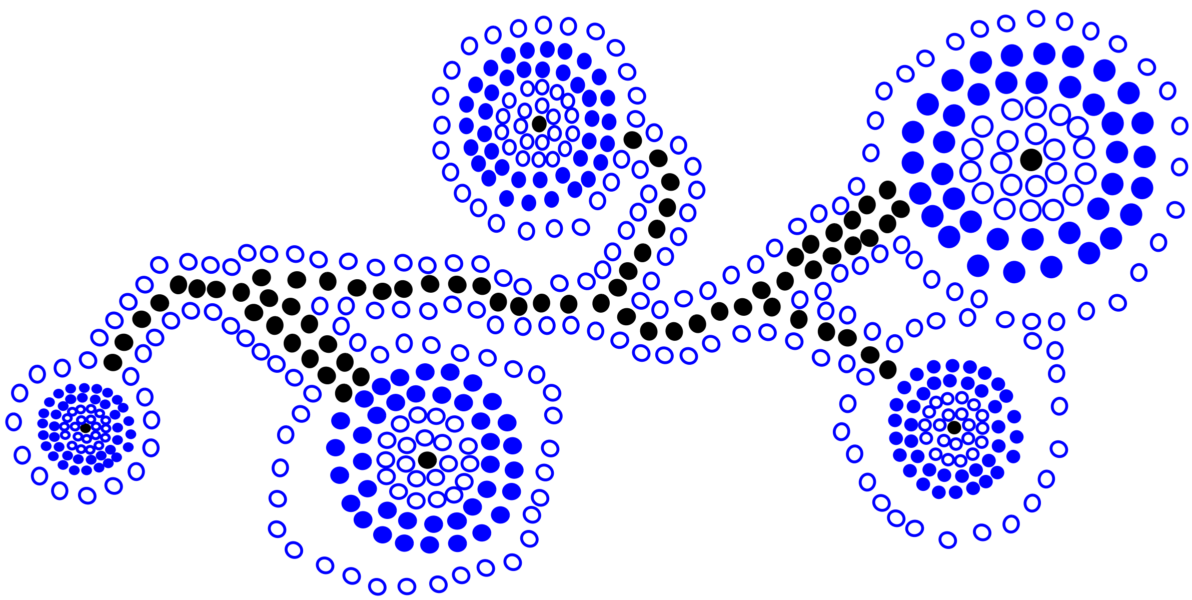
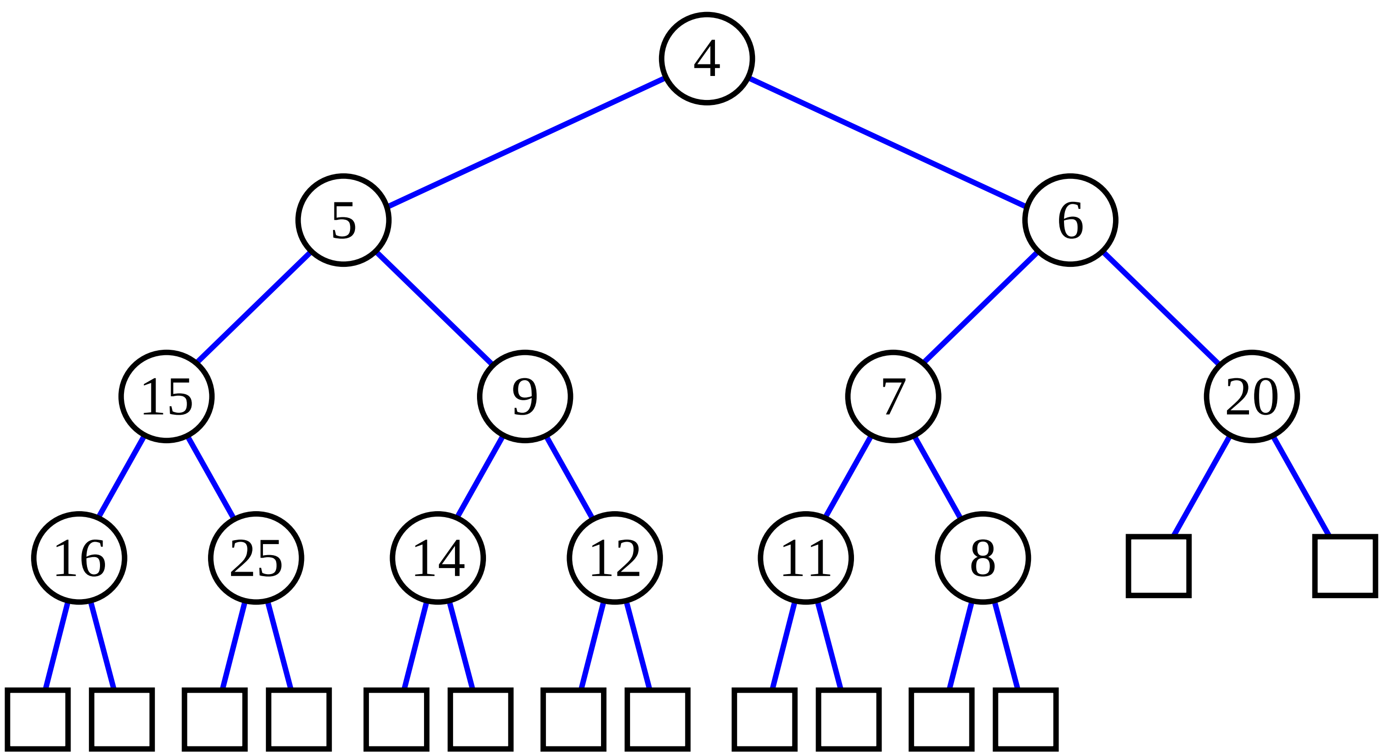
HEAPS I

