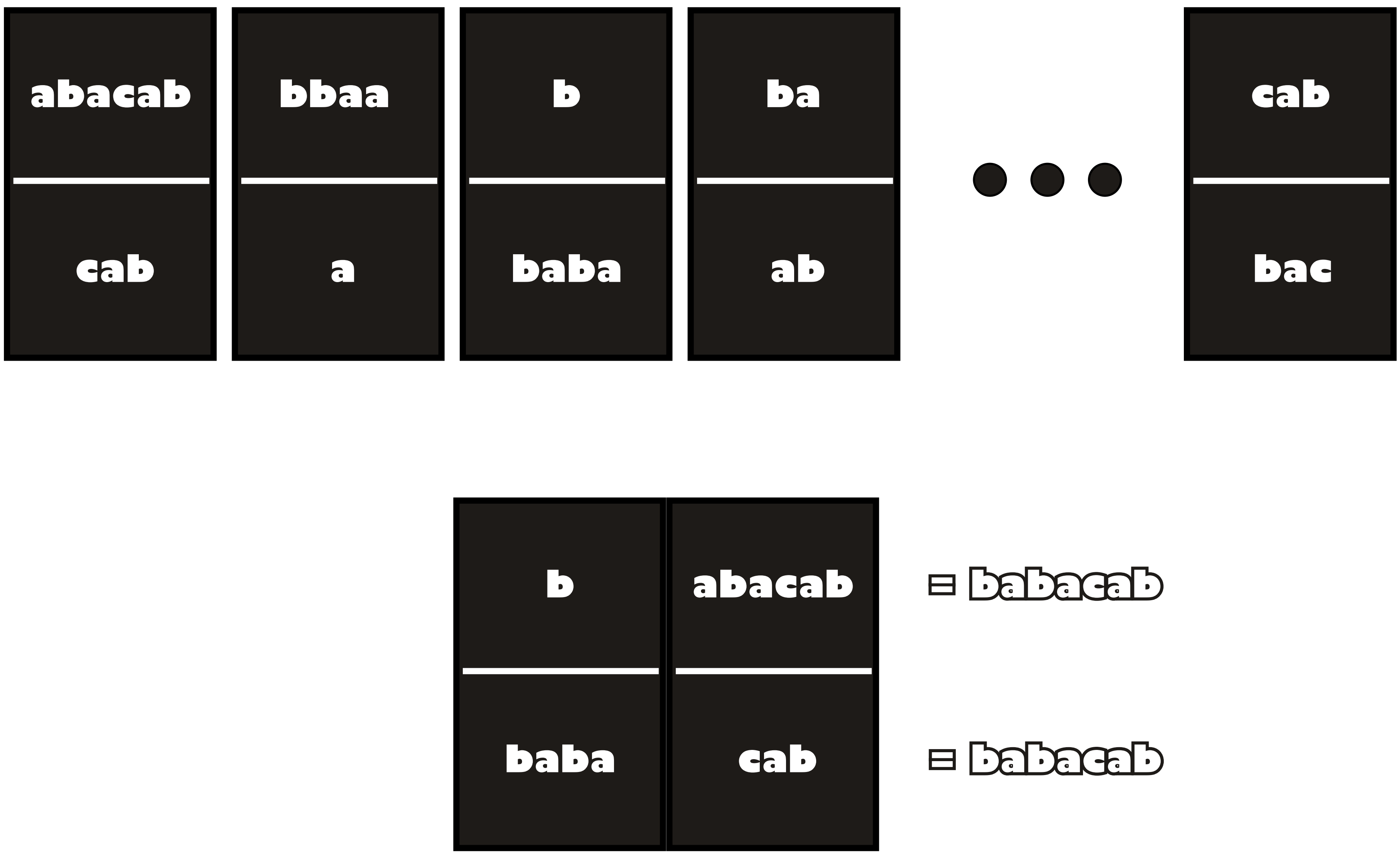
