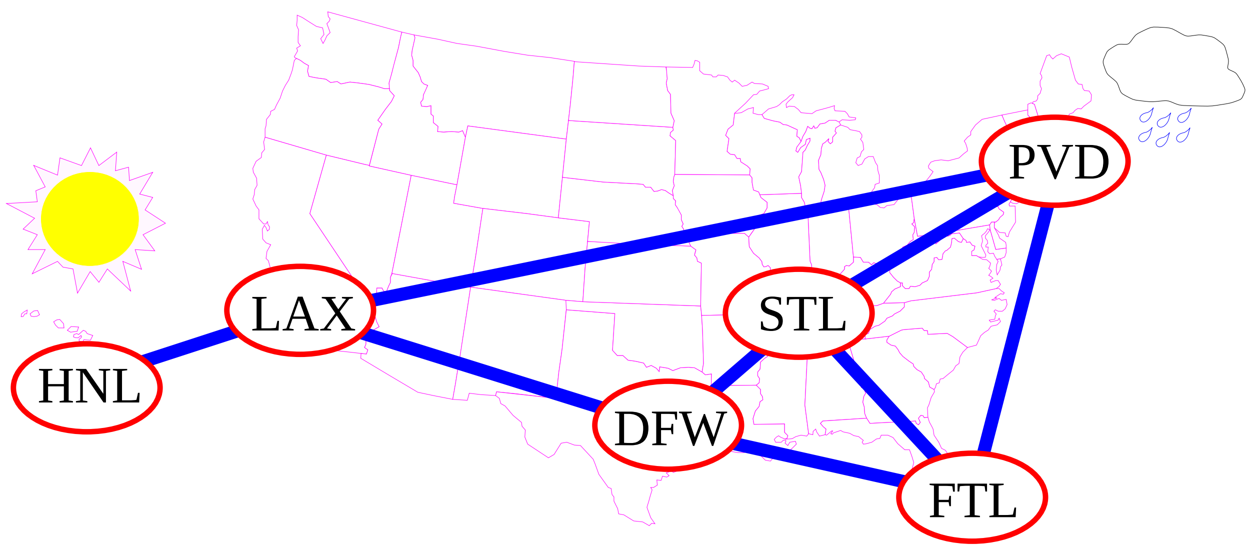
GRAPHS

- Definitions
- Examples
- The Graph ADT



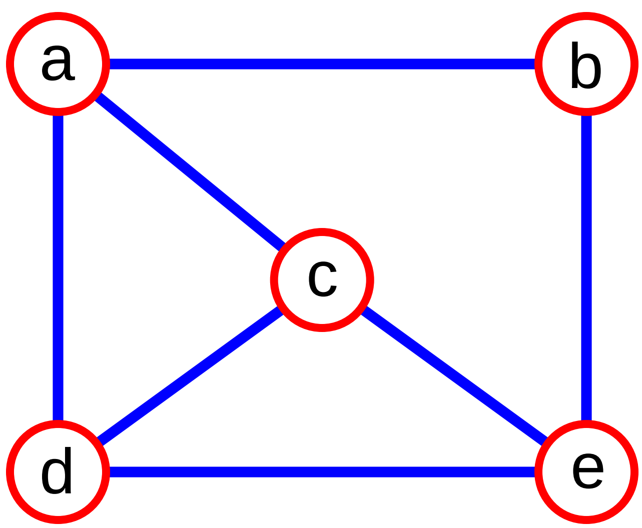
What is a Graph?

- A **graph G** = (**V**,**E**) is composed of:

  V: set of *vertices*

  E: set of *edges* connecting the *vertices* in **V**

- An **edge e** = (u,v) is a pair of vertices
- Example:

