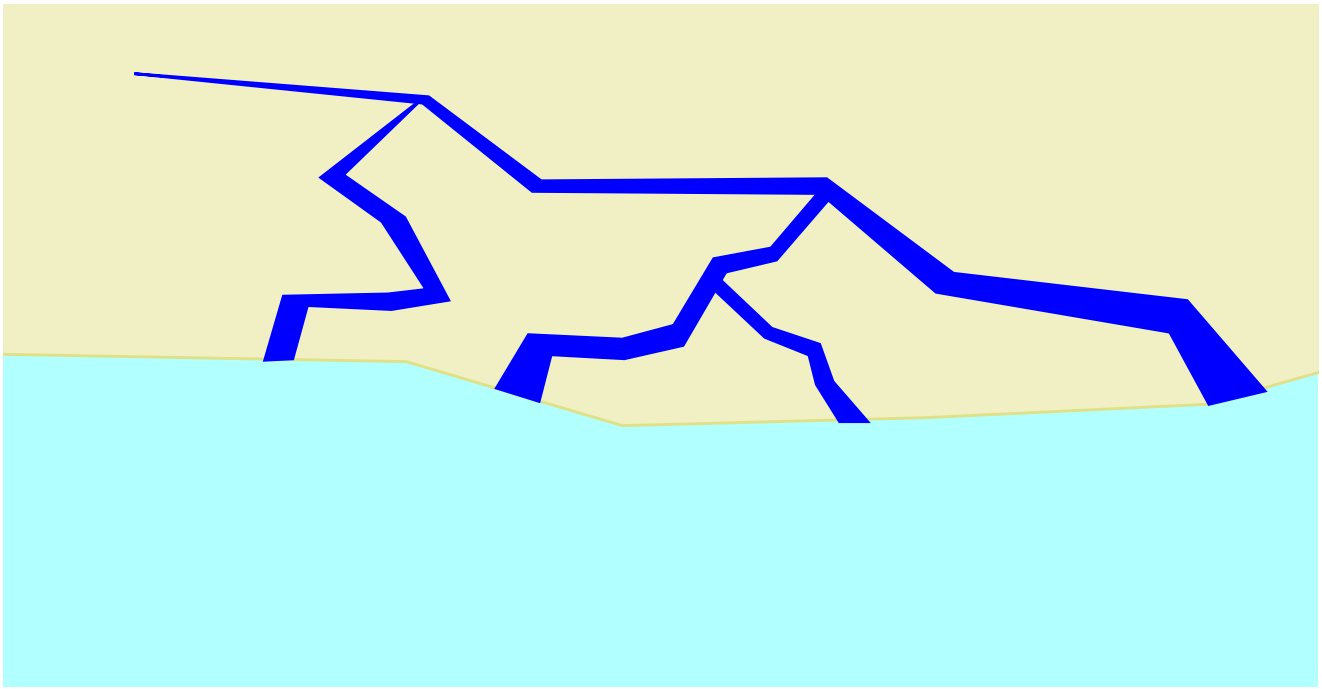
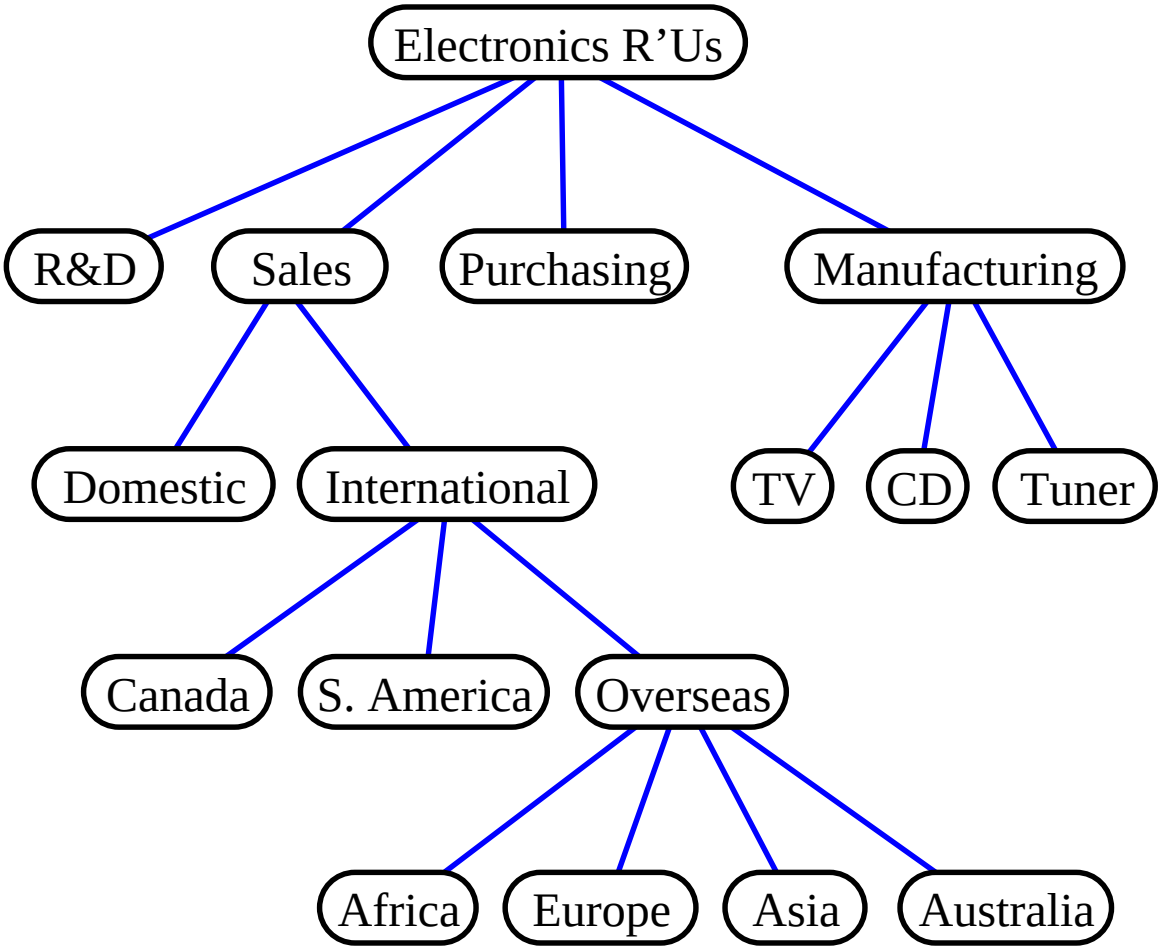
TREES

