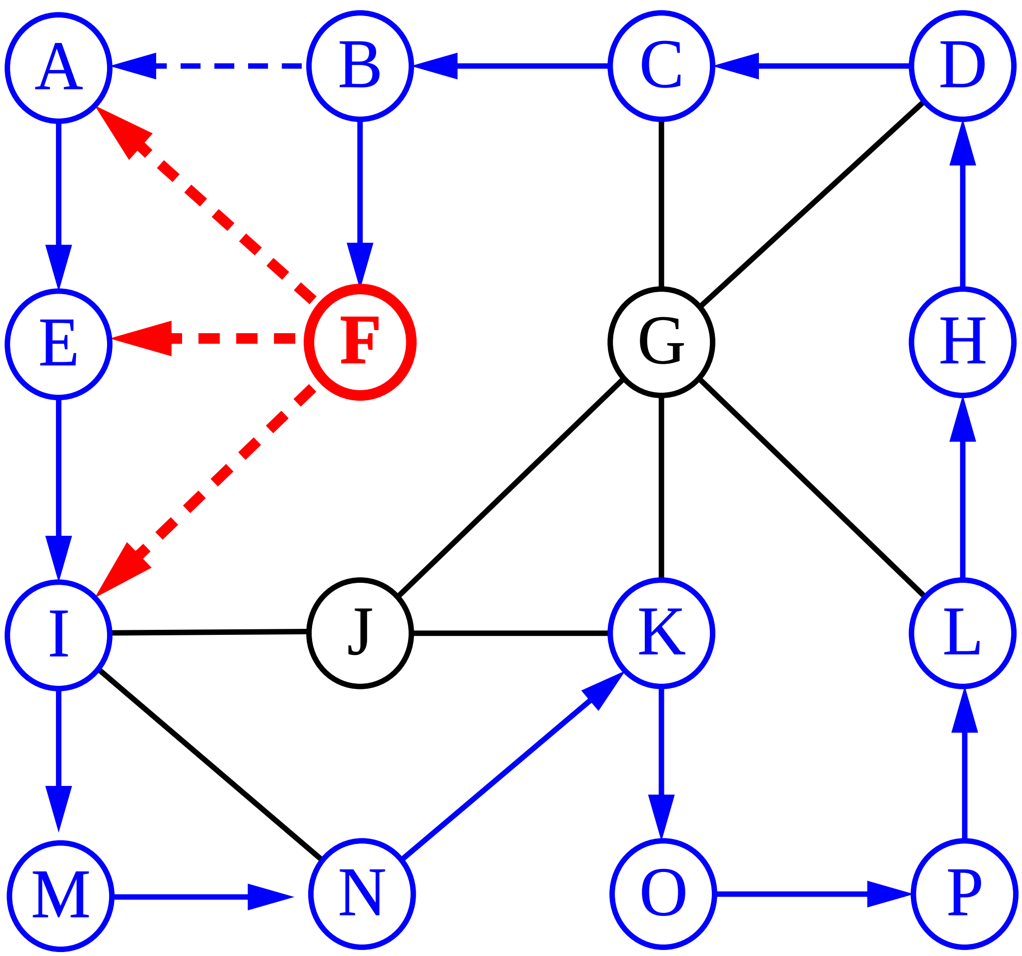
Depth-First Search

- Graph Traversals
- Depth-First Search



Exploring a Labyrinth Without Getting Lost

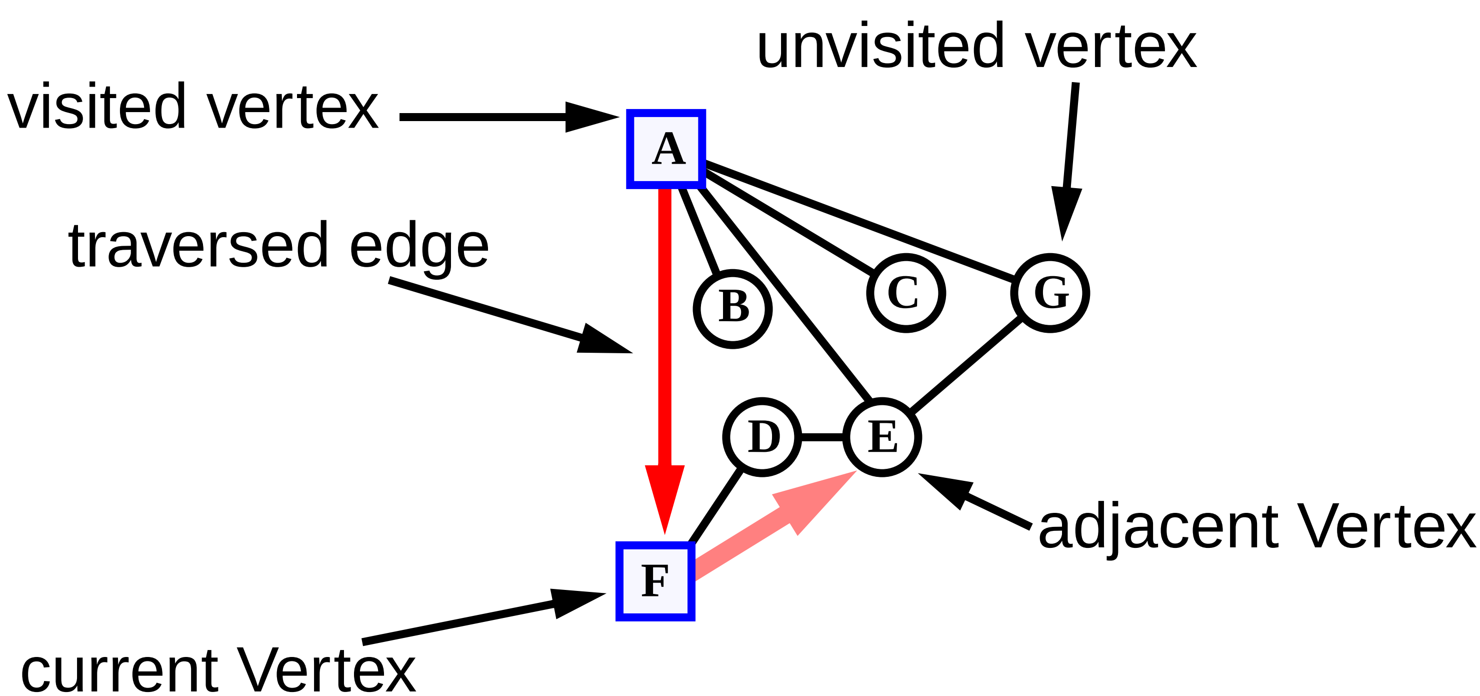
- A **depth-first search (DFS)** in an undirected graph G is like wandering in a labyrinth with a string and a can of red paint without getting lost.
- We start at vertex s, tying the end of our string to the point and painting s "visited". Next we label s as our current vertex called u.
- Now we travel along an arbitrary edge (u,v).
- If edge (u,v) leads us to an already visited vertex v we return to u.
- If vertex v is unvisited, we unroll our string and move to v, paint v "visited", set v as our current vertex, and repeat the previous steps.
- Eventually, we will get to a point where all incident edges on u lead to visited vertices. We then backtrack by unrolling our string to a previously visited vertex v. Then v becomes our current vertex and we repeat the previous steps.

Exploring a Labyrinth Without Getting Lost (cont.)

- Then, if we all incident edges on v lead to visited vertices, we backtrack as we did before. We continue to backtrack along the path we have traveled, finding and exploring unexplored edges, and repeating the procedure.
- When we backtrack to vertex *s* and there are no more unexplored edges incident on *s*, we have finished our DFS search.

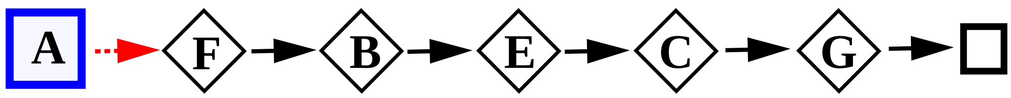
Depth-First Search

Algorithm DFS(v);
  Input: A vertex v in a graph
  Output: A labeling of the edges as “discovery” edges and “backedges”
  for each edge e incident on v **do**
    if edge e is unexplored **then**
      let w be the other endpoint of e
      if vertex w is unexplored **then**
        label e as a discovery edge
        recursively call **DFS**(w)
      else
        label e as a backedge



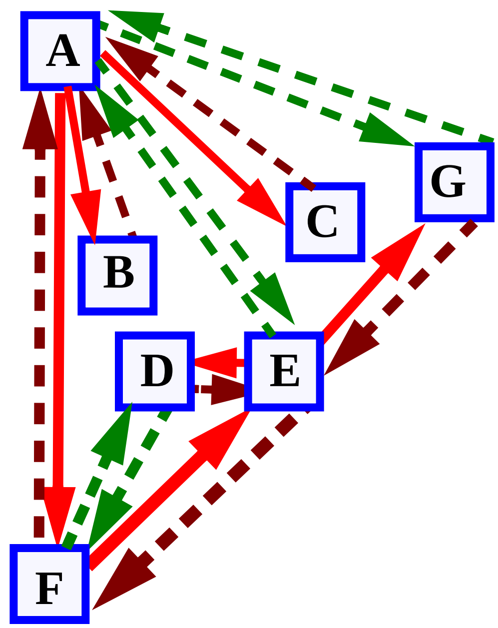
Determining Incident Edges

- DFS depends on how you obtain the incident edges.
- If we start at A and we examine the edge to F, then to B, then E, C, and finally G