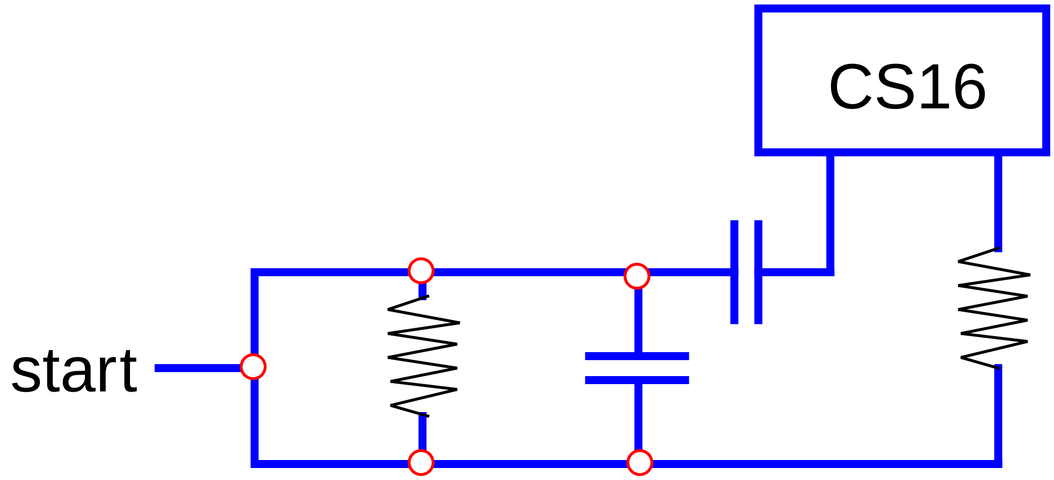


V= {a,b,c,d,e}

E= {(a,b),(a,c),(a,d), (b,e),(c,d),(c,e), (d,e)}

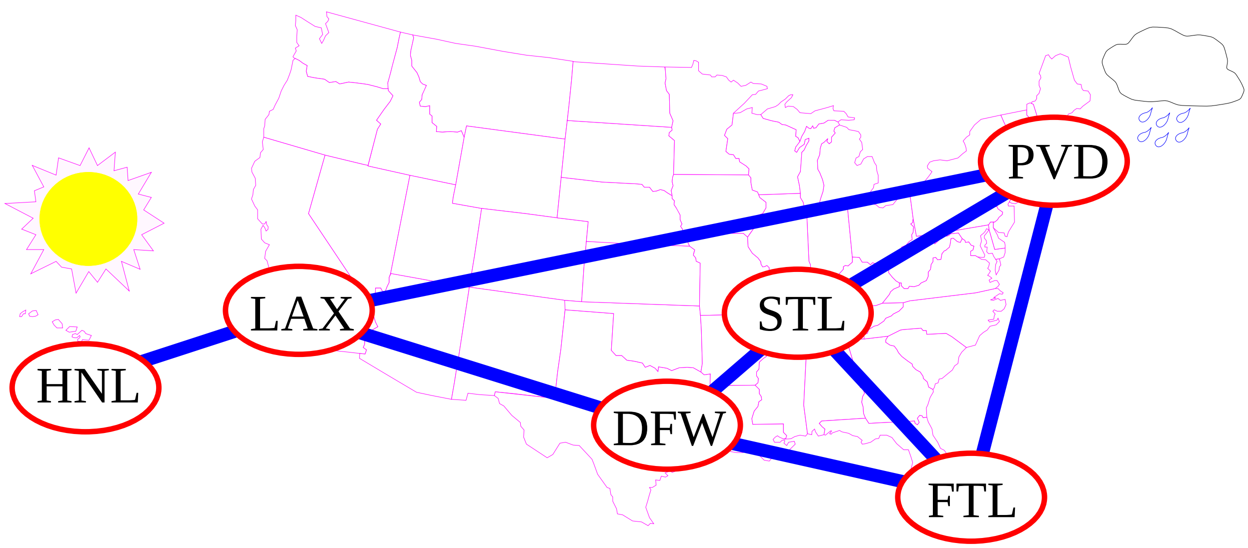
Applications

- electronic circuits



find the path of least resistance to CS16

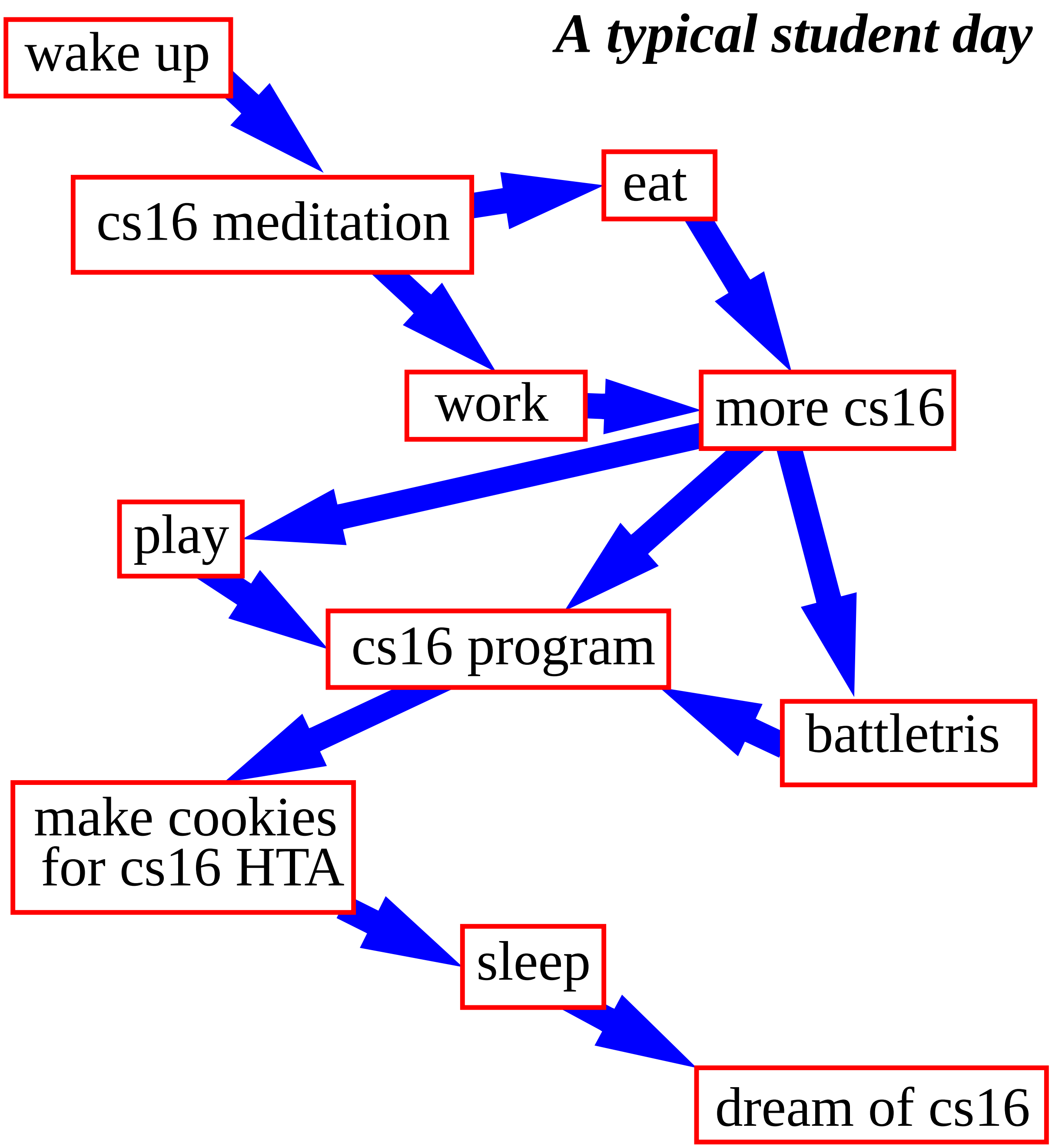
- networks (roads, flights, communications)