- trees
- binary trees
- traversals of trees
- template method pattern
- data structures for trees

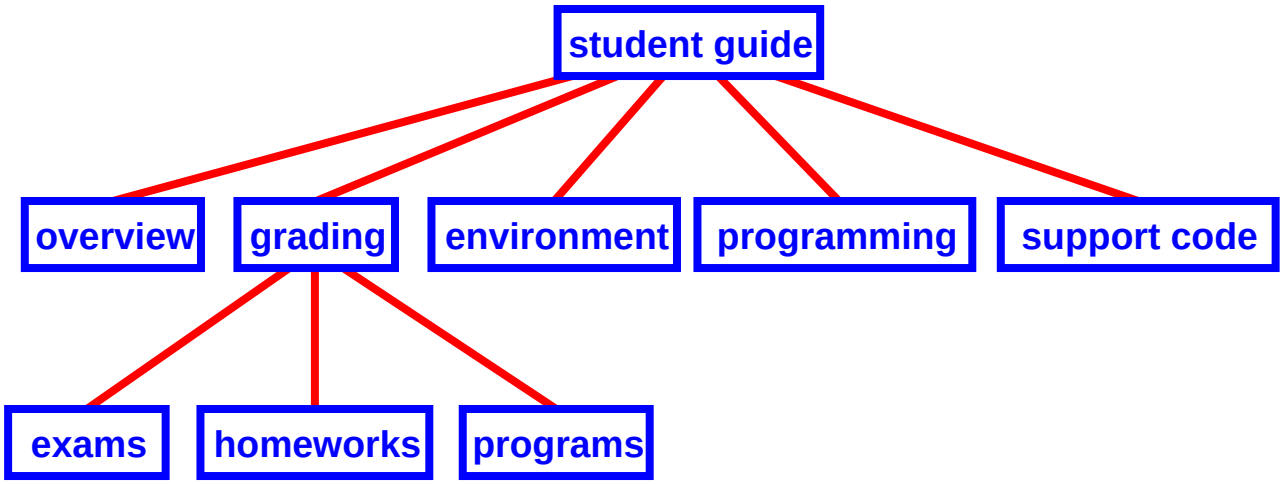


Trees

- a tree represents a hierarchy
  -organization structure of a corporation

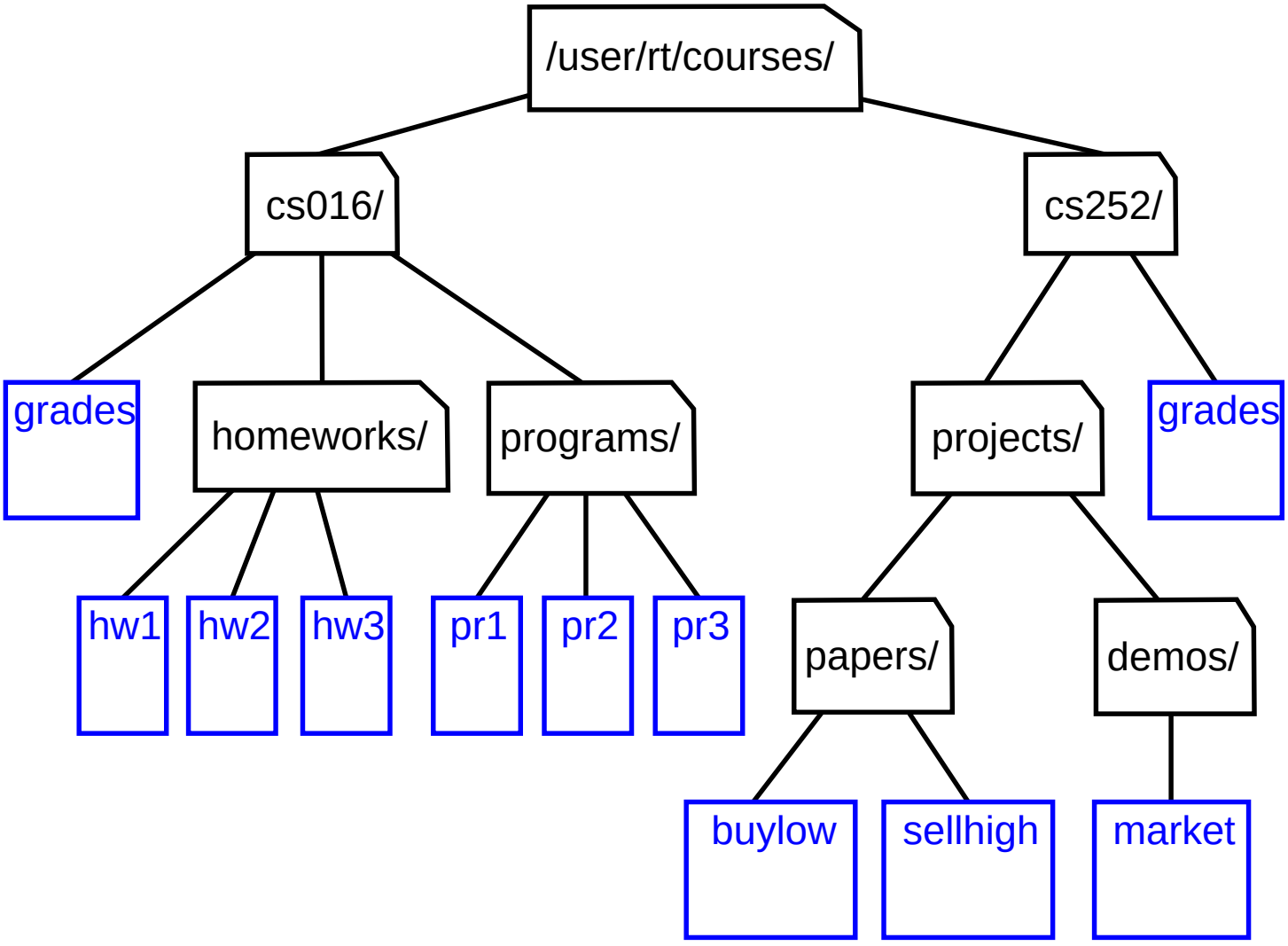


-table of contents of a book



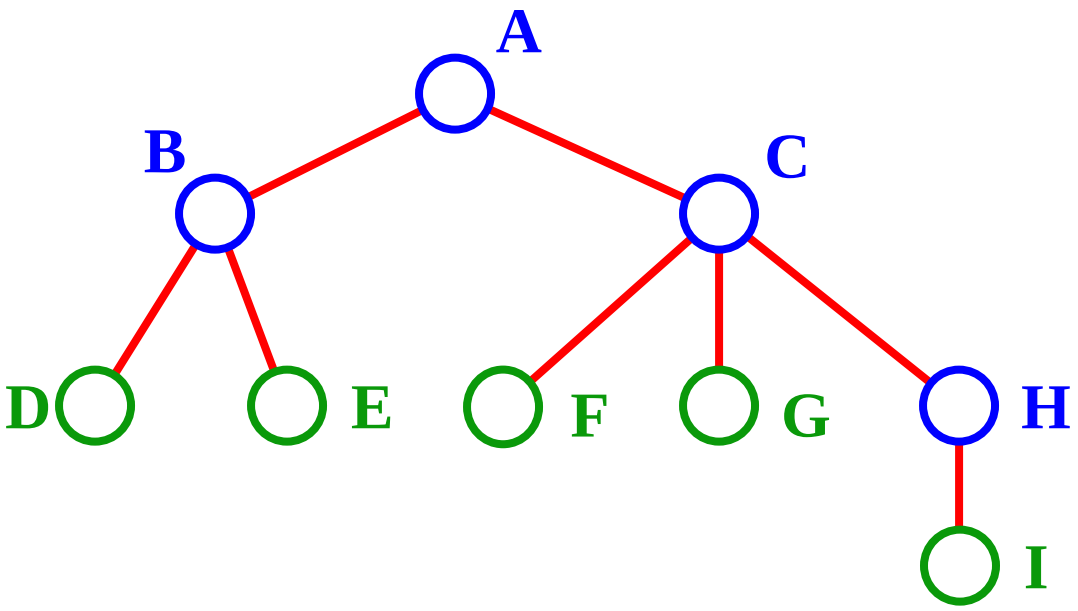
Another Example