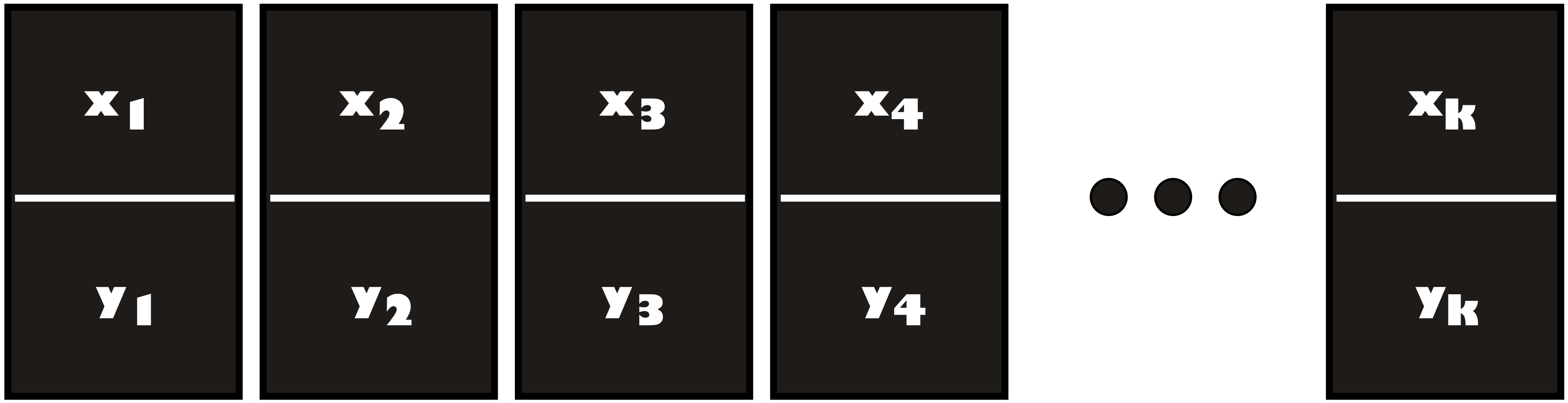
Post Correspondence Problem

