

Mathematical Induction

Let P be a predicate over the natural integers, $P: \mathbb{N} \rightarrow \{\text{true}, \text{false}\}$.

Let $a > 0$.

If

- (i) $P(a) = \text{true}$
- (ii) for any $n > a$, $P(n-1) \Rightarrow P(n)$

then for any $n \geq a$, $P(n) = \text{true}$.

Mathematical Induction

$$\sum_{i=1}^n i = n(n+1)/2, \text{ for all } n > 0$$

$$f_n = \begin{cases} n & \text{if } n=0,1 \\ f_{n-1} + f_{n-2} & \text{if } n>1 \end{cases}$$

$$f_n = (\phi^n - (-\phi)^{-n})/\sqrt{5}, \text{ for all } n \geq 0$$

where $\phi = (1 + \sqrt{5})/2$

$$\text{Collatz}_1(n) = \begin{cases} 1 & \text{if } n=1 \\ \text{Collatz}_1(n/2) & \text{even } n \\ \text{Collatz}_1(n+1) & \text{odd } n > 1 \end{cases}$$

$$\text{Collatz}_1(n)=1, \text{ for all } n > 0$$

$$\text{Mulàlarusse}(a,b)=a^*b, \text{ for all } a,b \geq 0$$

Generalized Mathematical Induction

Let P be a predicate over the natural integers, $P: \mathbb{N} \rightarrow \{\text{true}, \text{false}\}$.

Let $a > 0$.

If

- (i) $P(a) = \text{true}$
- (ii) for any $n > a$, $P(a), P(a+1), \dots, P(n-2), P(n-1) \Rightarrow P(n)$

then for any $n \geq a$, $P(n) = \text{true}$.

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Mulàlarusse(a,b)
If (b=0)
Then return 0 // a*b = a*0 = 0
Else If (b is even)
    Then return Mulàlarusse( a+a, b/2 ) // a*b = (2*a)*(b/2)
    Else return a + Mulàlarusse( a+a, (b-1)/2 ) // a*b = a + (2*a)*( (b-1)/2 )

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Let $P(b)$ = "for all $a \geq 0$, $\text{Mulàlarusse}(a,b) = a^b$ ".

If

- (i) $P(0)$ = "for all $a \geq 0$, $\text{Mulàlarusse}(a,0) = 0 = a^0$ " = true
- (ii) Let $b > 0$, assume $P(0) = P(1) = \dots = P(b-2) = P(b-1) = \text{true}$.

- If b is even then

$$\begin{aligned}
 \text{Mulàlarusse}(a,b) &= \text{Mulàlarusse}(2^*a, b/2) \\
 &= 2^*a^*b/2 && \text{by } P(b/2) \text{ since } 0 < b/2 < b \\
 &= a^b && (\Rightarrow P(b) = \text{true})
 \end{aligned}$$

- If b is odd then

$$\begin{aligned}
 \text{Mulàlarusse}(a,b) &= a + \text{Mulàlarusse}(2^*a, (b-1)/2) \\
 &= a + 2^*a^*(b-1)/2 && \text{by } P((b-1)/2), 0 \leq (b-1)/2 < b \\
 &= a^b && (\Rightarrow P(b) = \text{true})
 \end{aligned}$$

then for any $b \geq 0$, $P(b) = \text{true}$.