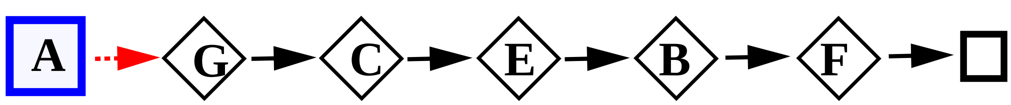


The resulting graph is:

discoveryEdge
backEdge
return from dead end

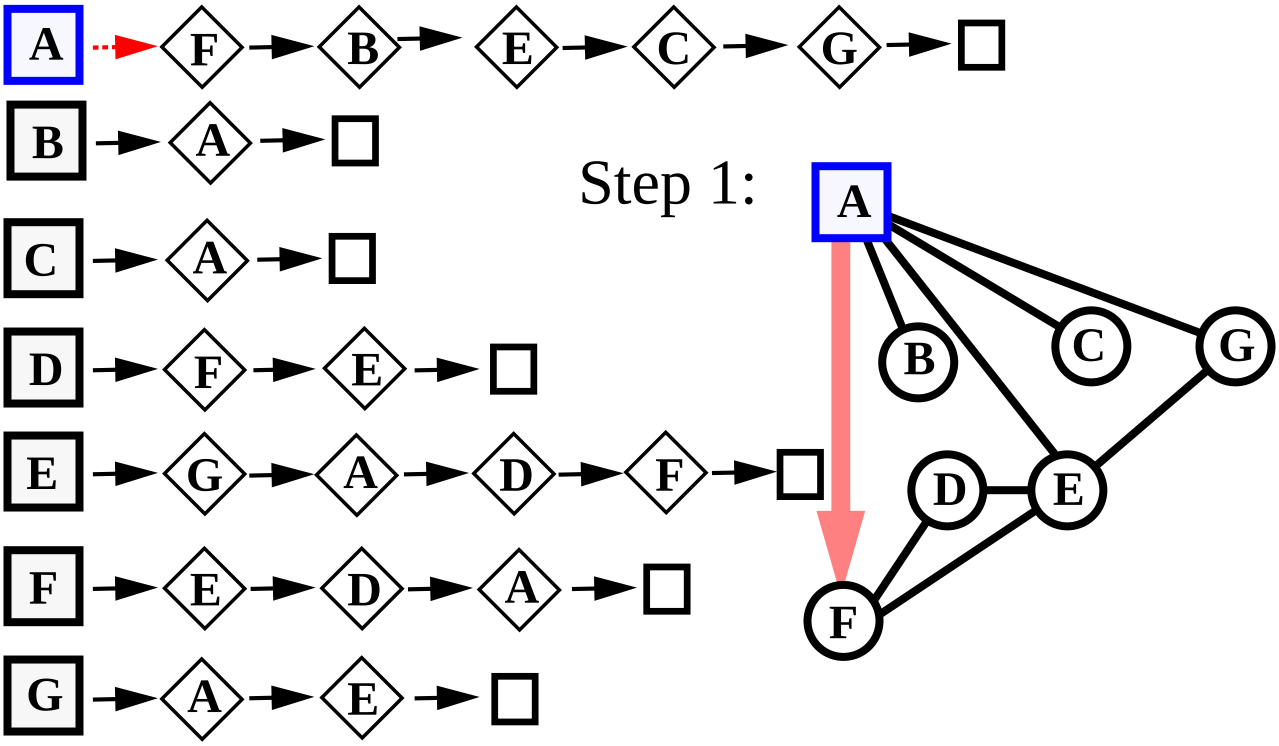


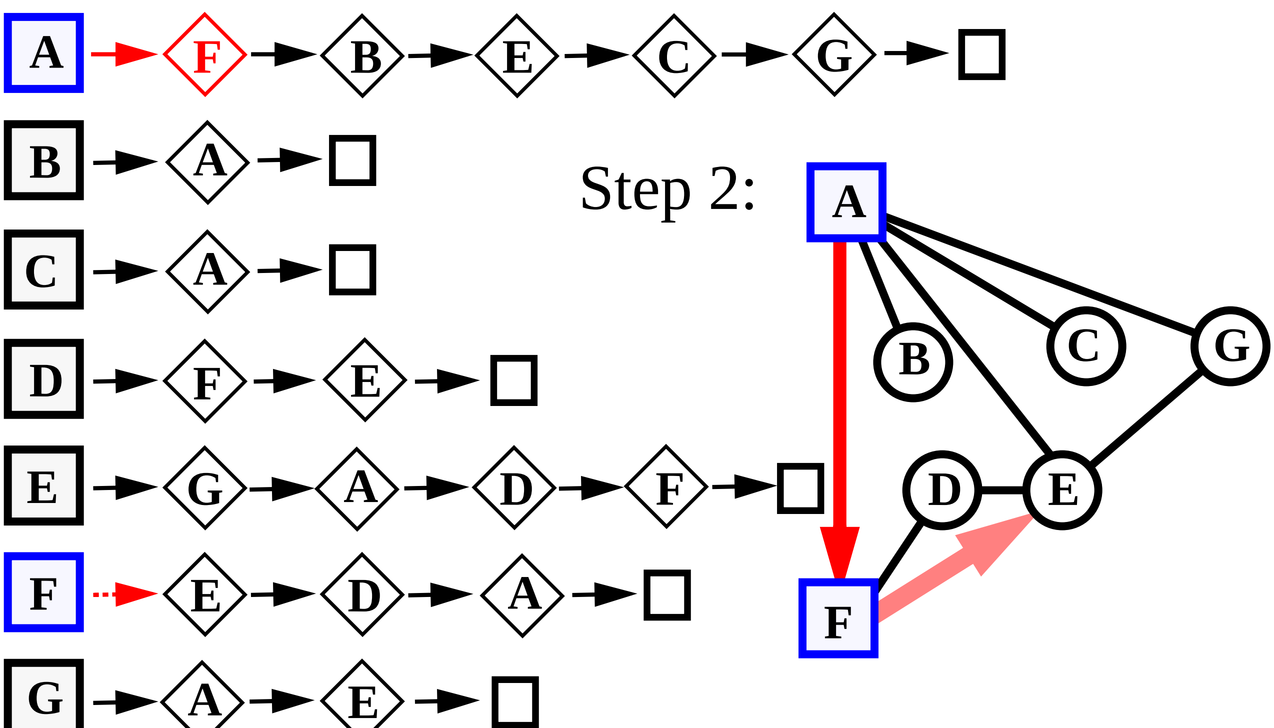
If we instead examine the tree starting at A and looking at F, the C, then E, B, and finally F,

