Quadratic-time Methods

Standard Multiplication and Multiplication à la russe

```
      981
X    1234
     3924
    2943
   1961
+ 981
  1210554
```

981 X	1234	1234
490	2468	
245	4936	4936
122	9872	
61	19744	19744
30	39488	
15	78976	78976
7	157952	157952
3	315904	315904
1	631808	+631808
		1210554

Multiply($a_n a_{n-1} \ldots a_0$, $b_m b_{m-1} \ldots b_0$)

FOR $k \leftarrow 1$ **TO** $m+n+1$

- $c_k \leftarrow 0$

FOR $i \leftarrow 0$ **TO** n

- carry $\leftarrow 0$
- **FOR** $j \leftarrow 0$ **TO** m
- · $k \leftarrow i+j$
- · $c_k \leftarrow c_k + \text{carry} + a_i * b_j$
- · carry $\leftarrow 0$
- · **WHILE** $c_k > 9$
- · · carry $\leftarrow$ carry + 1
- · · $c_k \leftarrow c_k - 10$
- $c_{i+m+1} \leftarrow$ carry

Divide-and-Conquer Multiplication

```
          [09][81]
X         [12][34]
      ([34]X[81])
   ([34]X[09])••
   ([12]X[81])••
+([12]X[09])••••
            2754
           306••
           972••
+          108••••
         1210554
```

```
        [3][4]
X       [8][1]
    ([1]X[4])
   ([1]X[3])•
   ([8]X[4])•
+ ([8]X[3])••
            4
           3.
          32.
+         24..
         2754
```

```
[1]
X [4]
  4

[1]
X [3]
  3

[8]
X [4]
  32

[8]
X [3]
  24
```

●●●

```
        [1][2]
X       [0][9]
    ([9]X[2])
   ([9]X[1])•
   ([0]X[2])•
+ ([0]X[1])••
           18
           9.
           0.
+          0..
          108
```

```
[9]
X [2]
  18

[9]
X [1]
  9

[0]
X [2]
  0

[0]
X [1]
  0
```

Another Quadratic-time Method !!!

Divide-and-Conquer Multiplication

[A+B]X[C+D]=AC+BD+[AD+BC]

```
                [09][81]
X               [12][34]
          ([34]X[81])
([34]X[09]+[12]X[81])••
+       ([12]X[09])••••
                2754
               1288••
+             108••••
             1210554
```

```
                [3][4]
X               [8][1]
          ([1]X[4])
([1]X[3]+[8]X[4])•
+       ([8]X[3])••
                  4
                 35•
+               24••
               2754
```

```
[1]
X [4]
  4

[8+1]
X [3+4]
  63-4-24=35

[8]
X [3]
  24
```

```
([34]X[09]+[12]X[81])=
[09+81]X[12+34]-[34]X[81]-[12]X[09]=
[90]X[46]-2754-108=
4140-2754-108=
1288
```

```
                [1][2]
X               [0][9]
          ([9]X[2])
([9]X[1]+[0]X[2])•
+       ([0]X[1])••
                 18
                  9•
+                0••
               108
```

```
[9]
X [2]
  18

[9+0]
X [1+2]
  27-18-0=9

[0]
X [1]
  0
```

A non Quadratic-time Method !!!