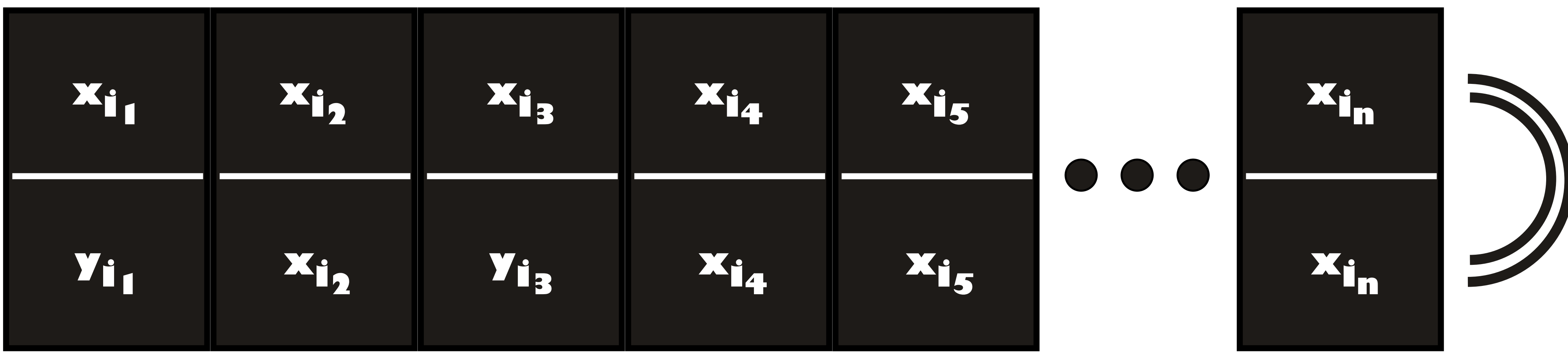


Post Correspondence Problem

Given words $x_1, x_2, \ldots, x_k$ and $y_1, y_2, \ldots, y_k$



Is there a sequence of indexes $i_1, i_2, i_3, \ldots, i_n$



such that $x_{i_1} x_{i_2} \ldots x_{i_n} = y_{i_1} y_{i_2} \ldots y_{i_n}$?