- Heaps
- Properties
- Insertion and Deletion



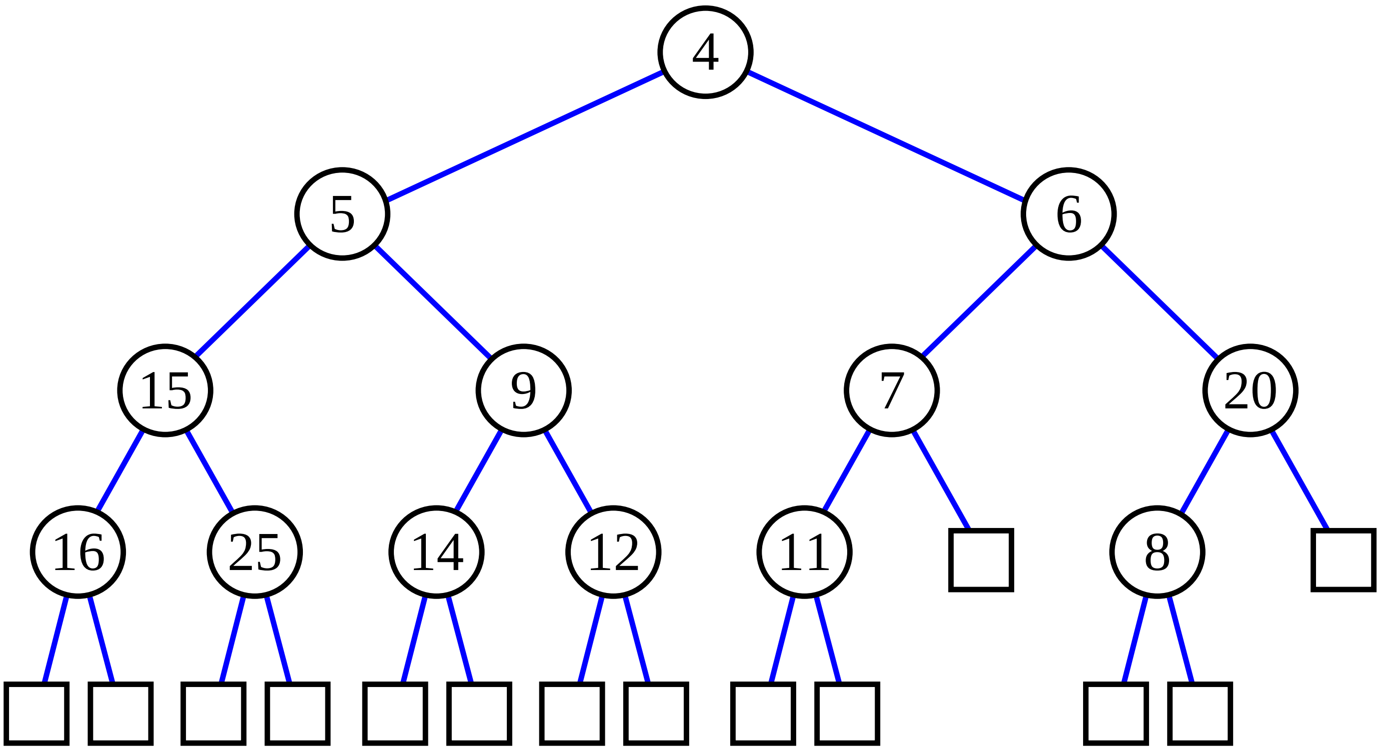
Heaps

- A ***heap*** is a binary tree *T* that stores a collection of keys (or key-element pairs) at its internal nodes and that satisfies two additional properties:
  - ***Order Property:*** key(parent) $\leq$ key(child)
  - ***Structural Property***: all levels are full, except the last one, which is left-filled (***complete binary tree***)

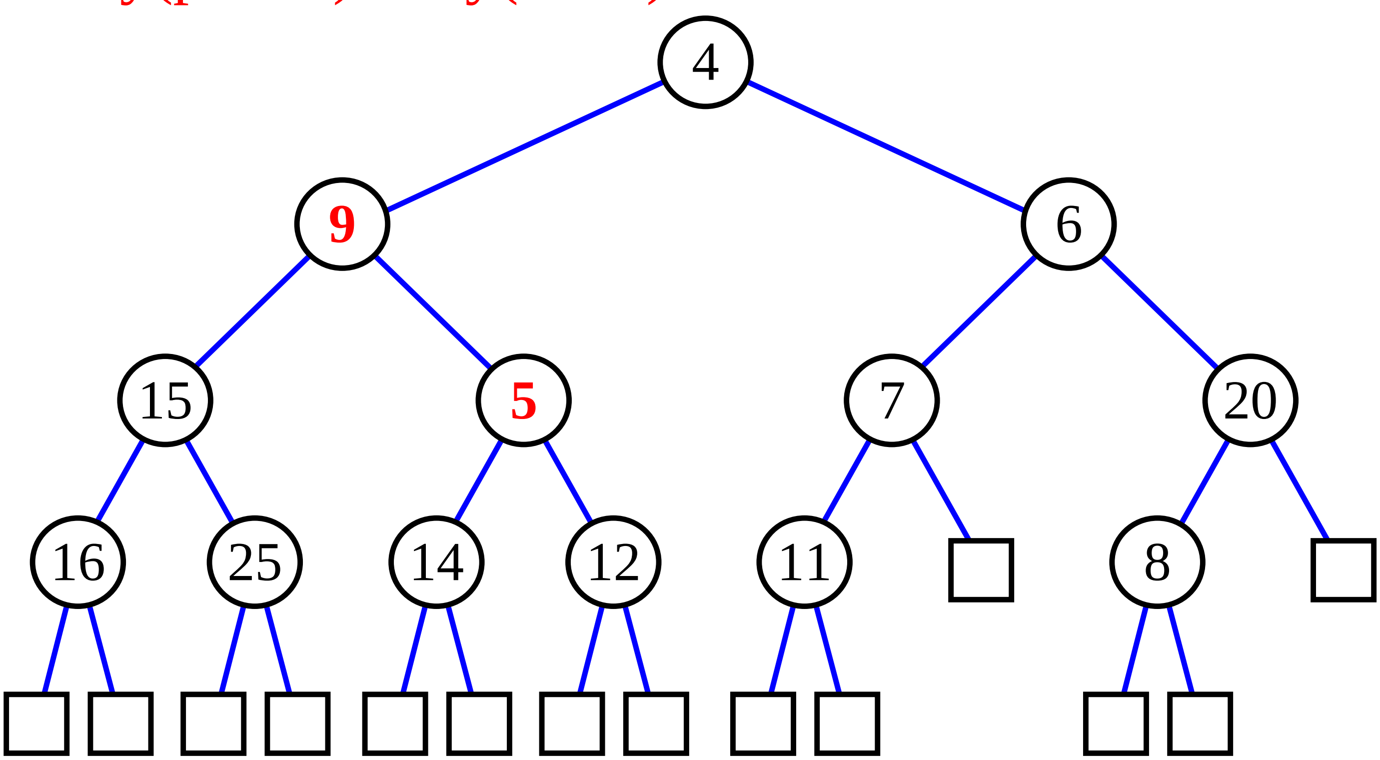


Not Heaps

- bottom level is not left-filled



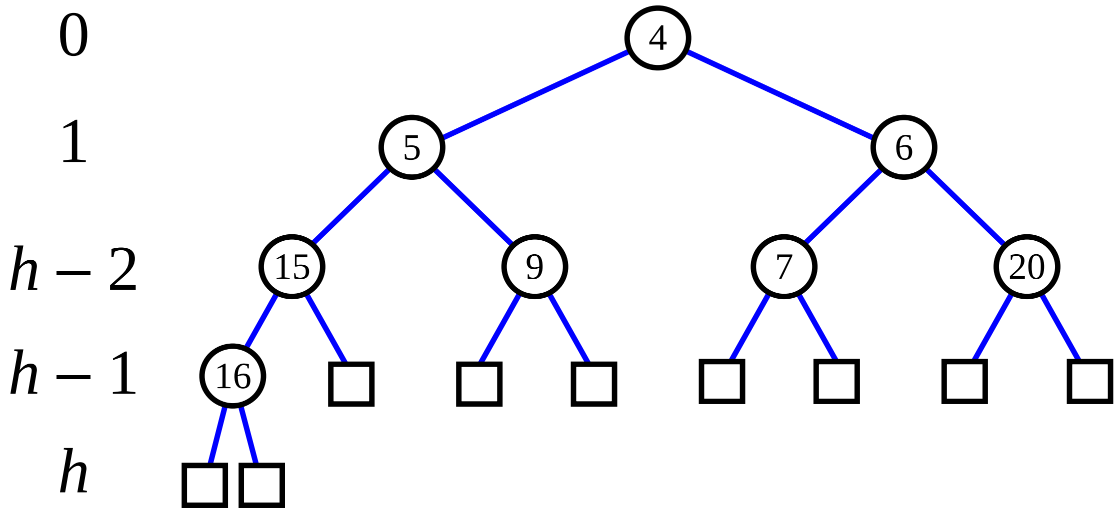
- key(parent)> key(child)



