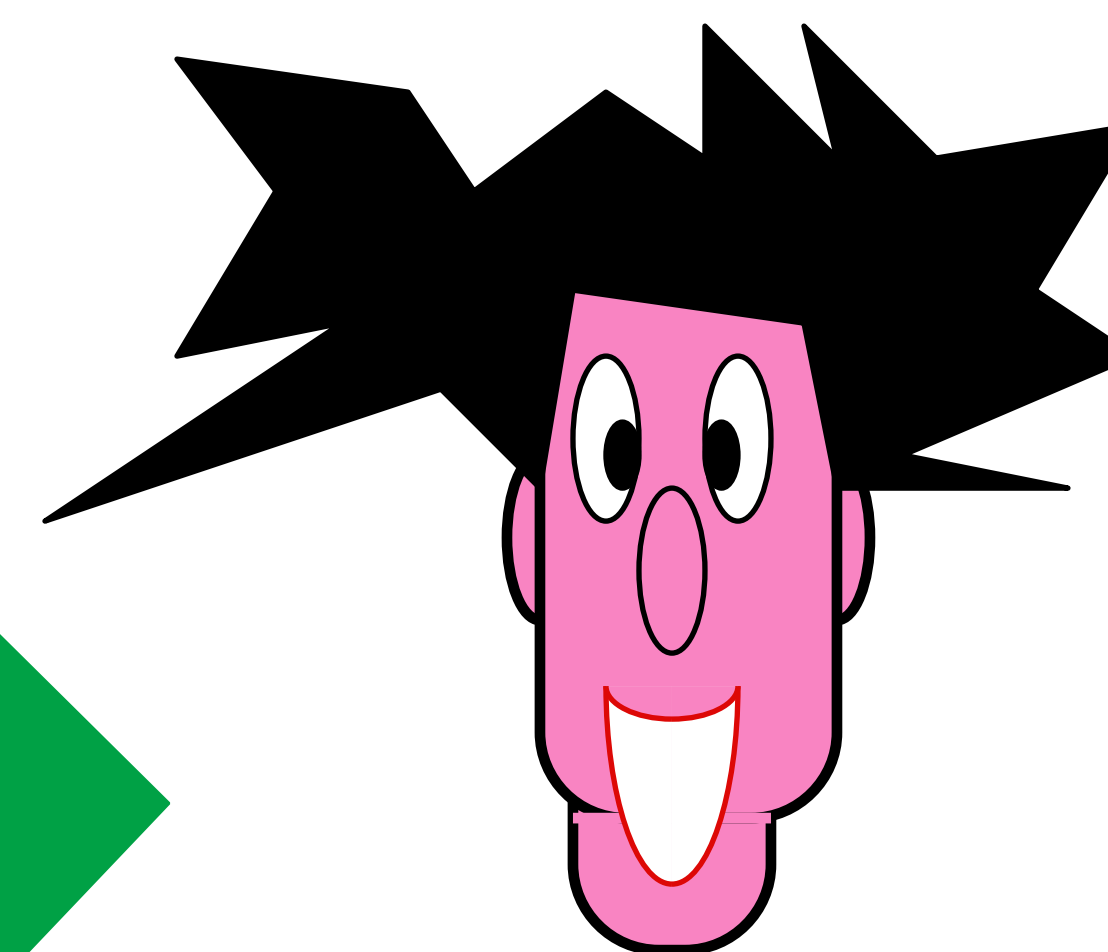
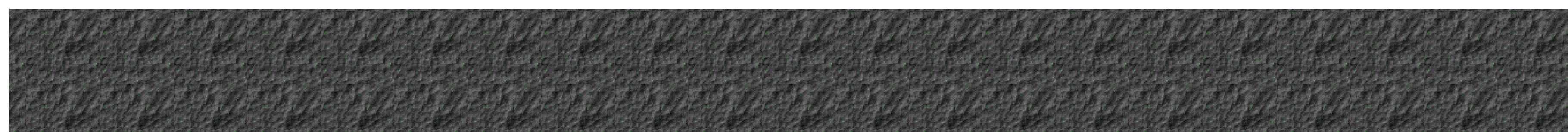
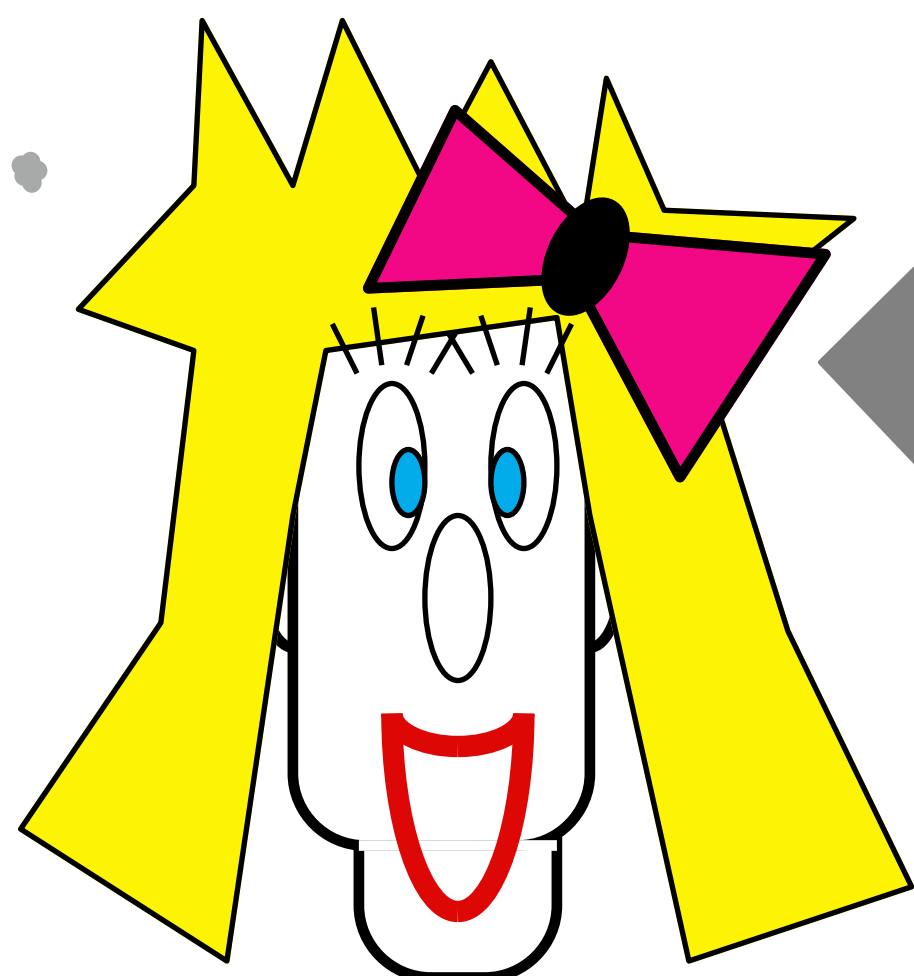


Ben-Or, Goldwasser, Kilian, Wigderson



$$z = x \quad \text{if } b = 0$$

$$z = x \oplus y \quad \text{if } b = 1$$



$$x \oplus z = b \cdot y?$$

Ben-Or, Goldwasser, Kilian, Wigderson

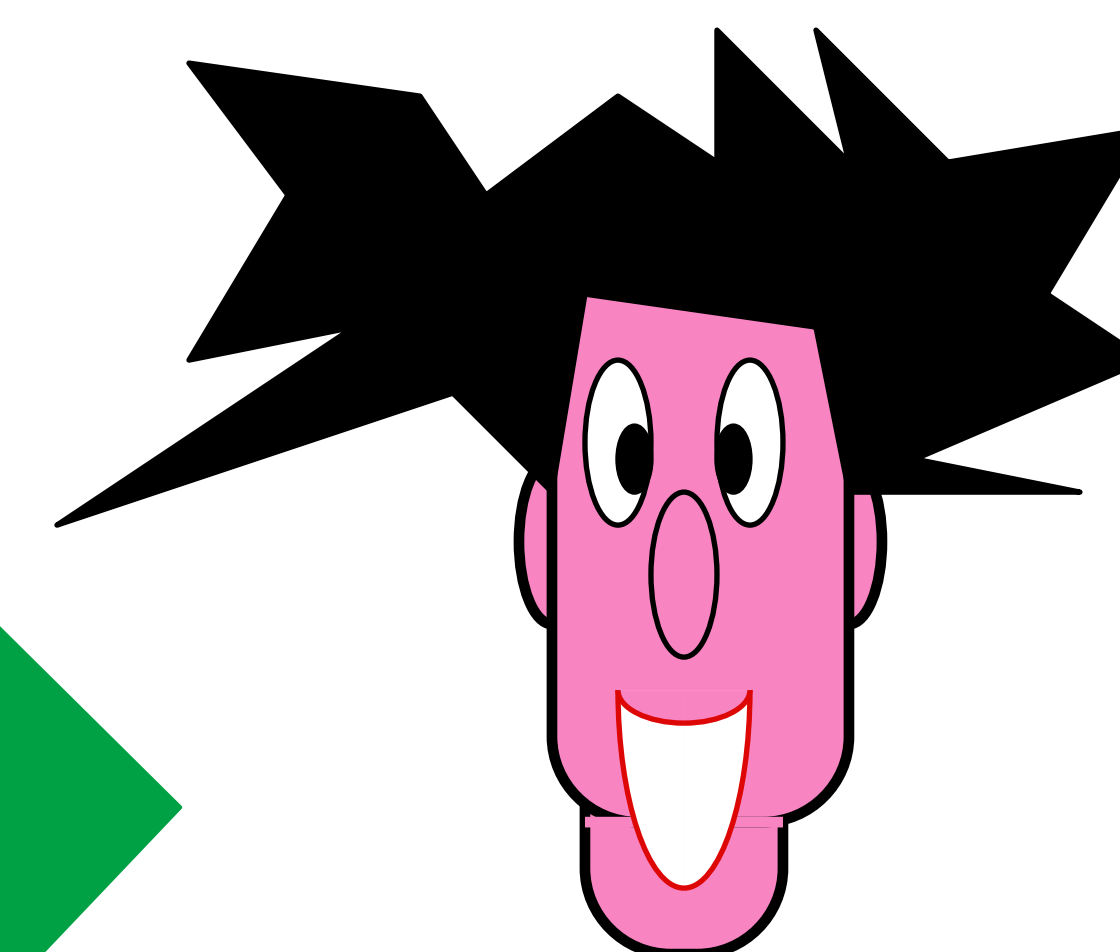
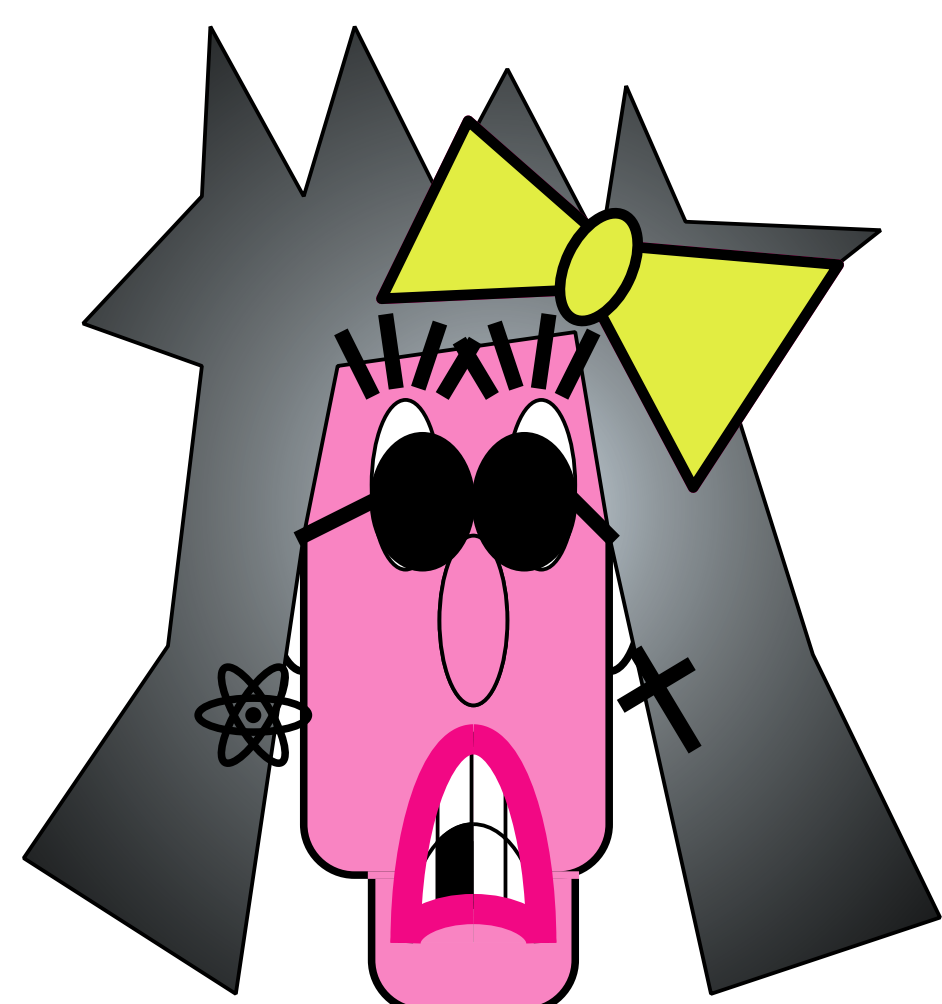
**SECURE**

$$x_0 \oplus z = 0 \cdot y = 0$$

$$x_1 \oplus z = 1 \cdot y = y$$

$$x_0 \oplus x_1 = (x_0 \oplus z) \oplus (x_1 \oplus z) = y$$

possible with prob. at most  $2^{-n}$



Ben-Or, Goldwasser, Kilian, Wigderson

## EPISODE 0

Multi-Prover Interactive Proofs:  
How to Remove Intractability AssumptionsMichael Ben-Or\*  
Hebrew UniversityShafi Goldwasser†  
MITJoe Kilian‡  
MITAvi Wigderson§  
Hebrew University**Abstract**

Quite complex cryptographic machinery has been developed based on the assumption that one-way functions exist, yet we know of only a few possible such candidates. It is important at this time to find alternative foundations to the design of secure cryptography. We introduce a new model of generalized interactive proofs as a step in this direction. We prove that all NP languages have perfect zero-knowledge proof-systems in this model, without making any intractability assumptions.

The generalized interactive-proof model consists of two computationally unbounded and untrusted provers, rather than one, who jointly agree on a strategy to convince the verifier of the truth of an assertion and then engage in a polynomial number of message exchanges with the verifier in their attempt to do so. To believe the validity of the assertion, the verifier must make sure that the two provers can not communicate with each other during the course of the proof process. Thus, the complexity assumptions made in previous work, have been traded for a physical separation between the two provers.

\*Supported by Alon Fellowship.

†Supported in part by NSF grant 865727-CCR, ARO grant DAAL03-86-K-017, and US-Israel BSF grant 86-00301, Jerusalem, Israel.

‡Supported by a Fannie and John Hertz Foundation fellowship.

§Supported by Alon Fellowship

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We call this new model the multi-prover interactive-proof model, and examine its properties and applicability to cryptography.

**1 Introduction**

The notion of randomized and interactive proof system, extending NP, was introduced in [GMR] and in [B]. An interactive proof-system consists of an all powerful prover who attempts to convince a probabilistic polynomial-time bounded verifier of the truth of a proposition. The prover and verifier receive a common input and can exchange upto a polynomial number of messages, at the end of which the verifier either accepts or rejects the input. Several examples of interactive proof-system for languages not known to be in NP (e.g graph non-isomorphism) are known.

In [GMW1] Goldreich, Micali and Wigderson show the fundamental result that that if "non-uniform" one-way functions exist (i.e no small circuits exist for the function inverse computation), then every NP language has a computationally zero-knowledge interactive proof system. This has far reaching implications concerning the secure design of cryptographic protocols. It also seems to be the strongest result possible. Results in [F] and [BHZ] imply that if perfect zero-knowledge interactive proof-systems for NP exist, (i.e which do not rely on the fact that the verifier is polynomial time bounded) then the polynomial time hierarchy would collapse to its second level. This provides strong evidence that it will be impossible (and at least very hard) to unconditionally show that NP has zero-knowledge interactive proofs.

In light of the above negative results, it is interesting to examine whether the definition of interactive proofs can be modified so as to still capture

## EPISODE 0

Multi-Prover Interactive Proofs:  
How to Remove Intractability AssumptionsMichael Ben-Or\*  
Hebrew UniversityShafi Goldwasser†  
MITJoe Kilian‡  
MITAvi Wigderson§  
Hebrew University**Abstract**

Quite complex cryptographic machinery has been developed based on the assumption that one-way functions exist, yet we know of only a few possible such candidates. It is important at this time to find alternative foundations to the design of secure cryptography. We introduce a new model of generalized interactive proofs as a step in this di-

mial number of message exchanges with the verifier in their attempt to do so. To believe the validity of the assertion, the verifier must make sure that the two provers can not communicate with each other during the course of the proof process. Thus, the complexity assumptions made in previ-

Thus, the complexity assumptions made in previous work, have been traded for a physical separation between the two provers.

\*Supported by Alon Fellowship.

†Supported in part by NSF grant 865727-CCR, ARO grant DAAL03-86-K-017, and US-Israel BSF grant 86-00301, Jerusalem, Israel.

‡Supported by a Fannie and John Hertz Foundation fellowship.

§Supported by Alon Fellowship

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We call this new model the multi-prover interactive-proof model, and examine its properties and applicability to cryptography.

**1 Introduction**

The notion of randomized and interactive proof system, extending NP, was introduced in [GMR]

active proof-system consists of a prover who attempts to convince a polynomial-time bounded verifier of a proposition. The prover and verifier receive the same input and can exchange up to a polynomial number of messages, at the end of which the verifier accepts or rejects the input. The complexity class of interactive proof-systems is known to be in NP (e.g. graph isomorphism is known to be in NP).

Frei, Miceli and Wigderson proved a negative result that if "non-uniform" circuits exist (i.e. no small circuits exist for the function inverse computation),

then every NP language has a computationally zero-knowledge interactive proof system. This has far reaching implications concerning the secure design of cryptographic protocols. It also seems to be the strongest result possible. Results in [F] and [BHZ] imply that if perfect zero-knowledge interactive proof-systems for NP exist, (i.e. which do not rely on the fact that the verifier is polynomial time bounded) then the polynomial time hierarchy would collapse to its second level. This provides strong evidence that it will be impossible (and at least very hard) to unconditionally show that NP has zero-knowledge interactive proofs.

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## Abstract

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grant DAAL03-86-K-017, and US-Israel BSF grant 86-00301, Jerusalem, Israel.

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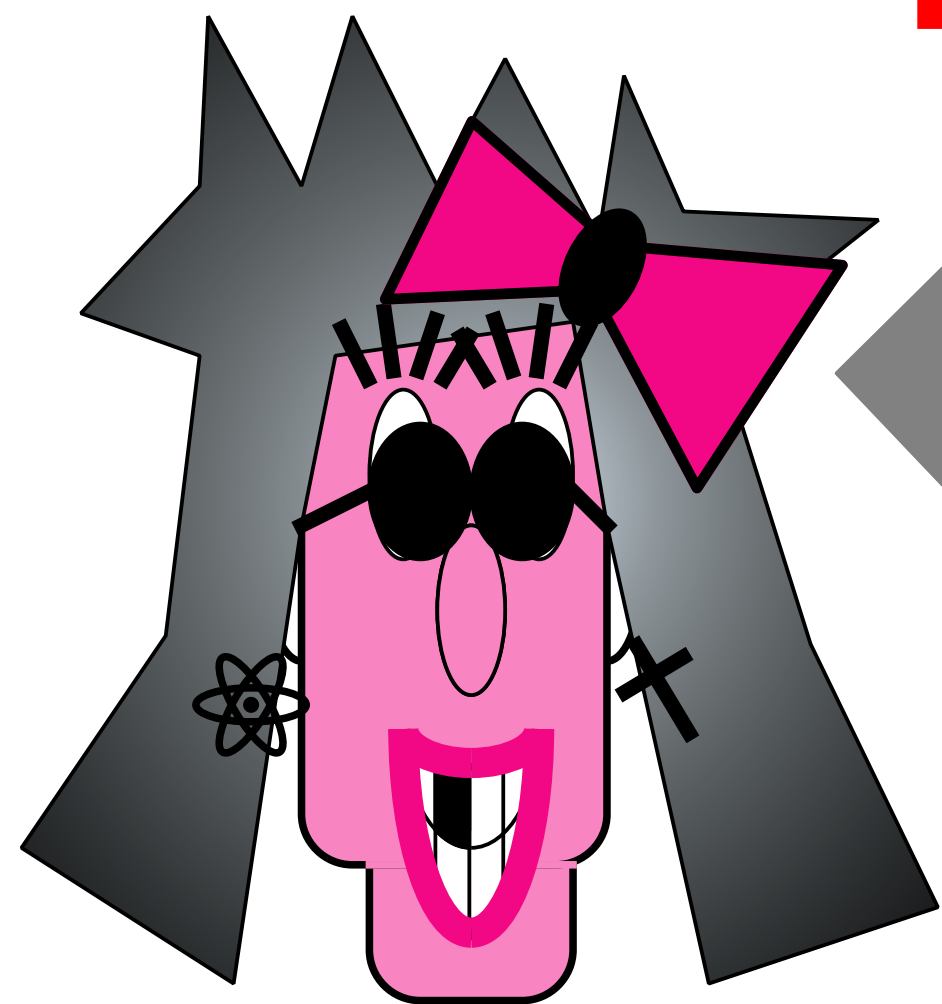
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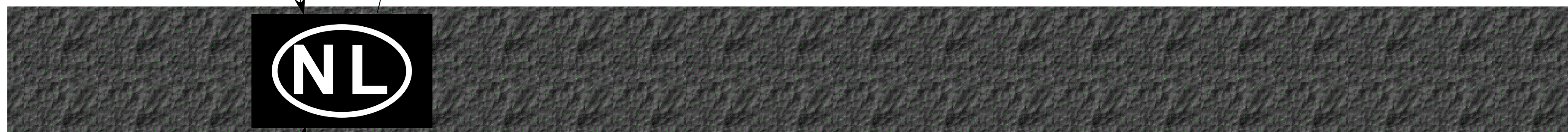
be the strongest result possible. Results in [F] and [BHZ] imply that if perfect zero-knowledge interactive proof-systems for NP exist, (i.e which do not rely on the fact that the verifier is polynomial time bounded) then the polynomial time hierarchy would collapse to its second level. This provides strong evidence that it will be impossible (and at least very hard) to unconditionally show that NP has zero-knowledge interactive proofs.