Post Correspondence Problem

Given words $x_1, x_2, \ldots, x_k$ and $y_1, y_2, \ldots, y_k$

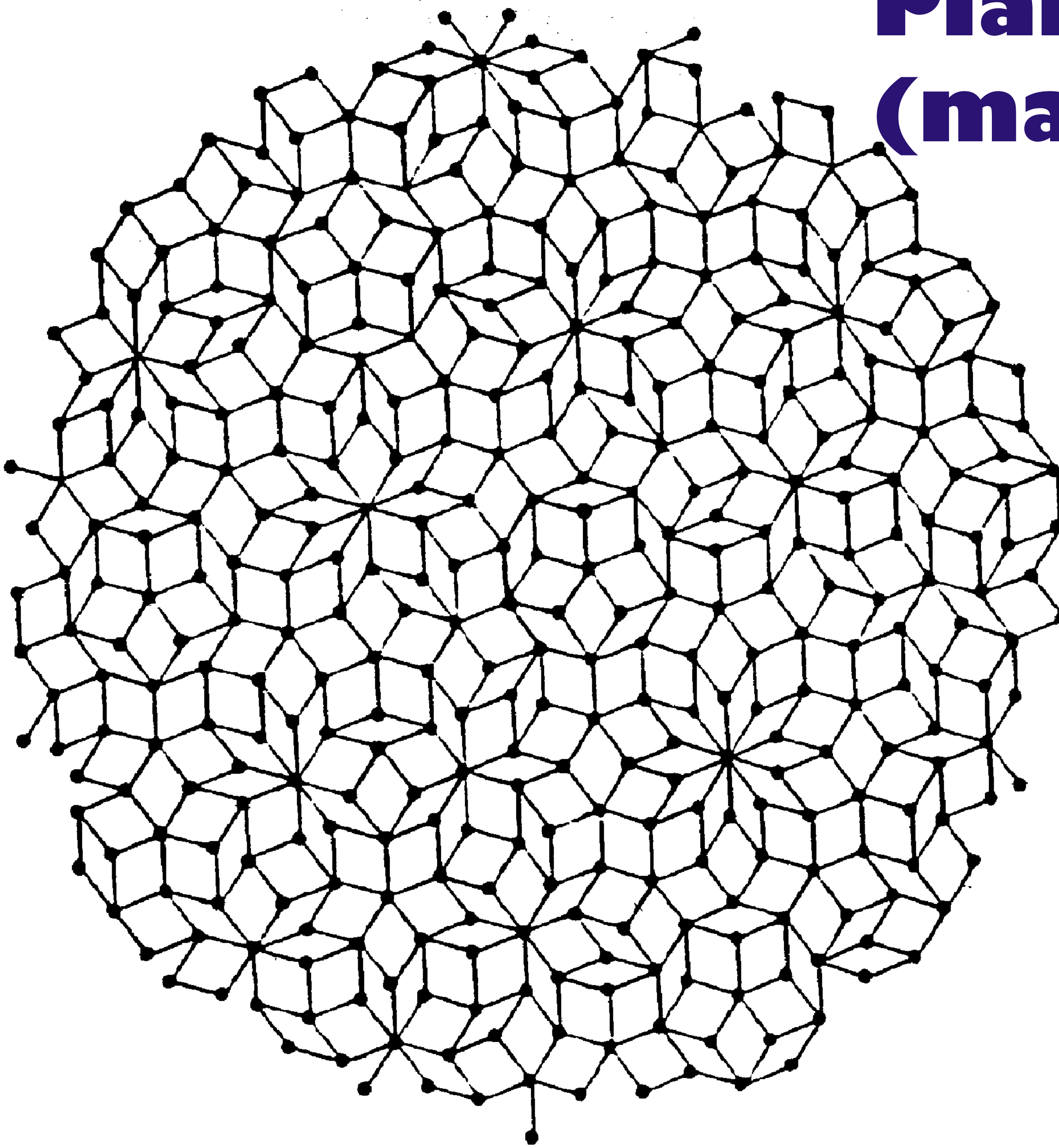
Is there a sequence of indexes $i_1, i_2, i_3, \ldots, i_n$
such that $x_{i_1} x_{i_2} \ldots x_{i_n} = y_{i_1} y_{i_2} \ldots y_{i_n}$?