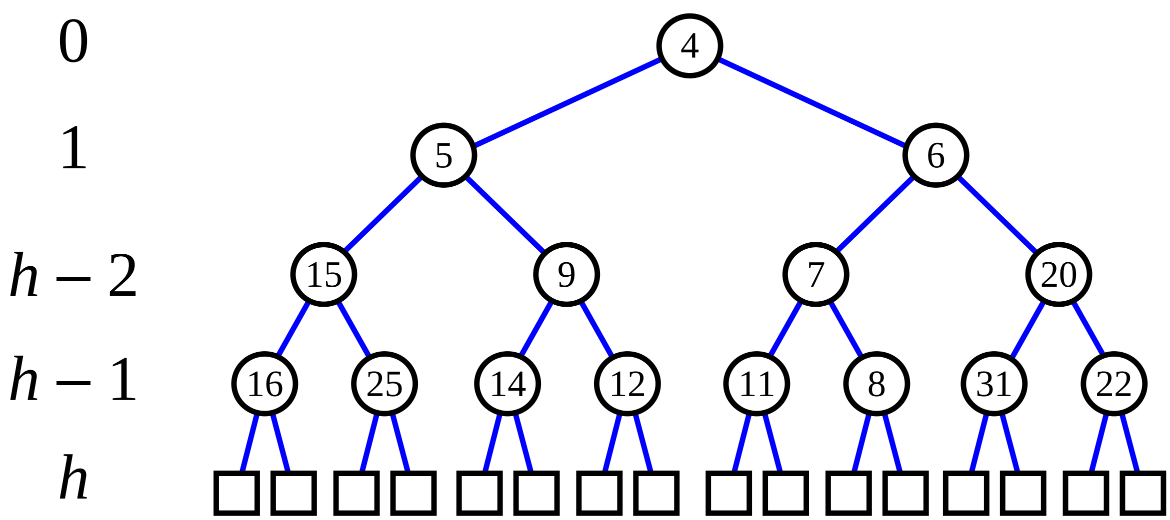
Height of a Heap

A heap T storing n keys has height $h = \lceil \log(n + 1) \rceil$, which is $O(\log n)$

- $n \geq 1 + 2 + 4 + \ldots + 2^{h-2} + 1 = 2^{h-1} - 1 + 1 = 2^{h-1}$



- $n \leq 1 + 2 + 4 + \ldots + 2^{h-1} = 2^h - 1$

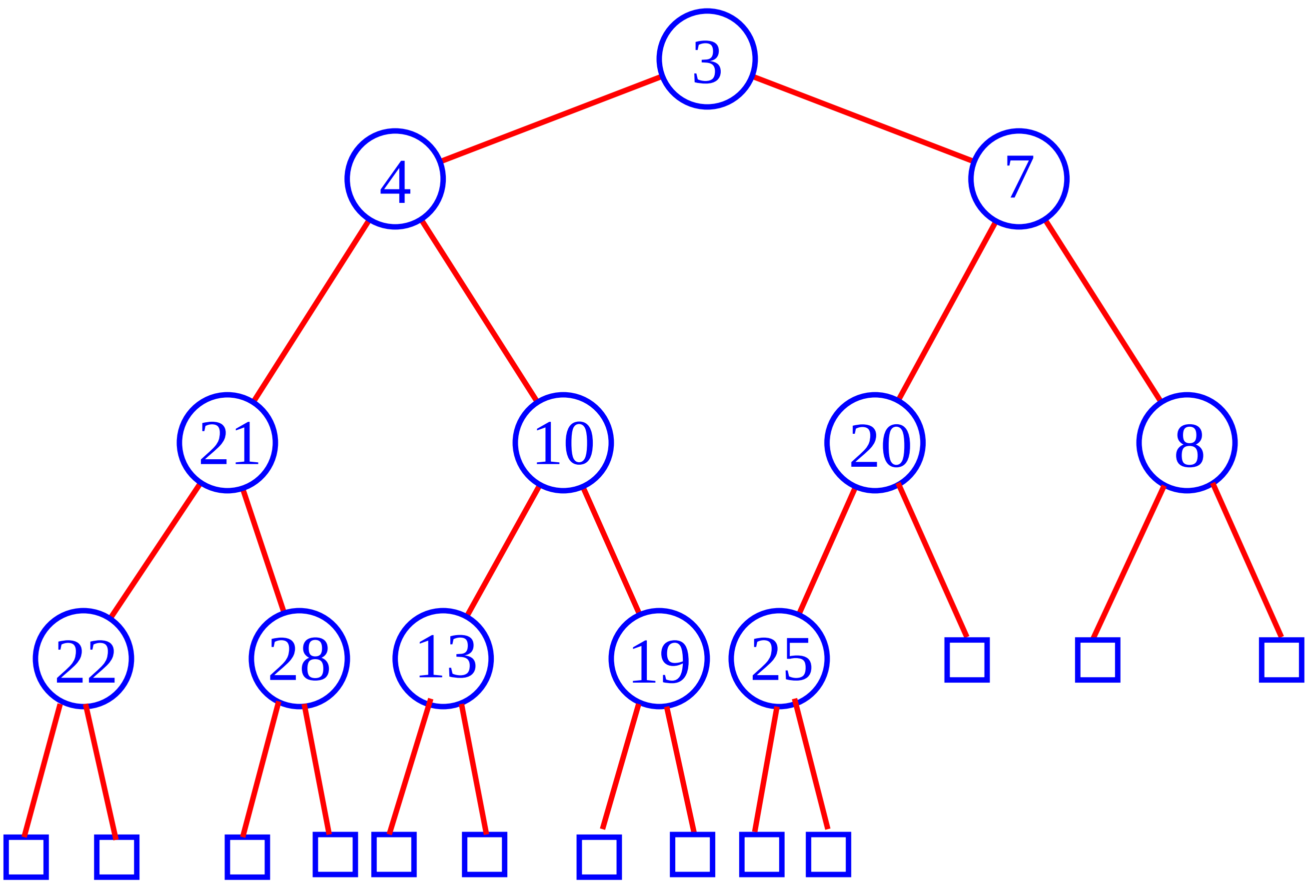


- Therefore $2^{h-1} \leq n \leq 2^h - 1$
- Taking logs, we get $\log(n + 1) \leq h \leq \log n + 1$
- Which implies $\boldsymbol{h = \lceil \log(n+1) \rceil}$