- Unix or DOS/Windows file system



Terminology

- ***A*** is the ***root*** node.
- ***B*** is the ***parent*** of D and E.
- ***C*** is the ***sibling*** of B
- ***D*** and ***E*** are the ***children*** of B
- ***D, E, F, G***, *I* are ***external nodes*****, or *leaves***
- ***A, B, C, H*** are ***internal nodes***
- The ***depth*** (***level***) of ***E*** is ***2***
- The ***height*** of the tree is ***3***
- The ***degree*** of node ***B*** is ***2***



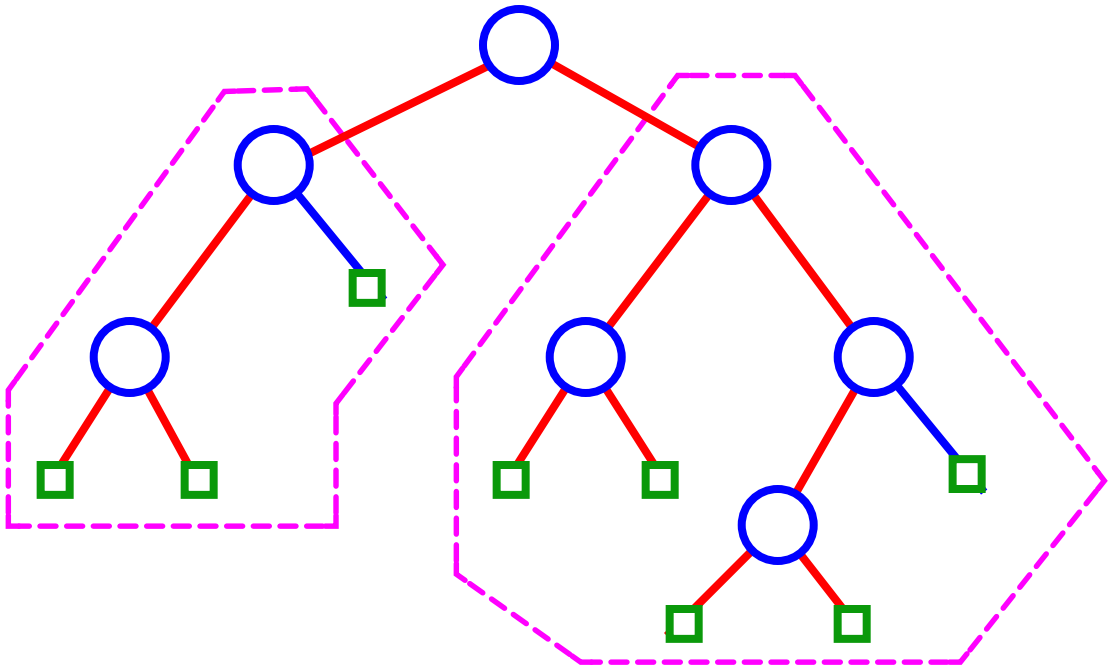
Property: (*# edges*) = (*#nodes*) − 1

Binary Trees

- ***Ordered tree:*** the children of each node are ordered.
- ***Binary tree:*** ordered tree with all internal nodes of ***degree 2***.