In light of the above negative results, it is interesting to examine whether the definition of interactive proofs can be modified so as to still capture

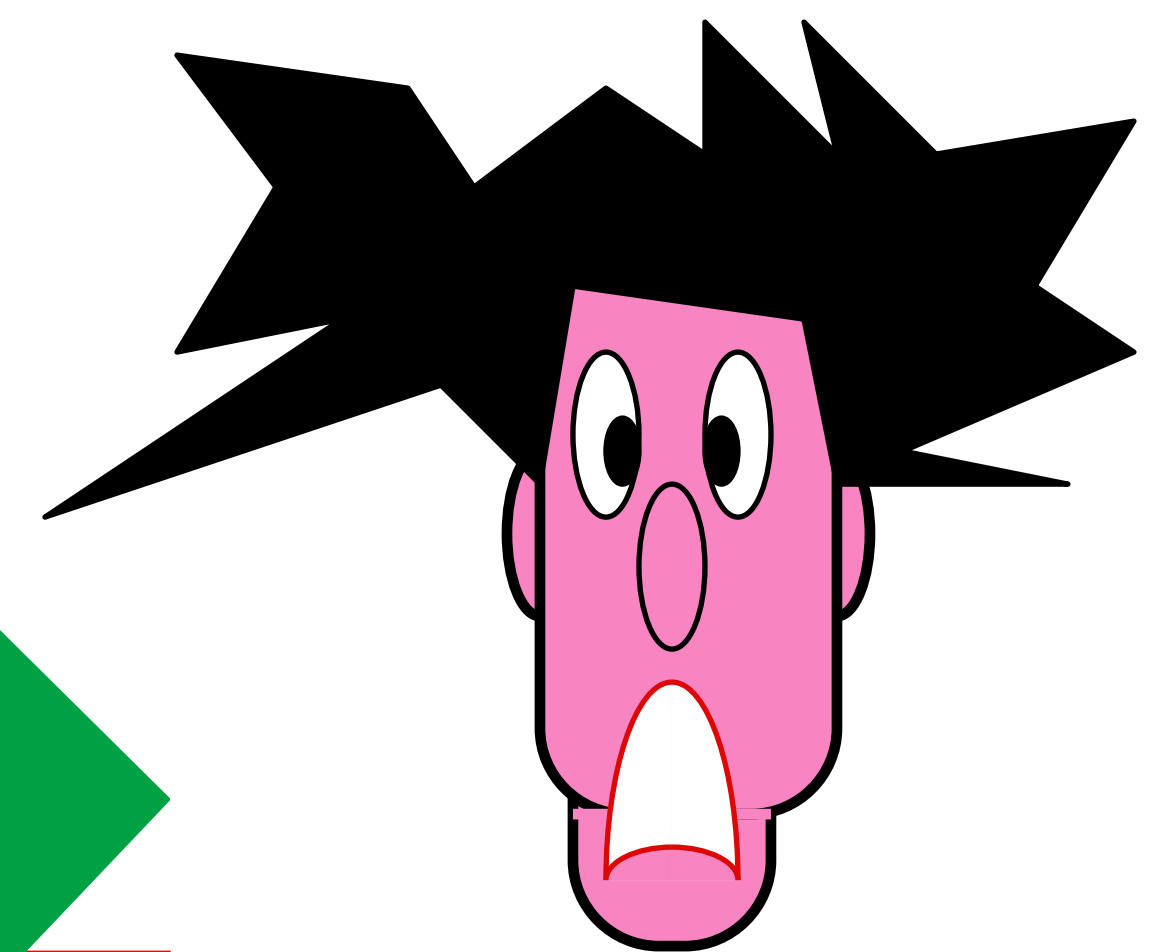
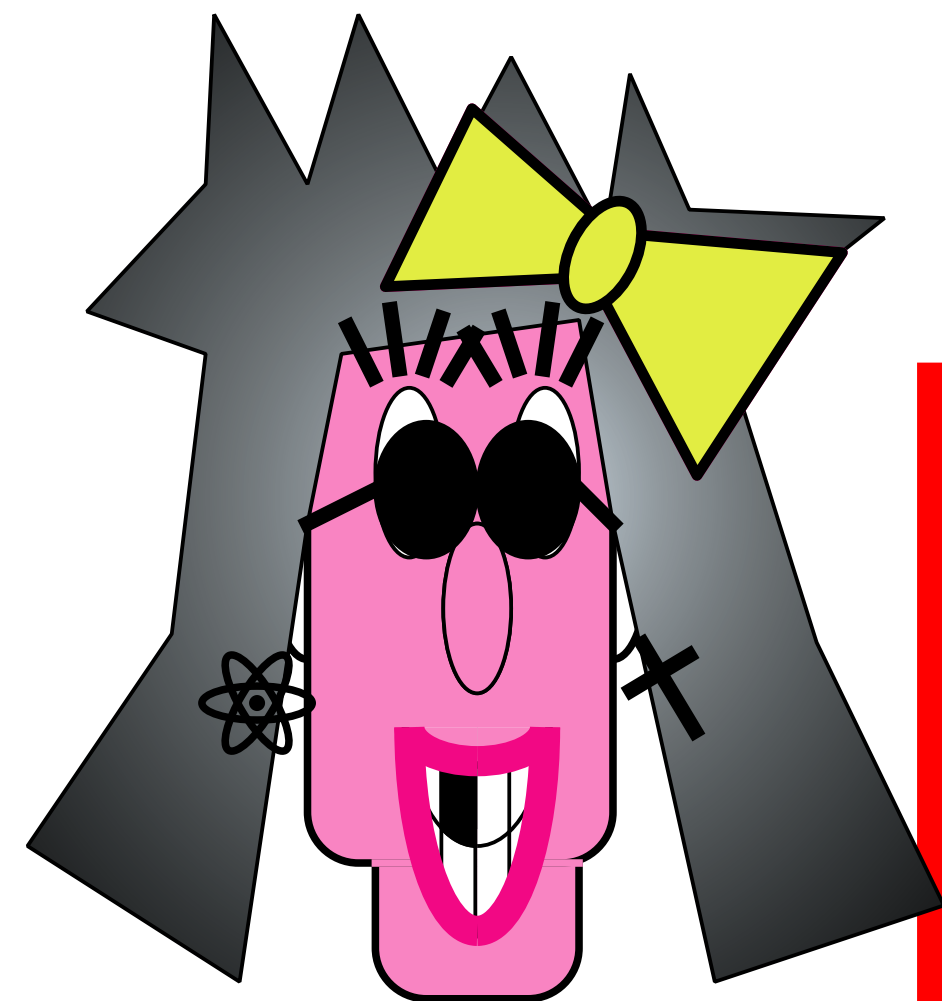
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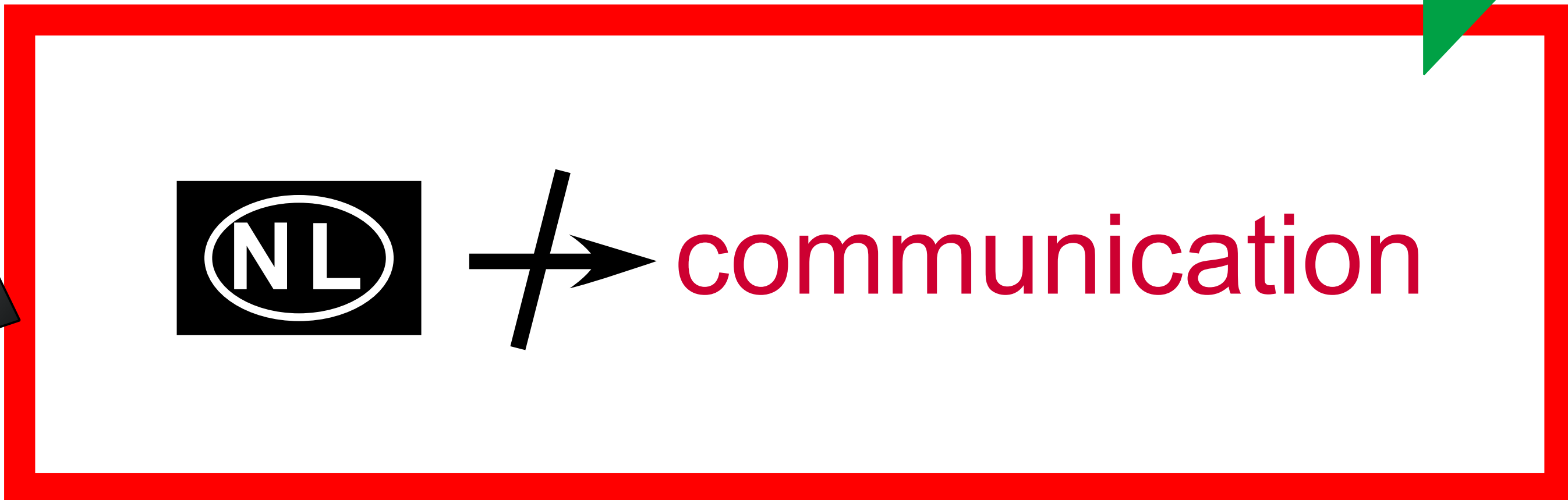
y z



b x

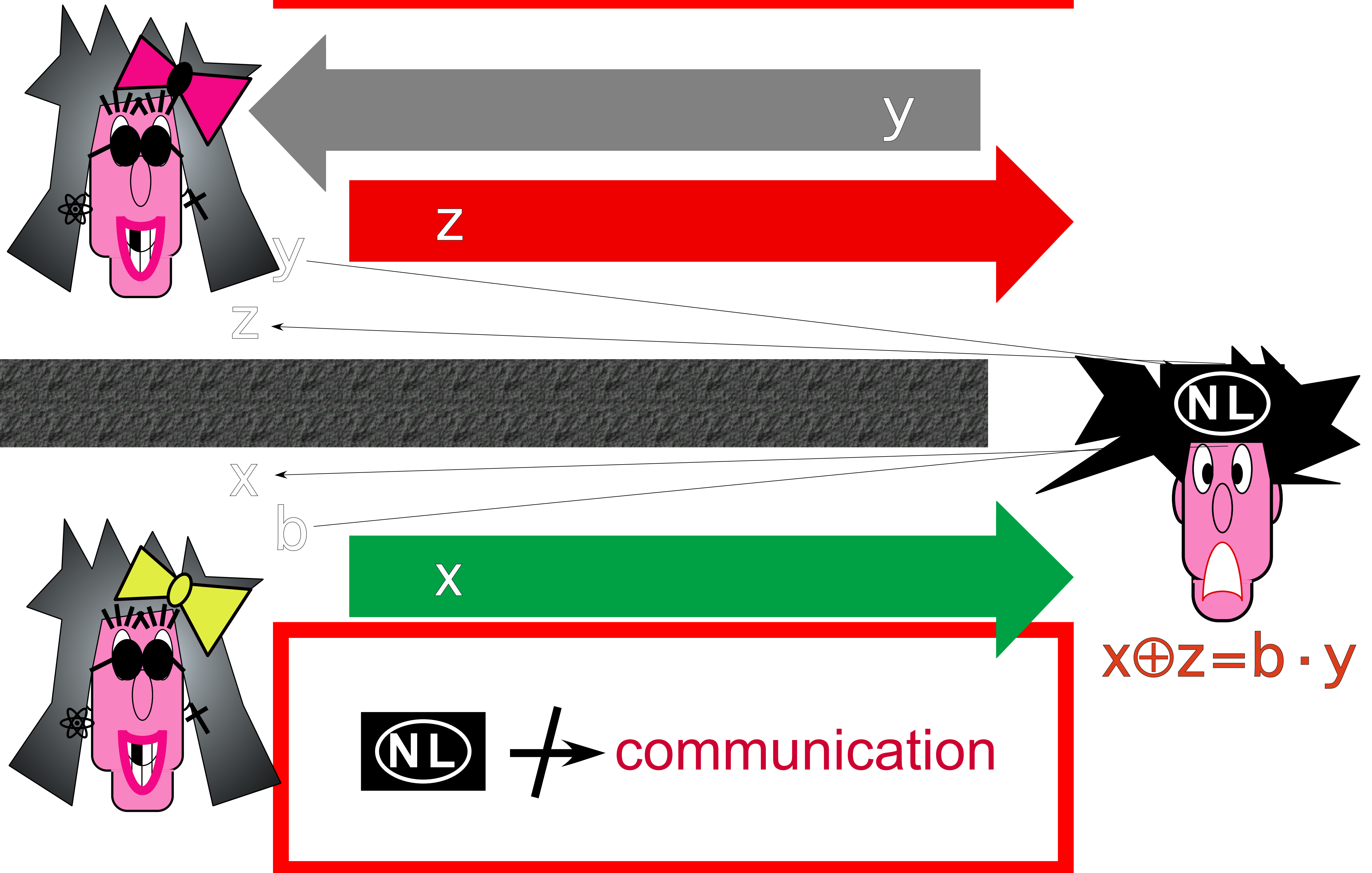


$$x \oplus z = b - y$$





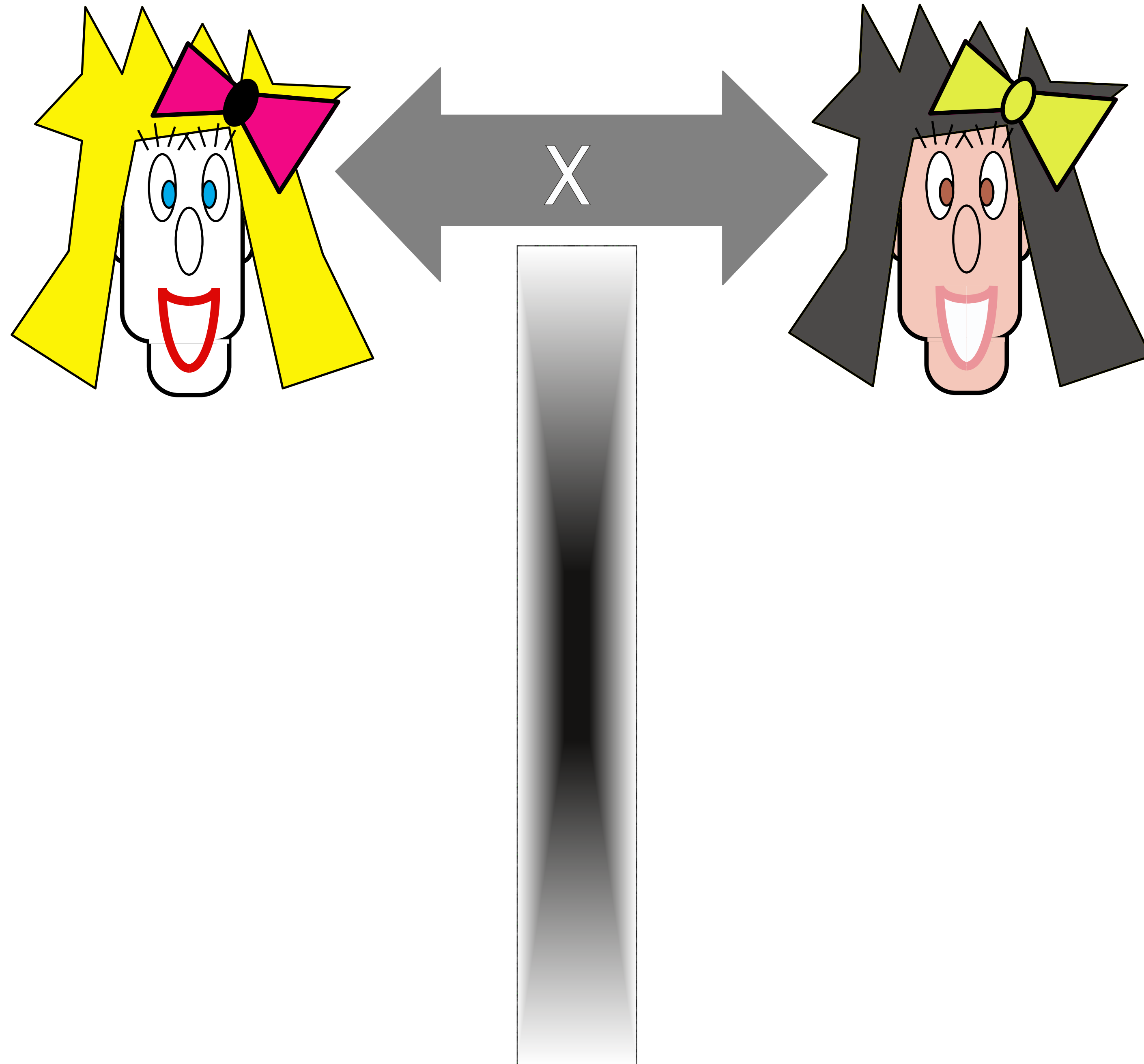
mial number of message exchanges with the verifier in their attempt to do so. To believe the validity of the assertion, the verifier must make sure that the two provers can not communicate with each other during the course of the proof process. Thus, the complexity assumptions made in previ-





