HEAPS II

- Implementation
- HeapSort
- Bottom-Up Heap Construction
- Locators
Implementation of a Heap

```java
public class HeapPriorityQueue implements PriorityQueue {
    BinaryTree T;
    Position last;
    Comparator comparator;
    ...
}
```
Implementation of a Heap (cont.)

- Two ways to find the insertion position $z$ in a heap:
Vector Based Implementation

- Updates in the underlying tree occur only at the “last element”

- A heap can be represented by a vector, where the node at rank $i$ has
  - left child at rank $2i$ and
  - right child at rank $2i + 1$

- The leaves do no need to be explicitly stored

- Insertion and removals into/from the heap correspond to `insertLast` and `removeLast` on the vector, respectively
Heap Sort

• All heap methods run in logarithmic time or better

• If we implement PriorityQueueSort using a heap for our priority queue, `insertItem` and `removeMin` each take $O(\log k)$, $k$ being the number of elements in the heap at a given time.

• We always have at most $n$ elements in the heap, so the worst case time complexity of these methods is $O(\log n)$.

• Thus each phase takes $O(n \log n)$ time, so the algorithm runs in $O(n \log n)$ time also.

• This sort is known as **heap-sort**.

• The $O(n \log n)$ run time of heap-sort is much better than the $O(n^2)$ run time of selection and insertion sort.

**In-Place Heap-Sort**

• Do not use an external heap

• Embed the heap into the sequence, using the vector representation
Bottom-Up Heap Construction

- build \((n + 1)/2\) trivial one-element heaps

- now build three-element heaps on top of them
Bottom-Up Heap Construction

- **downheap** to preserve the order property

- now form seven-element heaps
Bottom-Up Heap Construction (cont.)

Heaps II 6.8
Bottom-Up Heap Construction (cont.)

The End
Analysis of Bottom-Up Heap Construction

• **Proposition**: Bottom-up heap construction with $n$ keys takes $O(n)$ time.
  - Insert $(n + 1)/2$ nodes
  - Insert $(n + 1)/4$ nodes and downheap them
  - Insert $(n + 1)/8$ nodes and downheap them
  - ...
  - visual analysis:

  $n$ inserts, $n/2$ upheaps with total $O(n)$ running time
Locators

- Locators can be used to keep track of elements as they are moved around inside a container.

- A *locator* sticks with a specific element, even if that element changes positions in the container.

- The locator ADT supports the following fundamental methods:
  - `element()`: return the element of the item associated with the locator.
  - `key()`: return the key of the item associated with the locator.

- Using locators, we define additional methods for the priority queue ADT
  - `insert(k,e)`: insert \((k,e)\) into \(P\) and return its locator
  - `min()`: return the locator of an element with the smallest key
  - `remove(l)`: remove the element with locator \(l\)

- In the stock trading application, we return a locator when an order is placed. The locator allows to specify unambiguously an order when a cancellation is requested
Positions and Locators

• At this point, you may be wondering what the difference is between locators and positions, and why we need to distinguish between them.

• It’s true that they have very similar methods

• The difference is in their primary usage

• **Positions** abstract the specific implementation of accessors to elements (indices vs. nodes).

• **Positions** are defined relatively to each other (e.g., previous-next, parent-child)

• **Locators** keep track of where elements are stored. In the implementation of an ADT with locators, a locator typically holds the current position of the element.

• **Locators** associate elements with their keys
Locators and Positions at Work

• For example, consider the CS16 Valet Parking Service (started by the TA staff because they had too much free time on their hands).

• When they began their business, Andy and Devin decided to create a data structure to keep track of where exactly the cars were.

• Andy suggested having a position represent what parking space the car was in.

• However, Devin knew that the TAs were driving the customers’ cars around campus and would not always park them back into the same spot.

• So they decided to install a locator (a wireless tracking device) in each car. Each locator had a unique code, which was written on the claim check.

• When a customer demanded her car, the HTAs activated the locator. The horn of the car would honk and the lights would flash.

• If the car was parked, Andy and Devin would know where to retrieve it in the lot.

• Otherwise, the TA driving the car knew it was time to bring it back.