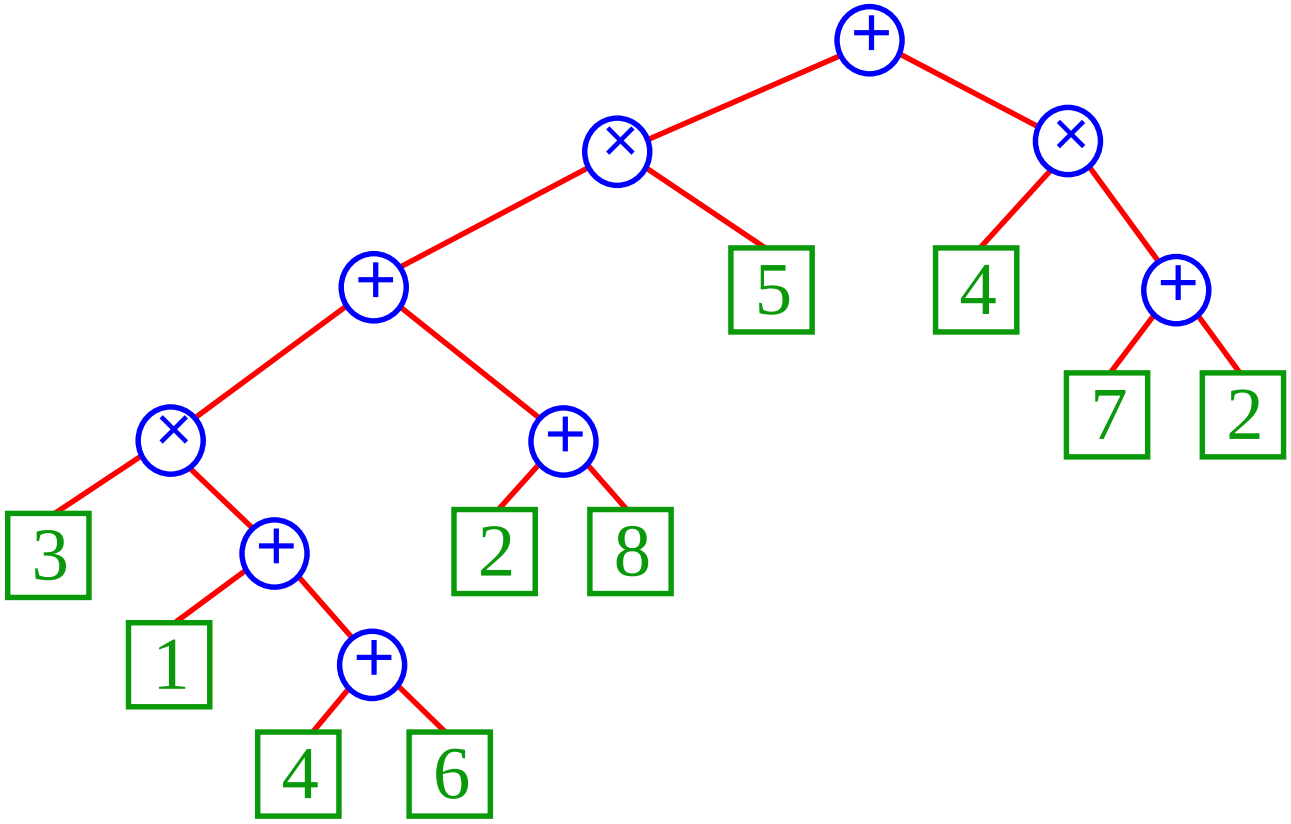
•Recursive definition of binary tree:

- A ***binary tree*** is either
  - a n external node (leaf), **or**
  - a n internal node (the ***root***) and two binary trees (***left subtree*** and ***right subtree***)