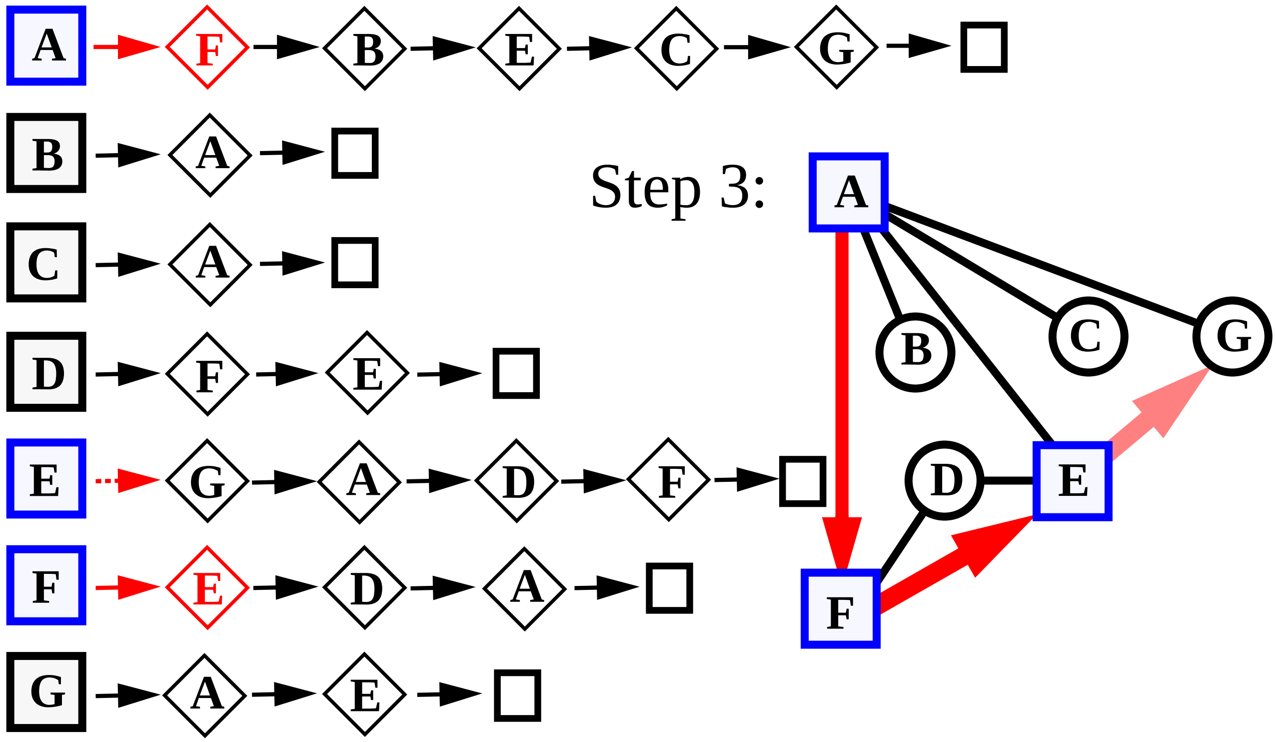


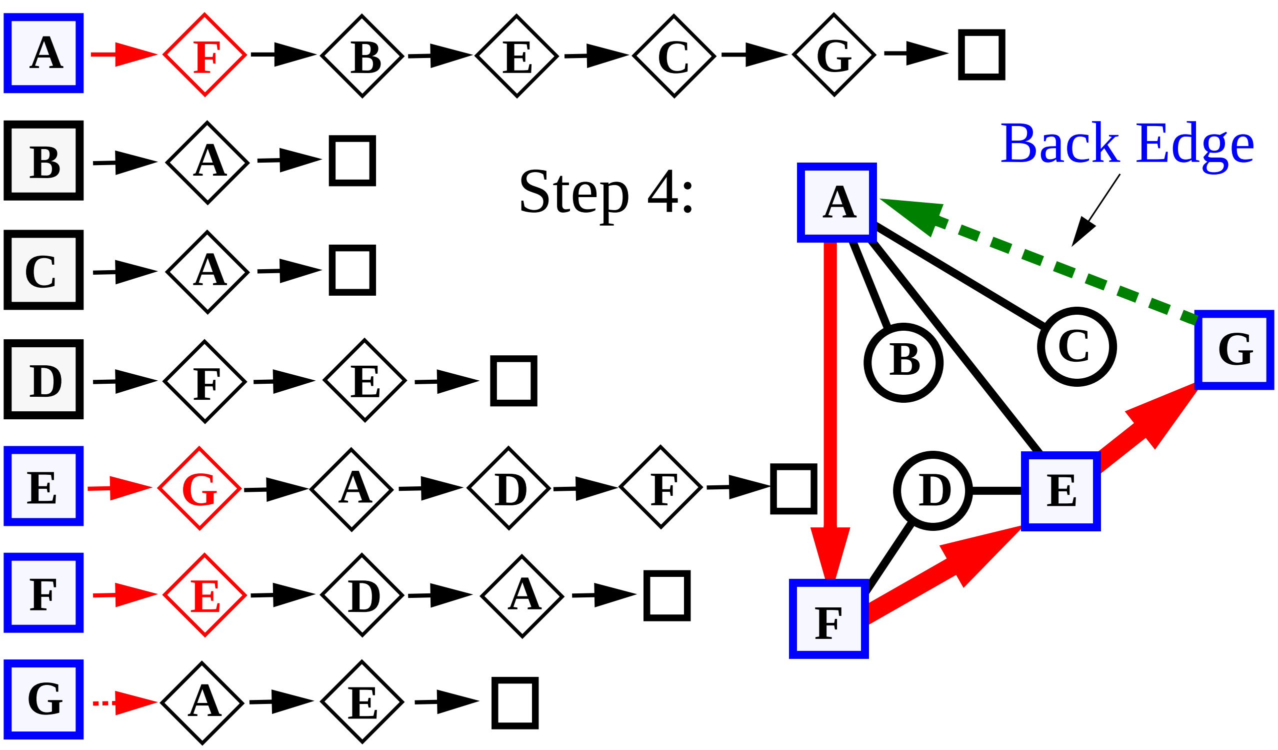
the resulting set of backEdges, discoveryEdges and recursion points is different.

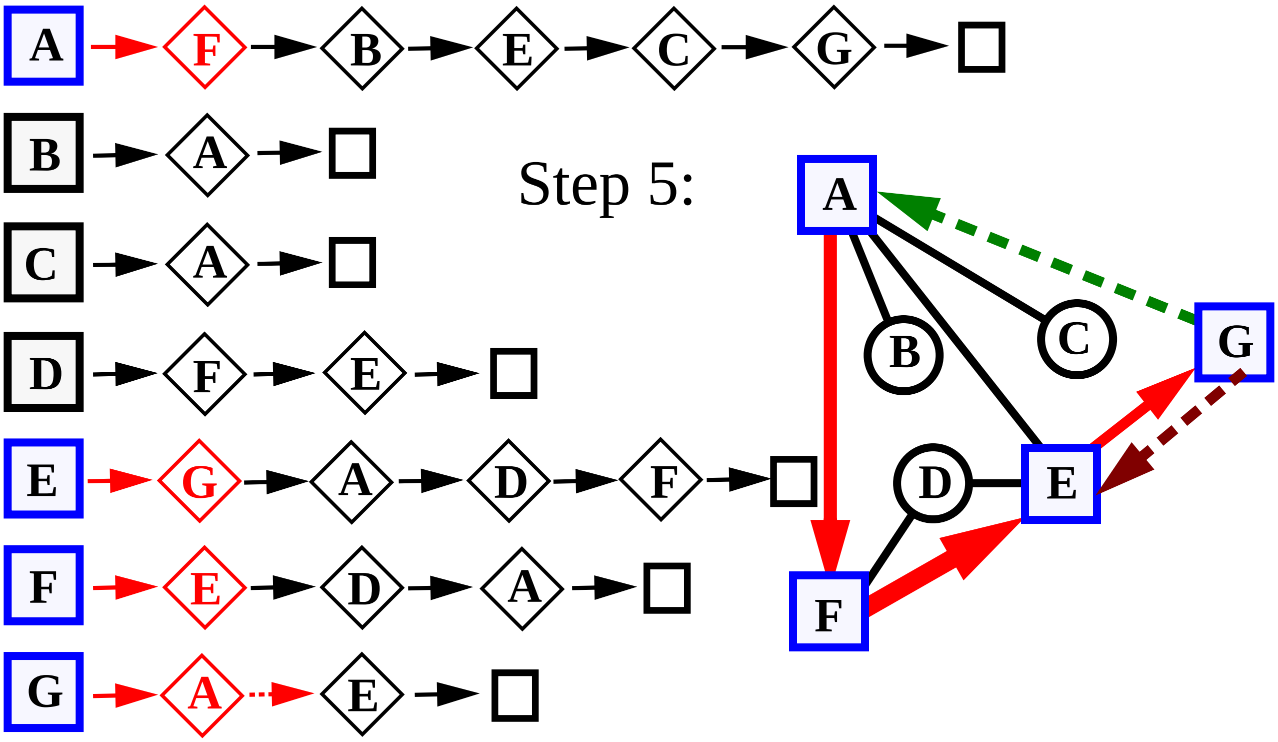
- Now an example of a DFS.