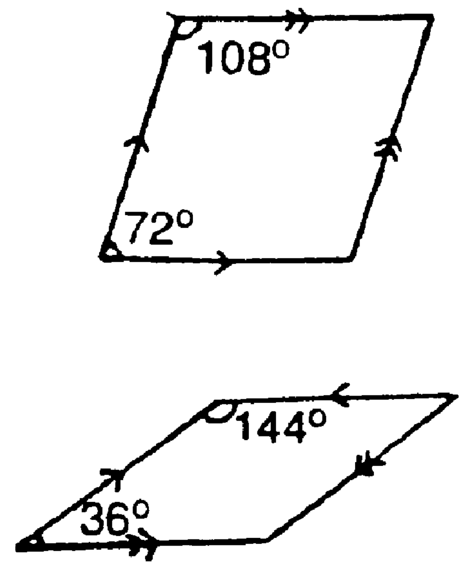
Theorem:

there cannot exist an algorithm that answers correctly on all instances of this problem.

PCP is undecidable

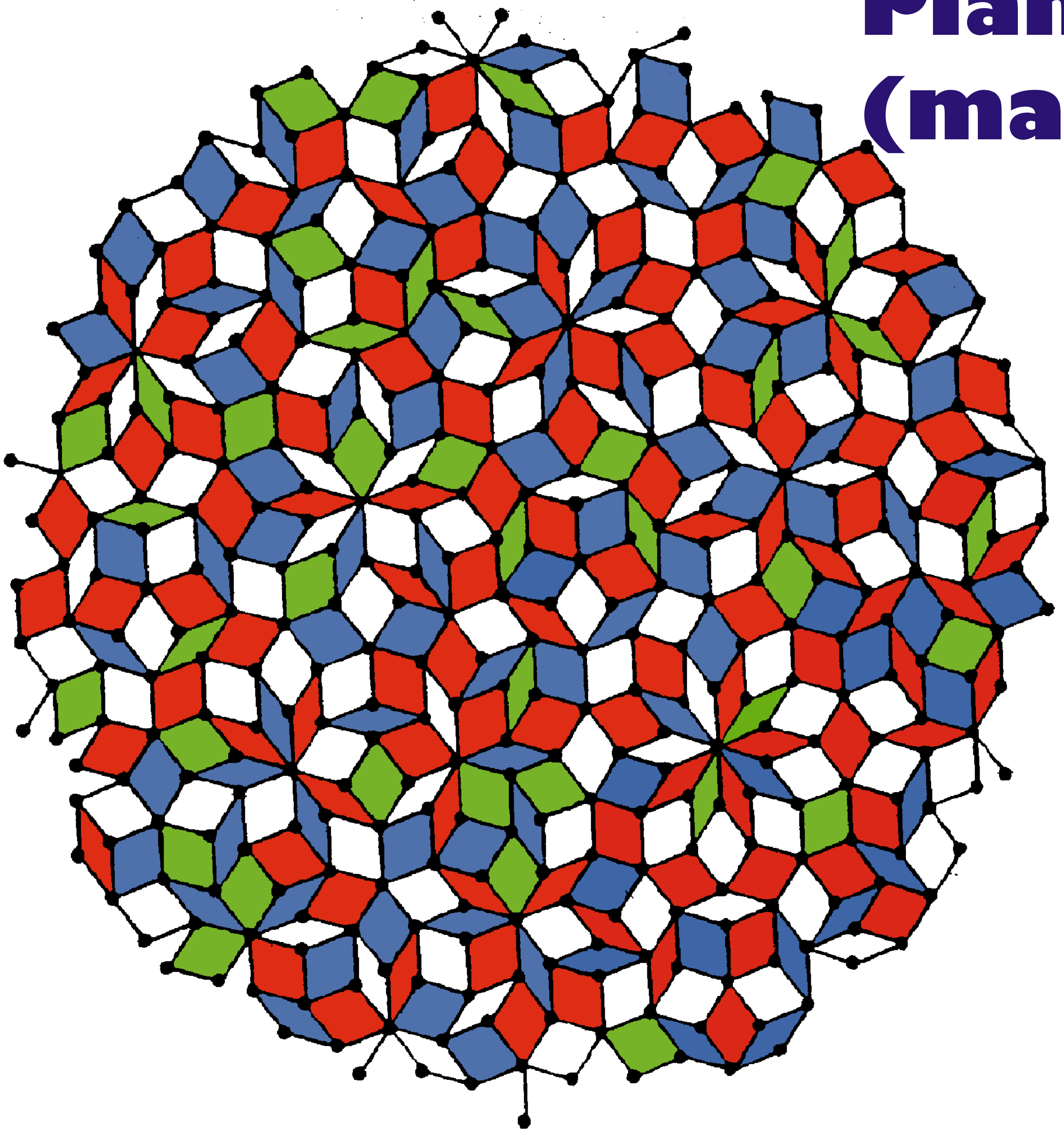
Plannar graphs (maps)