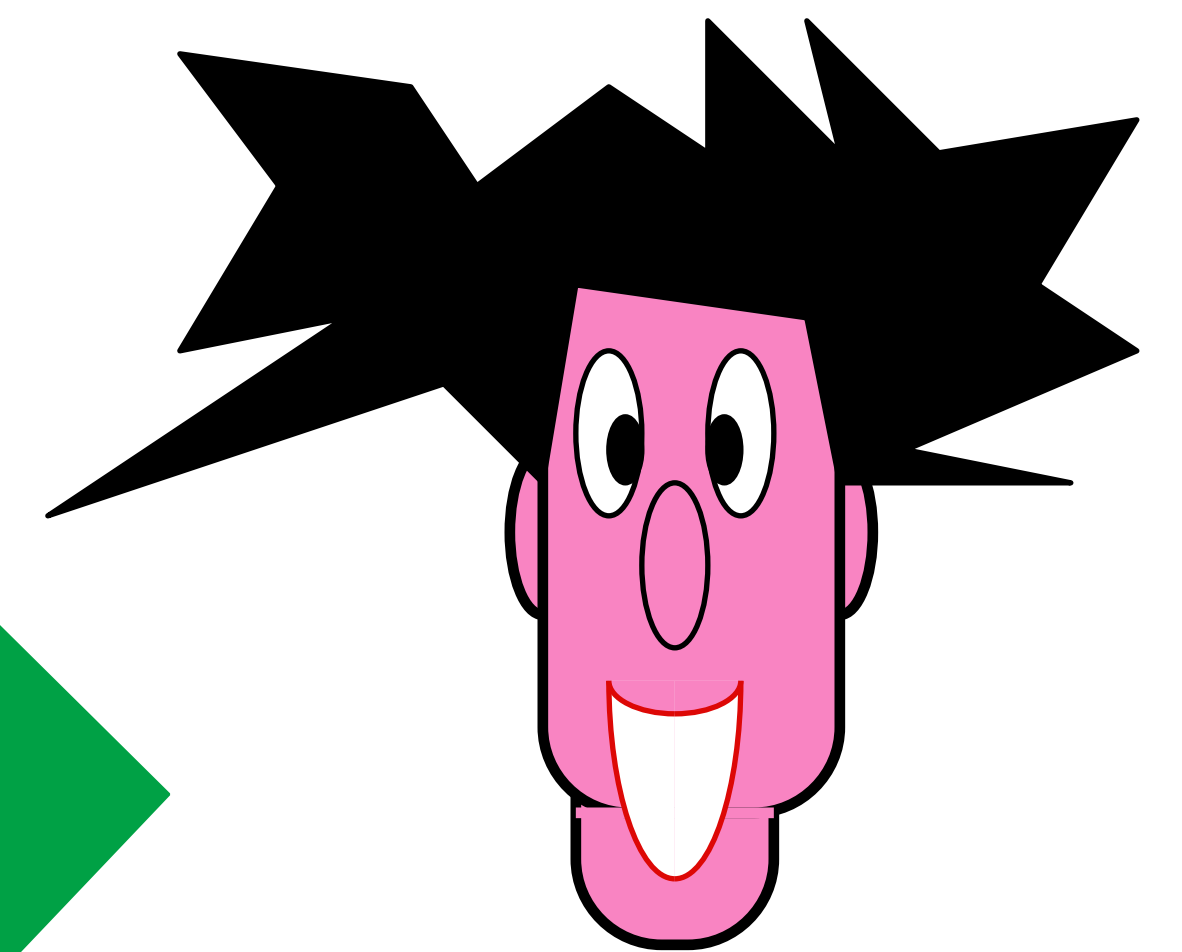
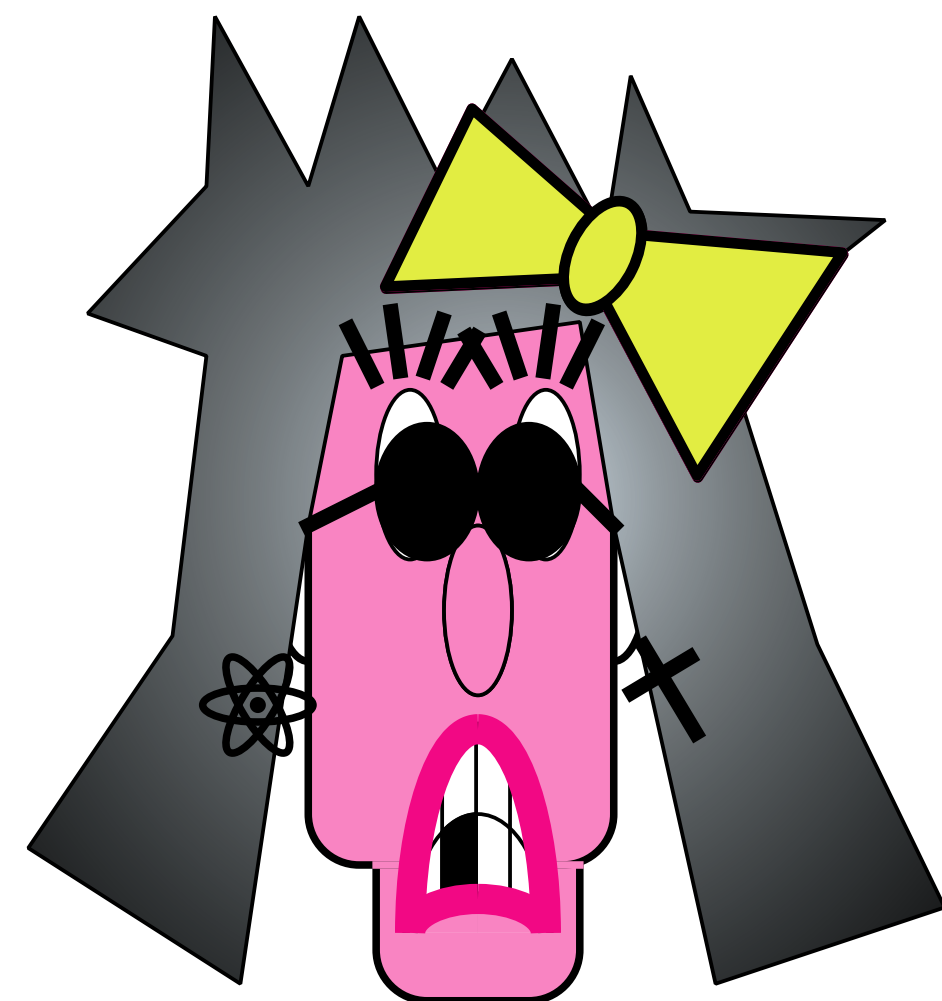
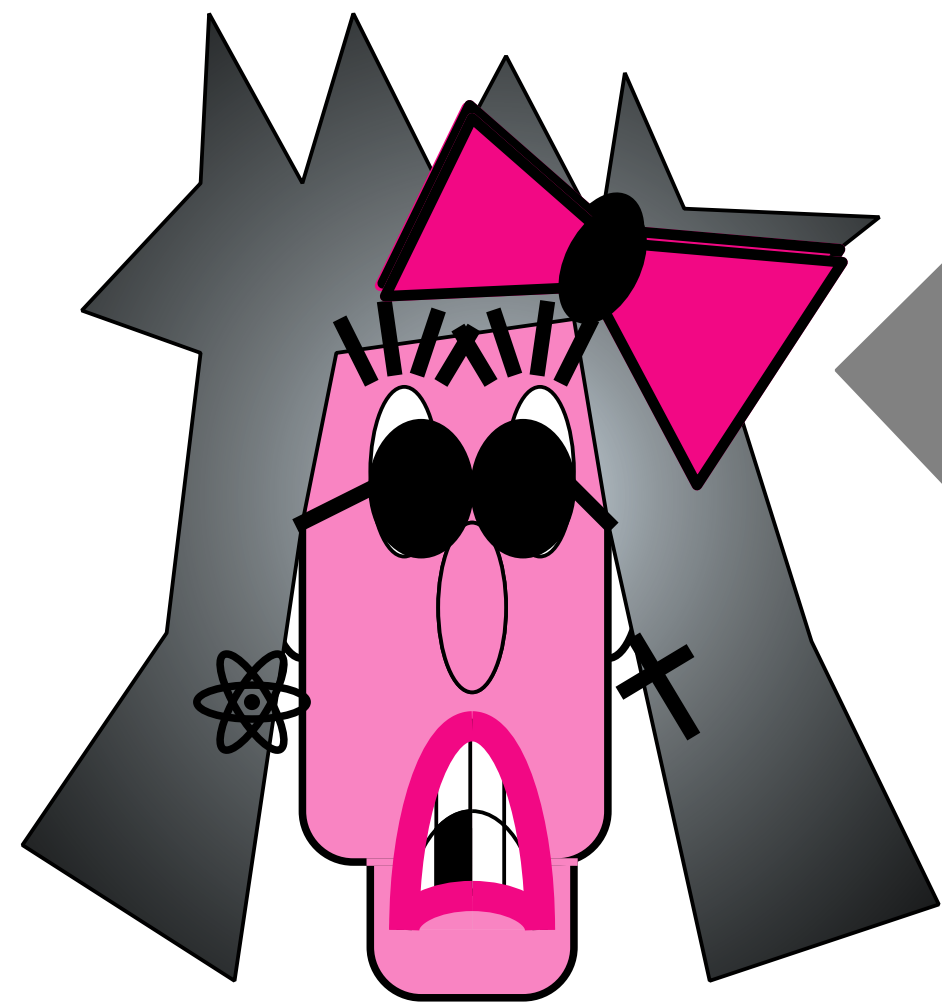
Yoda said:  
"Happens to every guy sometimes  
. this does"

# Classically



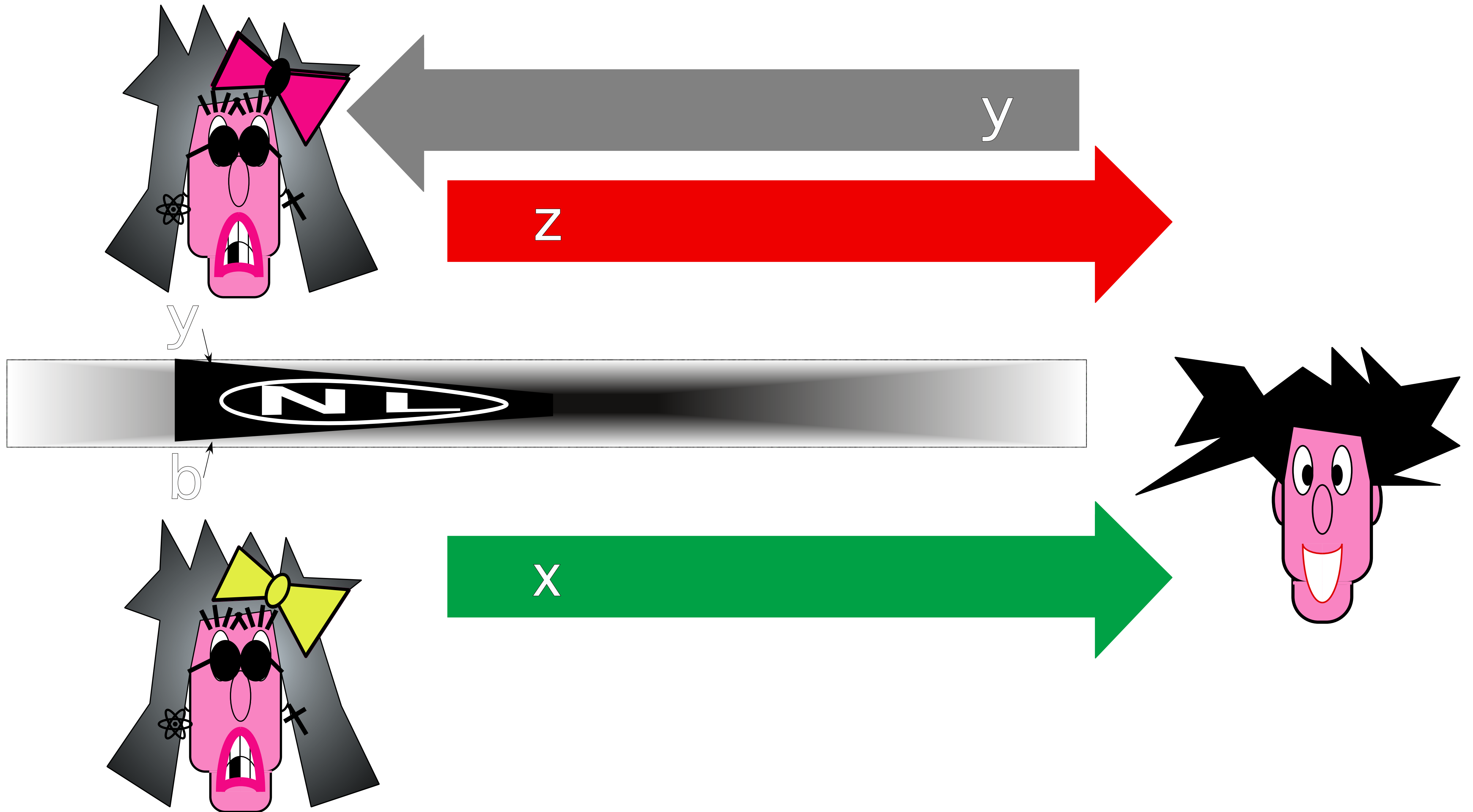
**STRONG BGKW**

# Classically



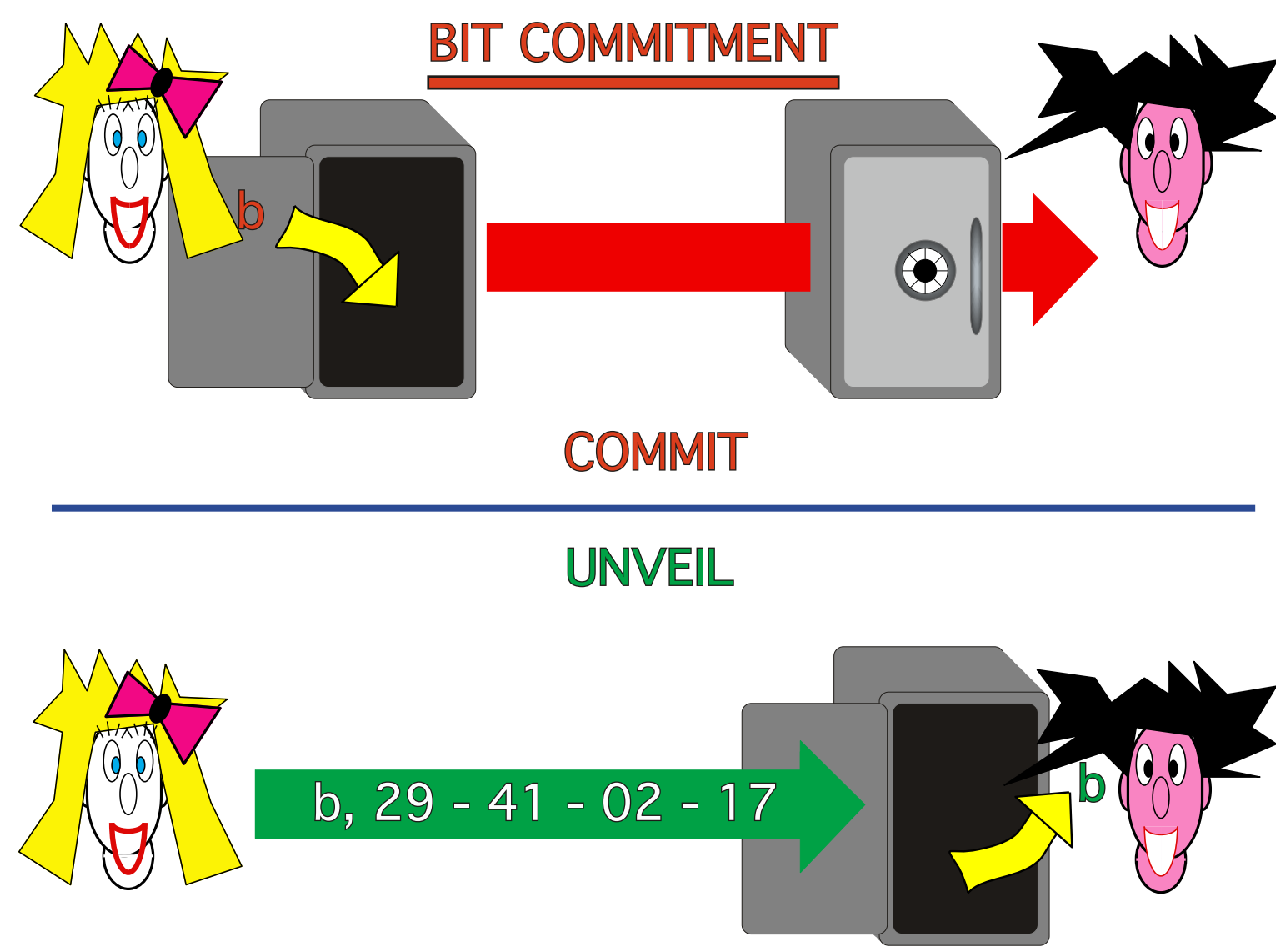
**STRONG BGKW**

# Classically

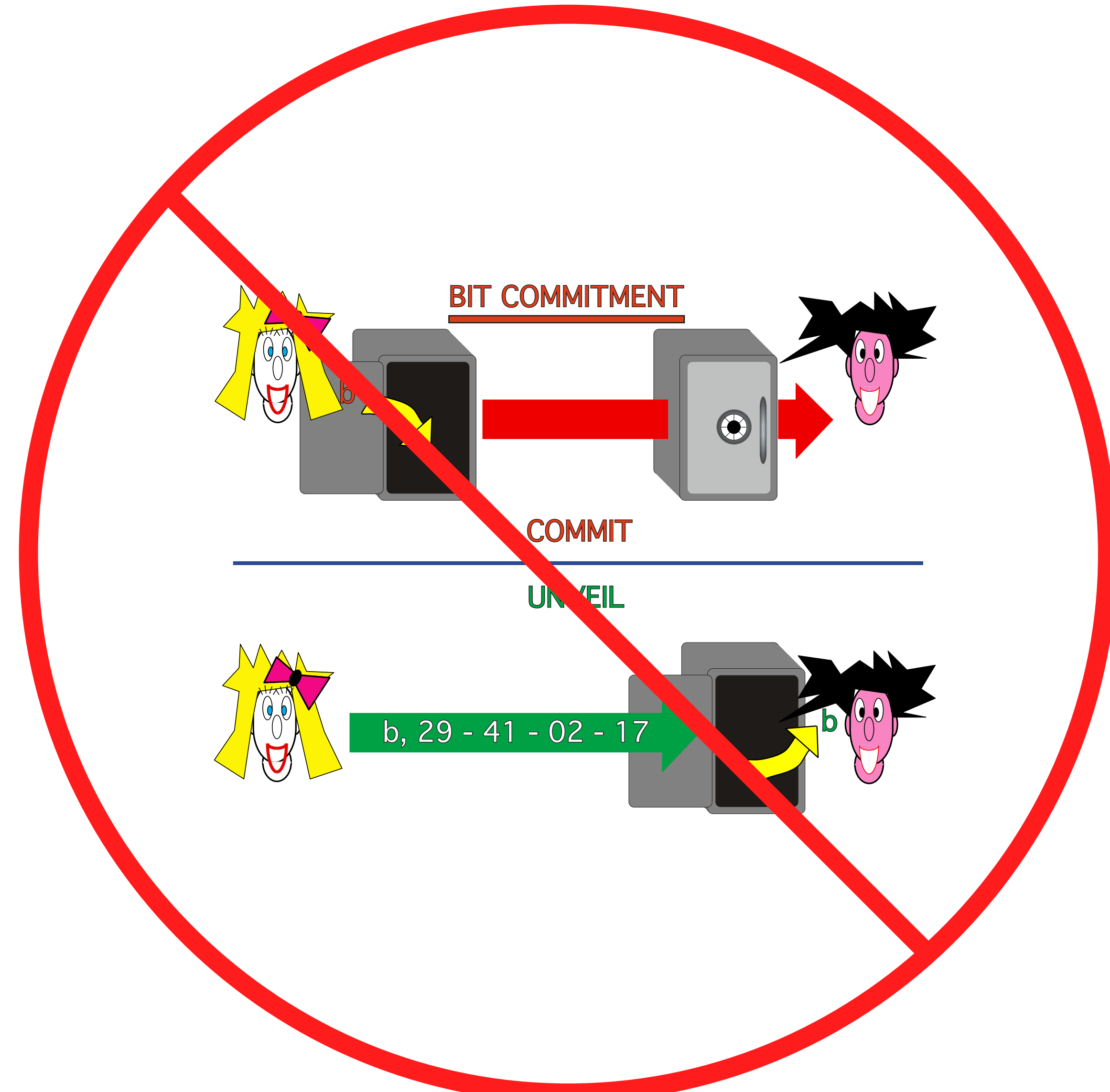


**STRONG BGKW**

# Quantumly



or



???