mo' better examples

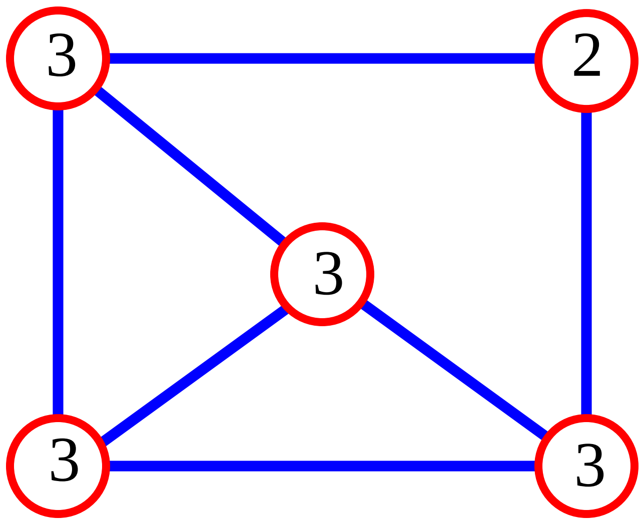
A Spike Lee Joint Production

- scheduling (project planning)



Graph Terminology

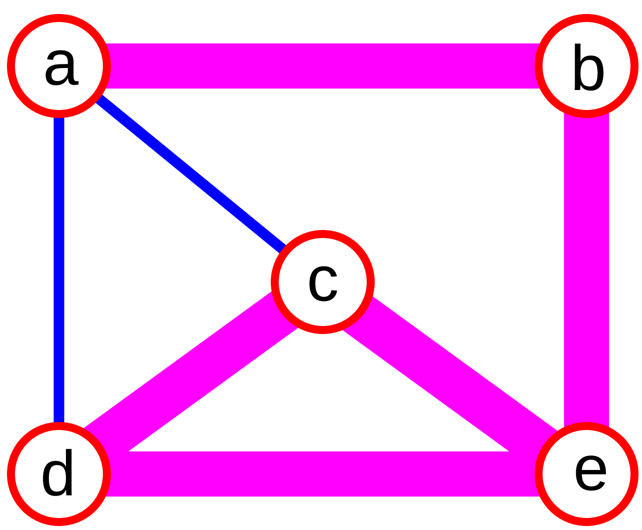
- adjacent vertices: connected by an edge
- degree (of a vertex): # of adjacent vertices



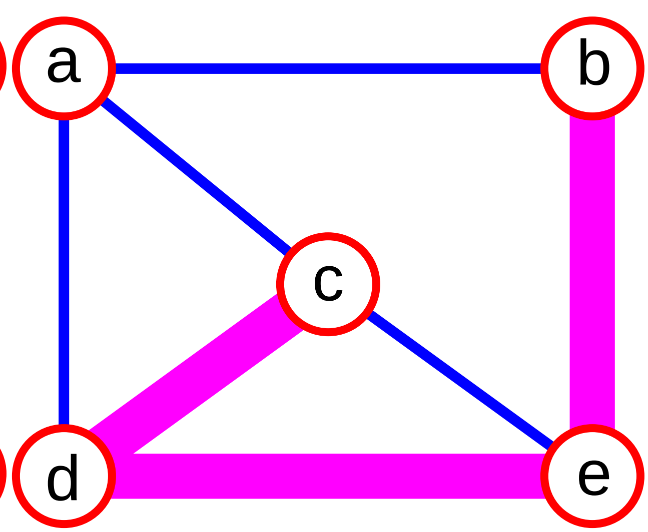
$$\sum_{v \in V} \deg(v) = 2(\#\ \text{edges})$$

- Since adjacent vertices each count the adjoining edge, it will be counted twice

path: sequence of vertices $v_1, v_2, \ldots v_k$ such that consecutive vertices v_i and v_{i+1} are adjacent.