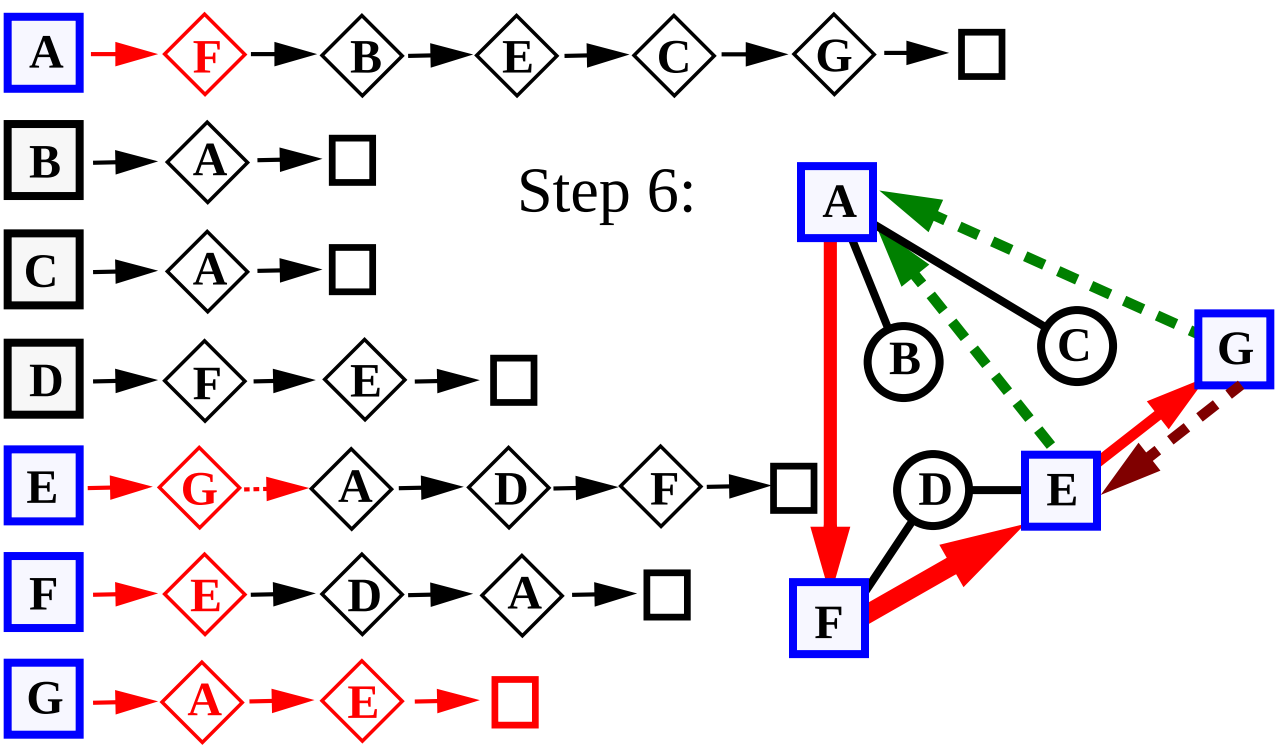

A F B E C G
B A
C A
D F E
E G A D F
F E D A
G A E
Step 1:
A B C G D E F

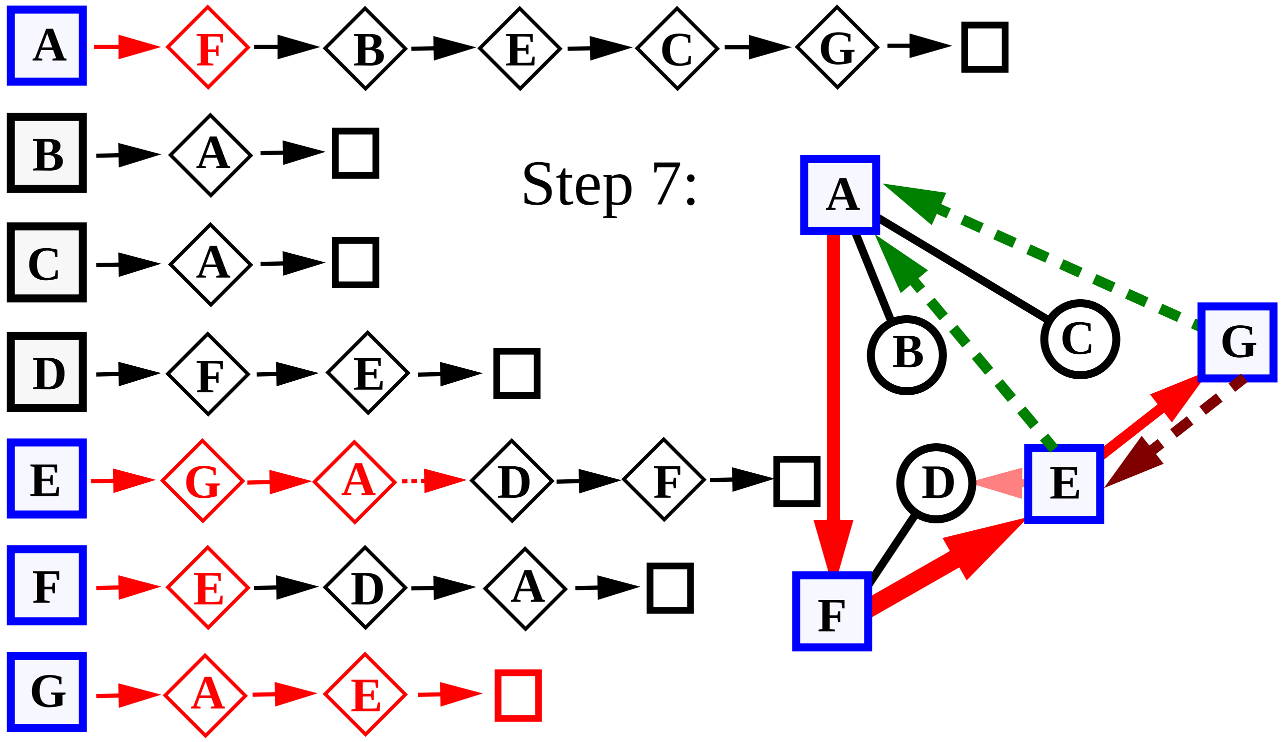

A F B E C G
B A
C A
D F E
E G A D F
F E D A
G A E
Step 2:
A B C G D E F

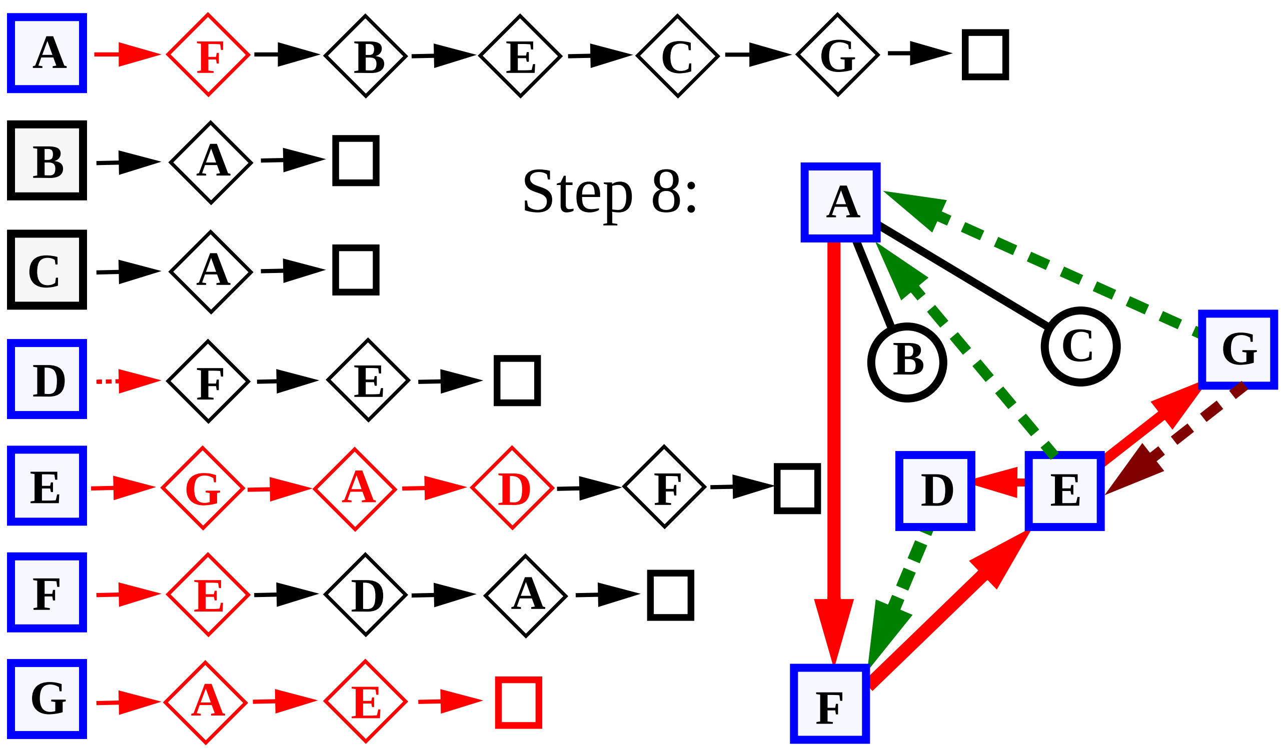

Step 3:

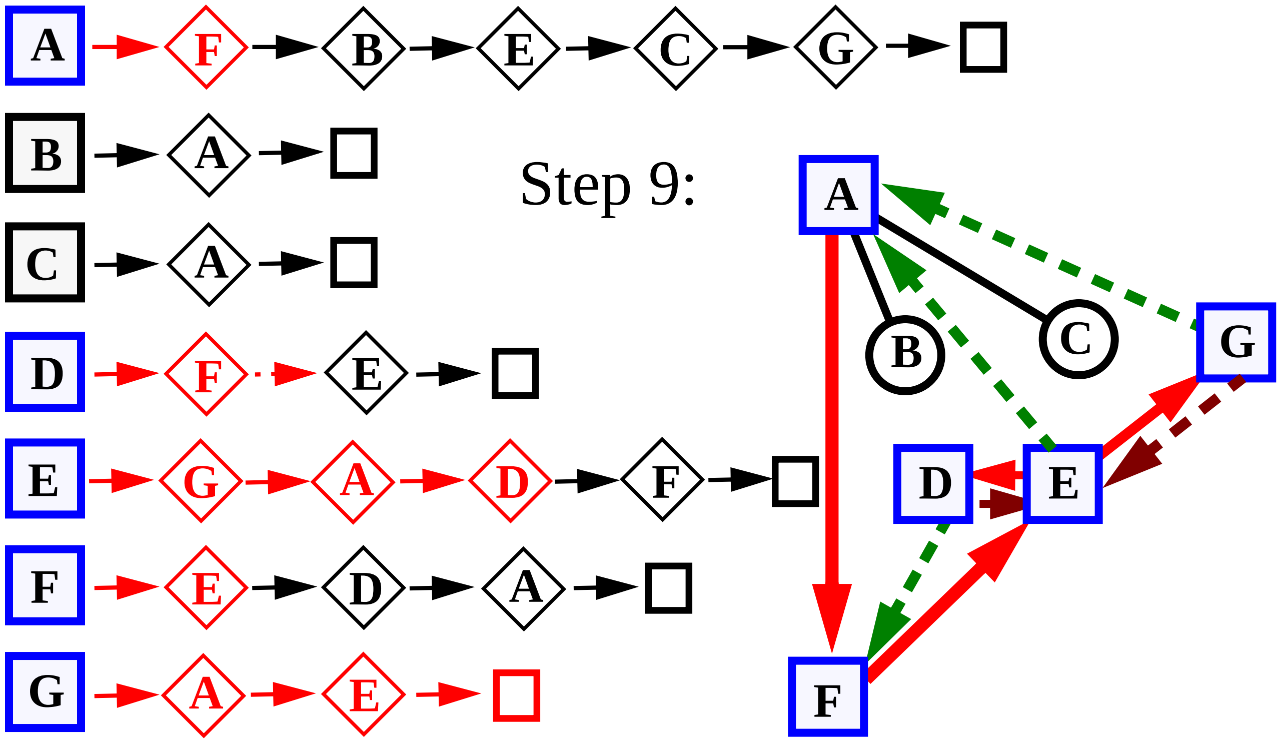

Step 4:
Back Edge

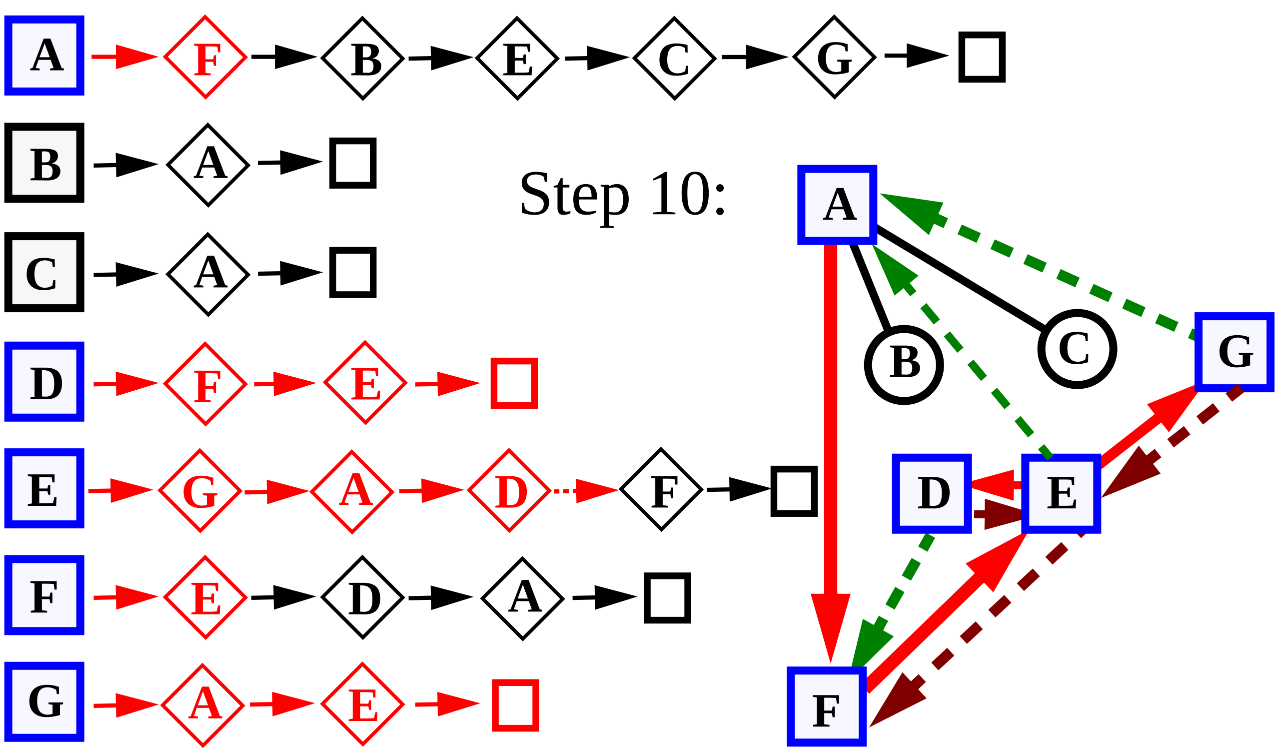

A F B E C G
B A
C A
D F E
E G A D F
F E D A
G A E
Step 5:
A B C D E F G