Heap Insertion

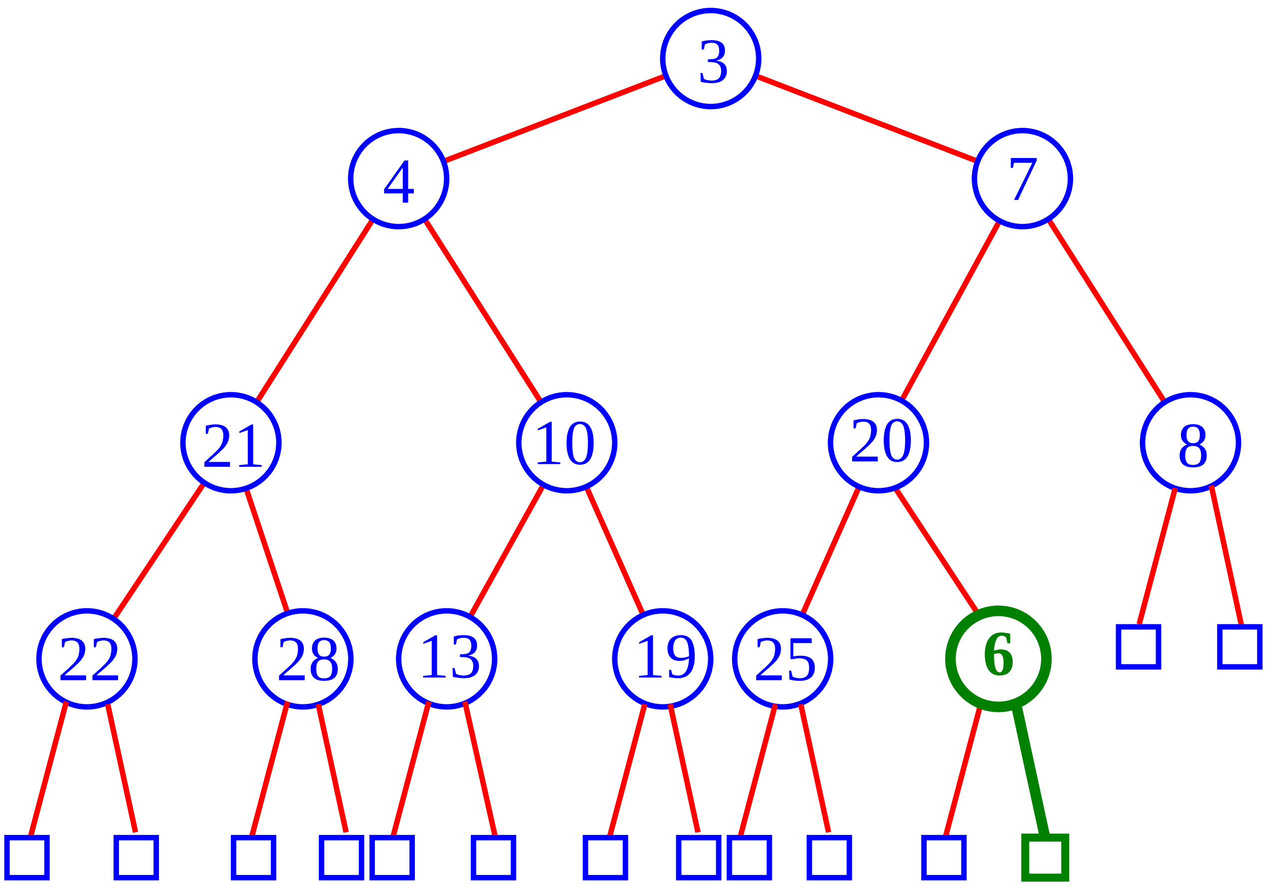
So here we go ...

The key to insert is **6**



Heap Insertion

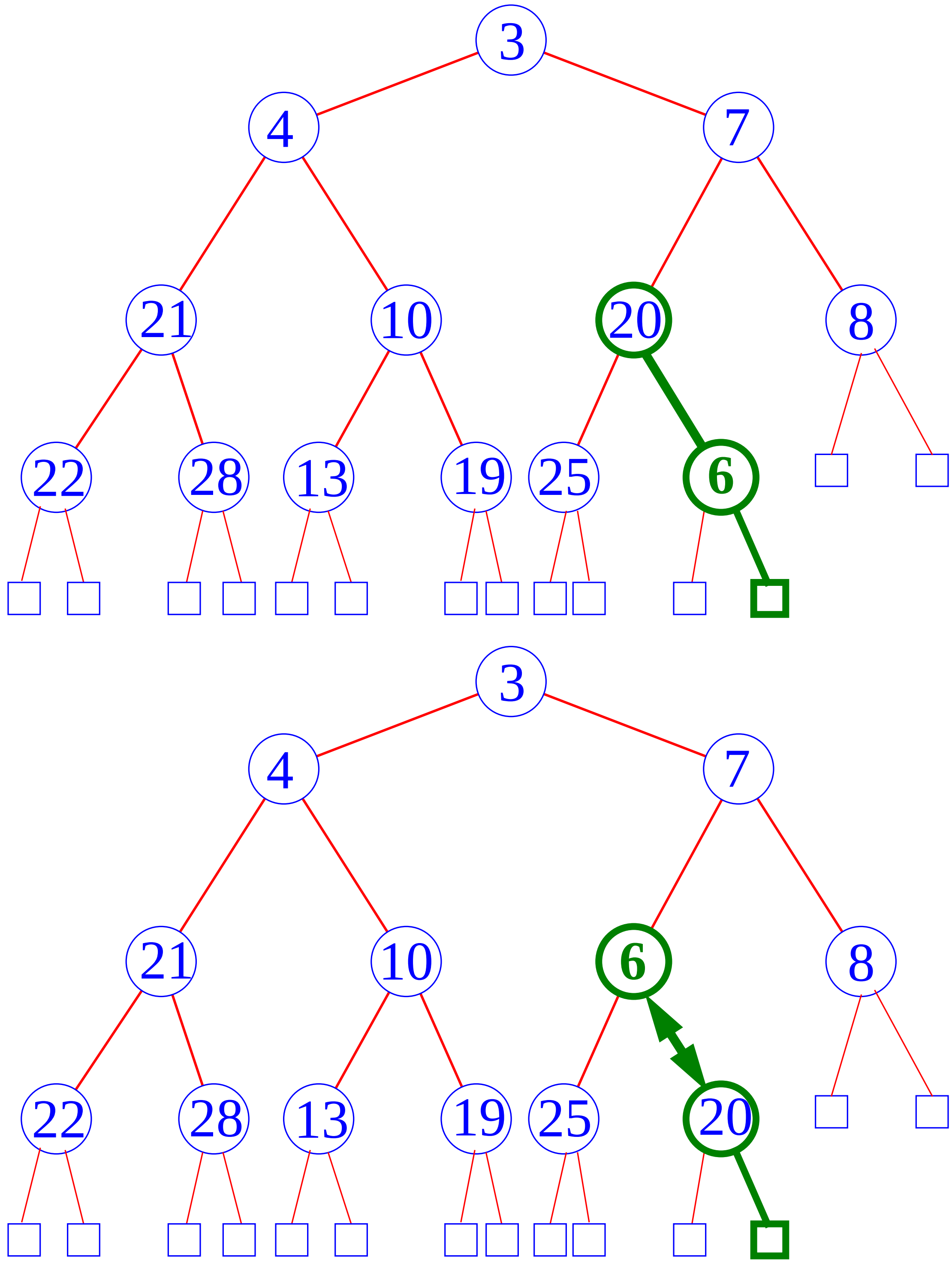
Add the key in the ***next available position*** in the heap.



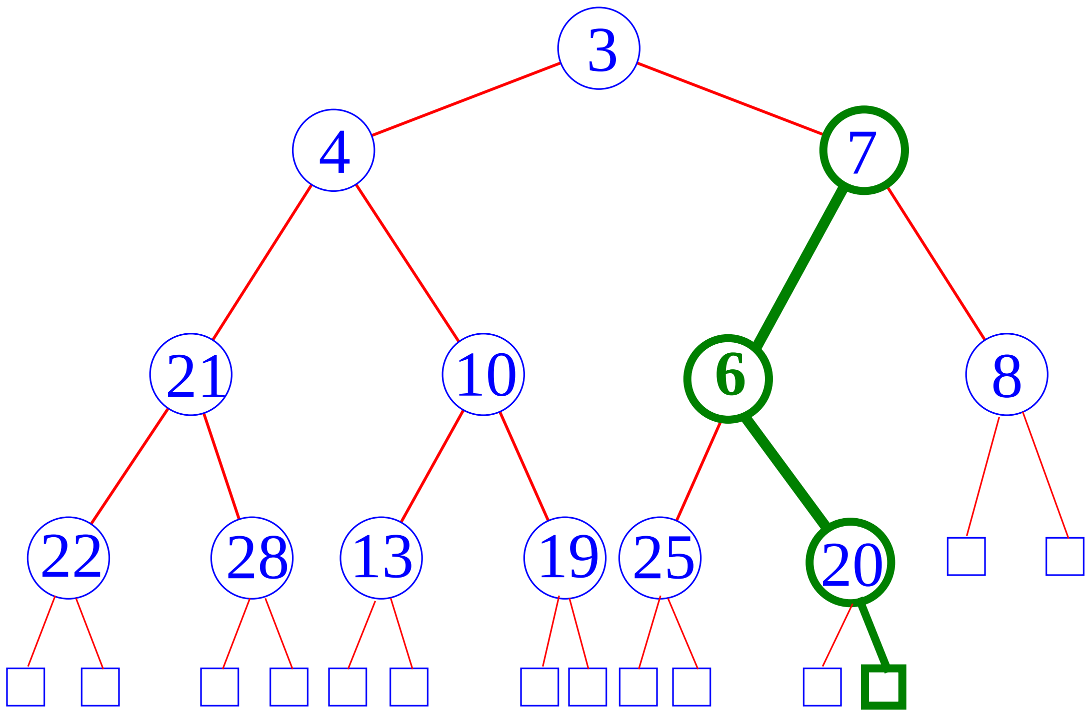
Now begin ***Upheap***.