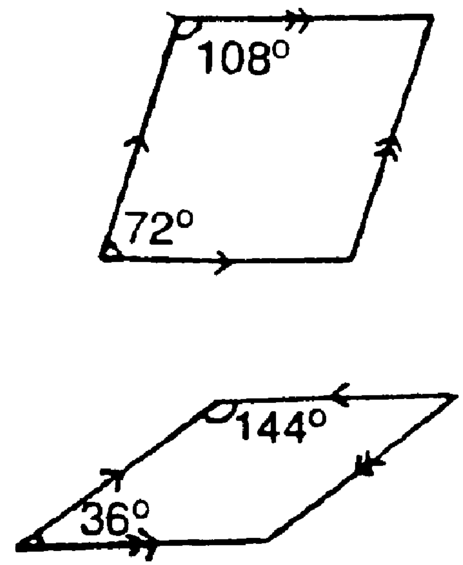




Is it k-colorable ?

Plannar graphs (maps)

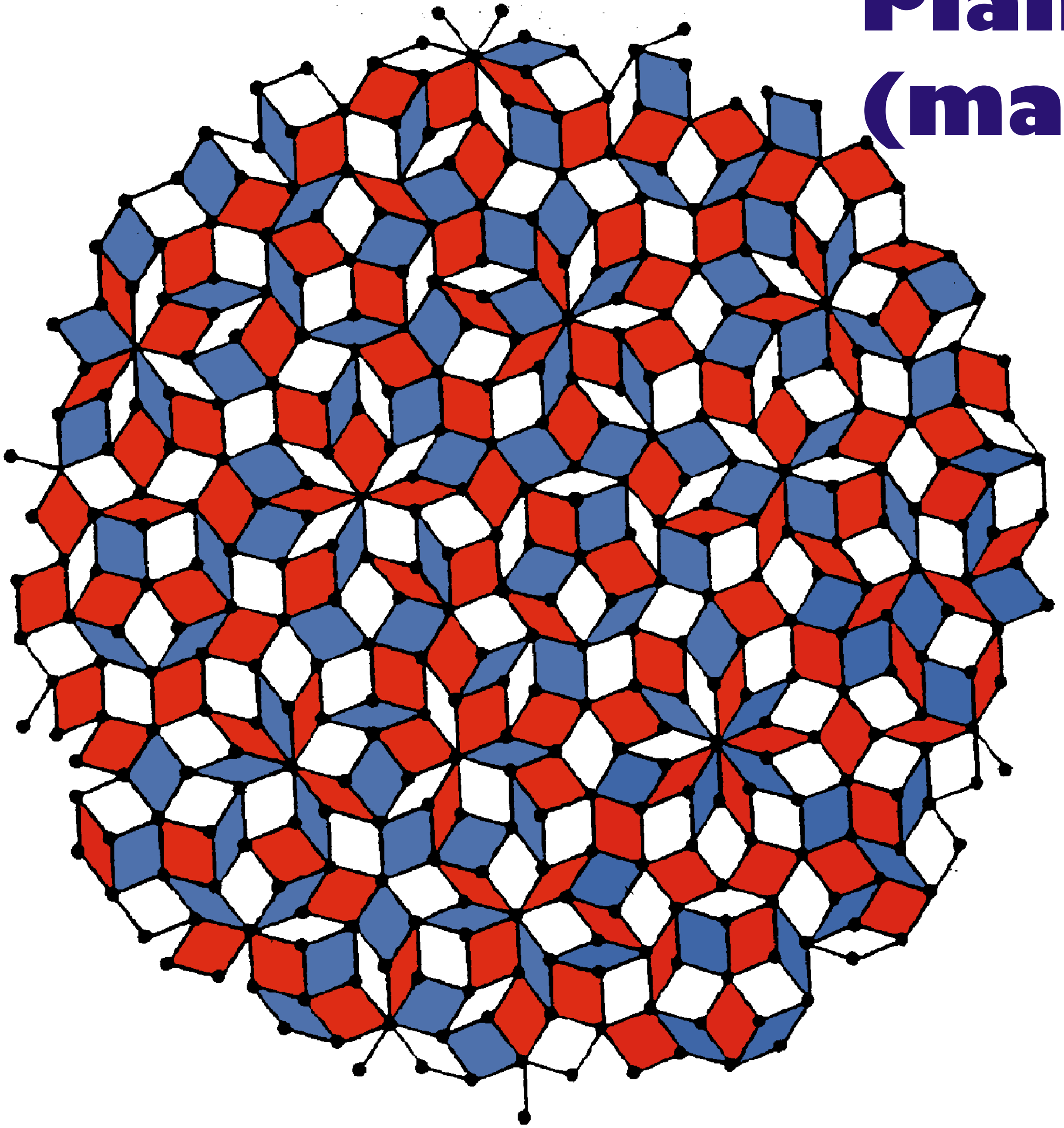


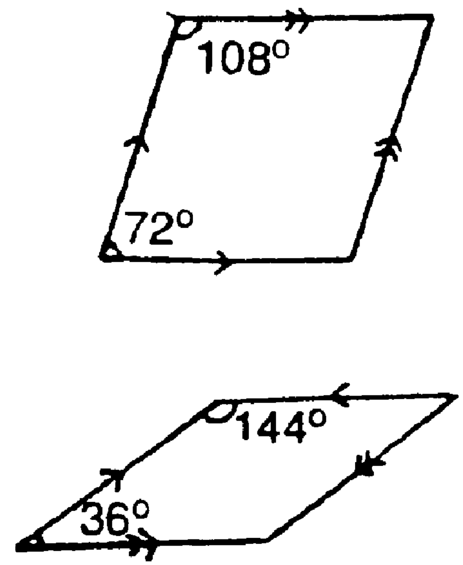


Is it 4-colorable ? yes, always !

(very hard theorem)

Plannar graphs (maps)