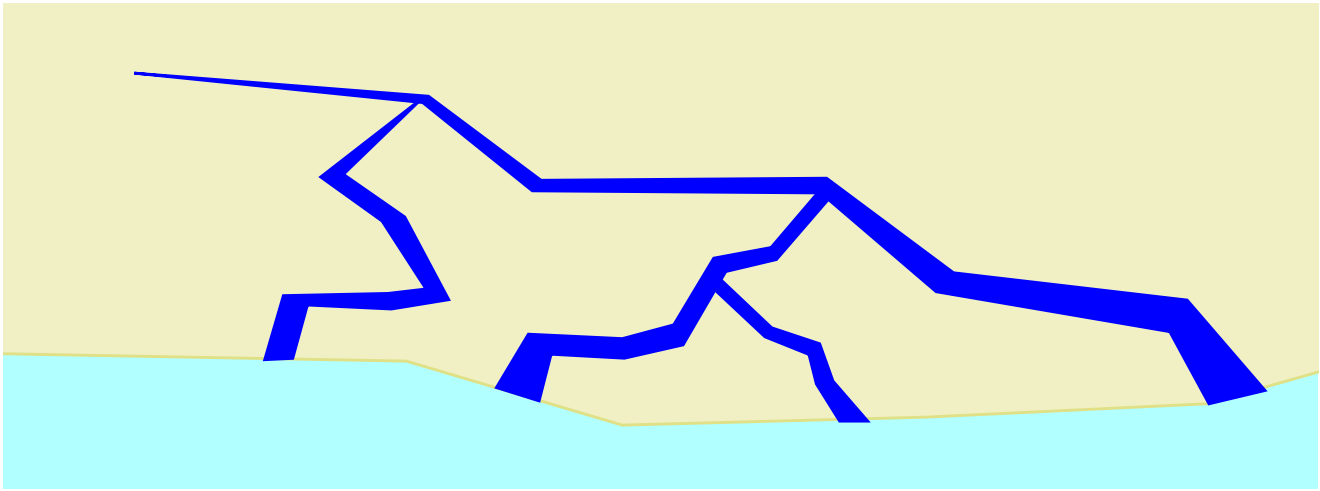
Examples of Binary Trees

•arithmetic expression



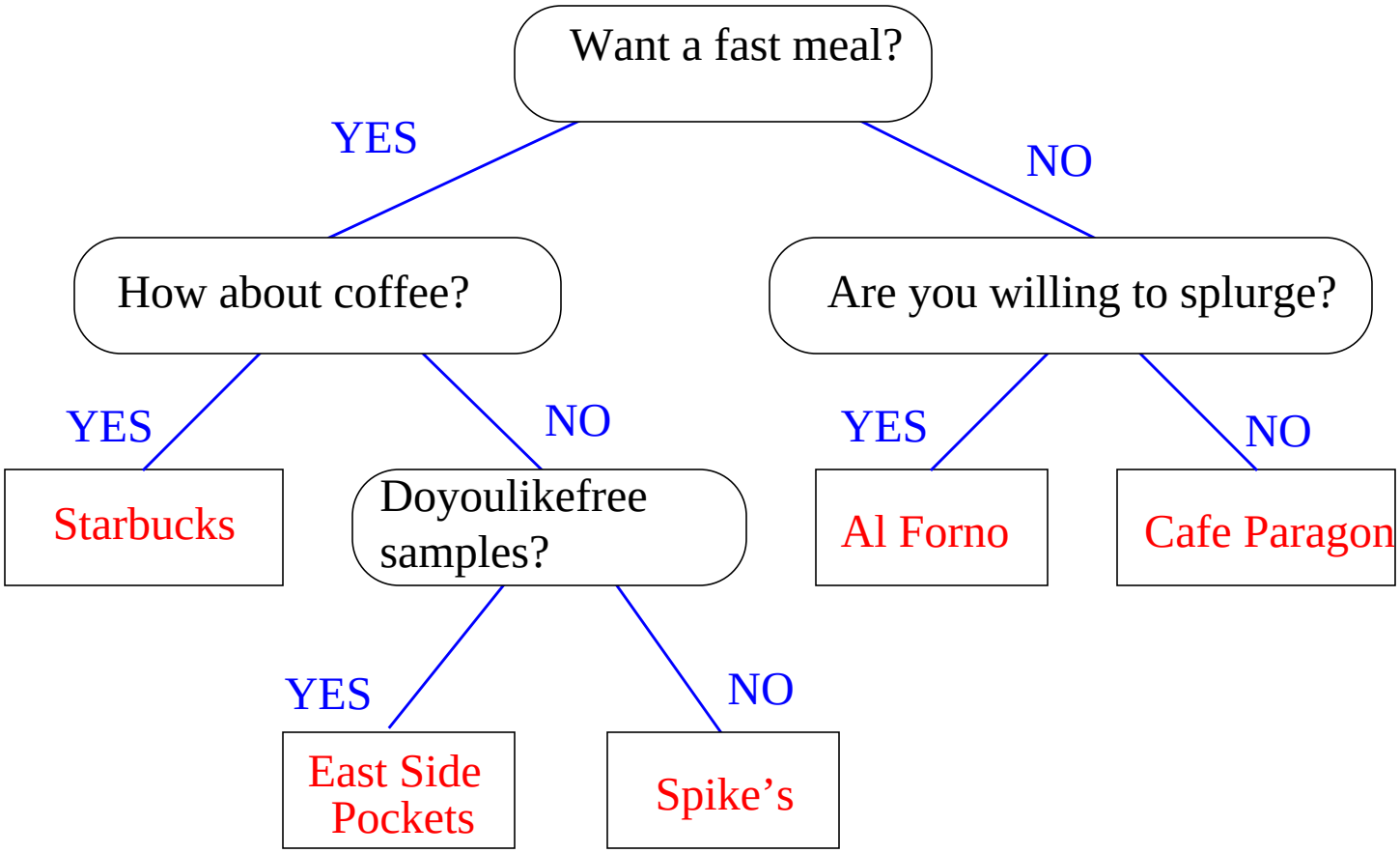
((((3 × (1 + (4 + 6))) + (2 + 8)) × 5) + (4 × (7 + 2)))