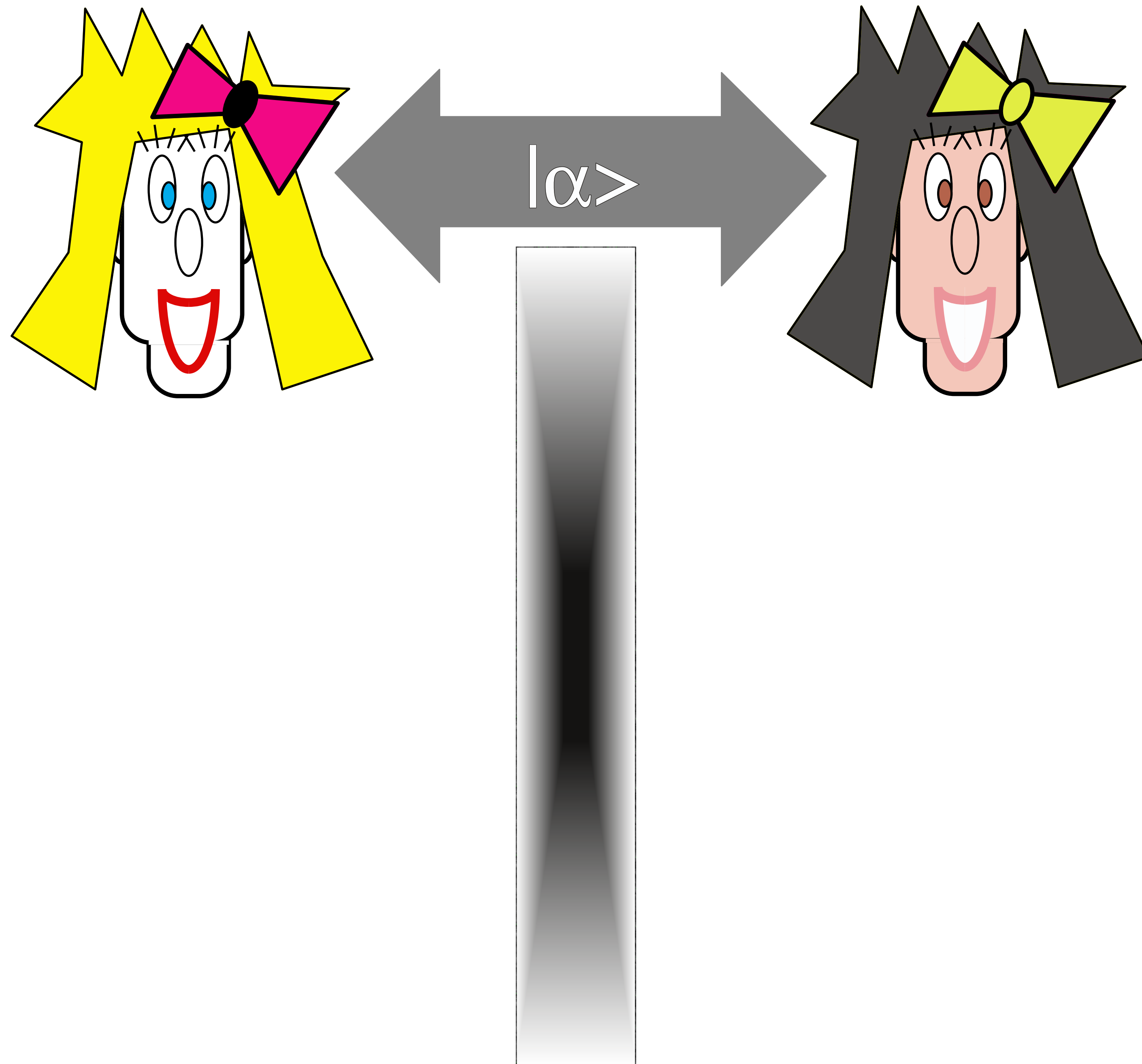
**(3)**

**two provers BC**

**Classically Secure**

**Quantumly Insecure**

# Quantumly

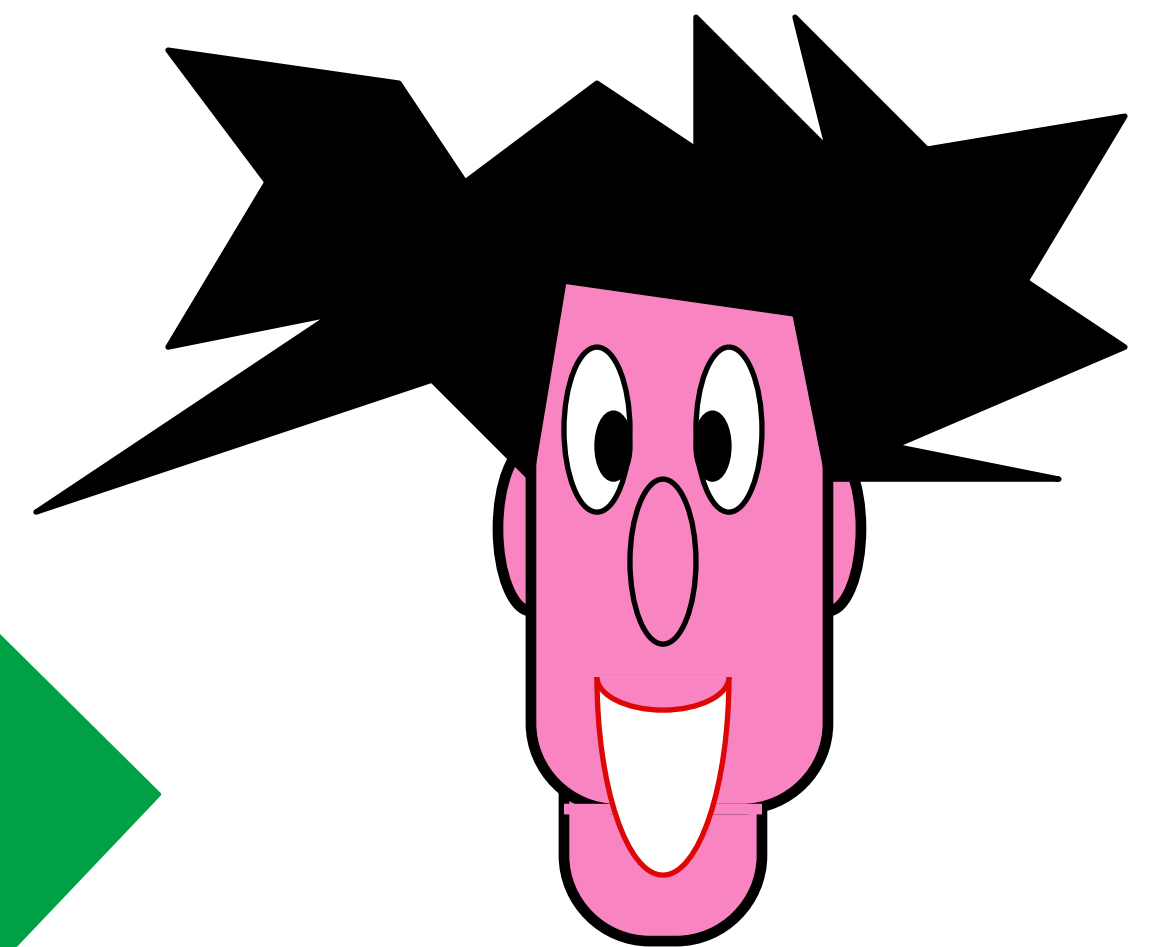
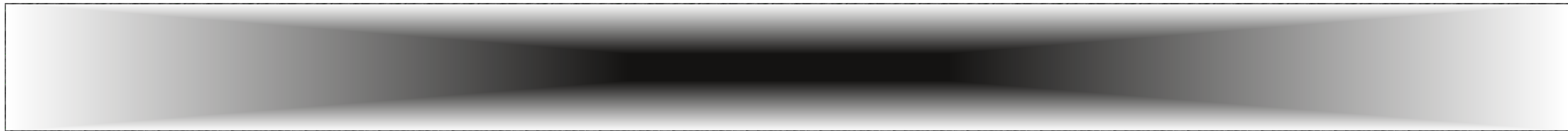
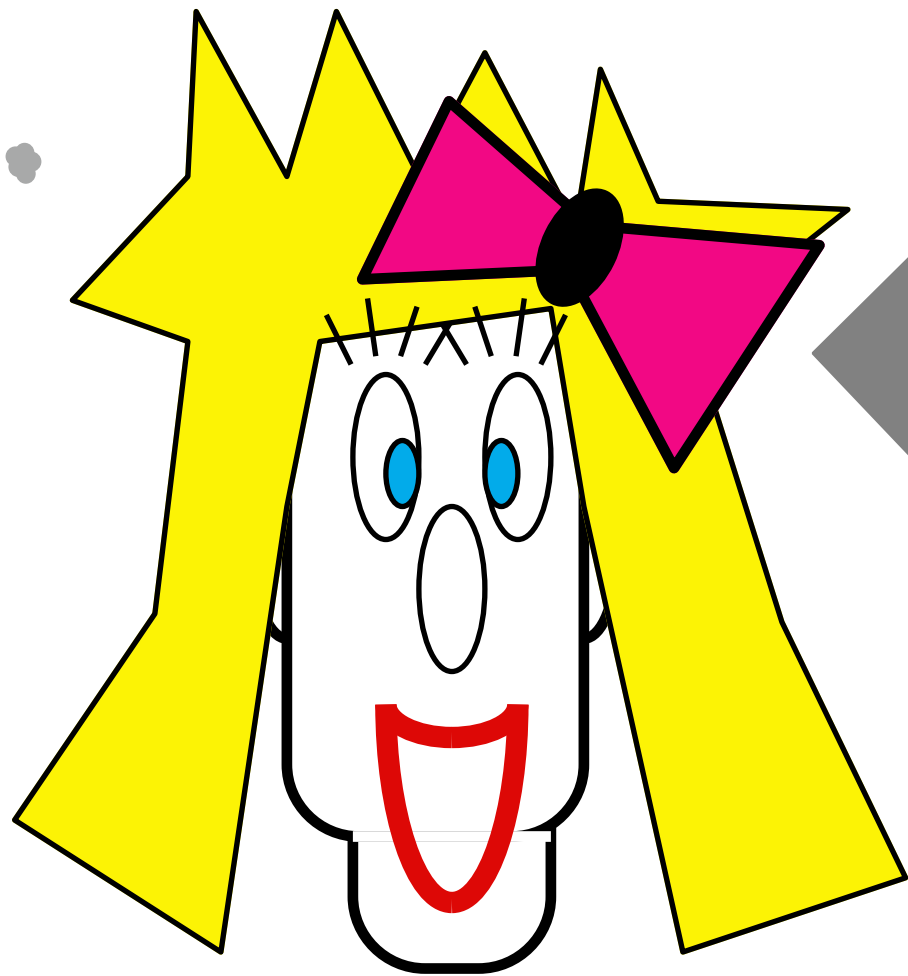
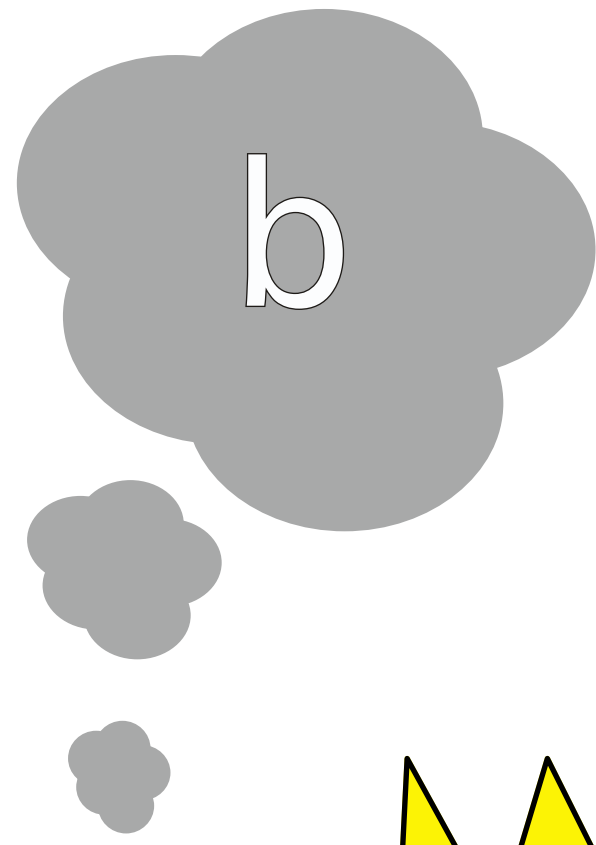


**STRONG Q-BGKW**



$$z = x \quad \text{if } b = 0$$

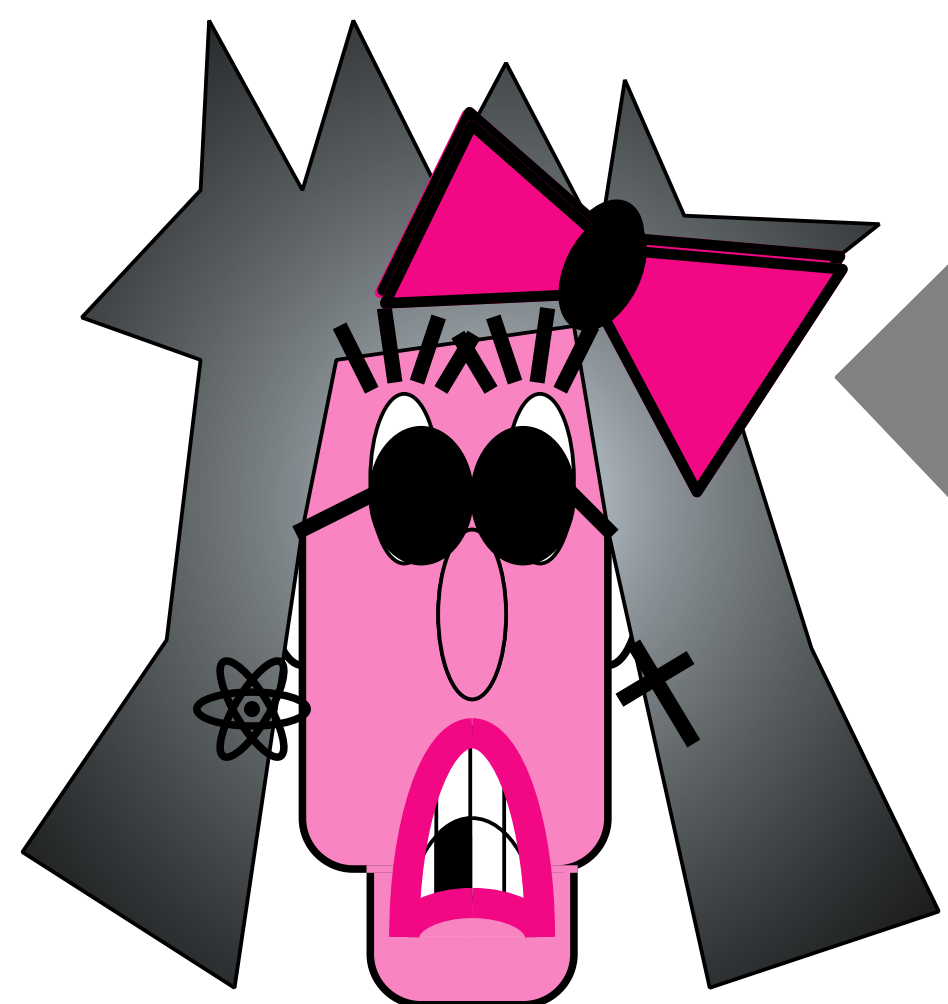
$$z = x \oplus y \quad \text{if } b = 1$$



$$D_H(x \oplus z, b \cdot y) < n/5?$$

STRONG Q-BGKW'

# Classically

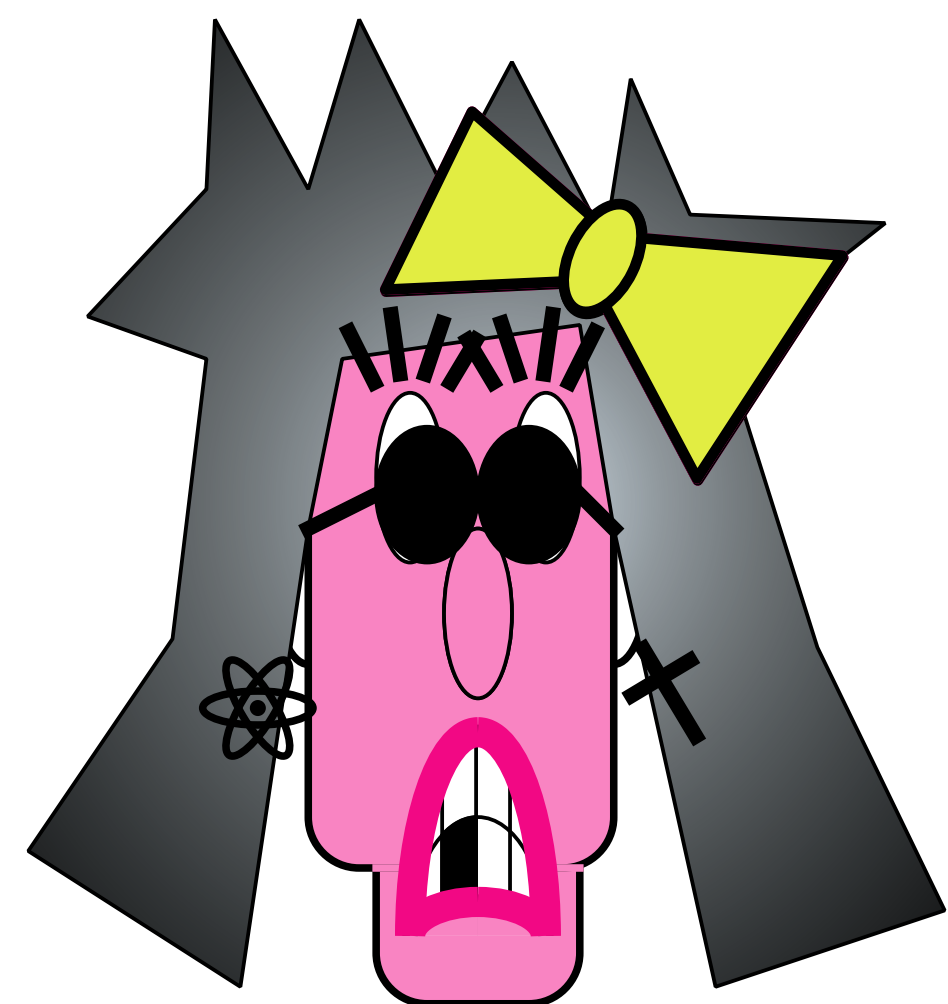
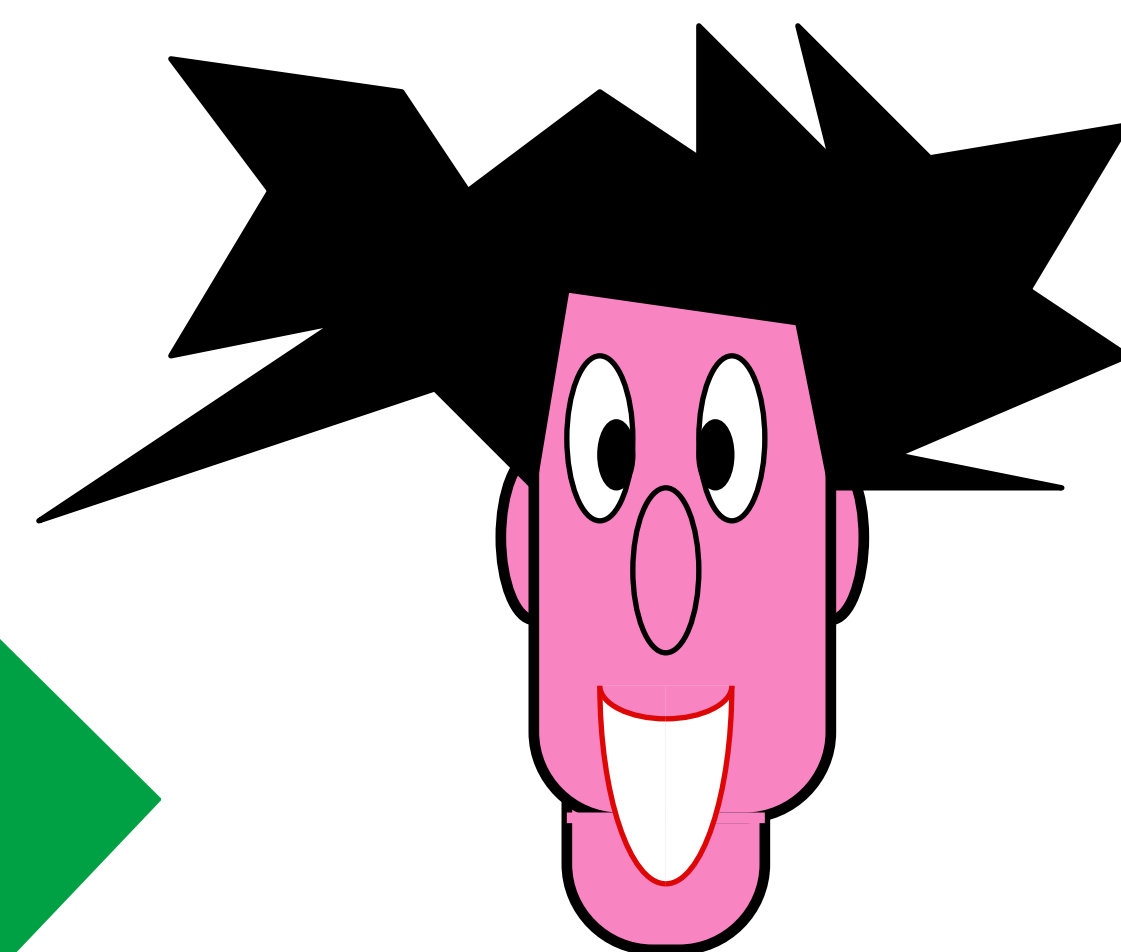