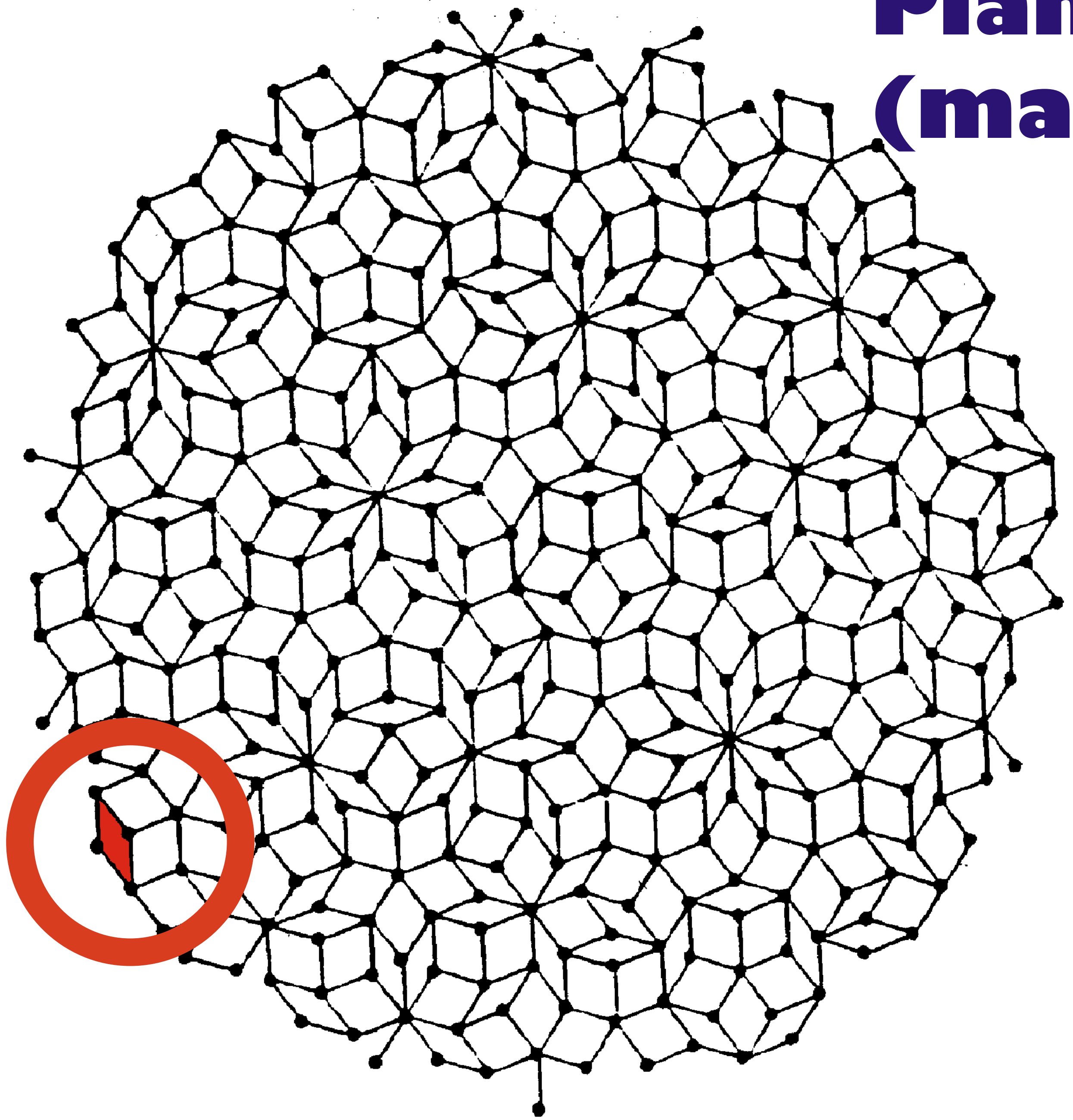


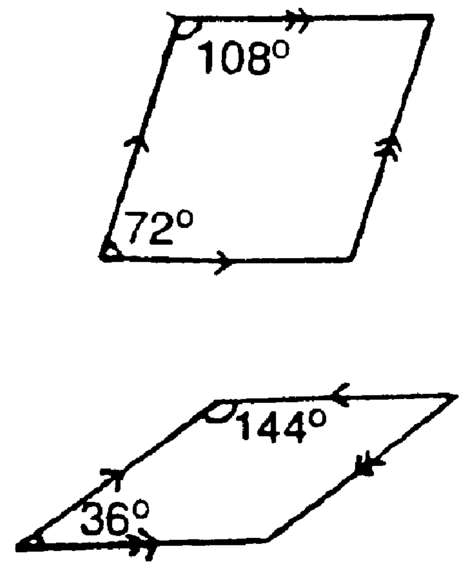


Is it 3-colorable ? yes, sometimes

hard to tell, but easy to check

Plannar graphs (maps)





Is it 2-colorable ? no, sometimes

easy to tell, and easy to check

Plannar graphs (maps)

Is it <u>2-colorable</u> ?

easy to tell, and easy to check

Plannar graphs 2-colorable in P

Is it <u>3-colorable</u> ?