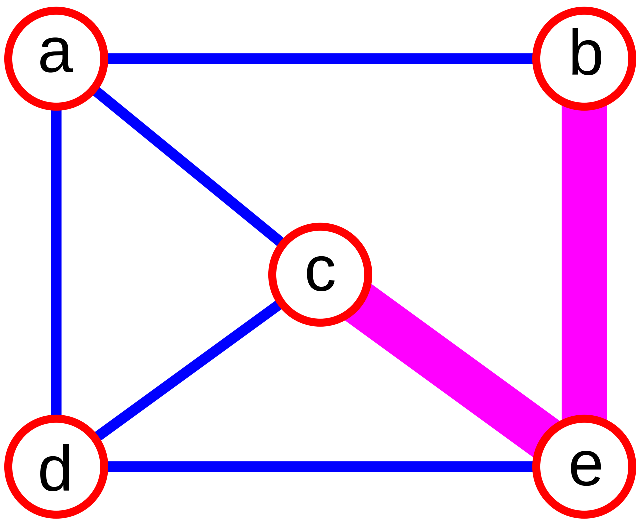
a b e d c



b e d c

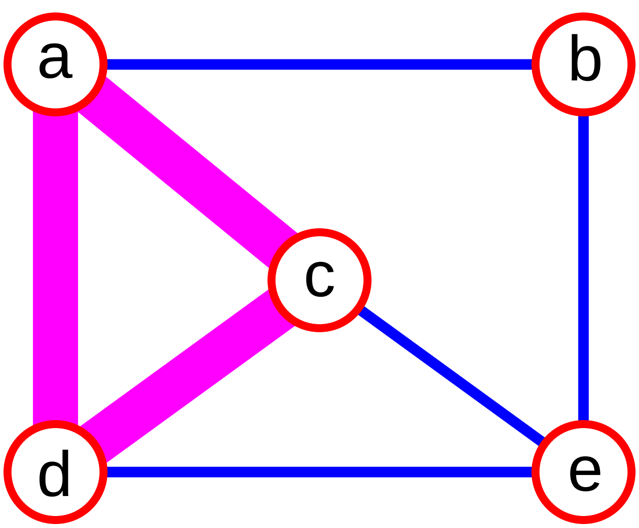
More Graph Terminology

- simple path: no repeated vertices

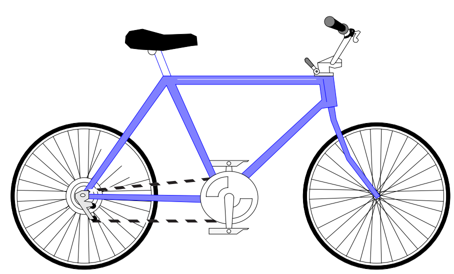


b e c

- cycle: simple path, except that the last vertex is the same as the first vertex