y z



b x

$$\rightarrow D_H(x \oplus z, b \cdot y) \approx 25\%n > n/5$$

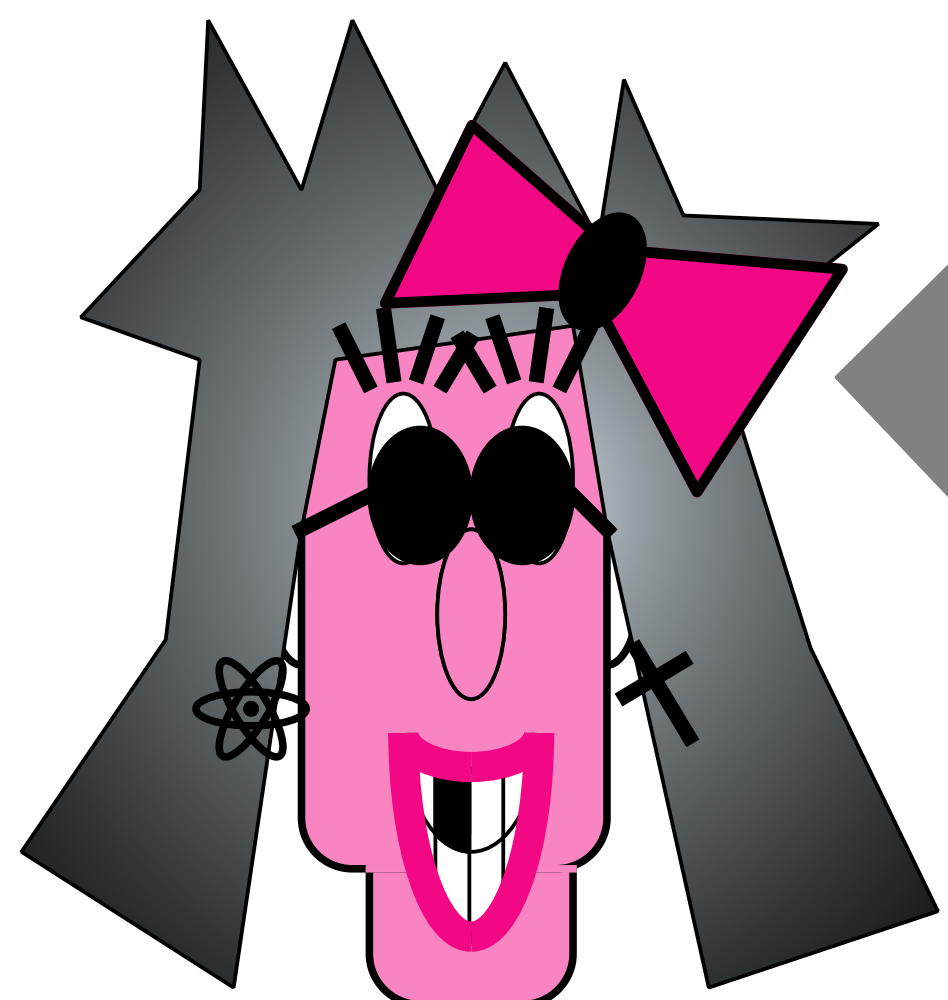


$$D_H(x \oplus z, b \cdot y) < n/5?$$

**STRONG Q-BGKW'**

# Quantumly

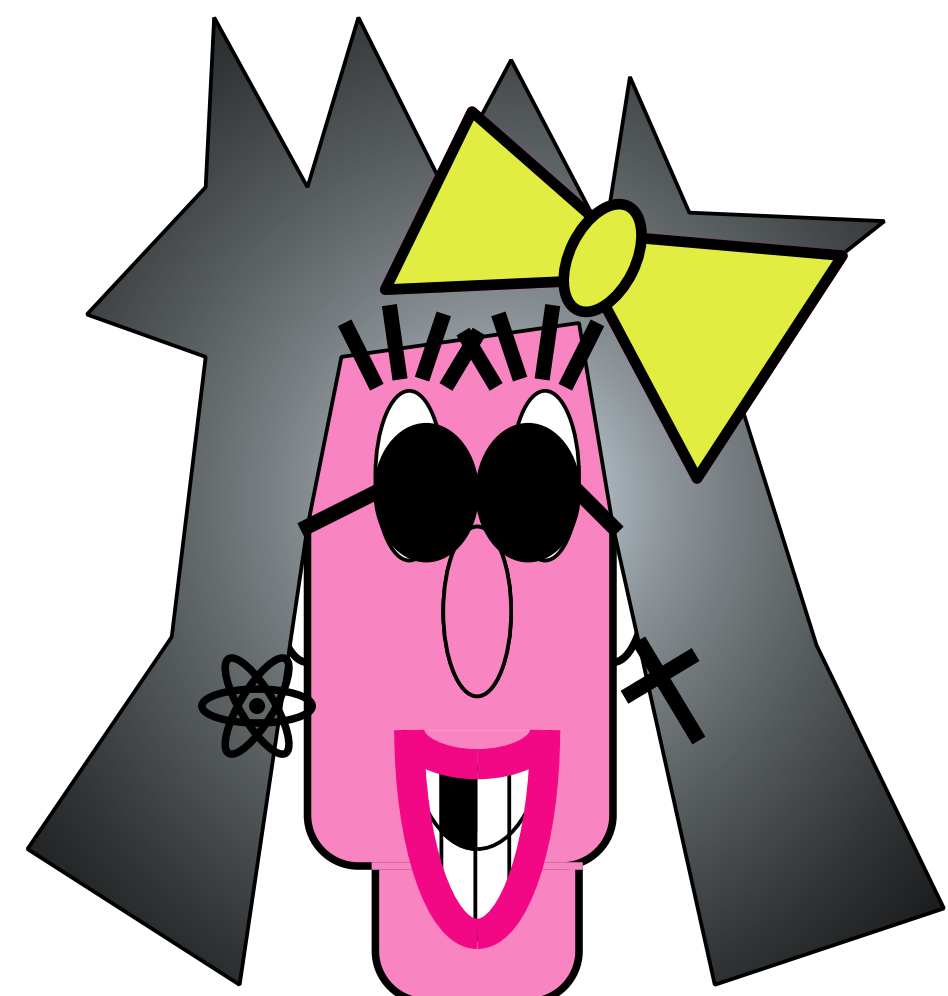
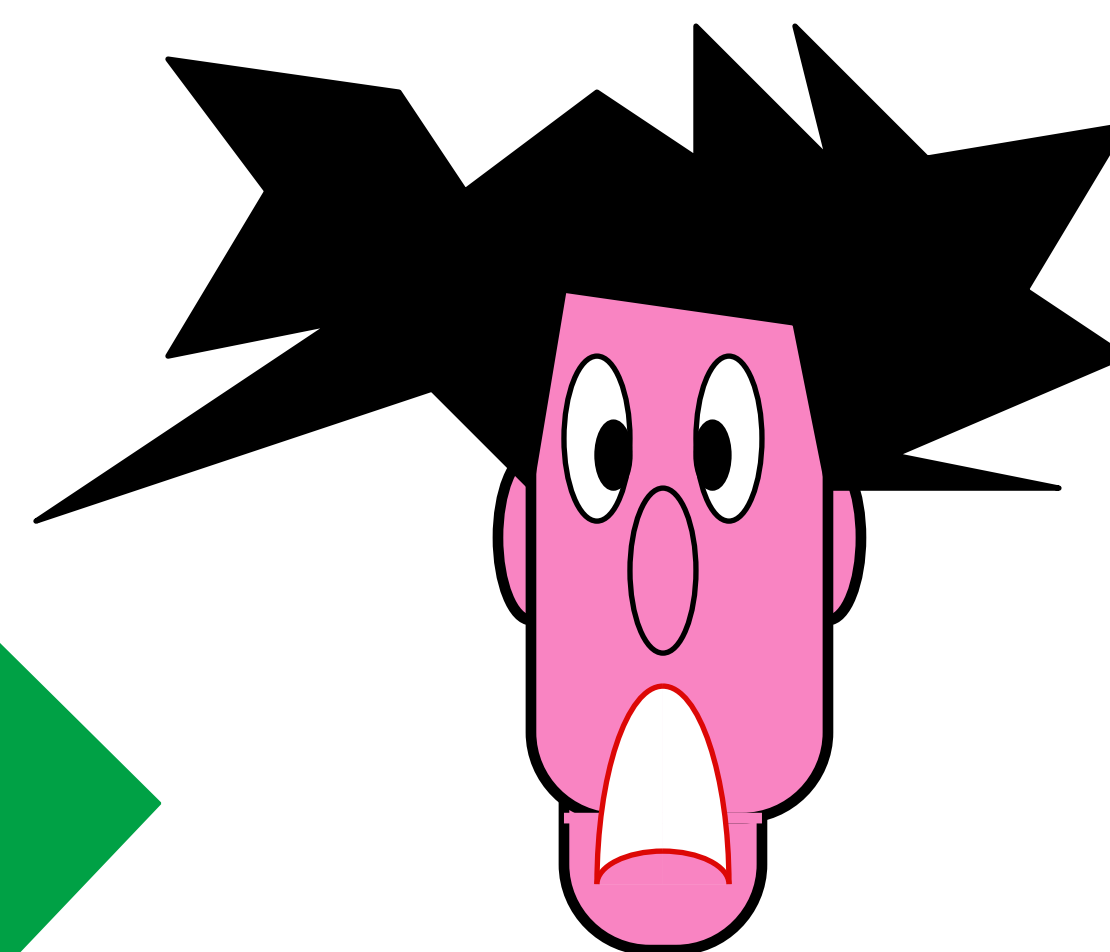
**INSECURE**



y z



->  $D_H(x \oplus z, b \cdot y) \approx 15\%n < n/5$



$D_H(x \oplus z, b \cdot y) < n/5?$

~~STRONG Q-BGKW'~~

# Classically

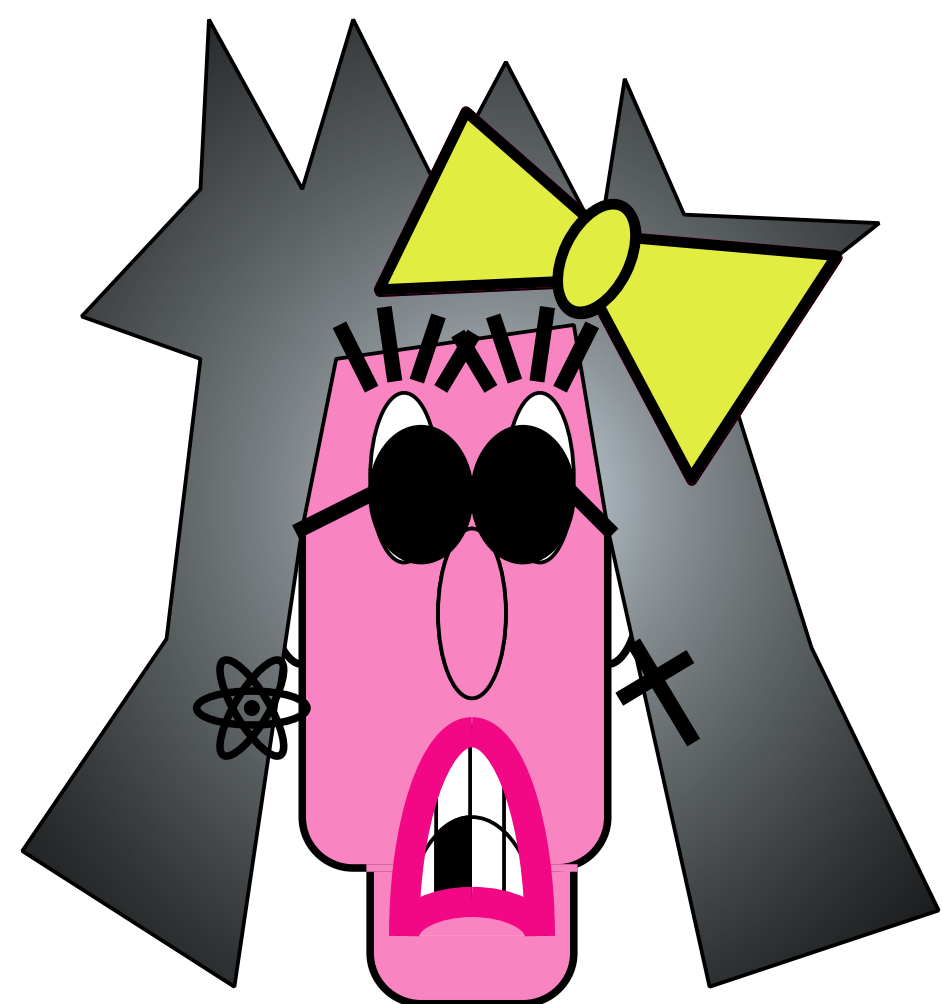
$$D_H(x_0 \oplus z, 0 \cdot y) = D_H(x_0 \oplus z, 0) < n/5$$

$$D_H(x_1 \oplus z, 1 \cdot y) = D_H(x_1 \oplus z, y) < n/5$$

$$D_H(x_0 \oplus x_1, y) = D_H((x_0 \oplus z) \oplus (x_1 \oplus z), 0 \oplus y) < 2n/5 < n/2$$

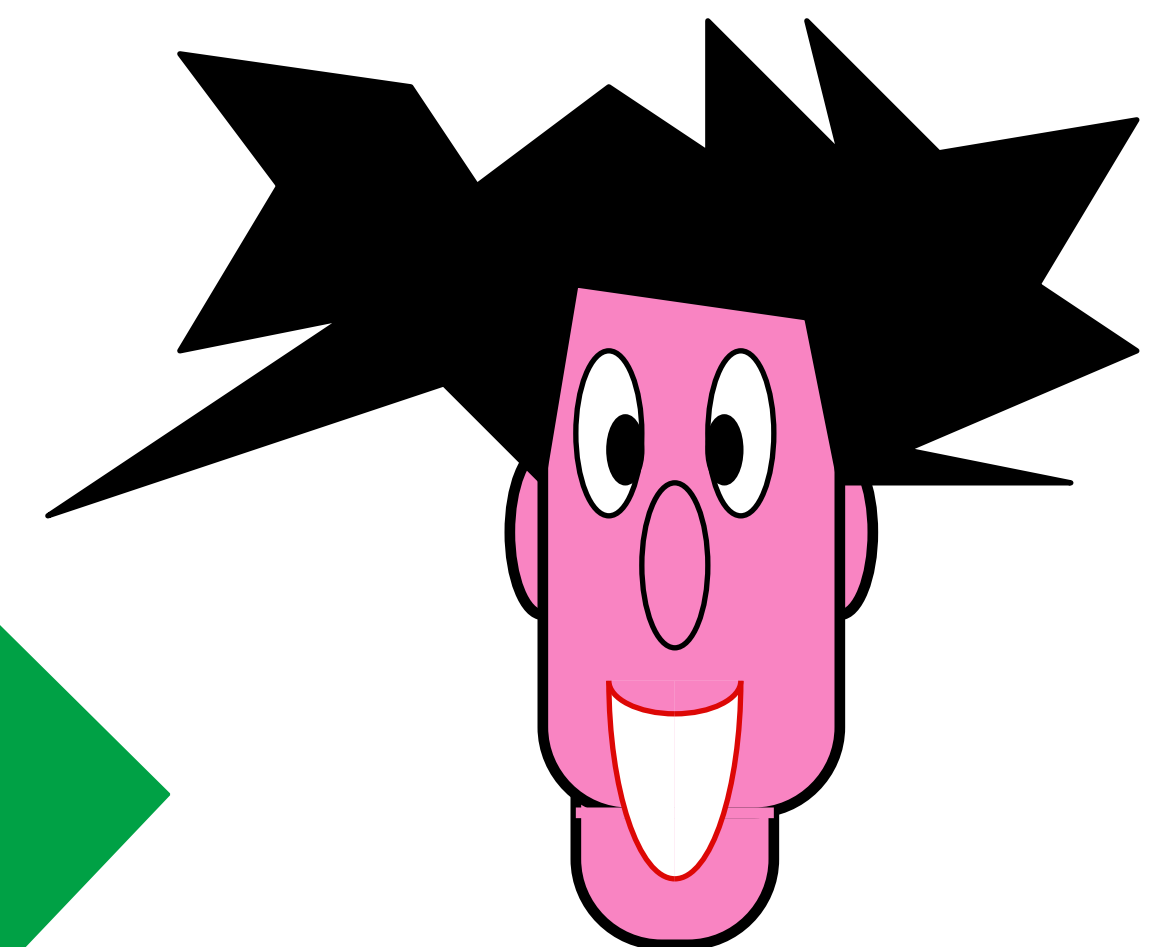
possible with prob. at most  $\varepsilon^{-n}$