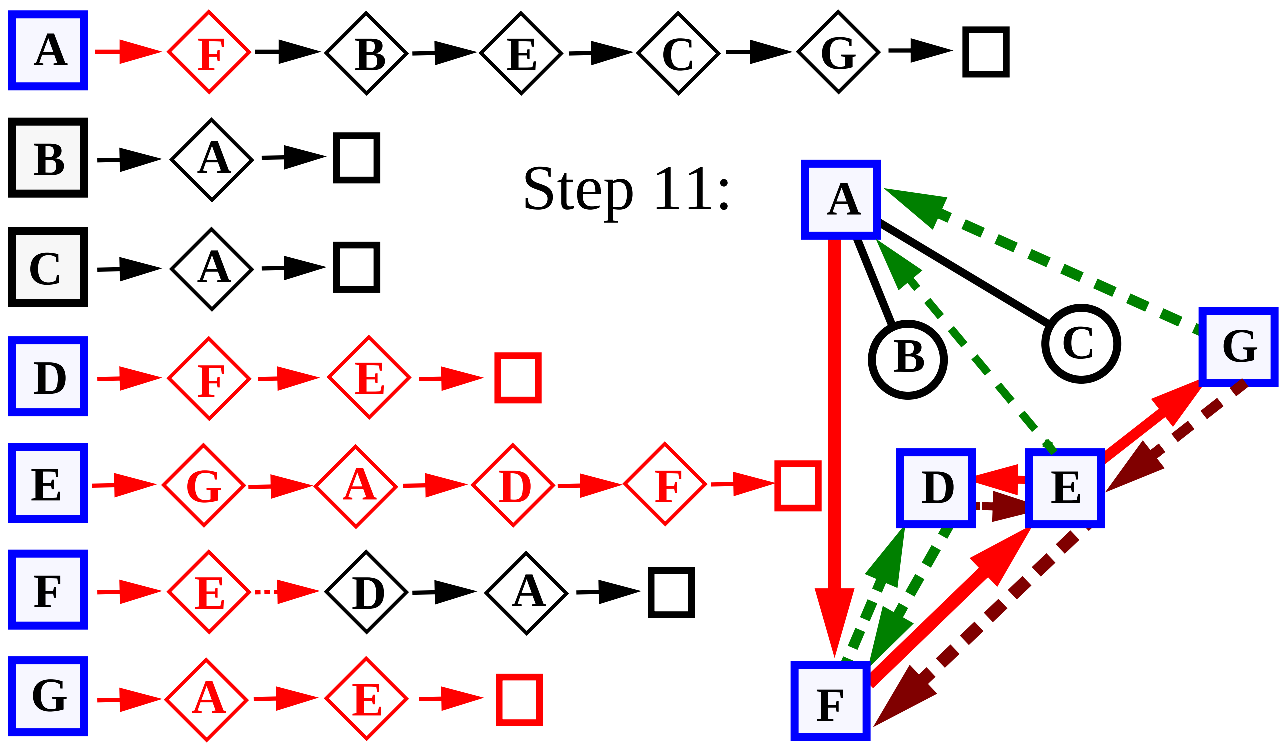

A F B E C G
B A
C A
D F E
E G A D F
F E D A
G A E
Step 6:
A B C D E F G

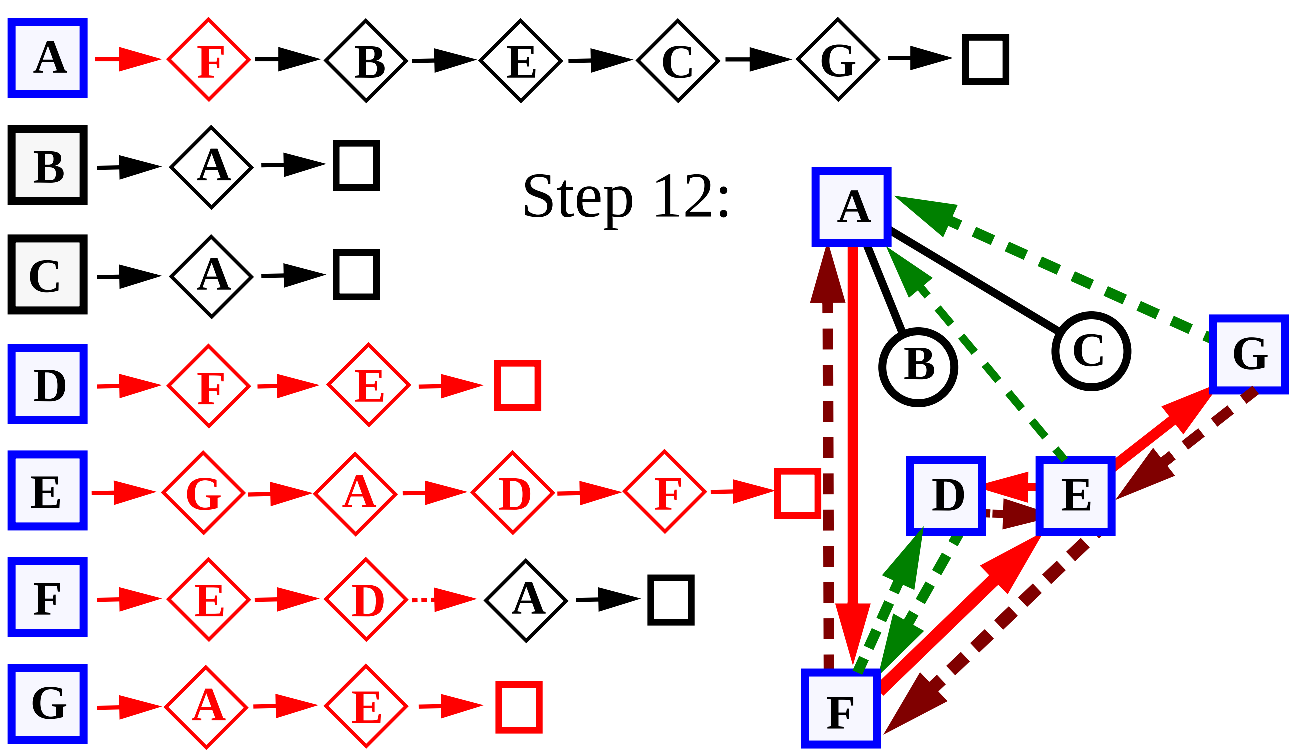

Step 7:

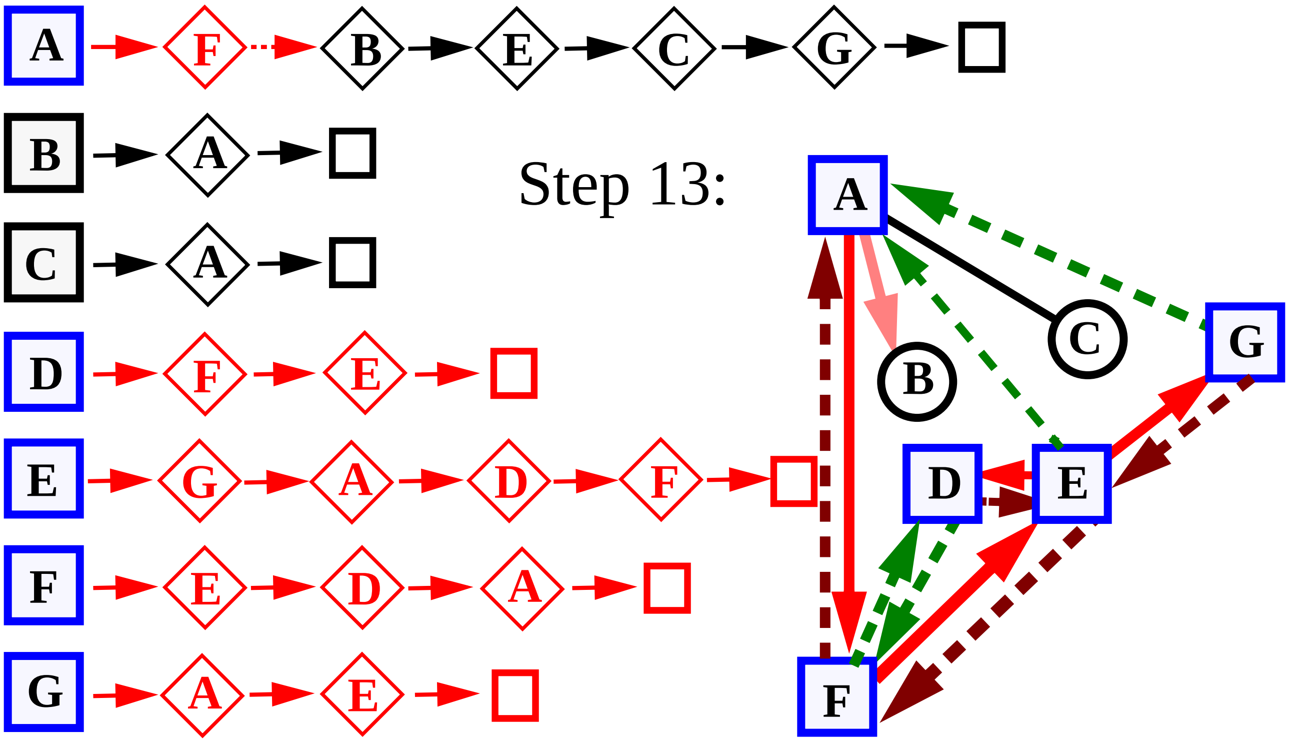

Step 8:

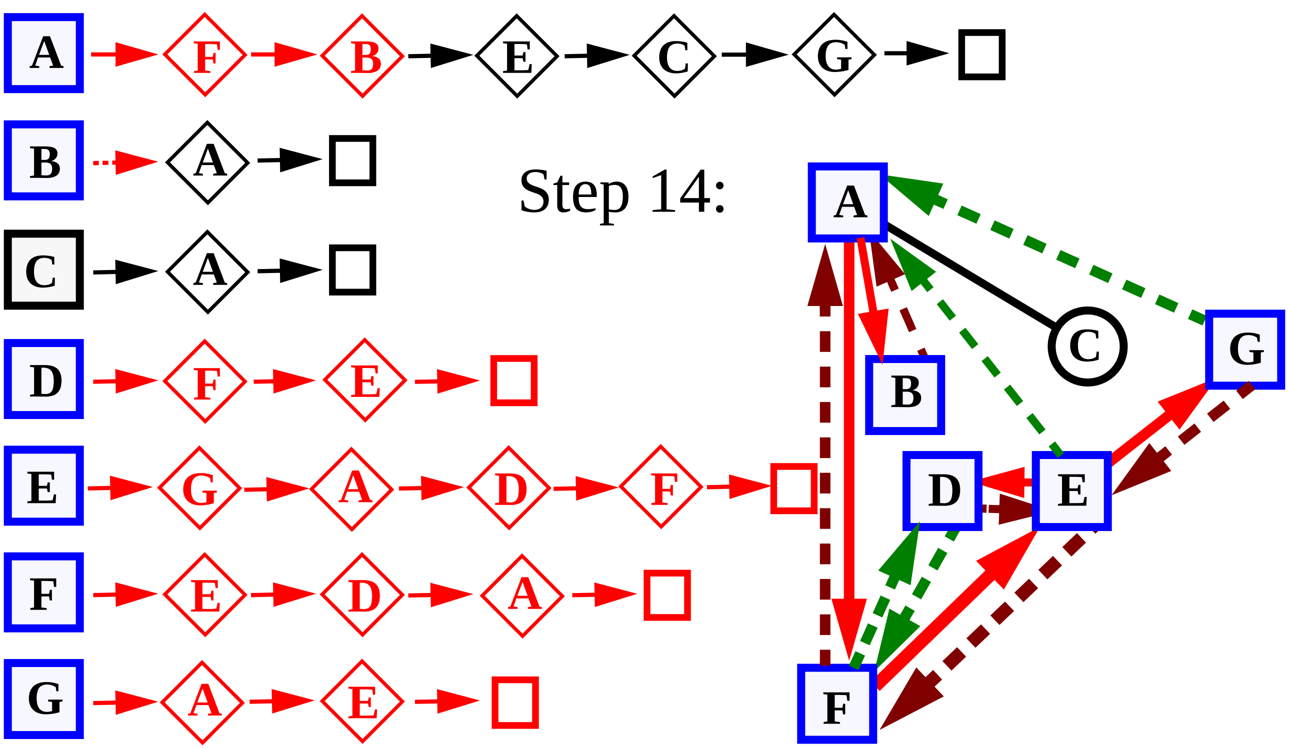

Step 9:
A F B E C G
B A
C A
D F E
E G A D F
F E D A
G A E
A B C G D E F

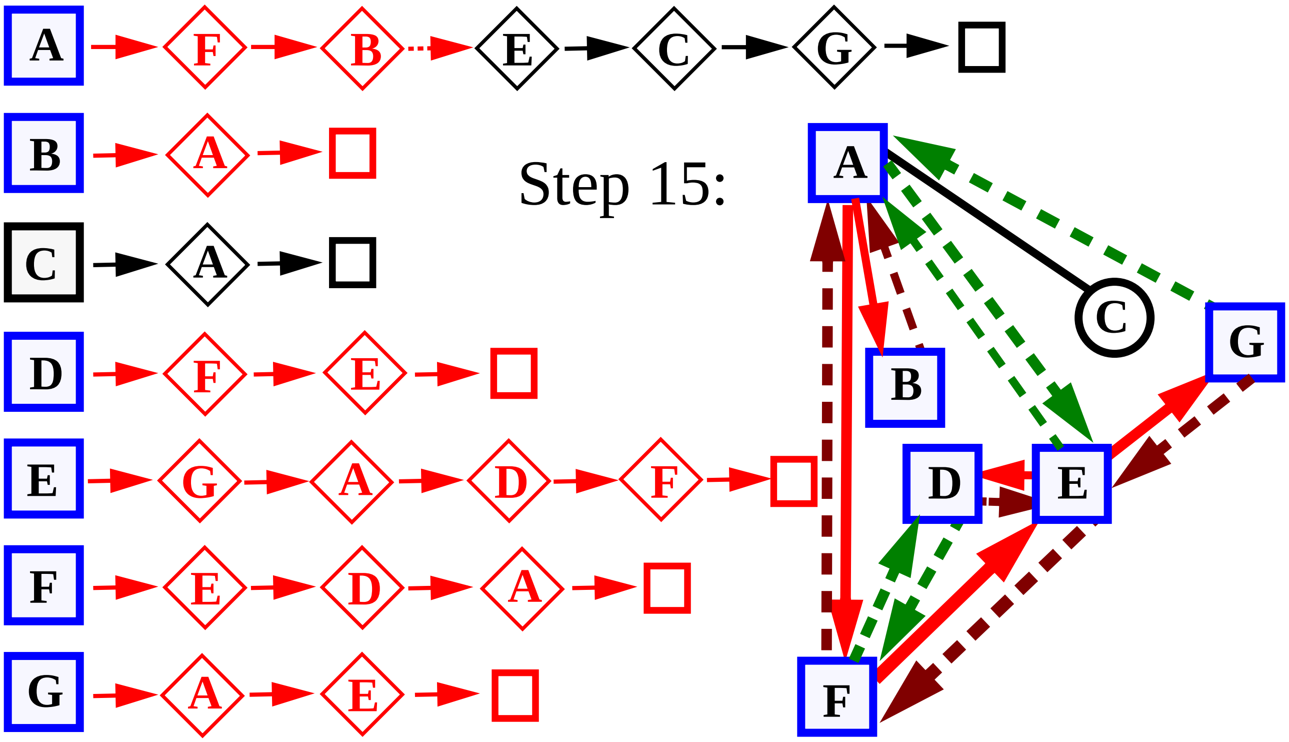

Step 10:
A F B E C G
B A
C A
D F E
E G A D F
F E D A
G A E
A B C G D E F

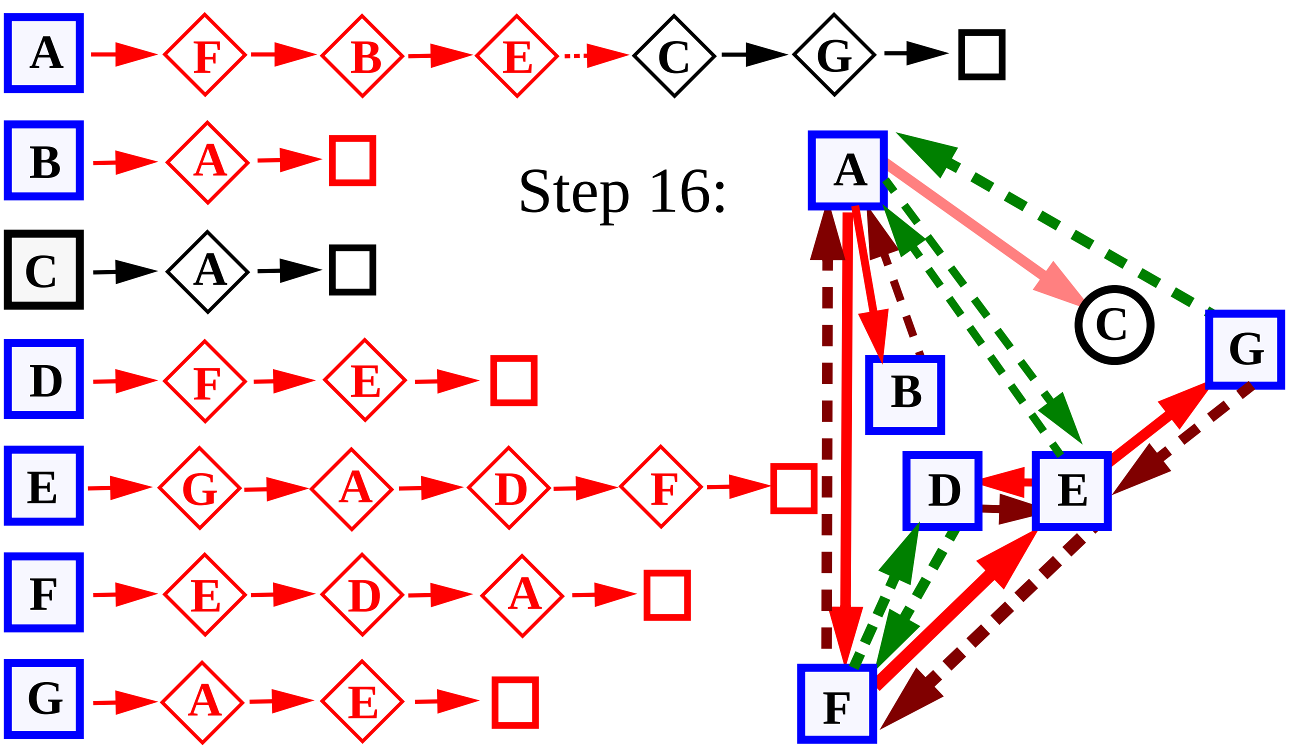

A F B E C G
B A
C A
D F E
E G A D F
F E D A
G A E
Step 11:
A
B
C
G
D
E
F

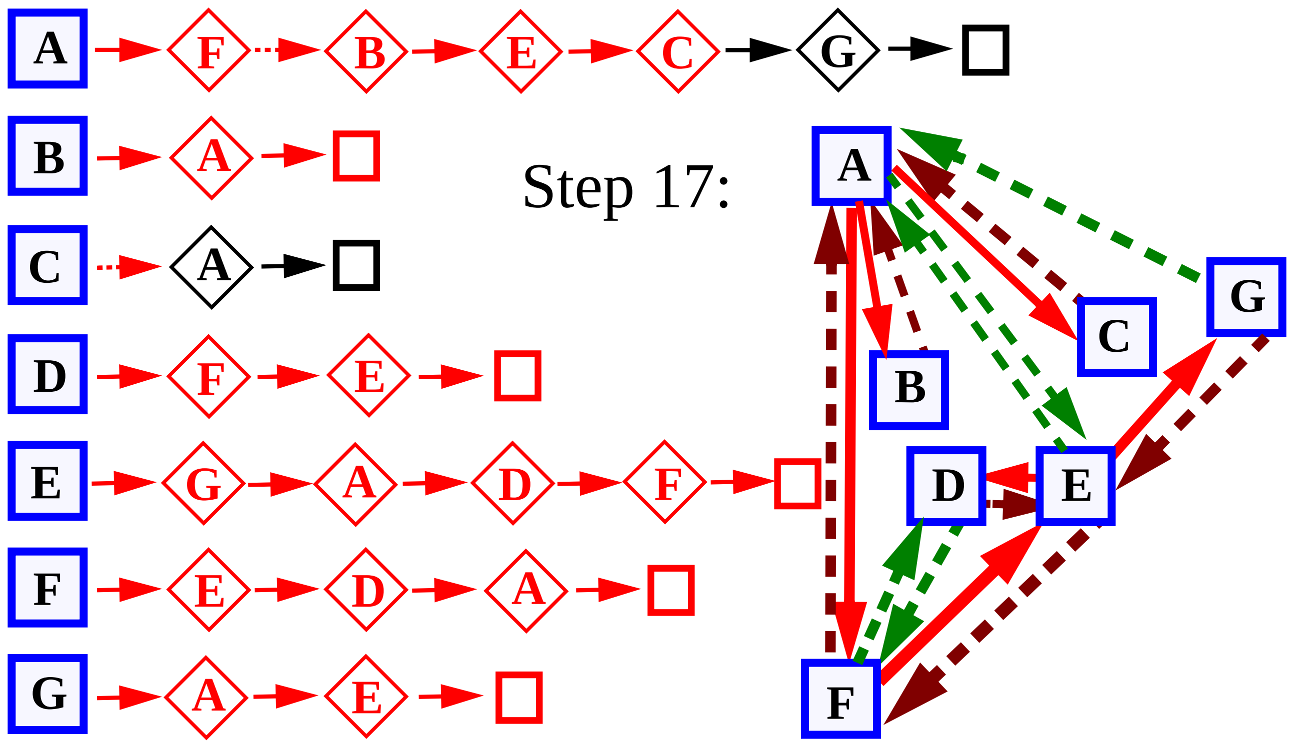

A F B E C G
B A
C A
D F E
E G A D F
F E D A
G A E
Step 12:
A
B
C
G
D
E
F

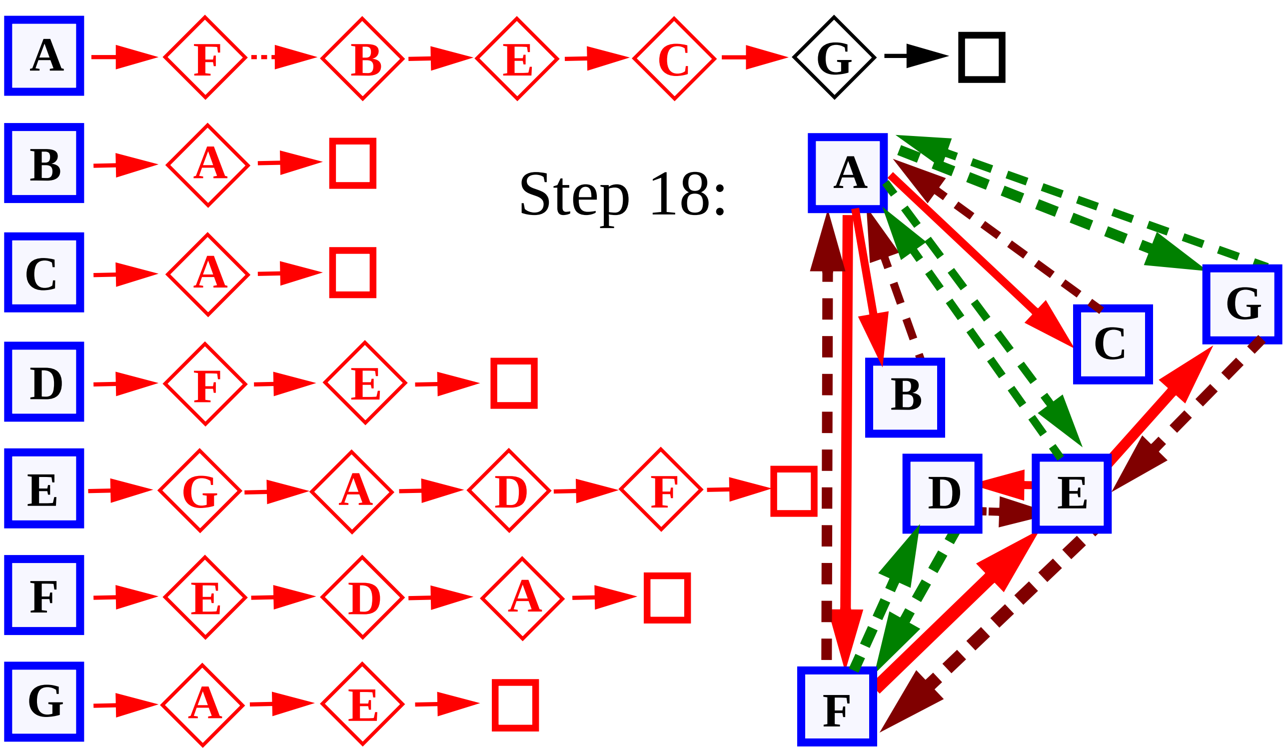

A F B E C G
B A
C A
D F E
E G A D F
F E D A
G A E
Step 13:
A
B
C
D
E
F
G

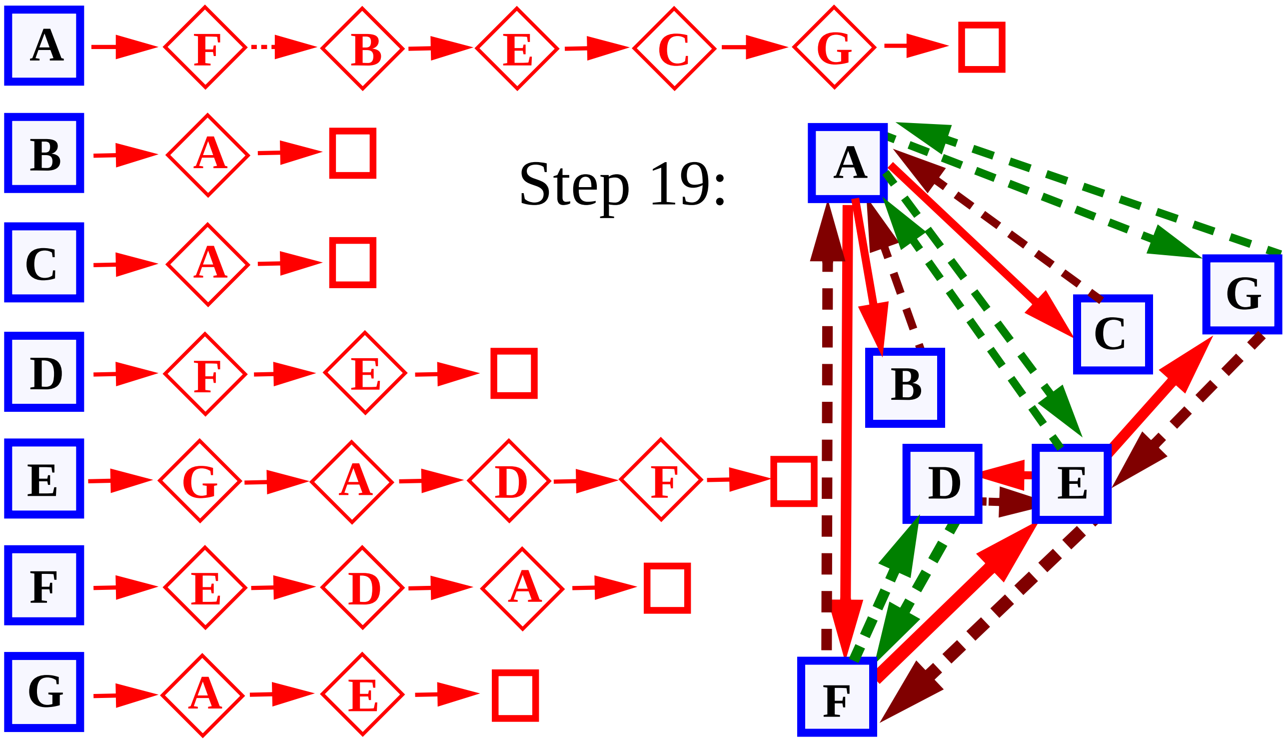

A F B E C G
B A
C A
D F E
E G A D F
F E D A
G A E
Step 14:
A
B
C
D
E
F
G


Step 15:
A F B E C G
B A
C A
D F E
E G A D F
F E D A
G A E


Step 16:
A F B E C G
B A
C A
D F E
E G A D F
F E D A
G A E


A F B E C G
B A
C A
D F E
E G A D F
F E D A
G A E
Step 17:
A B C D E F G


A F B E C G
B A
C A
D F E
E G A D F
F E D A
G A E
Step 18:
A B C D E F G



And we’re done!

DFS Properties

- Proposition 9.12 : Let G be an undirected graph on which a DFS traversal starting at a vertex *s* has been preformed. Then:
  1) The traversal visits all vertices in the connected component of *s*
  2) The discovery edges form a spanning tree of the connected component of *s*

- Justification of 1):
  - Let's use a contradiction argument: suppose there is at least on vertex *v* not visited and let *w* be the first unvisited vertex on some path from *s* to *v*.
