

Quadratic-time Methods

Standard Multiplication and Multiplication à la russe

$$\begin{array}{r} 981 \\ \times \underline{1234} \\ \hline \end{array}$$

3924

2943

1961

$$\begin{array}{r} + \underline{981} \\ \hline 1210554 \end{array}$$

$$\begin{array}{rrr} 981 & \times & 1234 \\ 490 & & 2468 \\ 245 & & 4936 \\ 122 & & 9872 \\ 61 & & 19744 \\ 30 & & 39488 \\ 15 & & 78976 \\ 7 & & 157952 \\ 3 & & 315904 \\ 1 & & 631808 \\ & & \underline{+631808} \\ & & 1210554 \end{array}$$

Multiply($a_n a_{n-1} \dots a_0$, $b_m b_{m-1} \dots b_0$)

FOR $k = 1$ **TO** $m+n+1$

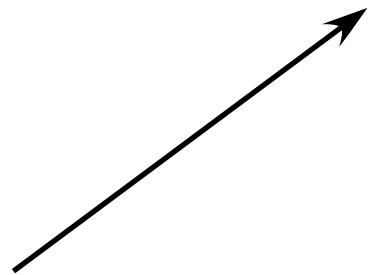
- $c_k = 0$

FOR $i = 0$ **TO** n

- carry = 0
- **FOR** $j = 0$ **TO** m
- . . $k = i+j$
- . . $c_k = c_k + \text{carry} + a_i * b_j$
- . . carry = 0
- . . **WHILE** $c_k > 9$
- . . . carry = carry + 1
- . . . $c_k = c_k - 10$
- $c_{i+m+1} = \text{carry}$

Divide-and-Conquer Multiplication

$$\begin{array}{r}
 [09][81] \\
 \times [12][34] \\
 \hline
 ([34]x[81]) \\
 ([34]x[09])\cdots \\
 ([12]x[81])\cdots \\
 + ([12]x[09])\cdots\cdots \\
 \hline
 2754 \\
 306\cdots \\
 972\cdots \\
 + 108\cdots\cdots \\
 \hline
 1210554
 \end{array}$$



$$\begin{array}{r}
 [3][4] \\
 \times [8][11] \\
 \hline
 ([1]x[4]) \\
 ([1]x[3])\cdots \\
 ([8]x[4])\cdots \\
 + ([8]x[3])\cdots\cdots \\
 \hline
 4 \\
 3 \\
 32 \\
 + 24\cdots \\
 \hline
 2754
 \end{array}$$

$$\begin{array}{r}
 \bullet\bullet\bullet \\
 [1][2] \\
 \times [0][9] \\
 \hline
 ([9]x[2]) \\
 ([9]x[1])\cdots \\
 ([0]x[2])\cdots \\
 + ([0]x[1])\cdots\cdots \\
 \hline
 18 \\
 9 \\
 0 \\
 + 0\cdots \\
 \hline
 108
 \end{array}$$

Another Quadratic-time Method !!!

Divide-and-Conquer Multiplication

$$[A+B][C+D] = AC + BD + [AD+BC]$$

$$\begin{array}{r}
 [09][81] \\
 [12][34] \\
 \hline
 ([34]X[81]) \\
 ([34]X[09]+[12]X[81])\cdots \\
 + ([12]X[09])\cdots\cdots \\
 \hline
 2754 \\
 1288\cdots \\
 \hline
 + 108\cdots\cdots \\
 \hline
 1210554
 \end{array}$$

$$\begin{array}{r}
 [3][4] \\
 \hline
 [8][1] \\
 ([1]X[4]) \\
 + ([8]X[3])\cdots \\
 \hline
 35\cdots \\
 + 24\cdots \\
 \hline
 2754
 \end{array}$$

$\xrightarrow{[1]X[4]}$
 $\xrightarrow{[8+1]X[3+1]}$
 $\xrightarrow{63-4-24=35}$
 $\xrightarrow{[8]X[31]}$
 $\xrightarrow{24}$

$$\begin{aligned}
 & ([34]X[09]+[12]X[81]) = \\
 & [09+81]X[12+34]-[34]X[81]-[12]X[09]= \\
 & [90]X[46]-2754-108= \\
 & 4140-2754-108= \\
 & 1288
 \end{aligned}$$

$$\begin{array}{r}
 [1][2] \\
 \hline
 [0][9] \\
 ([9]X[2]) \\
 + ([0]X[1])\cdots \\
 \hline
 18 \\
 9\cdots \\
 \hline
 0\cdots \\
 \hline
 108
 \end{array}$$

$\xrightarrow{[9]X[2]}$
 $\xrightarrow{[9+0]X[1+2]}$
 $\xrightarrow{27-18-0=9}$
 $\xrightarrow{[0]X[11]}$
 $\xrightarrow{0}$

A non Quadratic-time Method !!!