**SECURE**



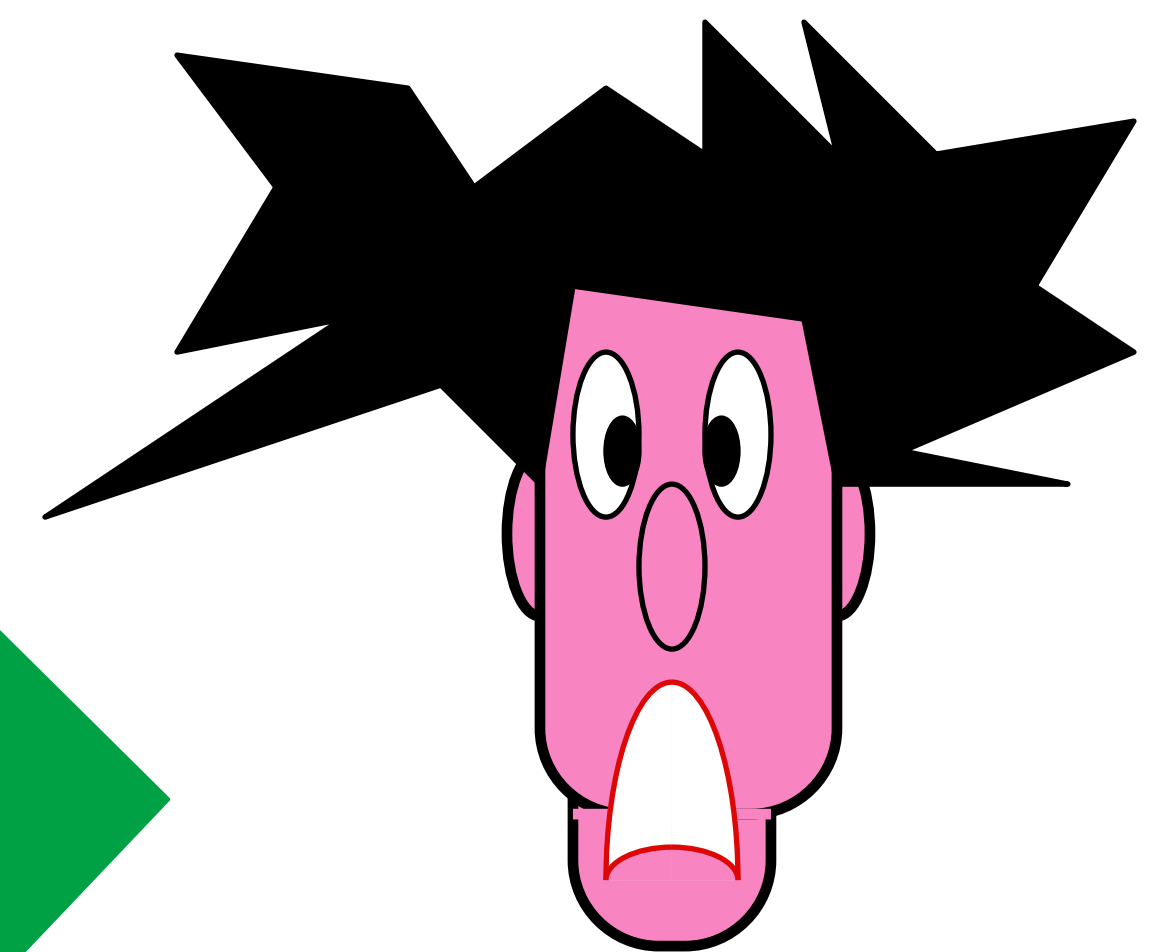
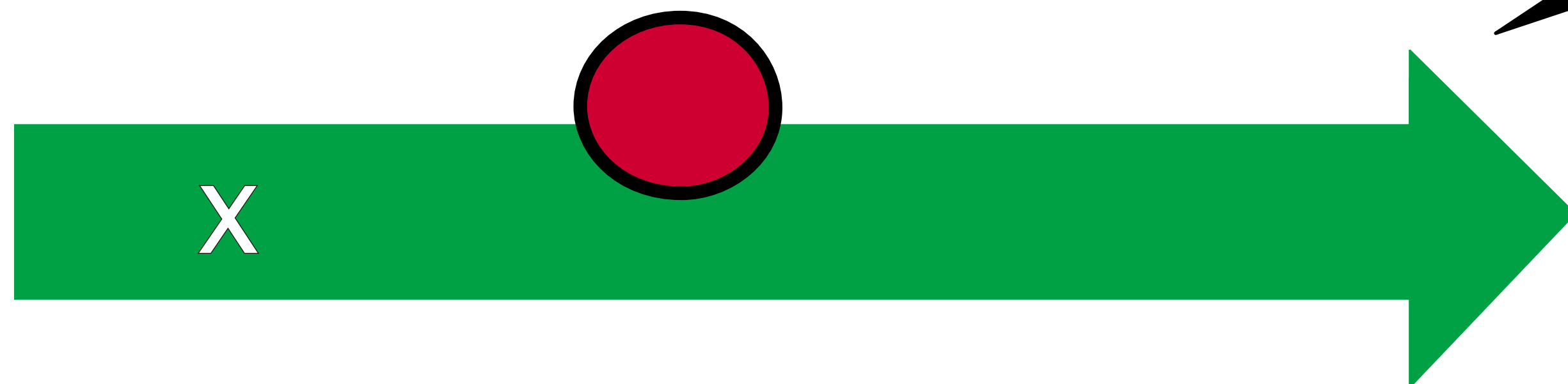
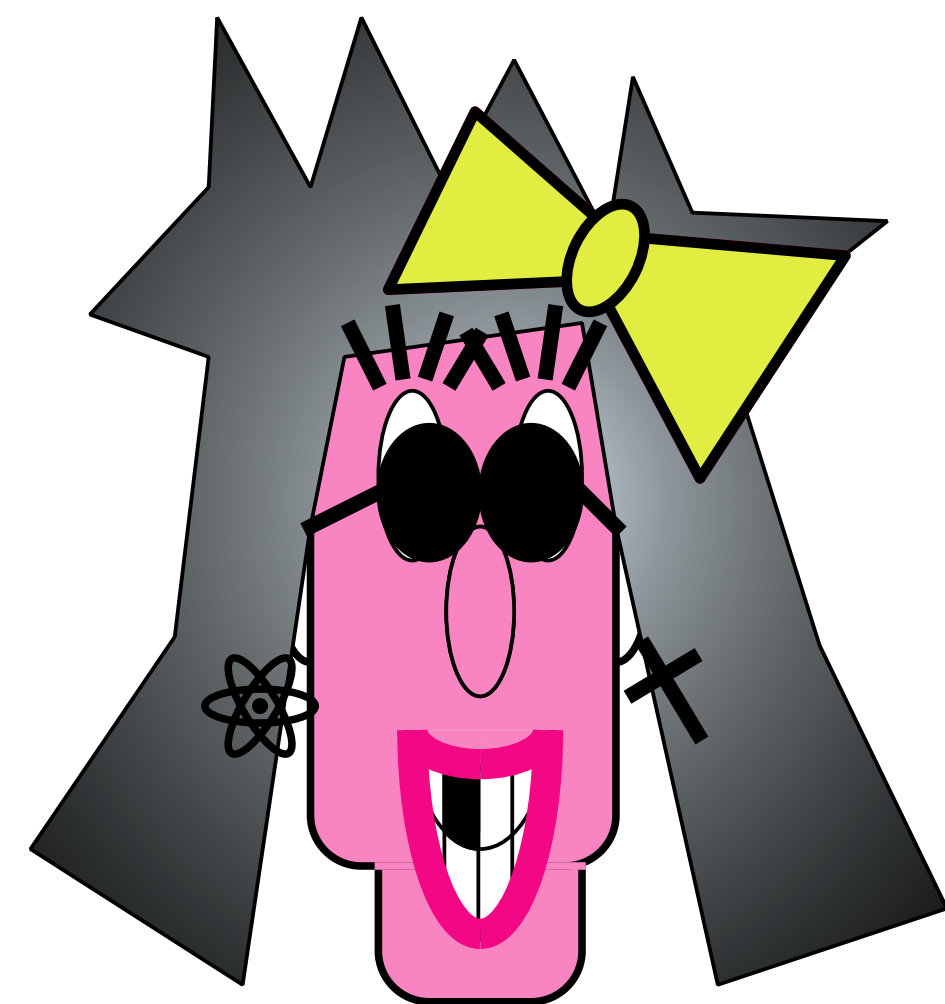
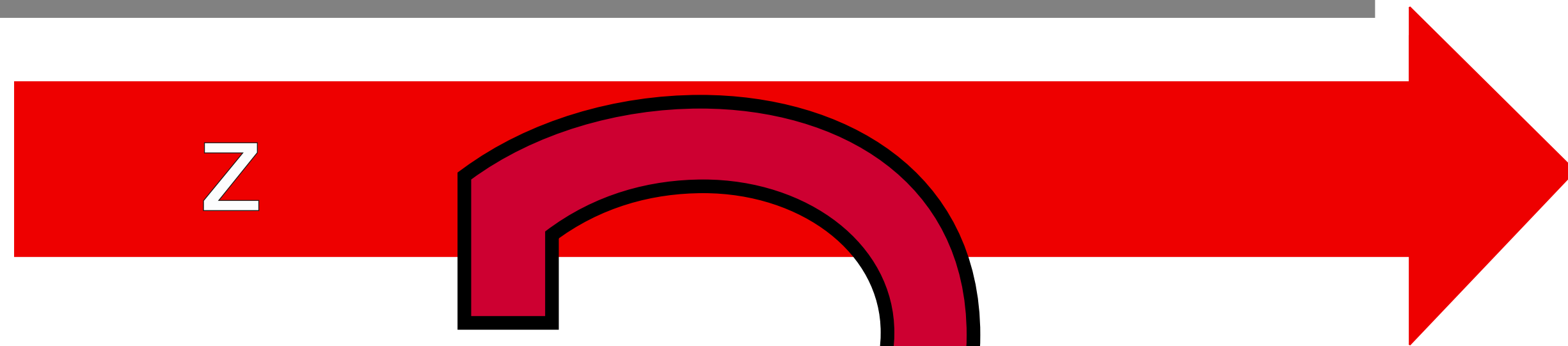
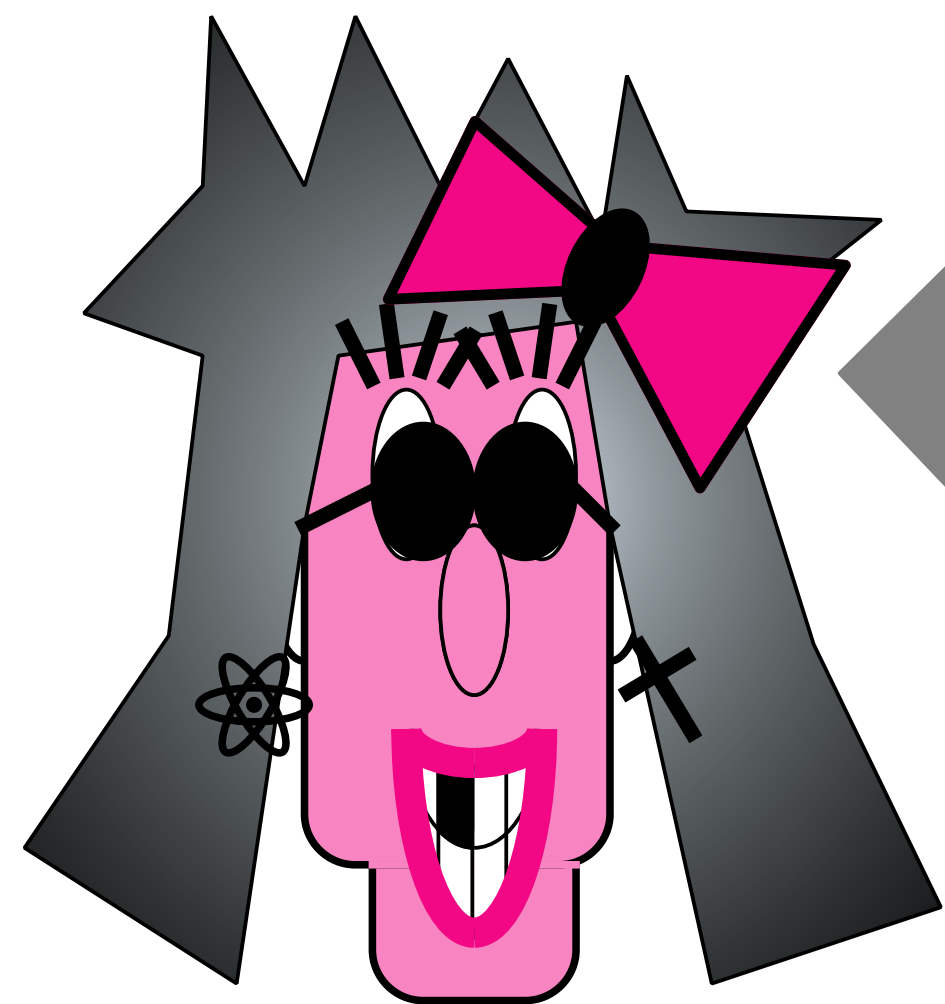
$x_0$

$x_1$



**STRONG Q-BGKW'**

# Quantumly



$$x \oplus z = b \cdot y$$

STRONG Q-BGKW

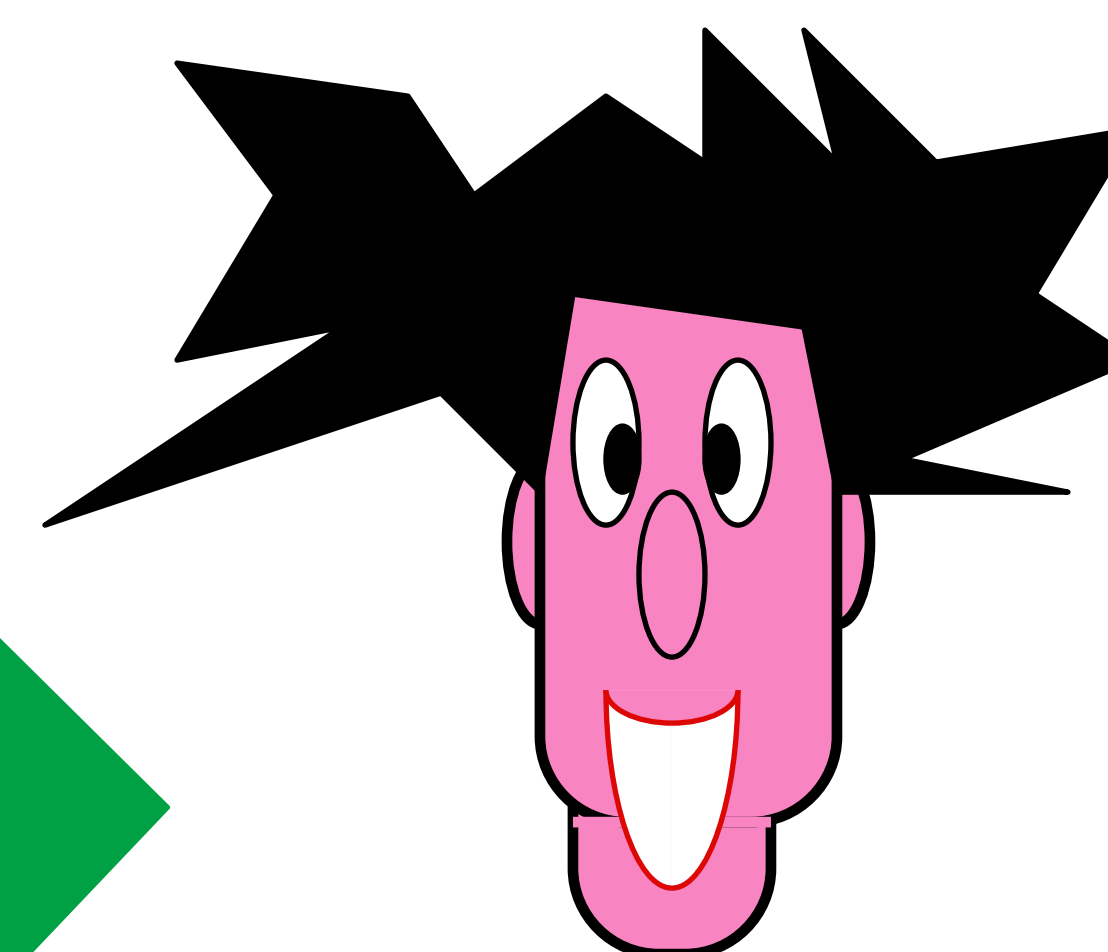
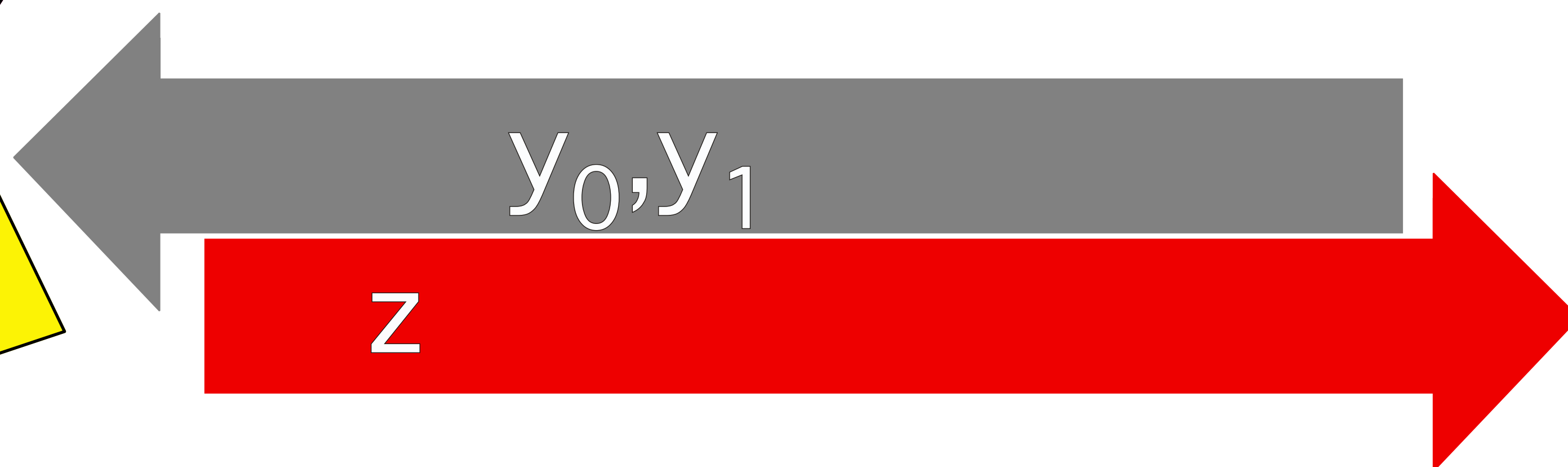
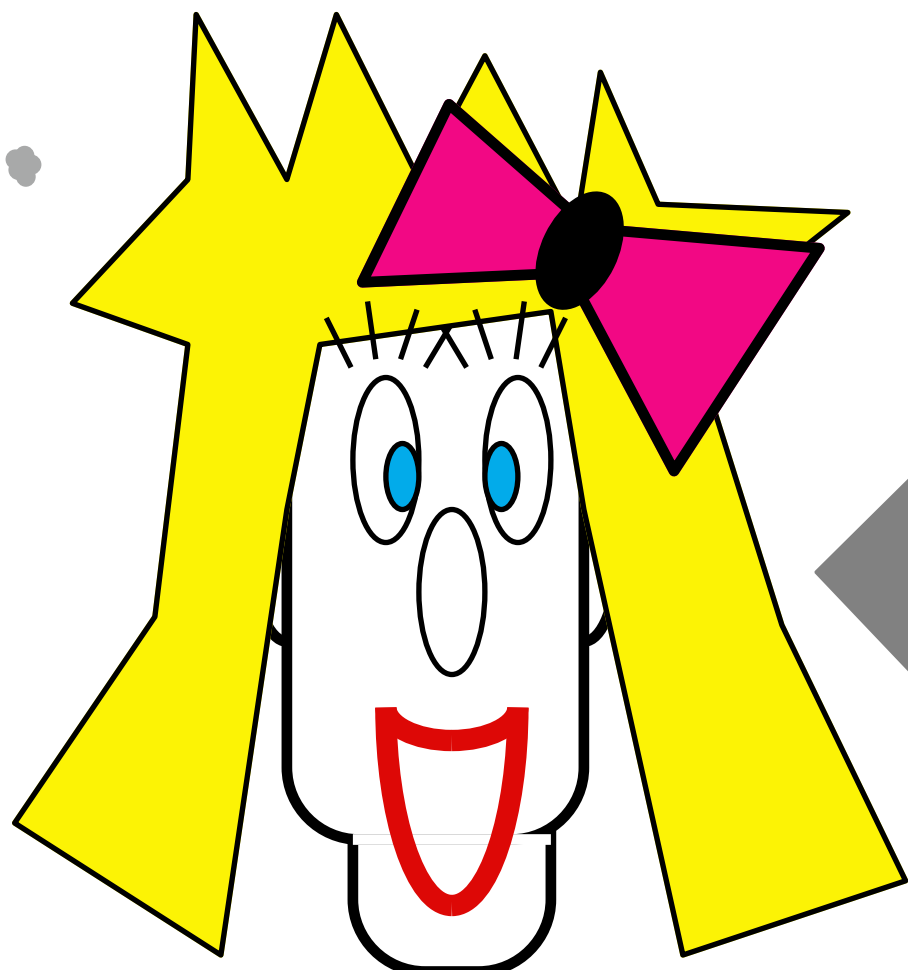
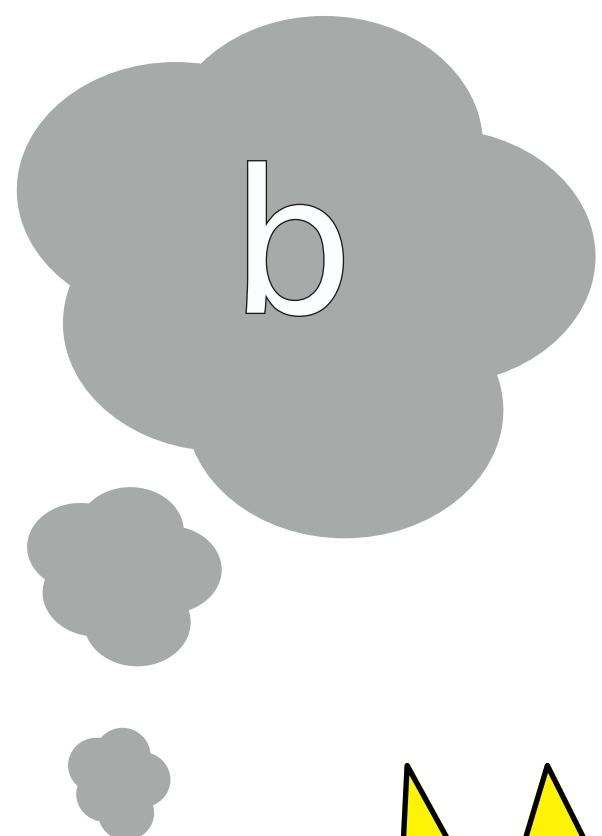
**(4)**

**two provers BC**

**Classically and**

**Quantumly Secure**

$$z = x \oplus y_b$$



$$x = z \oplus y_b ?$$

STRONG Q-BGKW"

# Classically

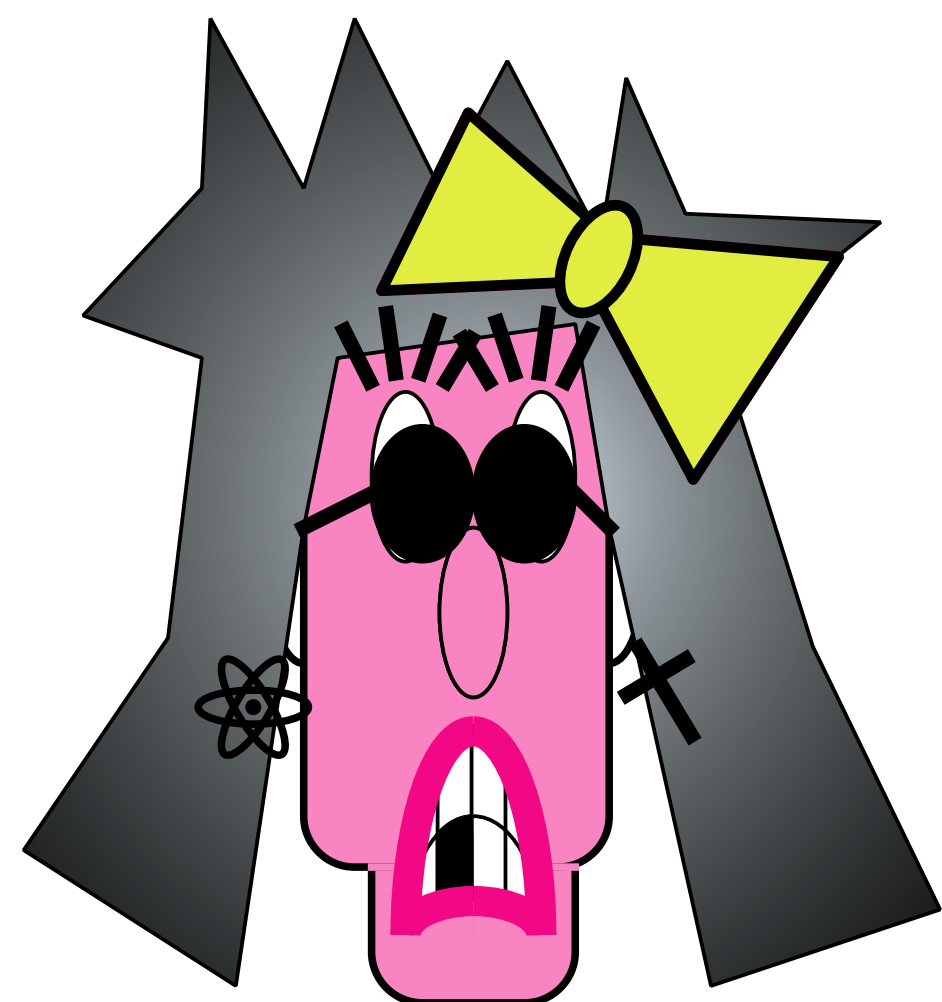


$$x_0 \oplus z = y_0$$

$$x_1 \oplus z = y_1$$

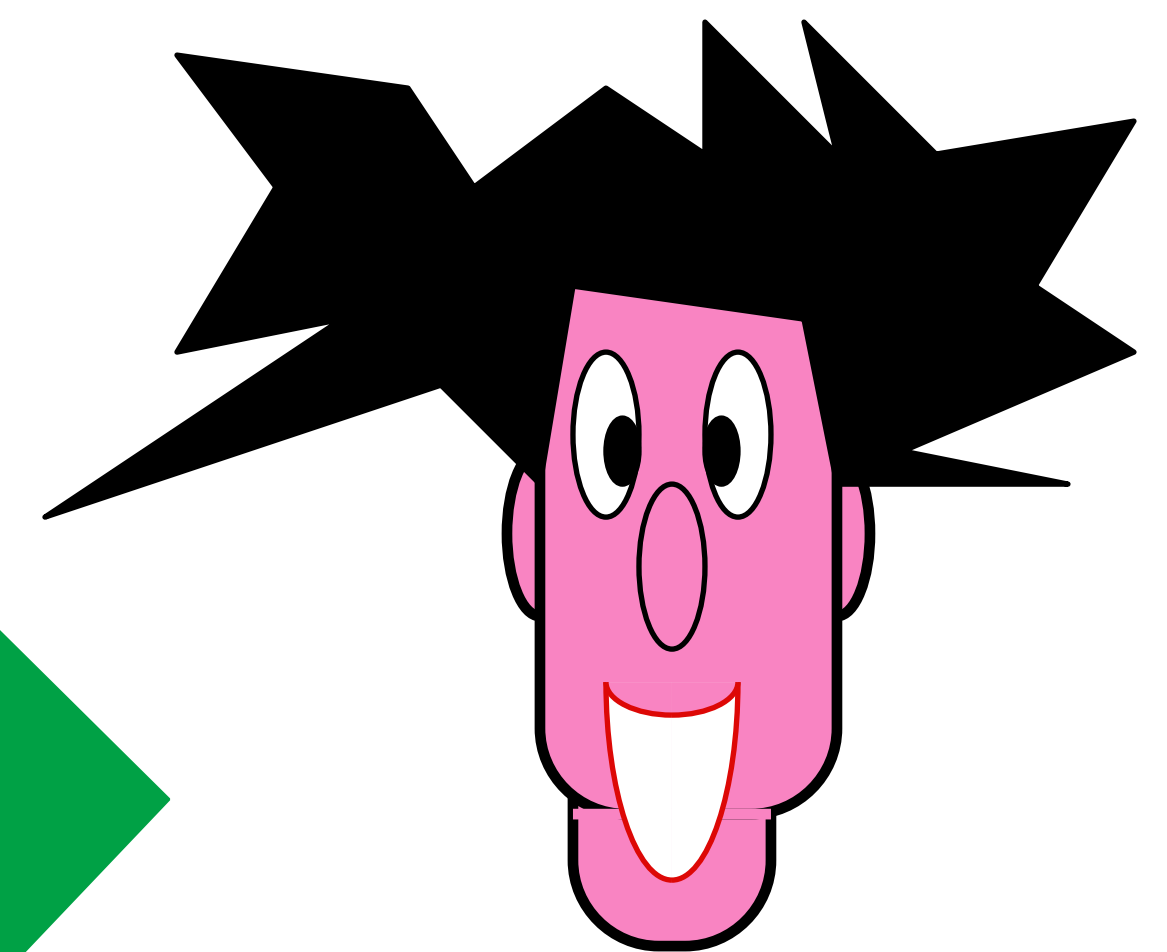
$$x_0 \oplus x_1 = (x_0 \oplus z) \oplus (x_1 \oplus z) = y_0 \oplus y_1$$