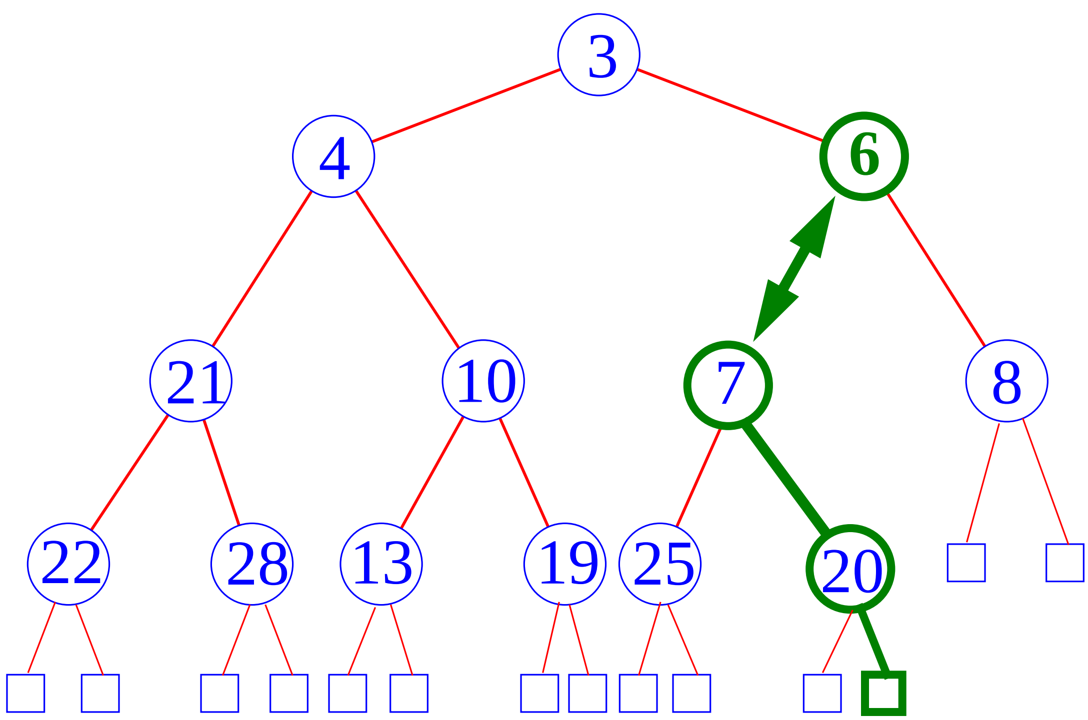
Upheap

- ***Swap parent-child keys out of order***

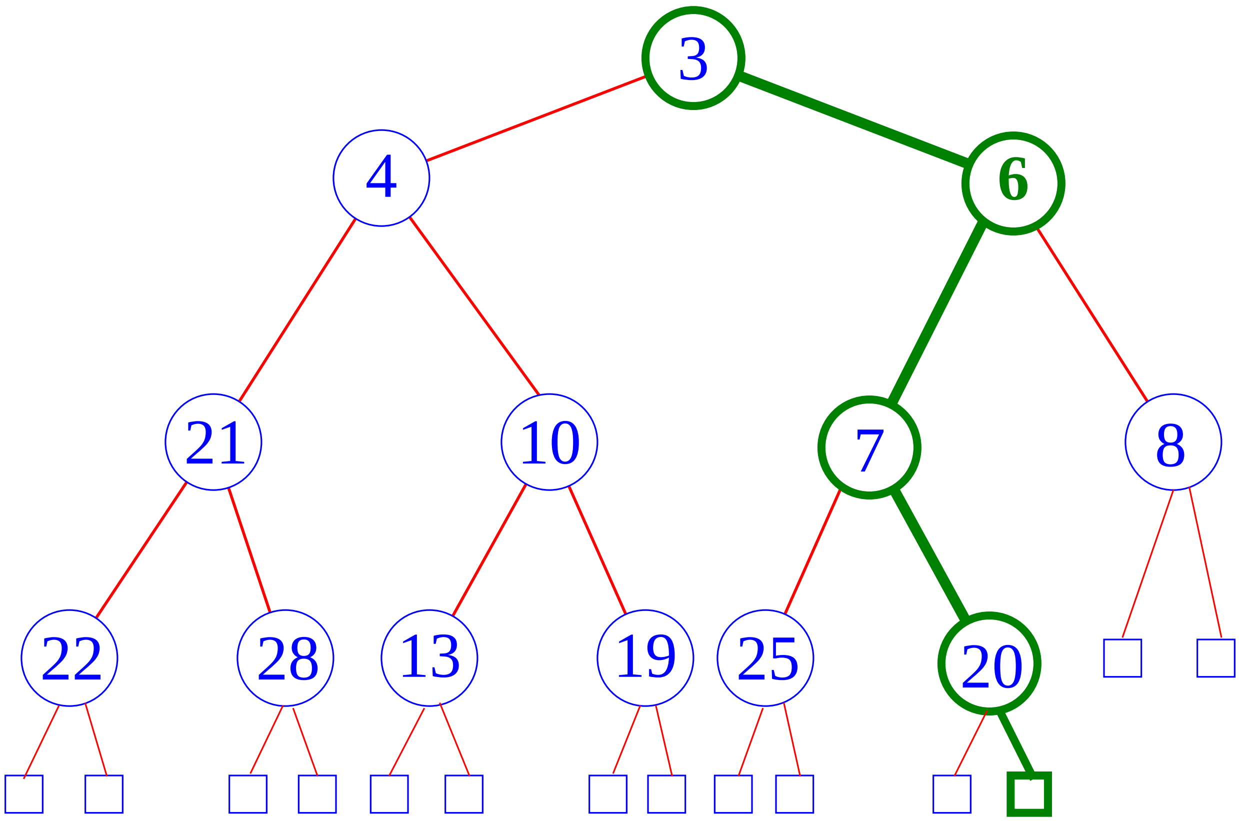


Upheap Continues





End of Upheap

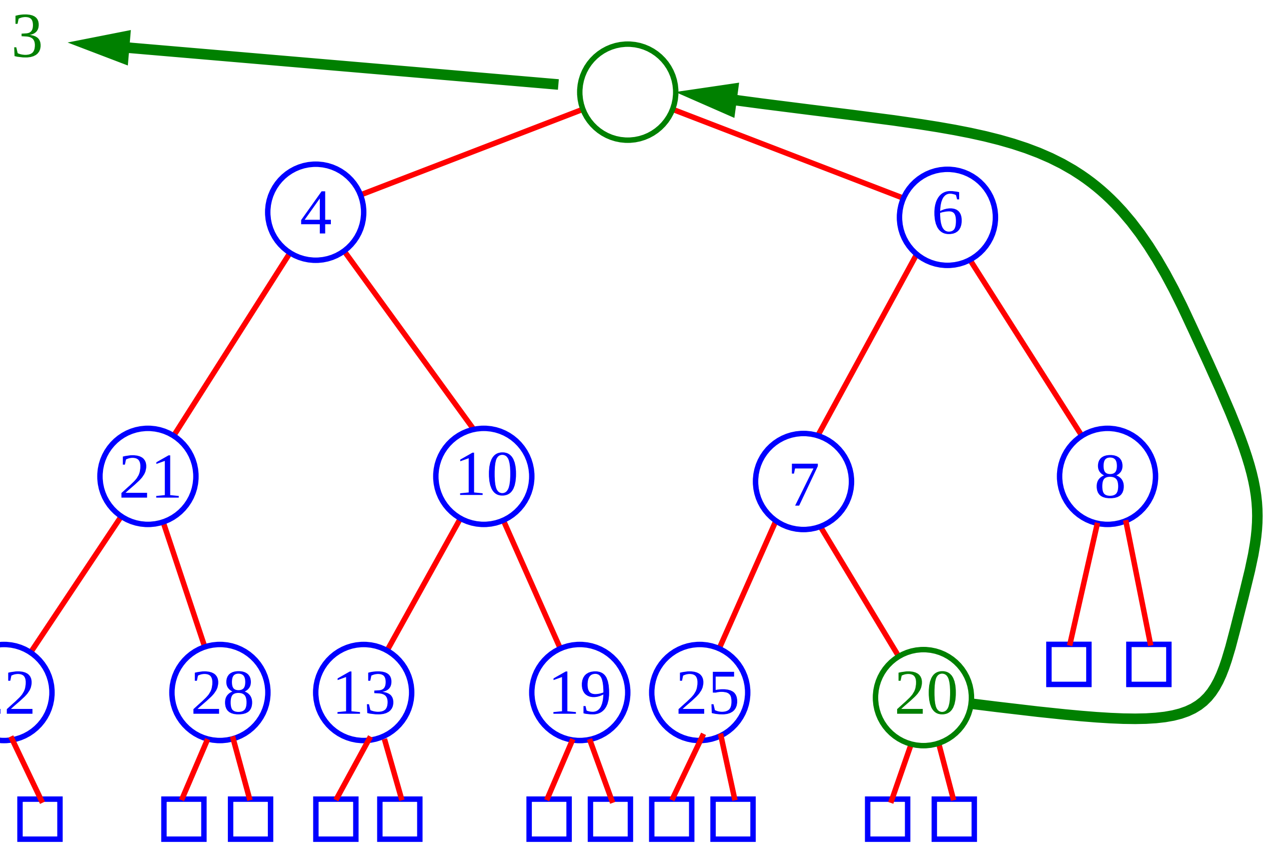