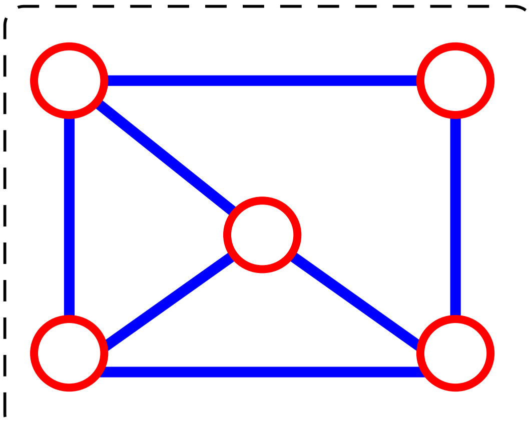


a c d a

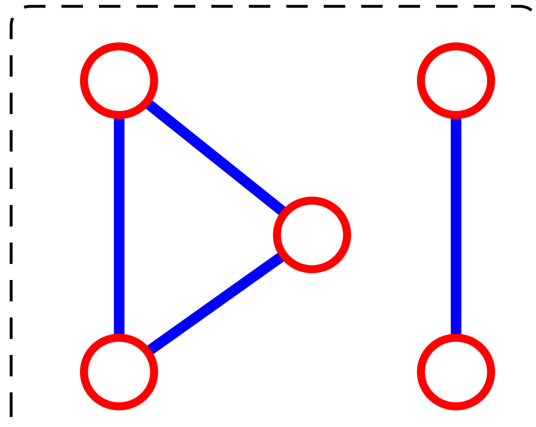


Even More Terminology

- connected graph: any two vertices are connected by some path



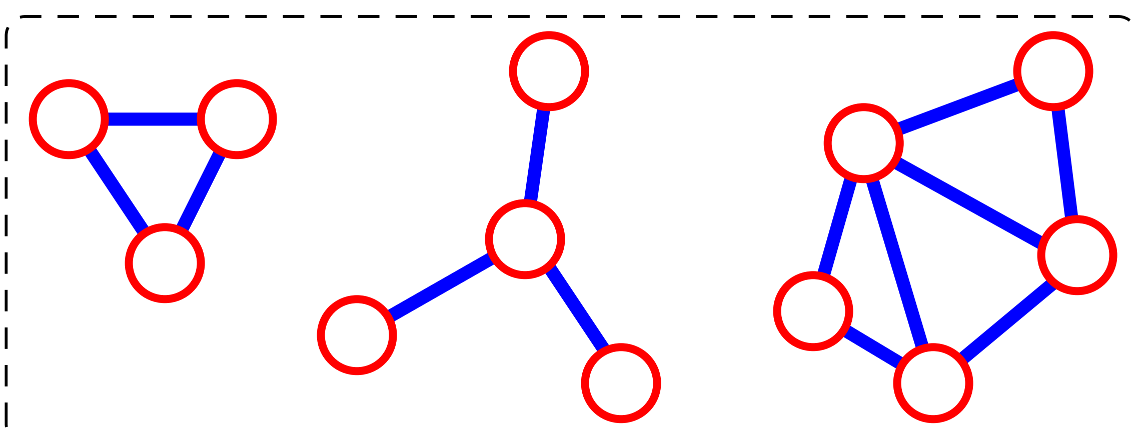
connected



not connected

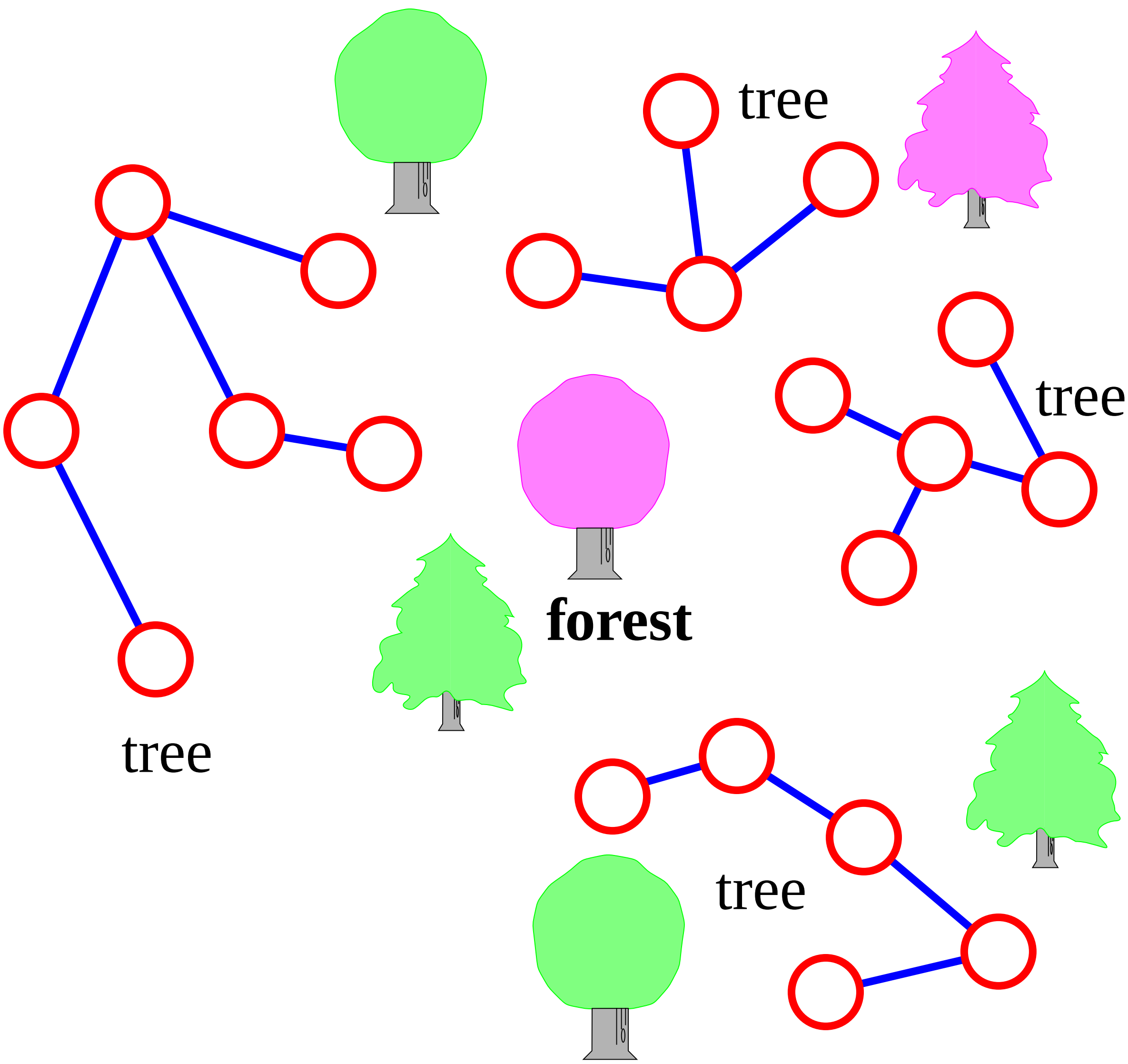
- subgraph: subset of vertices and edges forming a graph

- connected component: maximal connected subgraph. E.g., the graph below has 3 connected components.



¡Caramba! Another Terminology Slide!

- (free) tree - connected graph without cycles
- forest - collection of trees



Connectivity

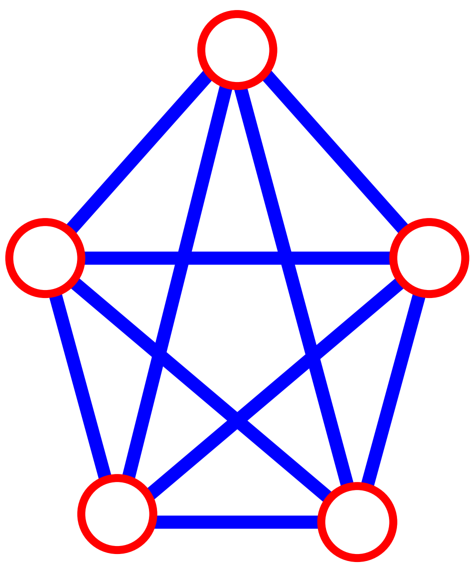
Let **n** = #vertices

m = #edges

- complete graph - all pairs of vertices are adjacent

$$\mathbf{m} = (1/2)\sum_{v \in \mathbf{V}} \deg(\mathbf{v}) = (1/2)\sum_{v \in V}(\mathbf{n} - 1) = \mathbf{n}(\mathbf{n}-1)/2$$

- Each of the **n** vertices is incident to **n** - 1 edges, however, we would have counted each edge twice!!! Therefore, intuitively, **m** = **n**(**n**-1)/2.