possible with prob. at most  $2^{-n}$



$x_0$

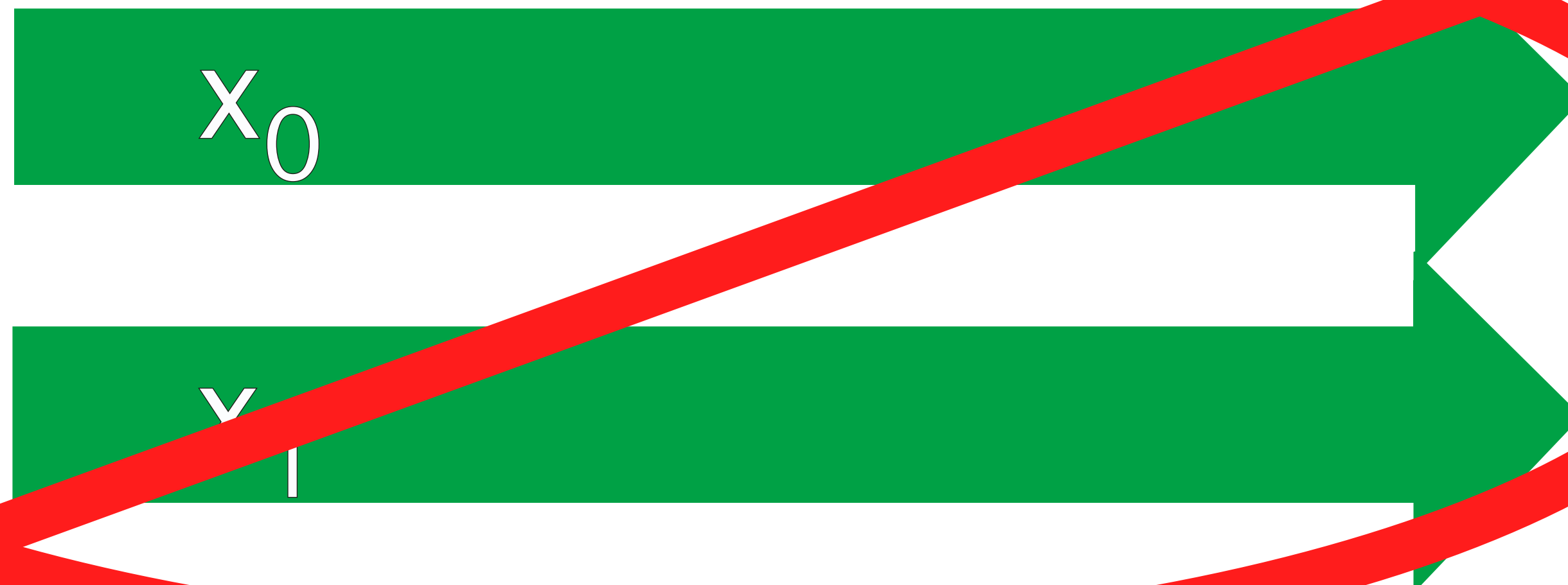
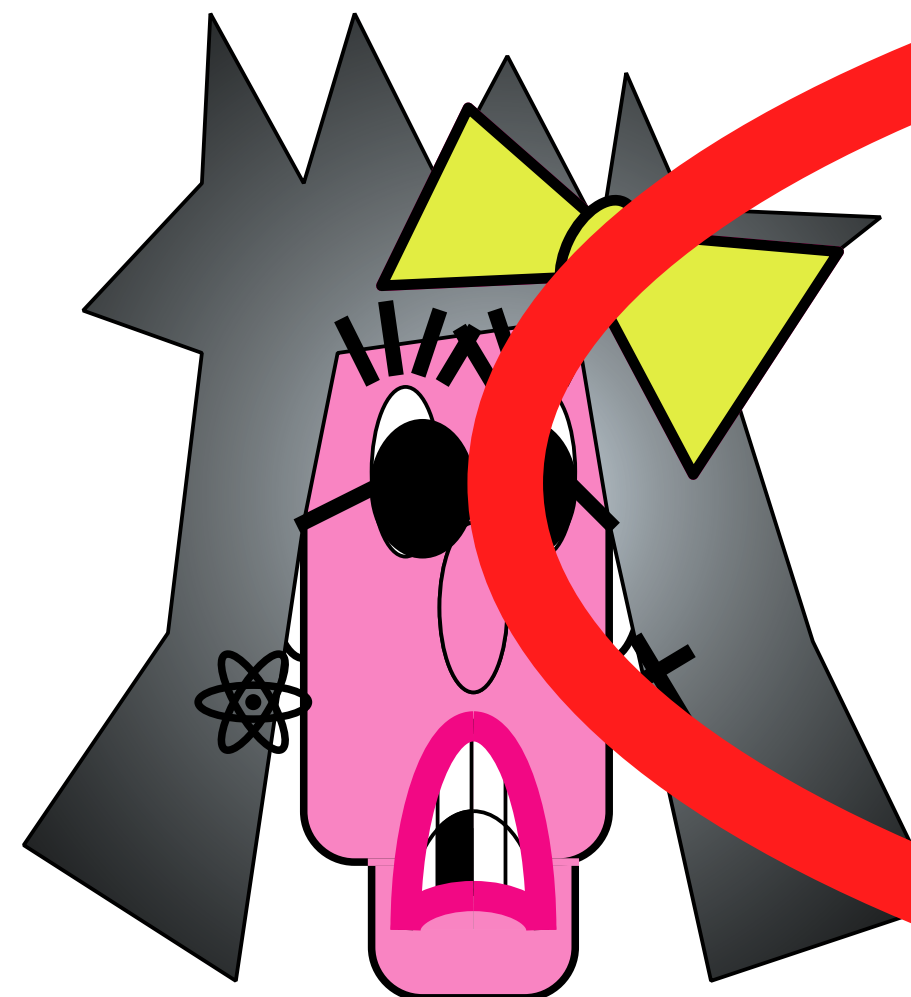
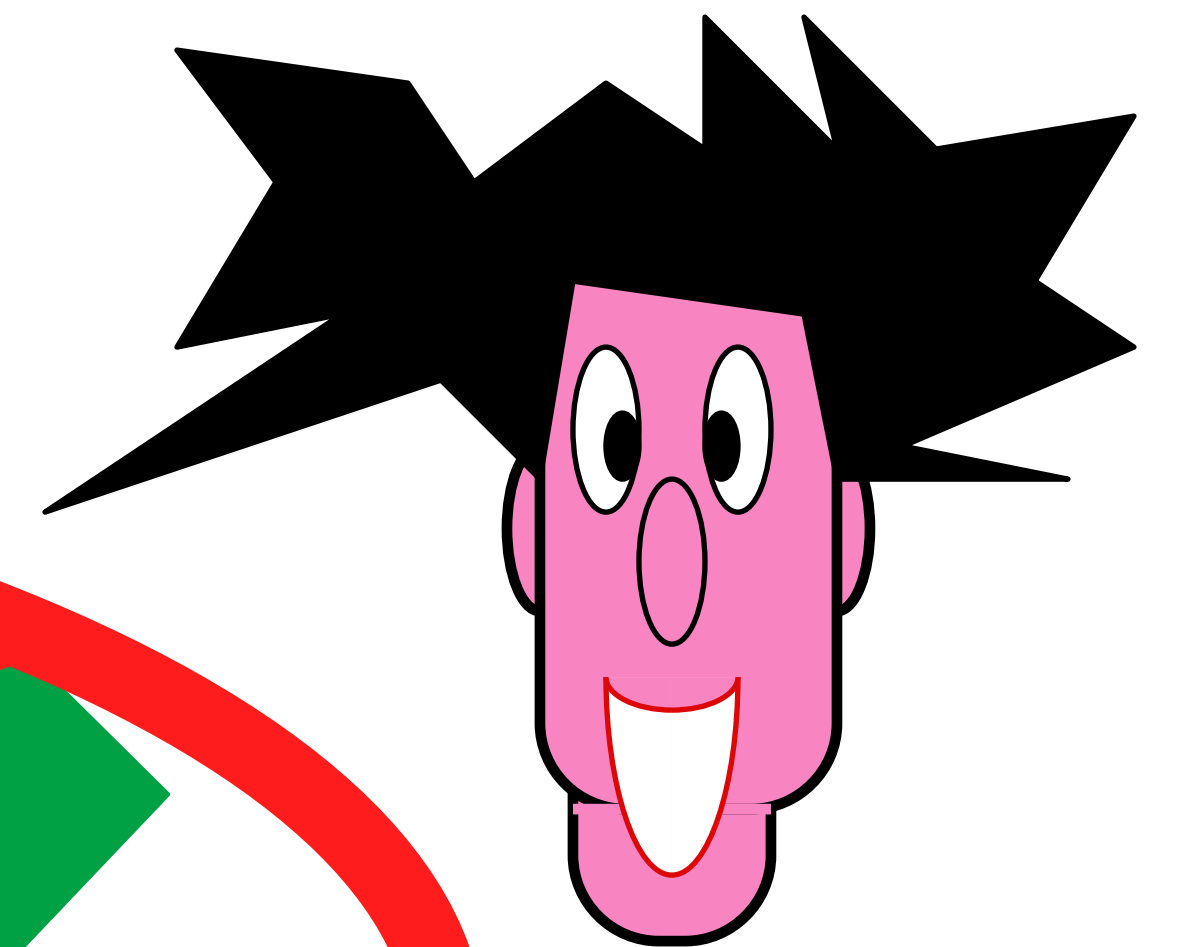
$x_1$



STRONG Q-BGKW"



# Quantumly



STRONG Q-BGKW”



# Quantumly

## MAIN THEOREM

Let  $\mathbf{0}$  and  $\mathbf{1}$  be POVMs such that outputs  $x_0$  and  $x_1$  one could obtain by applying one of them to the state shared among the two provers.

Suppose the success probability of unveiling is

$$p_0 + p_1 > 1 + \delta,$$

then the [prediction probability of  $y_0 \oplus y_1$ ]  $> \delta$ .

This prediction probability is achieved by first applying  $\mathbf{0}$  to the shared state followed by  $\mathbf{1}$  on the leftover system or the other way around.

**STRONG Q-BGKW''**



# Recently

## MAIN THEOREM

Let  $\mathcal{O}$  and  $\mathcal{I}$  be POVMs such that outputs  $x_0$  and  $x_1$  one could obtain by applying one of them to the state shared among the two provers.

Suppose the success probability of unveiling is

$$p_0 + p_1 > 1 + \delta,$$

then the [prediction probability of  $x_0 \oplus y_1$ ]  $> \delta$ .

This prediction probability is achieved by first applying  $\mathcal{O}$  to the shared state followed by  $\mathcal{I}$  on the leftover system or the other way around.

**STRONG Q-BGKW"**



# Yesterday



## MAIN THEOREM

Let  $\mathcal{O}$  and  $\mathcal{I}$  be POVMs such that outputs  $x_0$  and  $x_1$  one could obtain by applying one of them to the state shared among the two provers.

Suppose the success probability of unveiling is

$$p_0 + p_1 > 1 + \delta,$$

then the [prediction probability of  $y_0 \oplus y_1$ ]  $>$  **poly**( $\delta$ ).

This prediction probability is achieved by first applying  $\mathcal{O}$  to the shared state followed by  $\mathcal{I}$  on the leftover system (**after mostly undoing  $\mathcal{O}$** ) or the other way around.