• r i v e r



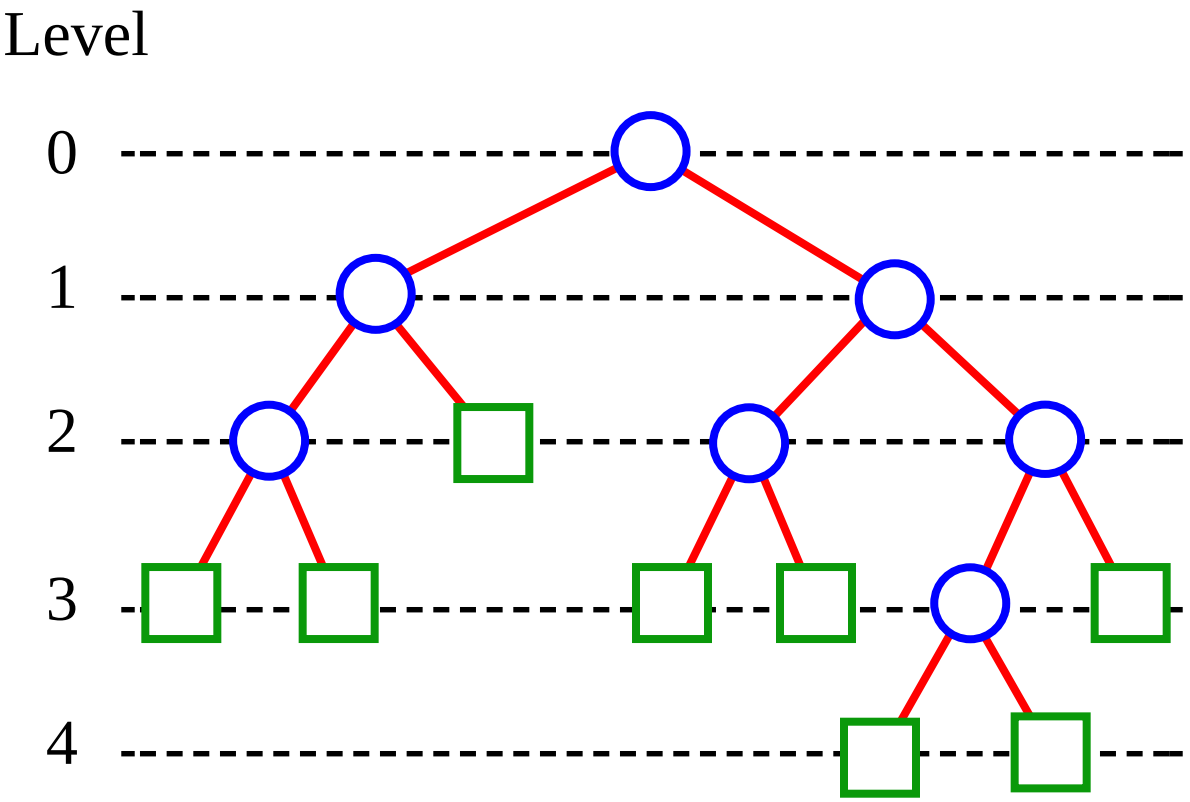
Examples of Binary Trees

•decision trees



Properties of Binary Trees

- (# external nodes) = (# internal nodes) + 1
- (# nodes at level i) $\leq 2^i$
- (# external nodes) $\leq 2^{(\text{height})}$
- (height) $\geq \log_2$ (# external nodes)
- (height) $\geq \log_2$ (# nodes) $-$ 1
- (height) $\leq$ (# internal nodes) = ((# nodes) $-$ 1)/2