$\mathbf{n} = 5$

$\mathbf{m} = (5 * 4)/2 = 10$

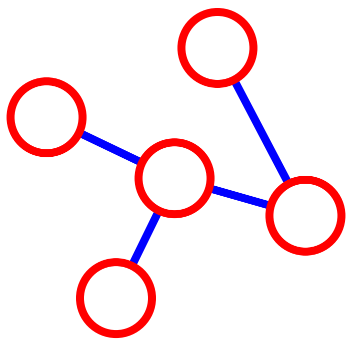
- Therefore, if a graph is ***not*** complete,

$$\mathbf{m} < \mathbf{n}(\mathbf{n}-1)/2$$

More Connectivity

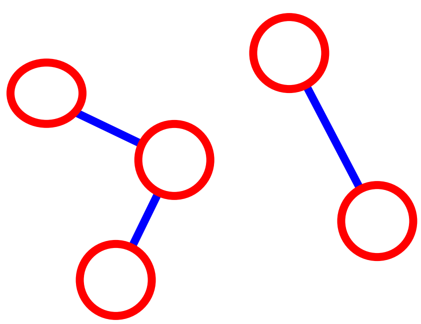
n = #vertices
m = #edges

- For a tree **m** = **n** - 1



n = 5
m = 4

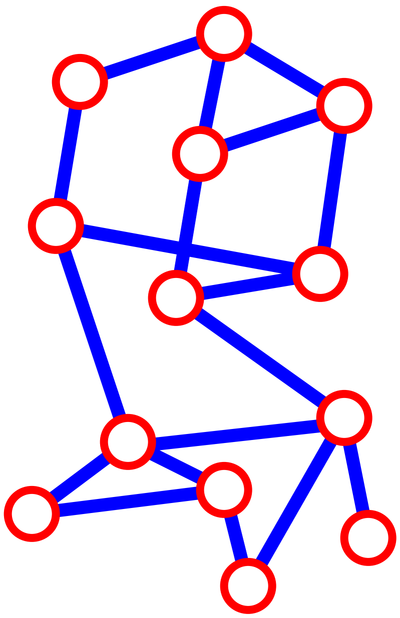
- If **m** < **n** - 1, G is not connected

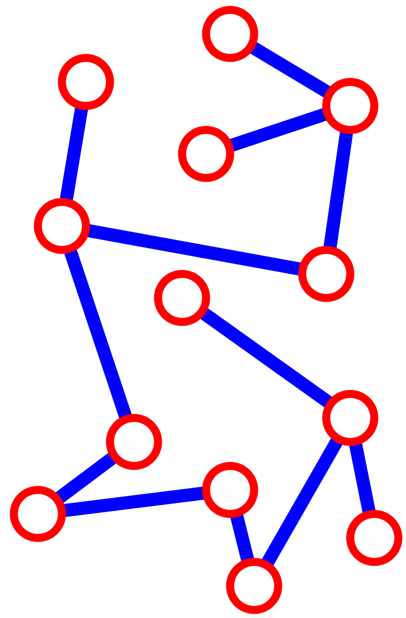


n = 5
m = 3

Spanning Tree

- A **spanning tree** of **G** is a subgraph which
  - is a tree
  - contains all vertices of **G**