**STRONG Q-BGKW"**

MUCH • T O U S A N D T A K • M I L L E S  
M E R C I S • G R A C I A S •  
T H A N K Y O U V E R Y

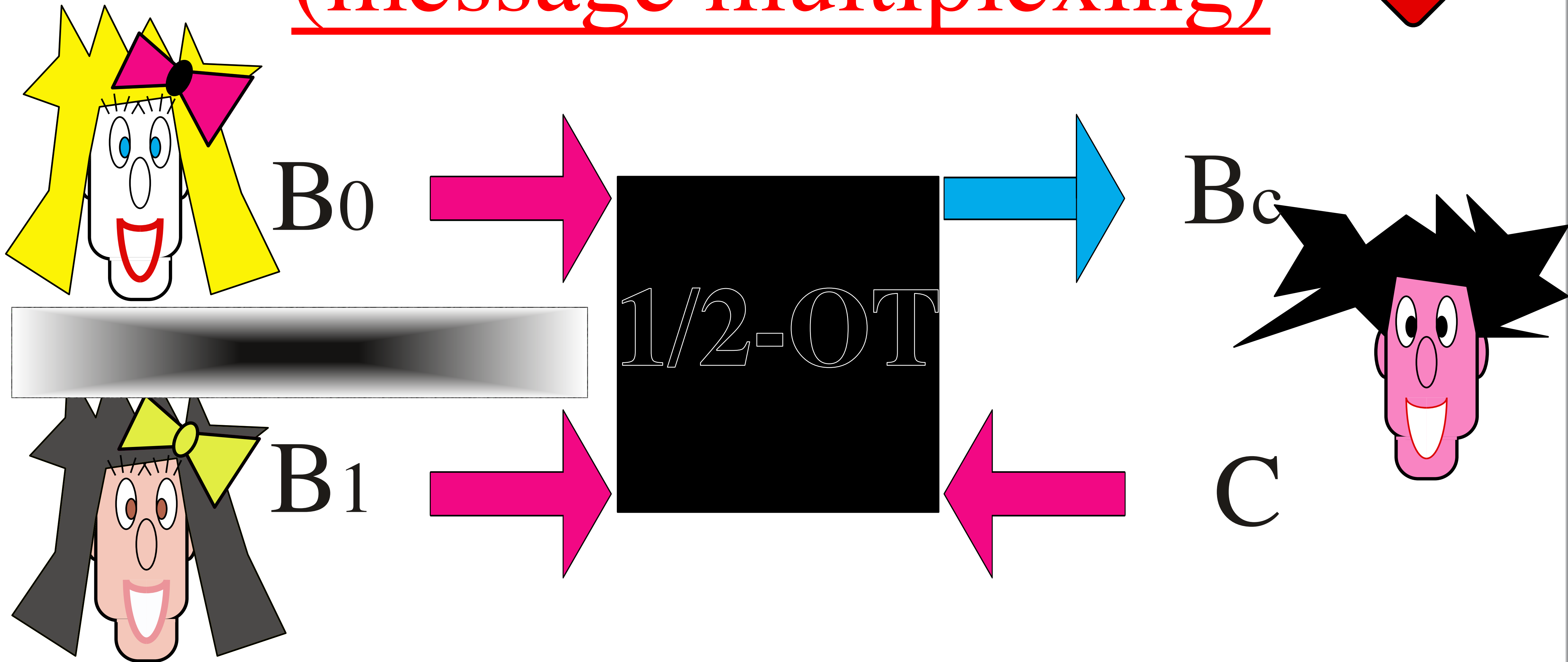
**SECURITE**



**STRONG Q-BGKW”**

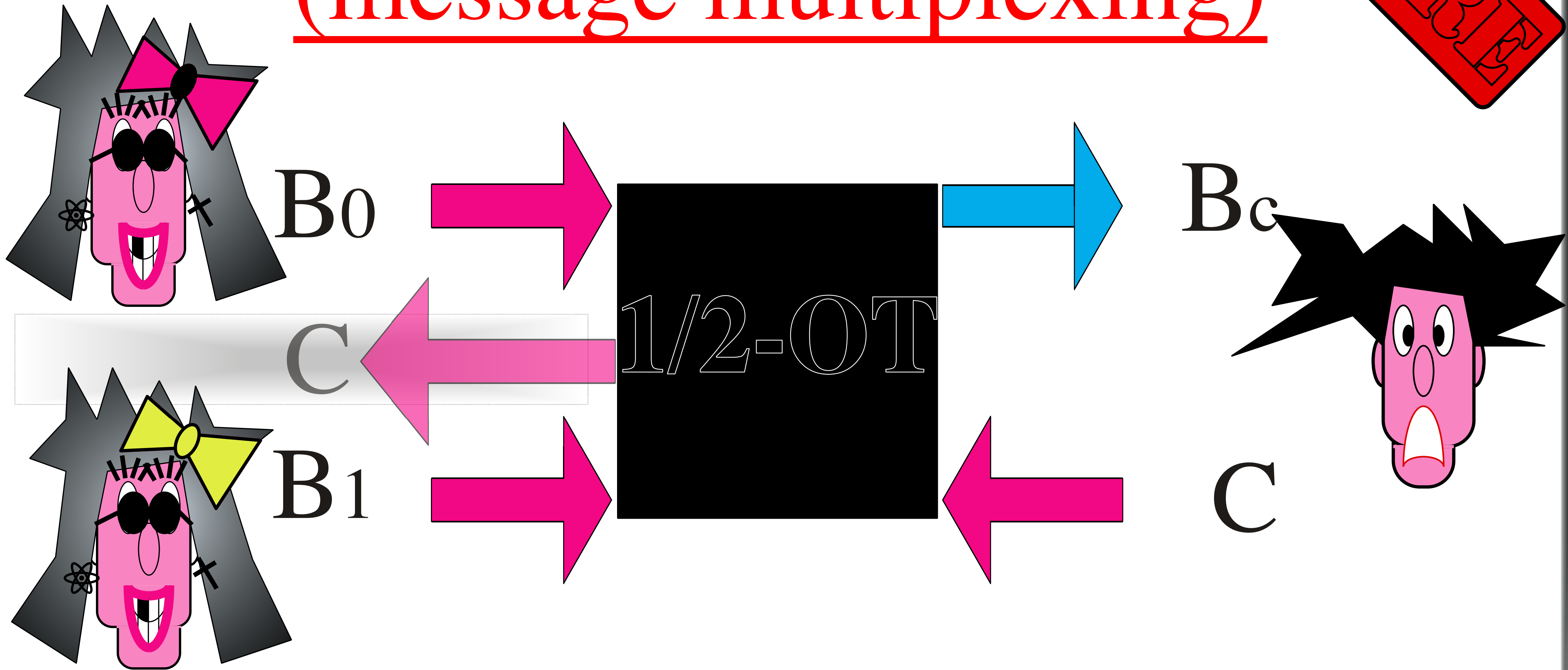
**(5)**  
**WARNING!**

# Oblivious Transfer (message multiplexing)



# Oblivious Transfer (message multiplexing)

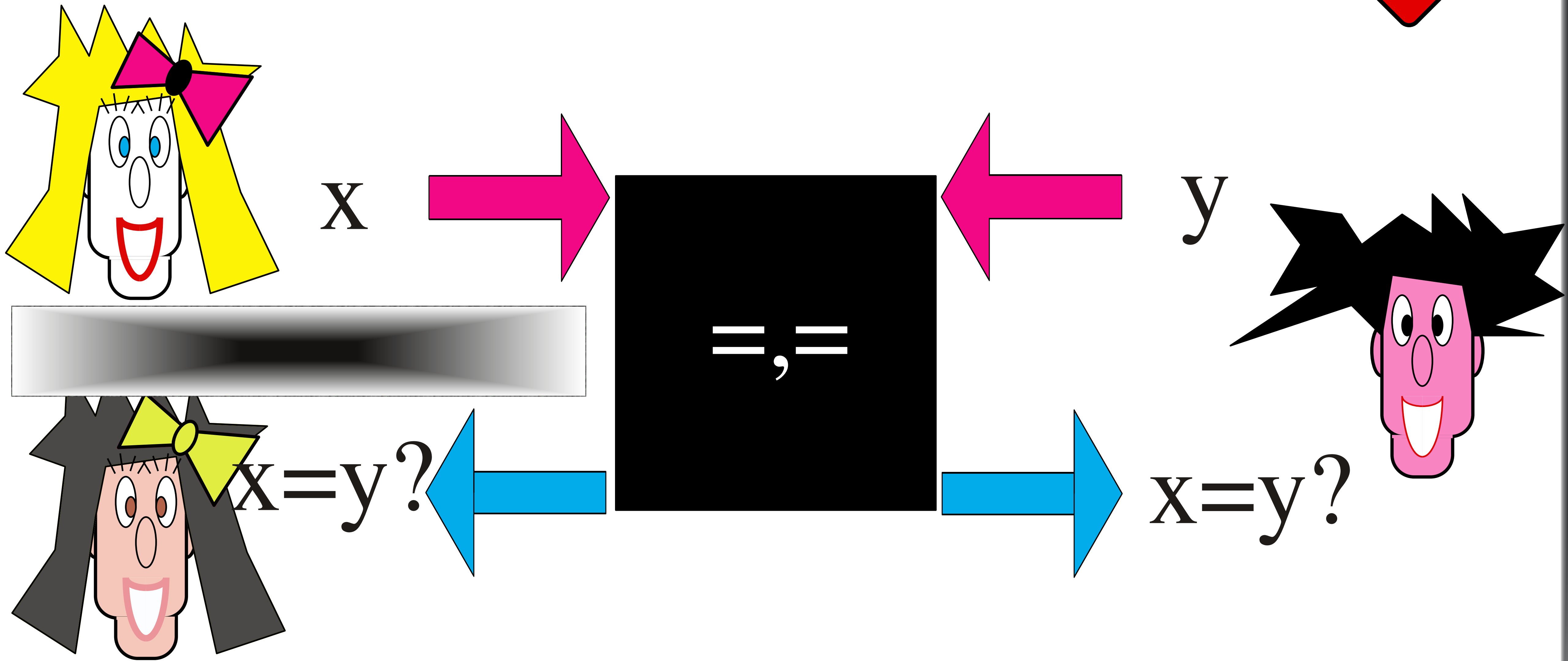
**INSECURE**



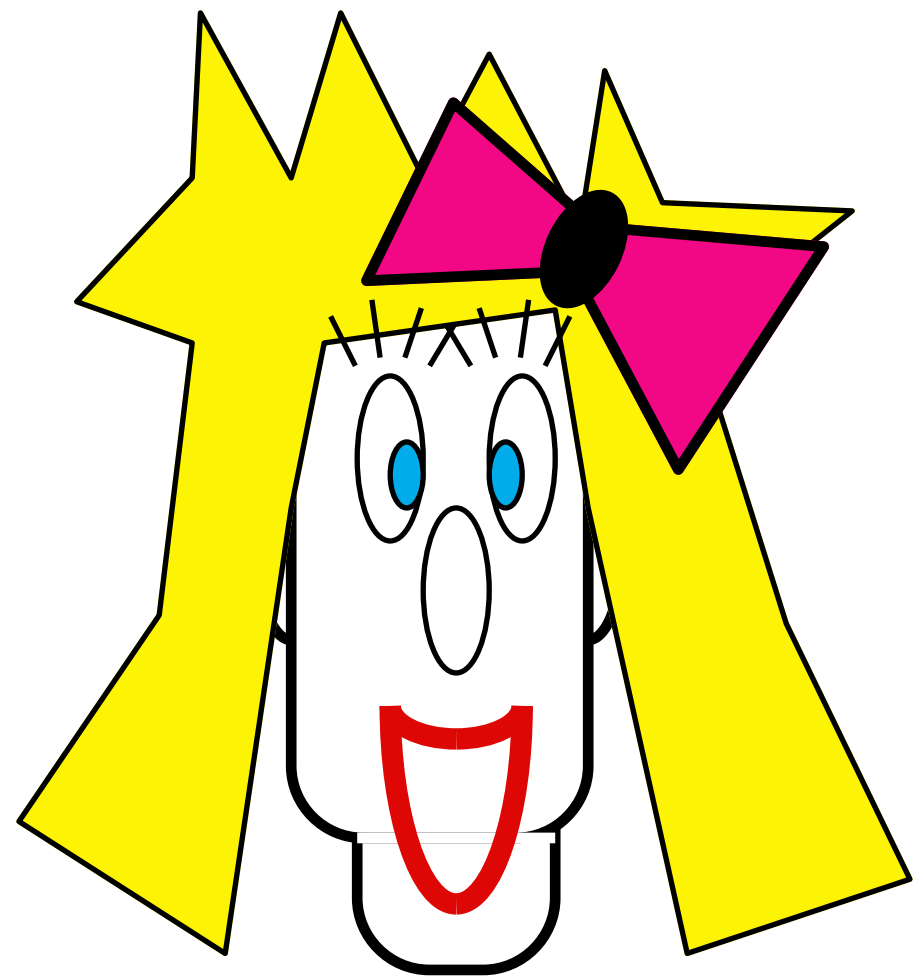
**Brassard, Crépeau, Mayers, Salvail 97**



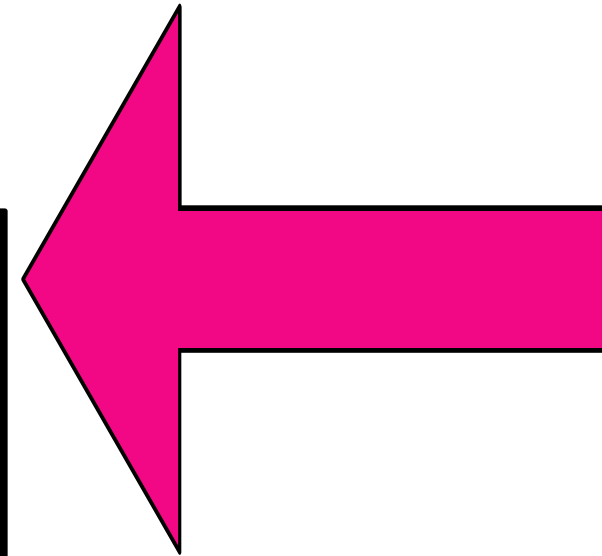
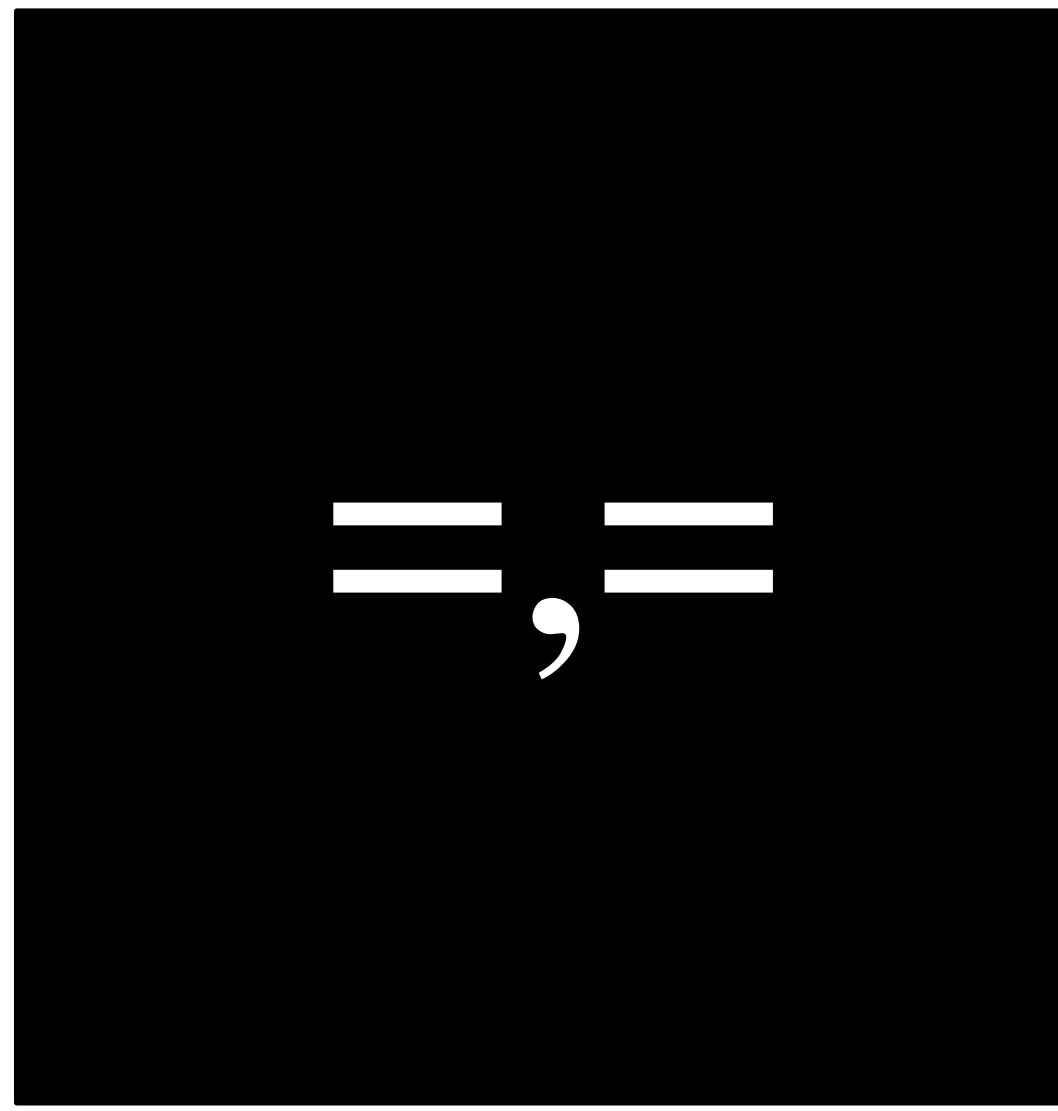
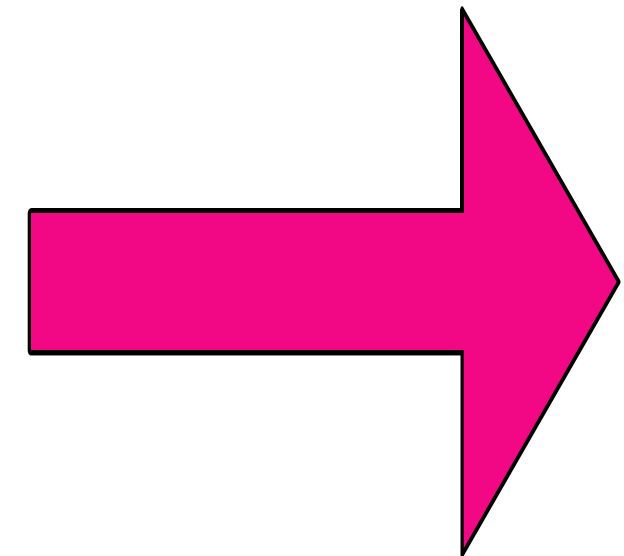
# Mutual Identification



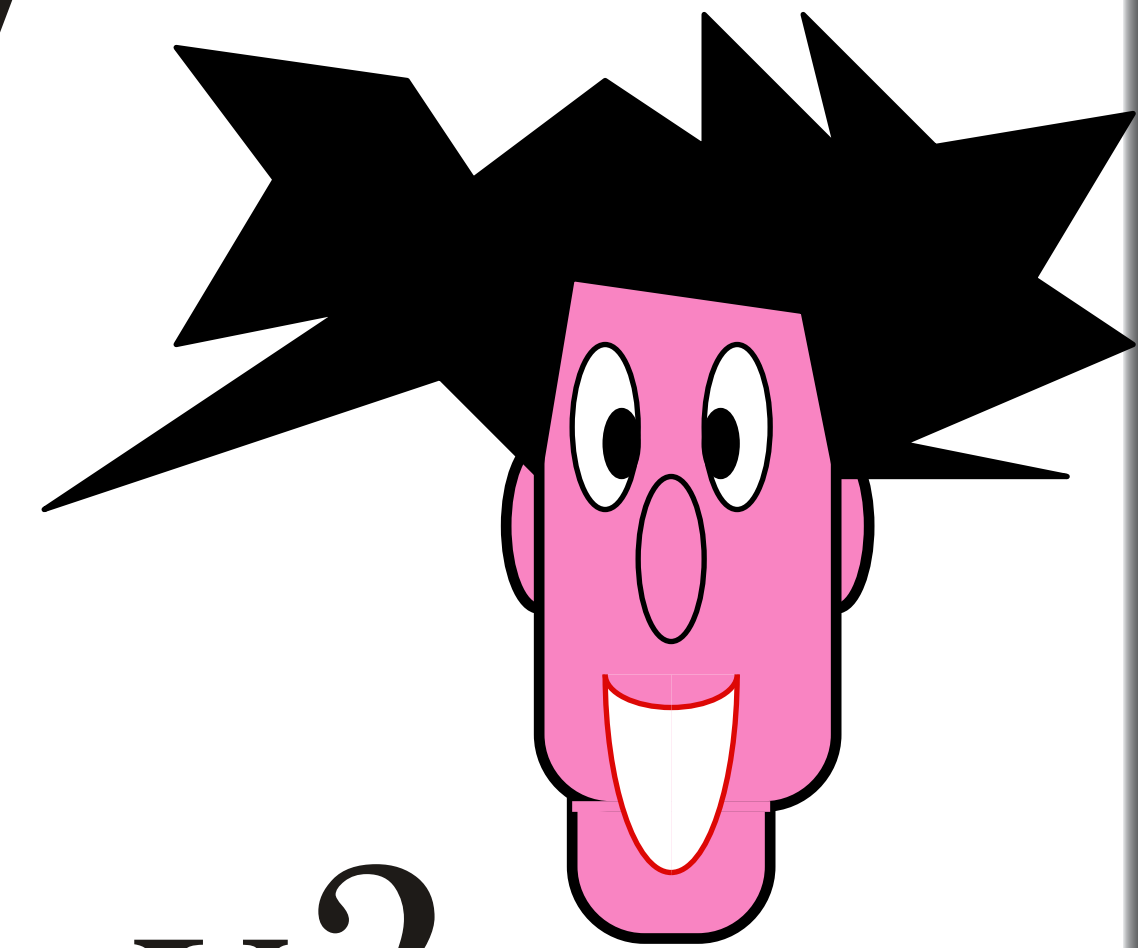
# Mutual Identification



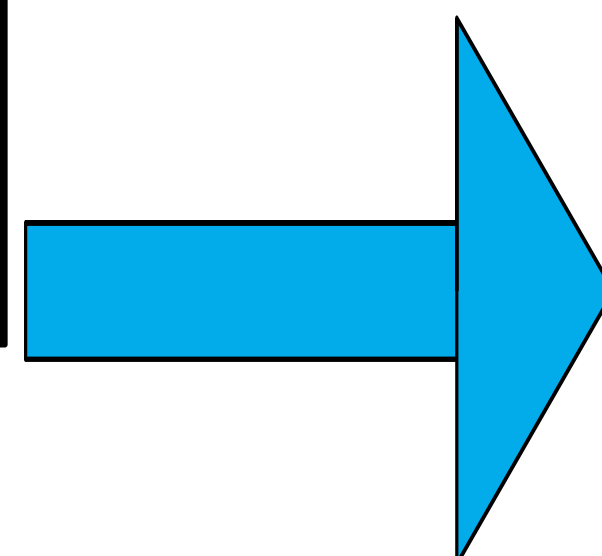
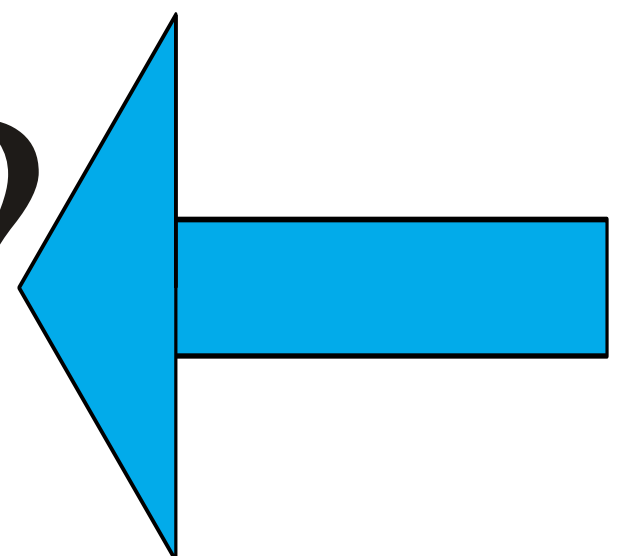
X



y



x=y?

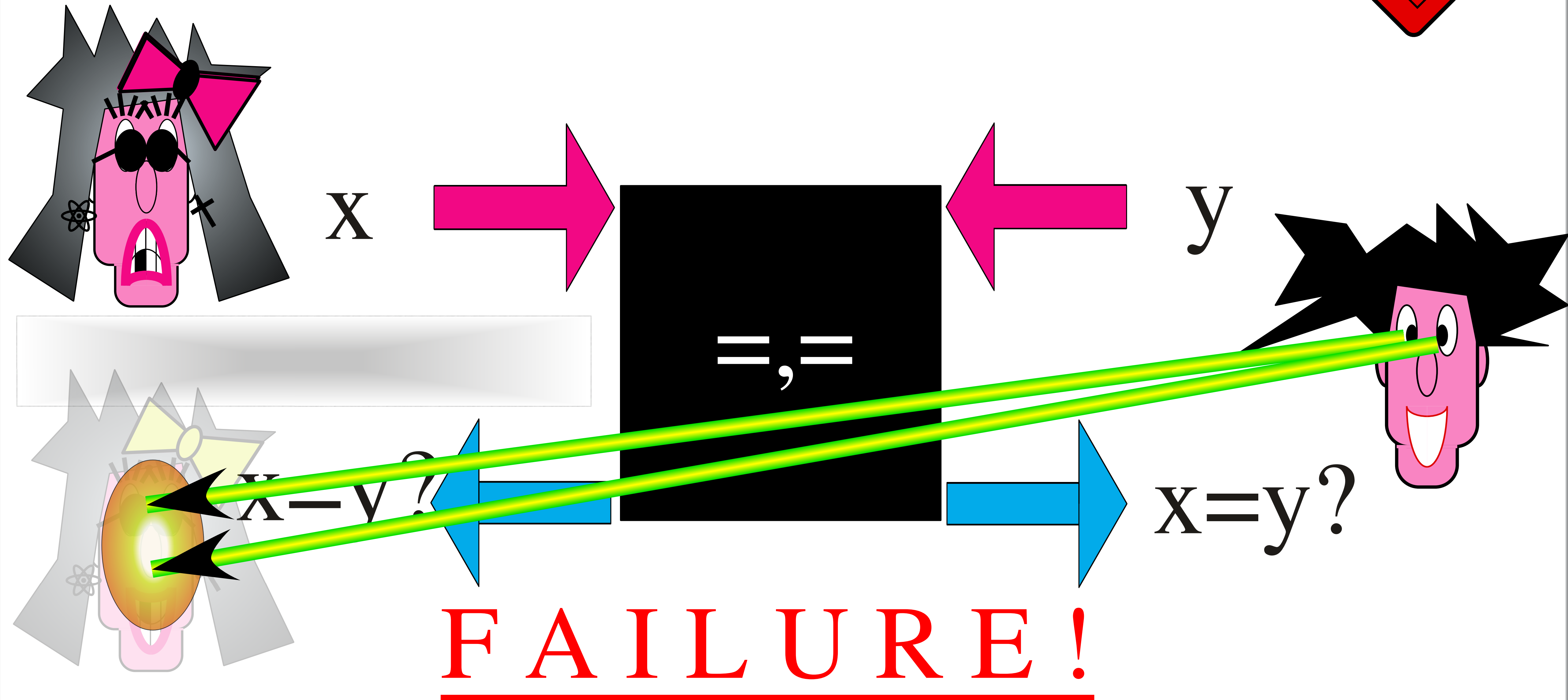


x=y?

S U C C E S S !

# Mutual Identification

**SECURE**



**FAILURE!**

# Bit Commitment Strikes Back

**Claude Crépeau**

**School of Computer Science  
McGill University**



**Joint work with J-R Simard and A Tapp**  
**L Salvail**