hard to tell, but easy to check

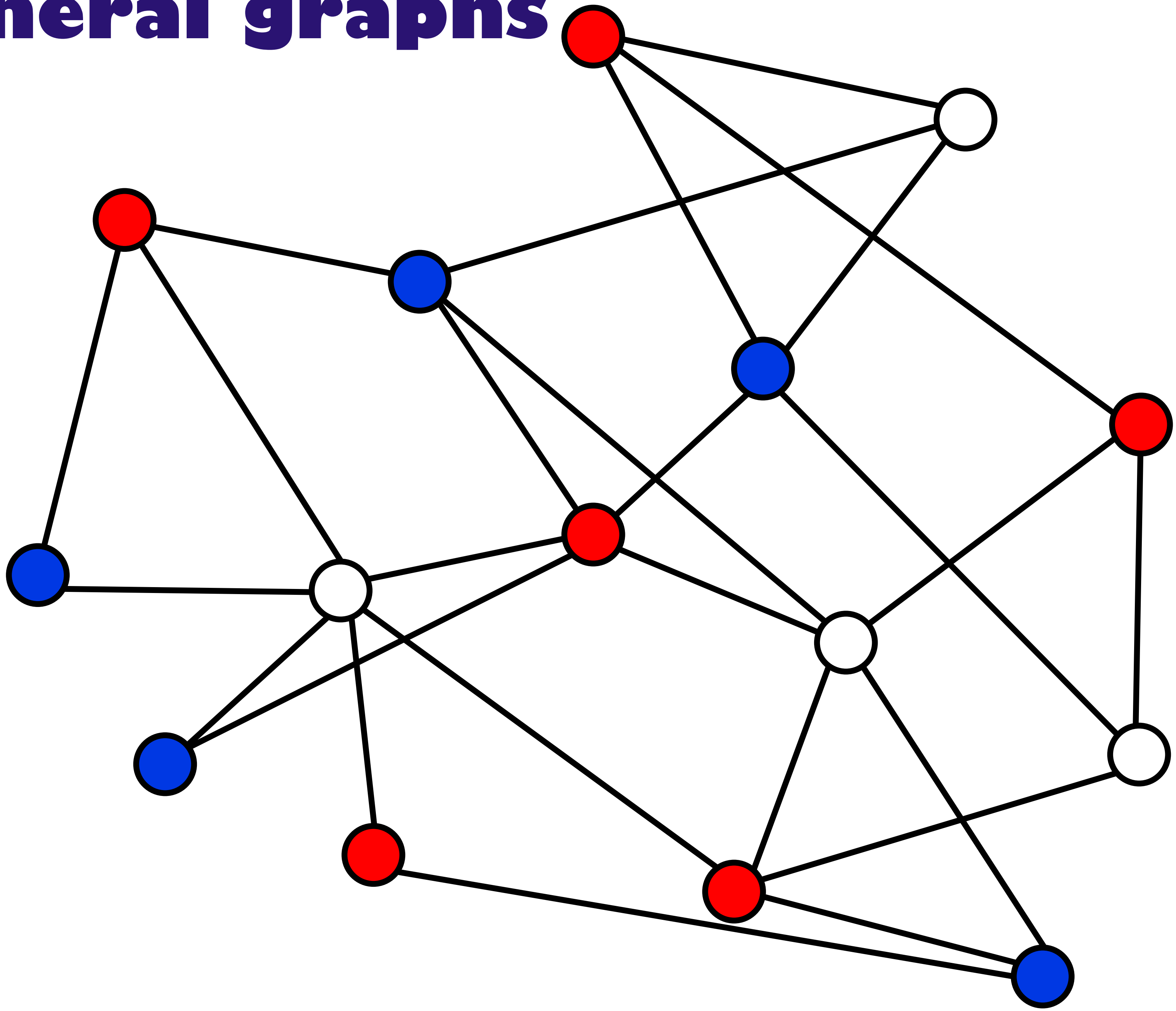
Plannar graphs 3-colorable in NP

Is it <u>K-colorable</u> ? K>3

easy to tell, and easy to check

Plannar graphs K-colorable in P

General graphs



General graphs

Is it <u>2-colorable</u> ?

easy to tell, and easy to check

General graphs 2-colorable in P

Is it <u>3-colorable</u> ?

hard to tell, but easy to check

General graphs 3-colorable in NP

Is it <u>K-colorable</u> ? K>3

hard to tell, but easy to check

General graphs K-colorable in NP

General graphs K-colorable, K>2
in NP

General graphs K-colorable, K>2
is NP-complete

meaning:
if
General graphs K-colorable, K>2
in P
then
every problem in NP is also in P.