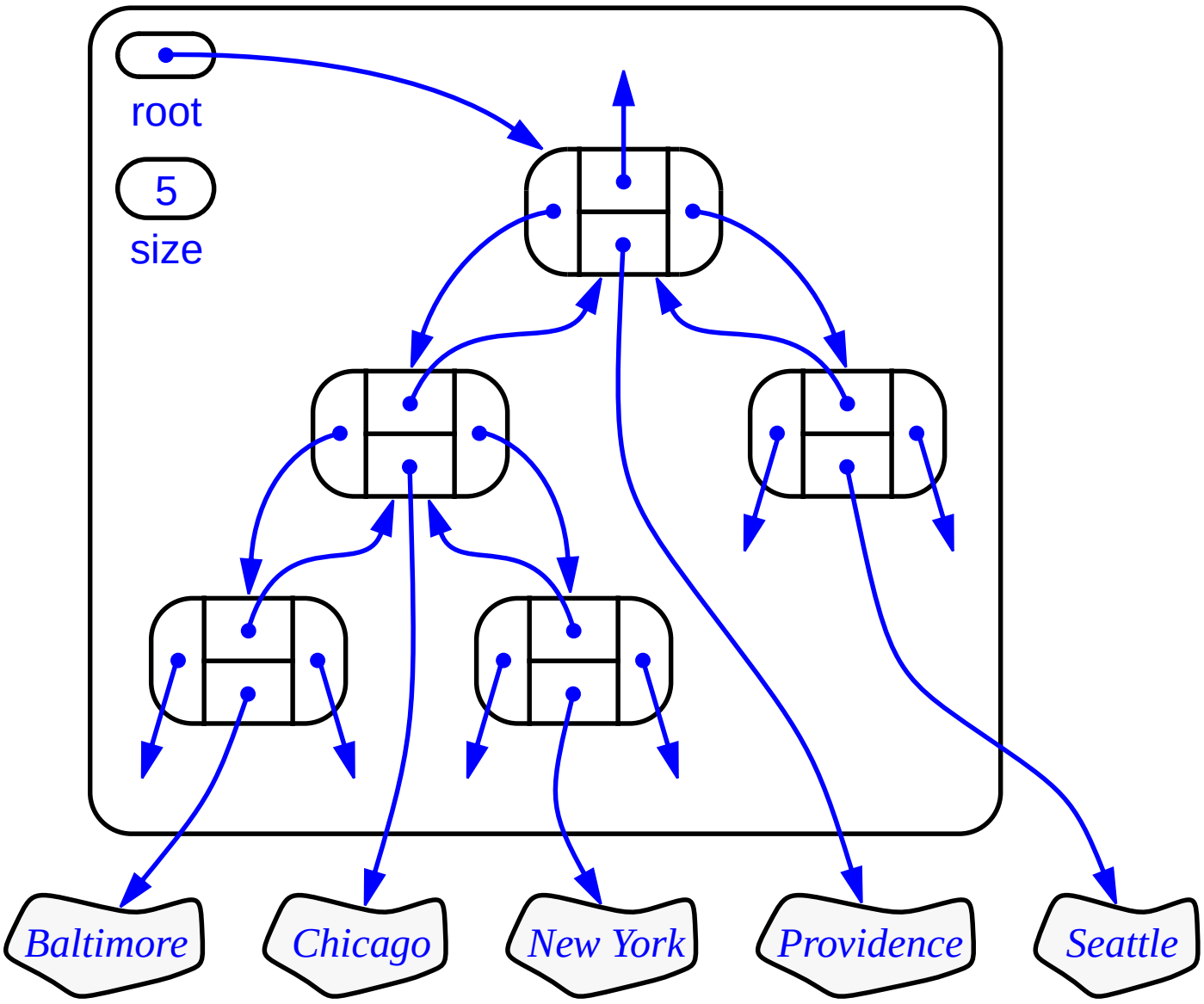
ADTs for Trees

- generic container methods
  - -size(), isEmpty(), elements()
- positional container methods
  - -positions(), swapElements(p,q), replaceElement(p,e)
- query methods
  - -isRoot(p), isInternal(p), isExternal(p)
- accessor methods
  - -root(), parent(p), children(p)
- update methods
  - -application specific

ADTs for Binary Trees

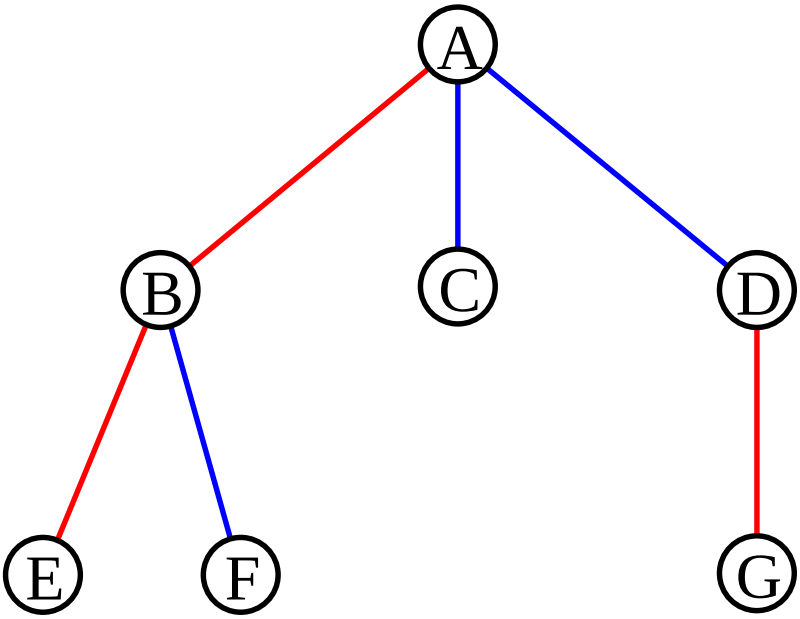
- accessor methods
  - -leftChild(p), rightChild(p), sibling(p)
- update methods
  - -expandExternal(p), removeAboveExternal(p)
  - -other application specific methods

Linked Data Structure for Binary Trees



Representing General Trees

•tree T



•binary tree T' representing T