- ***Upheap*** terminates when new key is greater than the key of its parent **or** the top of the heap is reached

- (total #swaps) $\leq$ $(h - 1)$, which is $O(\log n)$

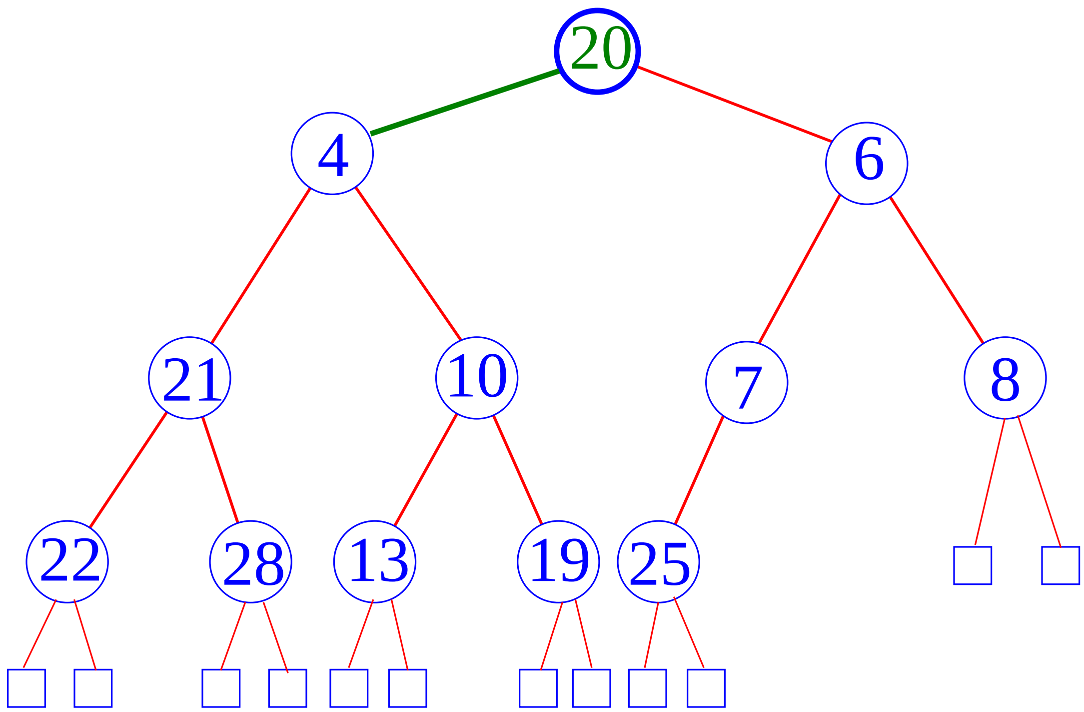
Removal From a Heap

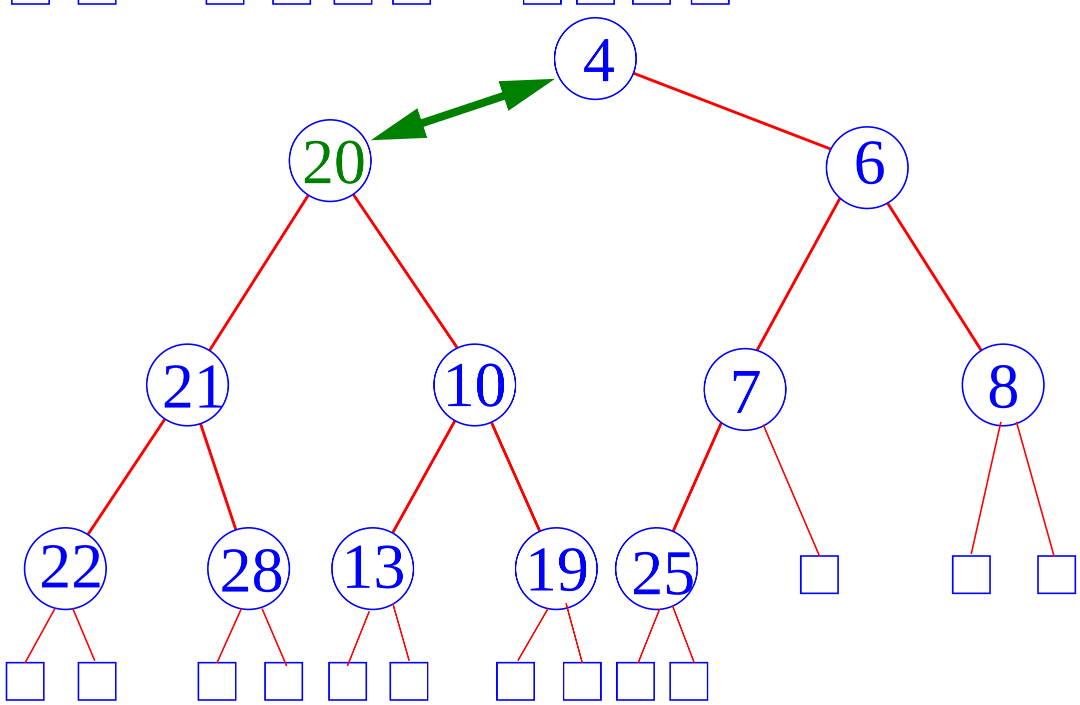
RemoveMin()



