

Interactive  
Zero-Knowledge Proofs  
and other Two-Party  
Cryptographic Protocols

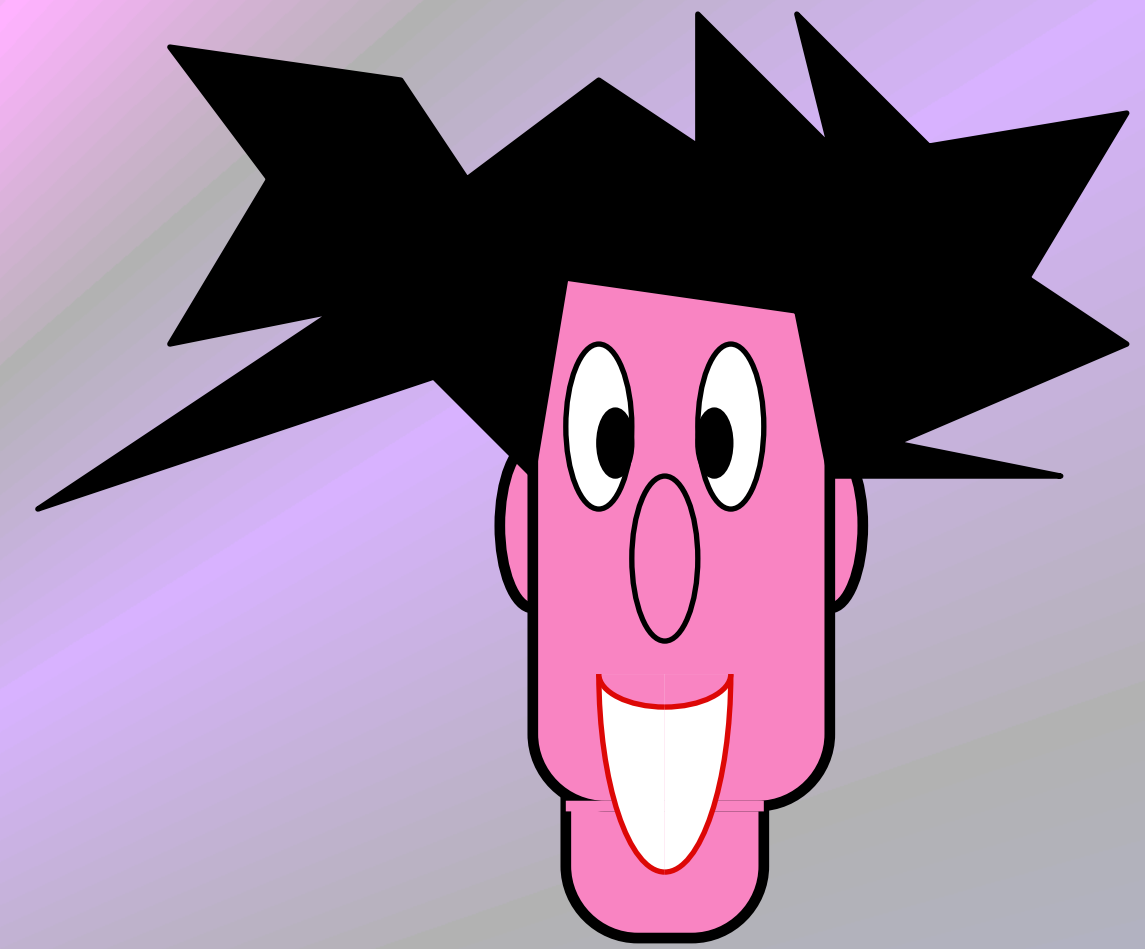
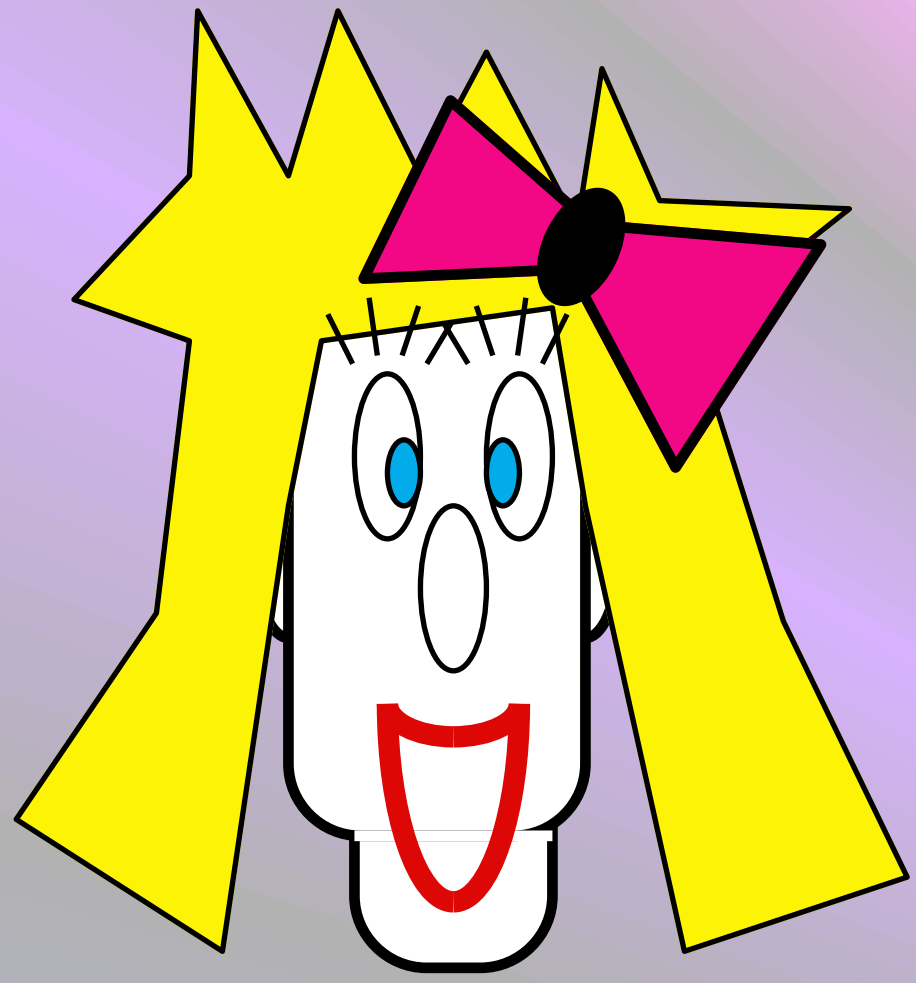
**Claude Crépeau**