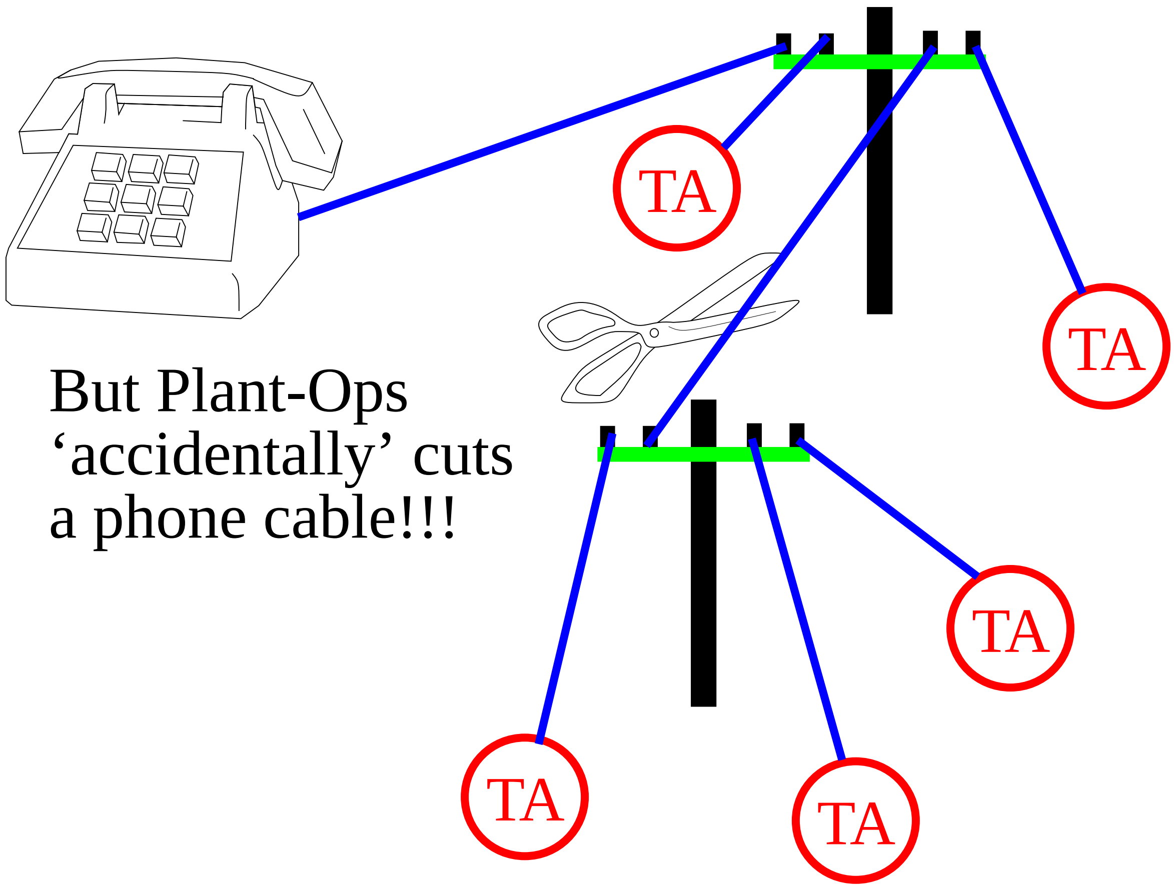
G spanning tree of **G**

- Failure on any edge disconnects system (least fault tolerant)

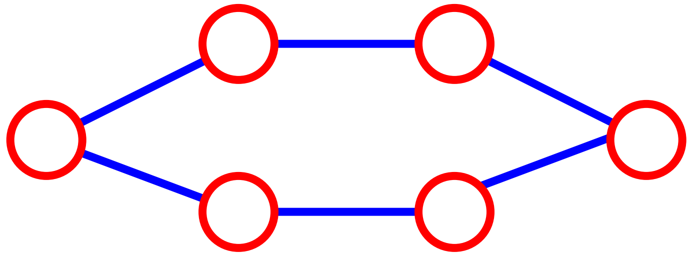
AT&T vs. RT&T

(Roberto Tamassia & Telephone)

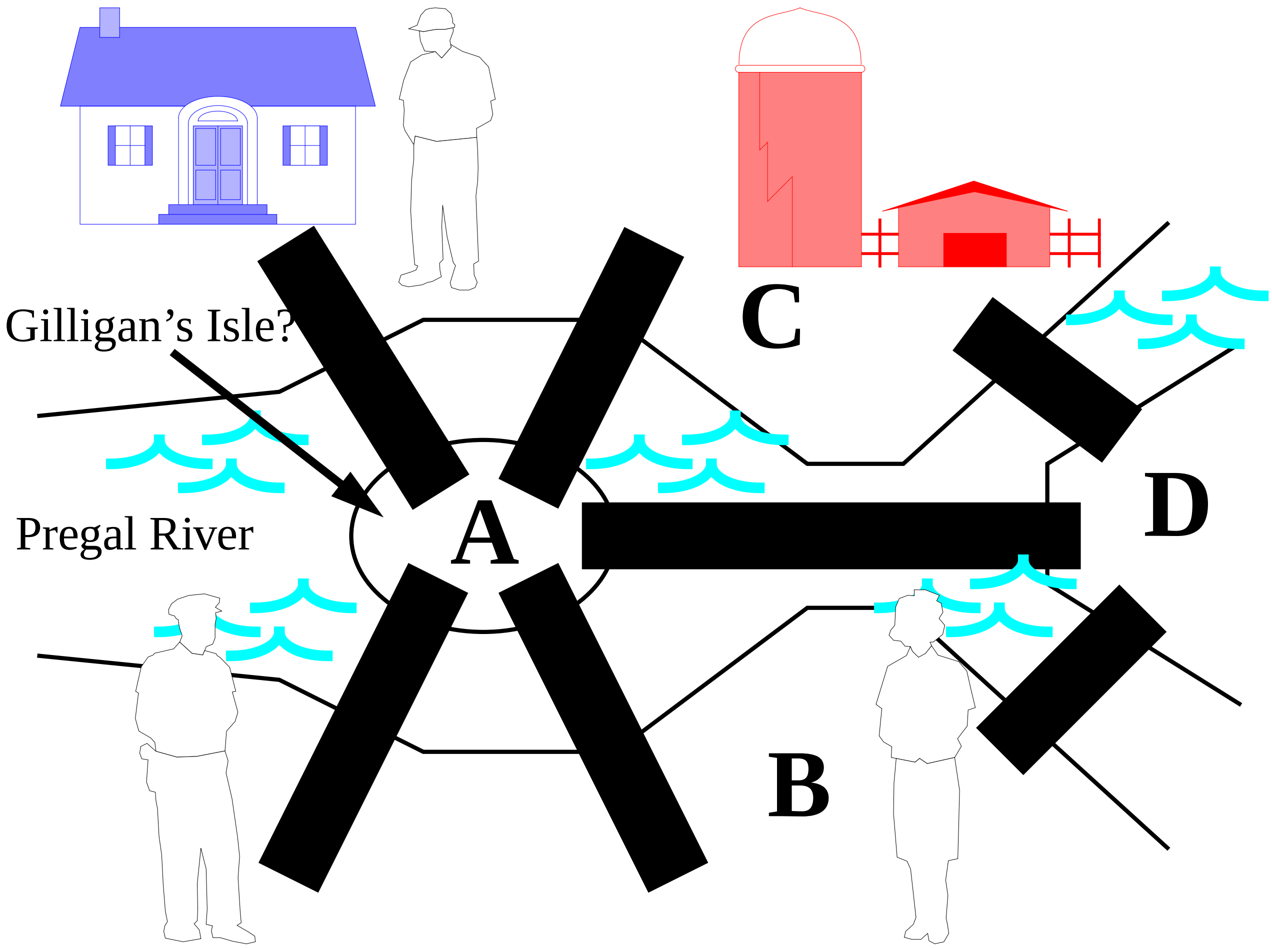
- Roberto wants to call the TA's to suggest an extension for the next program...