- The removal of the top key leaves a hole
- We need to fix the heap
- First, replace the hole with the last key in the heap
- Then, begin ***Downheap***

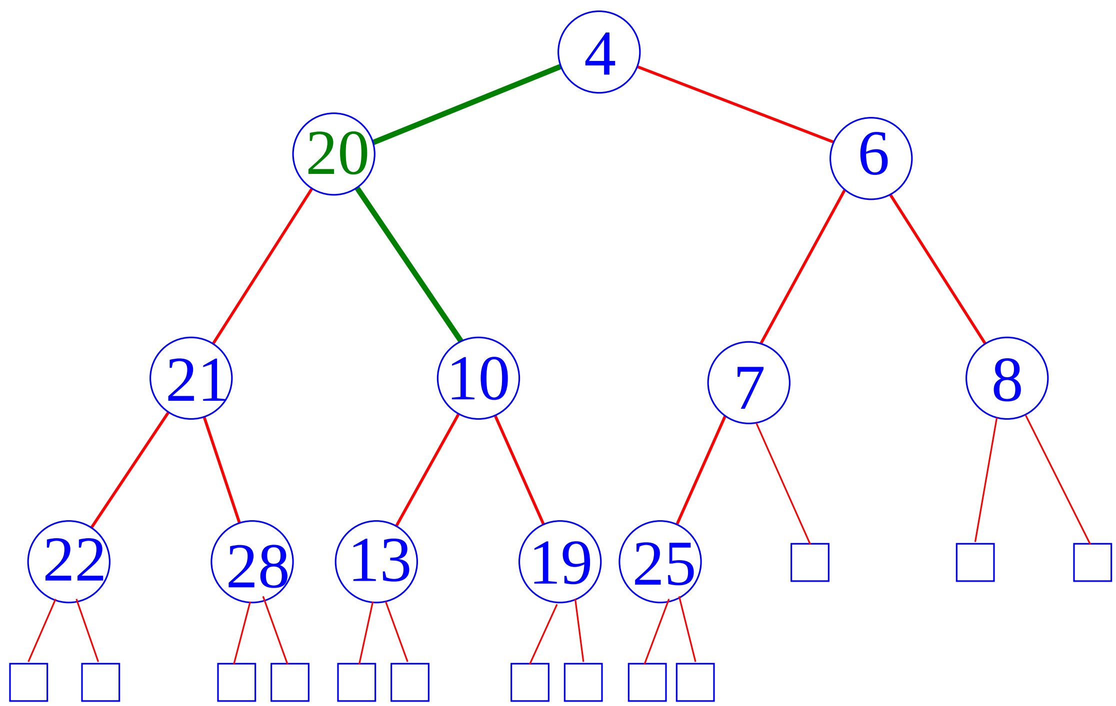
Downheap

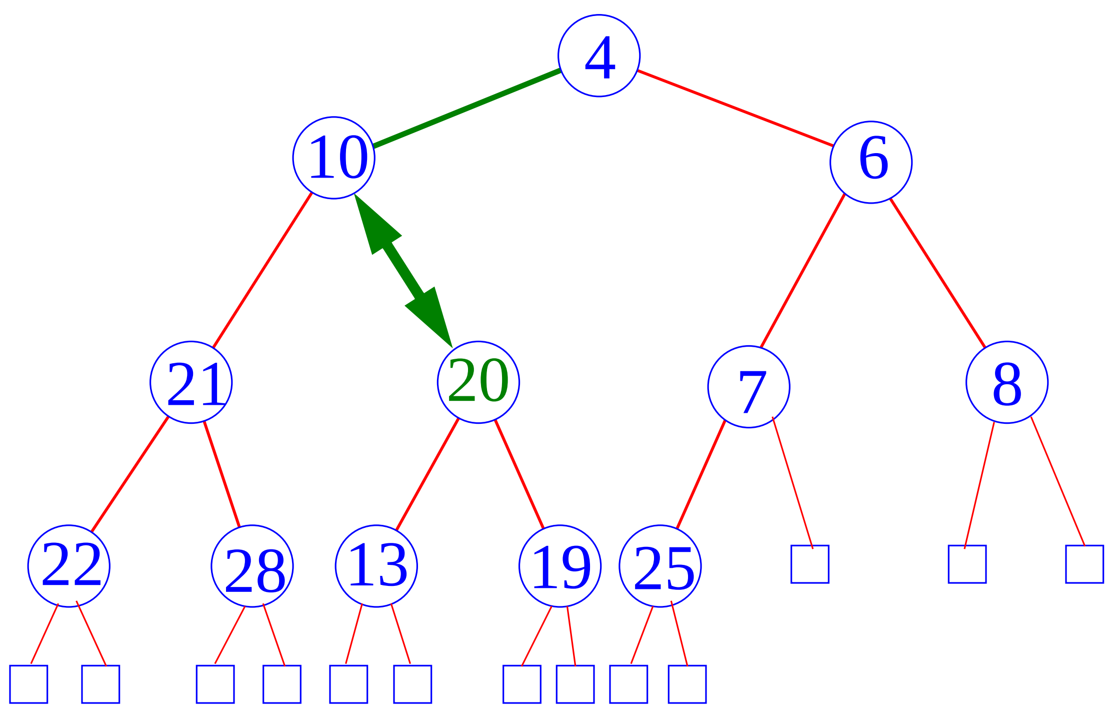




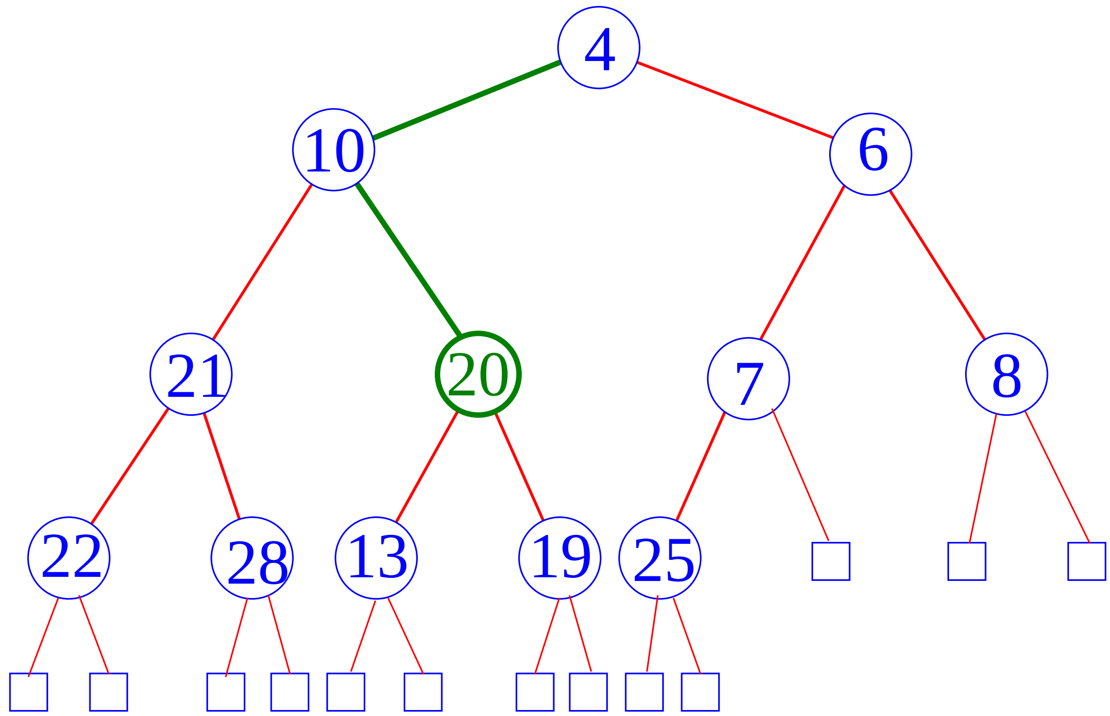
Downheap compares the parent with the smallest child. If the child is smaller, it switches the two.

