Faculty of Science  
Final Examination  

Computer Science 308-250B section I  
Introduction to Computer Science  

Examiner: Prof. Claude Crépeau  
Date: April 28, 2000  
Associate Examiner: Prof. Nathan Freidman  
Time: 9:00 – 12:00  

INSTRUCTIONS:  
• This examination is worth 60% of your final grade.  
• The total of all (sub-)questions is 65 points.  
• Each (sub-)question is assigned a value found in brackets next to it.  
• OPEN • BOOKS •/• OPEN • NOTES  
• Faculty standard calculator permitted only.  
• This examination consists of 3 pages including title page.  
• This examination consists of 6.  

SUGGESTION : read all the questions and their values before you start answering.
1) Consider the following pattern: "barbados".
   - Construct the failure function of the Knuth-Morris-Pratt algorithm for this pattern.
   - Draw a FSA that accepts exactly this pattern.

2) HEAPS
   a) What is the largest number of distinct integers that we can put in a binary tree that satisfies both the definitions (explain your answers)
      - of a (min)-heap and of a binary search tree?
      - of a (max)-heap and of a binary search tree?
   b) Consider the following list of numbers: {3, 19, 17, 11, 7, 9, 21}.
      Give two distinct ways of putting these elements in a (max)-heap.
      
      \[
      \begin{array}{c}
      \text{Reproduce these two trees in your exam book and fill them in with the appropriate numbers to form two distinct (max)-heaps.}
      \end{array}
      \]

3) Consider the following (Java and graphical) definition of a node

   ```java
   Class node
   {
   Object content;
   node next;
   }
   ```

   Let \( h \) be an object of type node, the head of a linked list of nodes.

   Give the pseudo-code description of an algorithm "reverse(node \( h \))" that reverses the nodes in the list without actually moving the contents.

   Make sure that your algorithm works even if the list loops on itself at the start, such as:

   ```java
   h :
   ```
4) For each statement, say if it is true or false.

| Right = +2pts, Wrong = -1pt, Blank = 0pt. |

(a) In order to compute the fibonacci number $f_n$, any algorithm requires at least time $\Theta(n)$.

(b) The regular expressions $a(b+a)^*$ and $a+a(a+b)^*$ generate the same strings.

(c) When you calculate a simple closed path using the algorithm seen in class around the vertices of an $n$-sided polygon, you can determine in time $O(\log n)$ if a given point is a vertex of the polygon or not.

(d) The convex hull of a rectangle is always the rectangle itself.

(e) Every planar graph is 4-colorable.

5) Suppose you are given a (single) queue $Q$ containing distinct integers.

- Find an algorithm (in pseudo-code) to sort these numbers using nothing else than a fixed number of extra `int` variables (no array, no stack, no other queue) and queue operations on $Q$.

- Estimate the worst-case running time of your algorithm and express this time function using the Big-O notation.

- Argue that this would be impossible with a single stack $S$ instead of a queue.

6) Consider the following binary tree. List the sequence of nodes visited according to a

- Depth-first search starting at (3).

- Breath-first search starting at (3).

Notice that this is a binary search tree.

- What is the successor of (2) ?

- Draw the resulting tree after removing (6).