•1) a) Show that for all integer \( r > 0 \), \( \sum_{i=1}^{n} i^r \) is \( \Theta(n^{r+1}) \).
   b) Show that \( \binom{n}{n/2} \) is \( O(2^n) \).
   c) Show that \( \binom{n}{n/2} \) is not \( \Omega(2^n) \).
   HINT: use Stirling’s formulae.

•2) Develop an algorithm and write a corresponding Java class that reads a String from the input and evaluates it as a mathematical expression made of quintits (“0”, “1”, “2”, “3”, “4”), and operators such as addition “+”, multiplication “*”, subtraction “-”, division “/”, and parentheses “(”, “)” . The operations “+”, “-” and “*” are to be interpreted mod 5, while division is to be interpreted as “ a/b ” = “ a*b’ ”, where b’ is the unique quintit such that b*b’ = 1. All priorities should be implemented according to the same rules as in JAVA. You will be provided with a JAVA object STACK that you may use in your programs. All our test examples will be syntactically valid, i.e. you do not have to recognize incorrect examples. However, you must return an error message if division by “0” is attempted.

Examples:

Input: “3+(4*2)”
Output: “1” since 4*2 mod 5 = 3, and 3+3 mod 5 = 1.

Input: “1+2*2”
Output: “0” since 2*2 mod 5 = 4, and 1+4 mod 5 = 0

Input: “3/(1+1)”
Output: “4” since 1+1 mod 5 = 2, and 3/2 mod 5 = 4, as 2*3 mod 5 = 1.

Input: “2*4/(2+1)/(3-2*4)+3”
Output: “DIVISION BY ZERO”

Suggestion: to analyze the input String use two Stacks; one to store (quintits) operands and one to store operators.