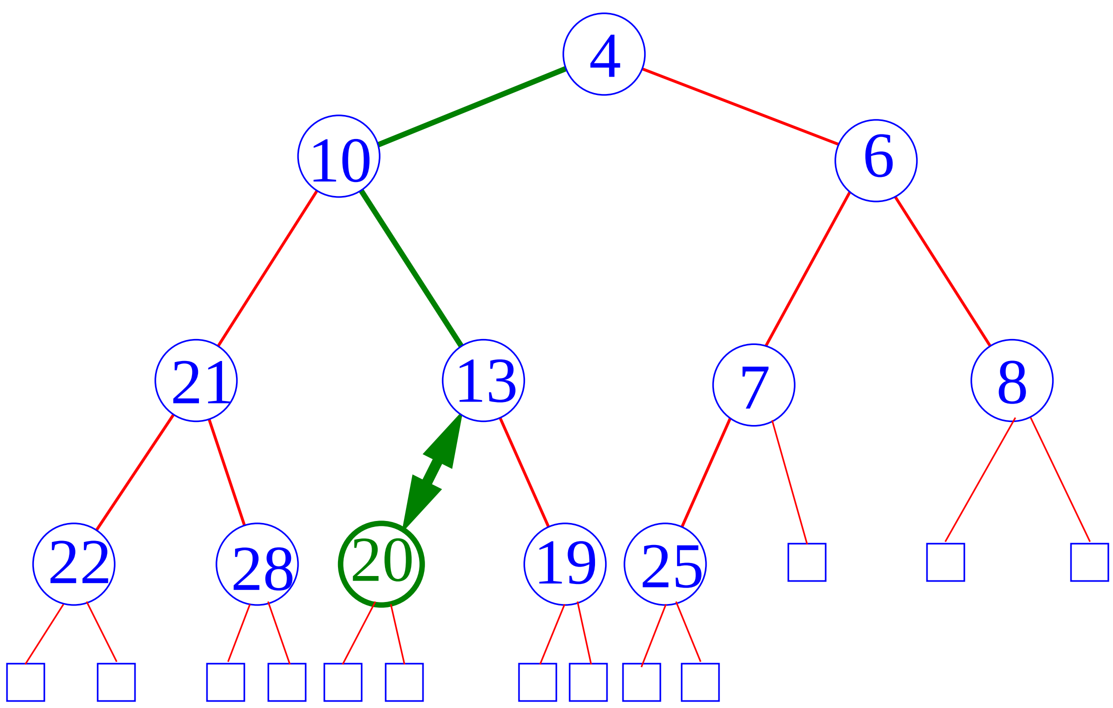
Downheap Continues



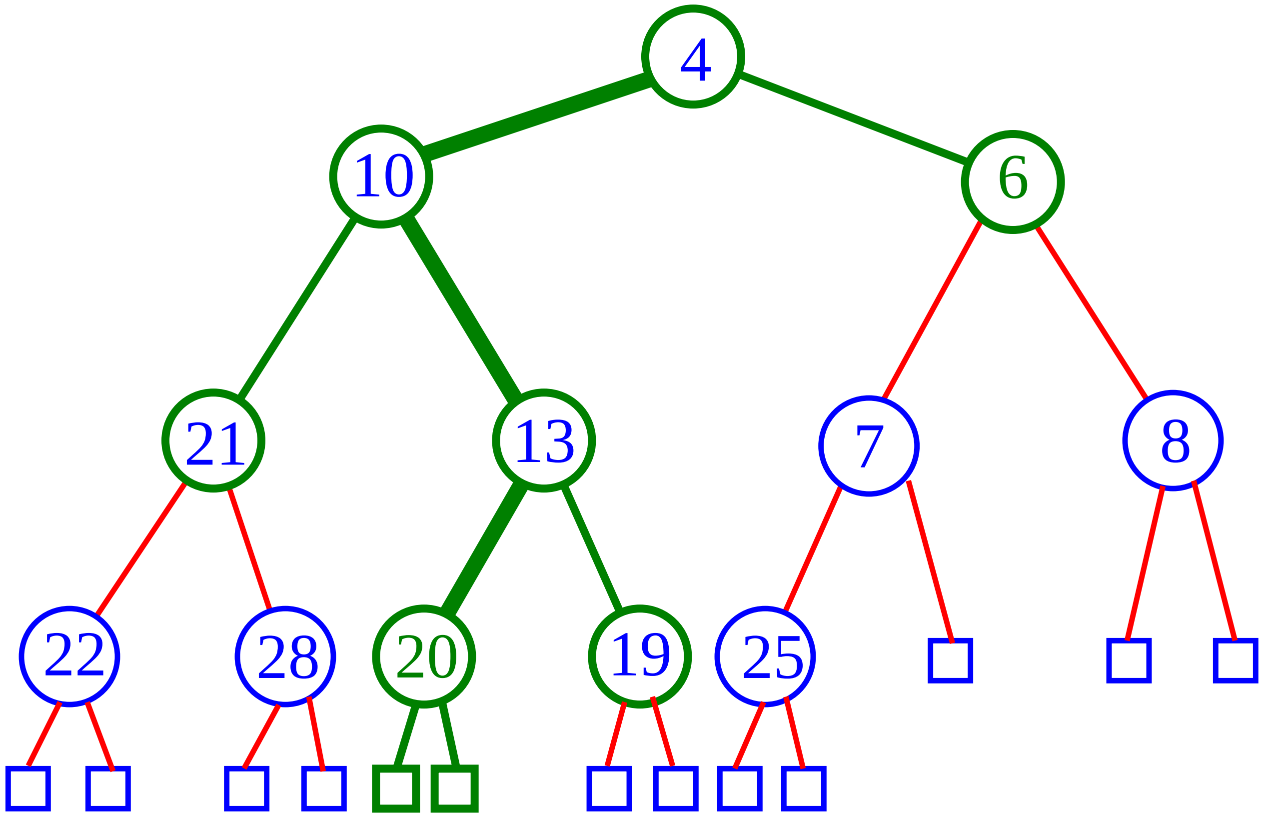


Downheap Continues





End of Downheap



- ***Downheap*** terminates when the key is greater than the keys of both its children **or** the bottom of the heap is reached.

- (total #swaps) $\leq$ $(h - 1)$, which is O(log n)