  - Because *w* was the first unvisited vertex on the path, there is a neighbor *u* that has been visited.
  - But when we visited *u* we must have looked at edge(u, w). Therefore *w* must have been visited.
  - and justification

- Justification of 2):
  - We only mark edges from when we go to unvisited vertices. So we never form a cycle of discovery edges, i.e. discovery edges form a tree.
  - This is a spanning tree because DFS visits each vertex in the connected component of *s*

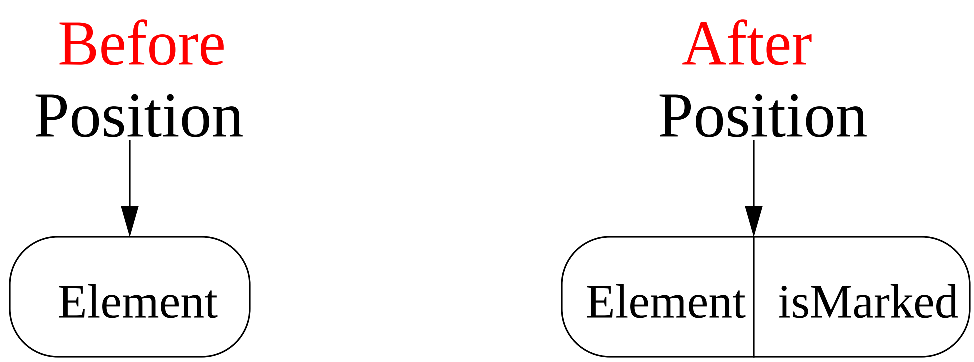
Running Time Analysis

- Remember:
  - DFS is called on each vertex exactly once.
  - Every edge is examined exactly twice, once from each of its vertices

- For n_s vertices and m_s edges in the connected component of the vertex s, a DFS starting at s runs in $O(n_s + m_s)$ time if:
  - The graph is represented in a data structure, like the adjacency list, where vertex and edge methods take constant time
  - Marking a vertex as explored and testing to see if a vertex has been explored takes O(degree)
  - By marking visited nodes, we can systematically consider the edges incident on the current vertex so we do not examine the same edge more than once.

Marking Vertices

- Let's look at ways to mark vertices in a way that satisfies the above condition.
- Extend vertex positions to store a variable for marking



- Use a hash table mechanism which satisfies the above condition is the probabilistic sense, because is supports the mark and test operations in O(1) expected time