- One fault will disconnect part of graph!!
- A cycle would be more fault tolerant and only requires **n** edges



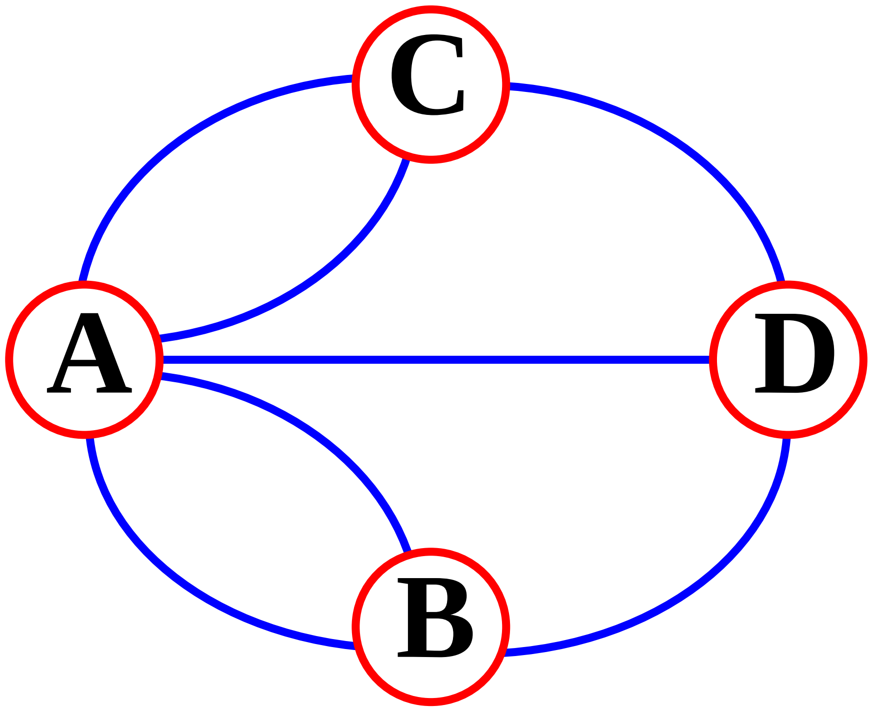
Euler and the Bridges of Koenigsberg



Can one walk across each bridge exactly once and return at the starting point?

- Consider if you were a UPS driver, and you didn't want to retrace your steps.
- In 1736, Euler proved that this is not possible

Graph Model(with parallel edges)



- Eulerian Tour: path that traverses every edge exactly once and returns to the first vertex
- Euler's Theorem: A graph has a Eulerian Tour if and only if all vertices have even degree

The Graph ADT

- The **Graph ADT** is a positional container whose positions are the vertices and the edges of the graph.

  - size() Return the number of vertices plus the number of edges of *G*.
  - isEmpty()
  - elements()
  - positions()
  - swap()
  - replaceElement()

Notation: Graph *G*; Vertices *v*, *w*; Edge *e*; Object *o*

  - numVertices() Return the number of vertices of *G*.
  - numEdges() Return the number of edges of *G*.
  - vertices() Return an enumeration of the vertices of *G*.
  - edges() Return an enumeration of the edges of *G*.

The Graph ADT (contd.)

- directedEdges()
  Return an enumeration of all directed edges in *G*.
- undirectedEdges()
  Return an enumeration of all undirected edges in *G*.
- incidentEdges(*v*)
  Return an enumeration of all edges incident on *v*.
- inIncidentEdges(*v*)
  Return an enumeration of all the incoming edges to *v*.
- outIncidentEdges(*v*)
  Return an enumeration of all the outgoing edges from *v*.
- opposite(*v*, *e*)
  Return an endpoint of *e* distinct from *v*
- degree(*v*)
  Return the degree of *v*.
- inDegree(*v*)
  Return the in-degree of *v*.
- outDegree(*v*)
  Return the out-degree of *v*.

More Methods ...

- adjacentVertices(*v*)

  Return an enumeration of the vertices adjacent to *v*.

- inAdjacentVertices(*v*)

  Return an enumeration of the vertices adjacent to *v* along incoming edges.

- outAdjacentVertices(*v*)

  Return an enumeration of the vertices adjacent to *v* along outgoing edges.

- areAdjacent(*v*,w)

  Return whether vertices *v* and w are adjacent.

- endVertices(*e*)

  Return an array of size 2 storing the end vertices of *e*.

- origin(*e*)

  Return the end vertex from which *e* leaves.

- destination(*e*)

  Return the end vertex at which *e* arrives.

- isDirected(*e*)

  Return true iff *e* is directed.

Update Methods

- makeUndirected(*e*)
  Set *e* to be an undirected edge.
- reverseDirection(*e*)
  Switch the origin and destination vertices of *e*.
- setDirectionFrom(*e*, *v*)
  Sets the direction of *e* away from *v*, one of its end vertices.
- setDirectionTo(*e*, *v*)
  Sets the direction of *e* toward *v*, one of its end vertices.
- insertEdge(*v*, *w*, *o*)
  Insert and return an undirected edge between *v* and *w*, storing *o* at this position.
- insertDirectedEdge(*v*, *w*, *o*)
  Insert and return a directed edge between *v* and *w*, storing *o* at this position.
- insertVertex(*o*)
  Insert and return a new (isolated) vertex storing *o* at this position.
- removeEdge(*e*)
  Remove edge *e*.