School of Computer Science  
McGill University



# Proofs

# Proofs



$x \in L$

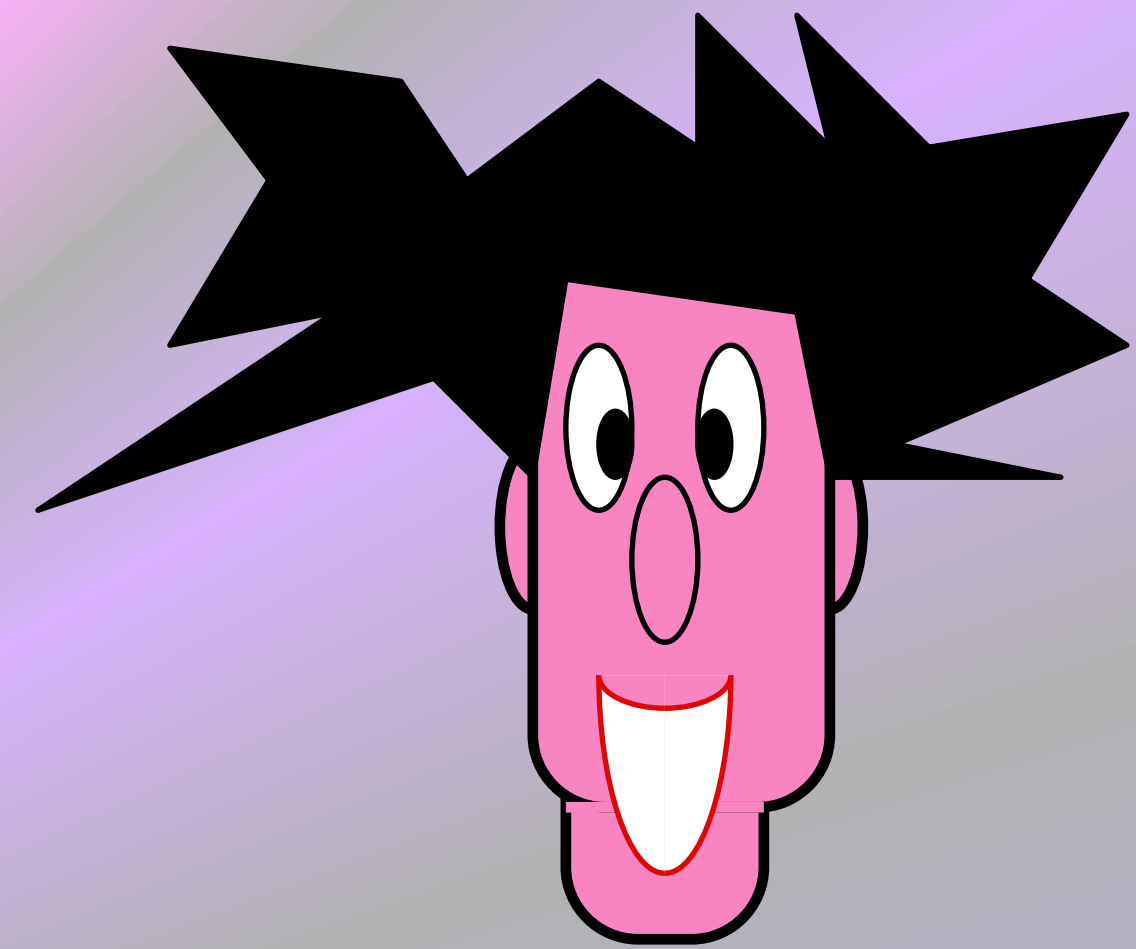
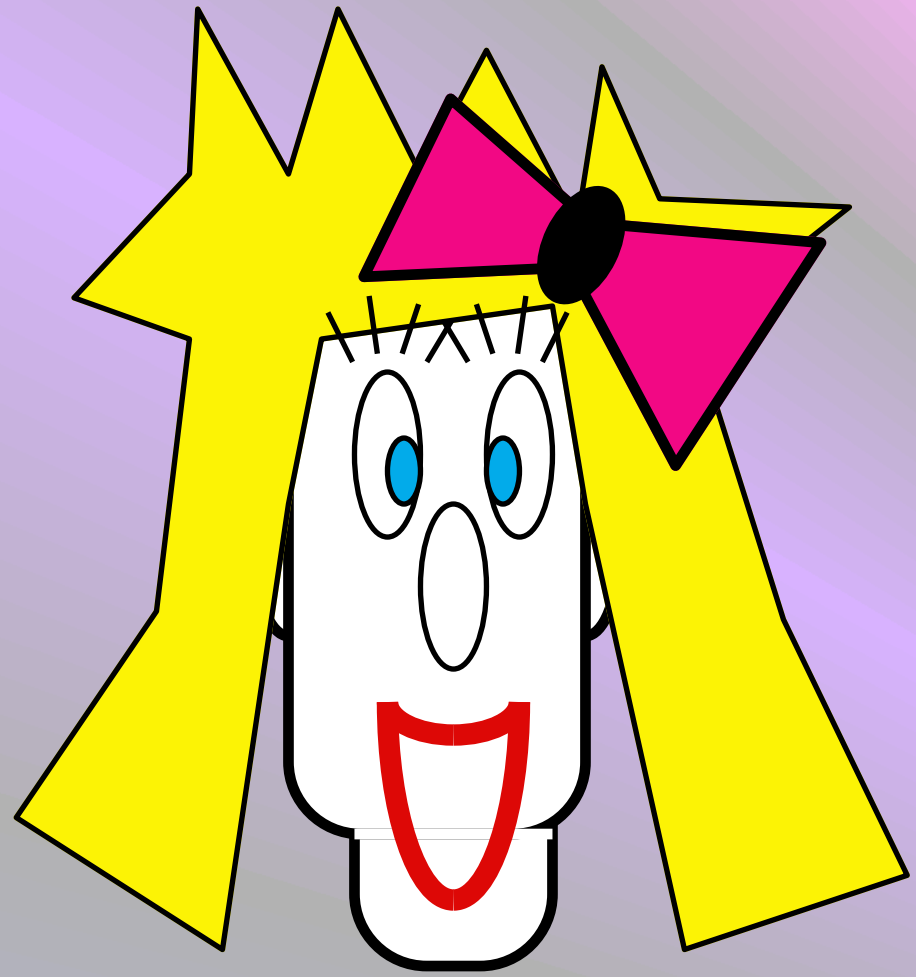
YES !

$w$



$$\forall x \in L \exists w \Pr( [\text{character}] (x, w) = \text{YES} ) = 1$$

# Proofs



$$(G_0, G_1) \in \text{ISO}$$
$$(G_0 = \pi(G_1))$$

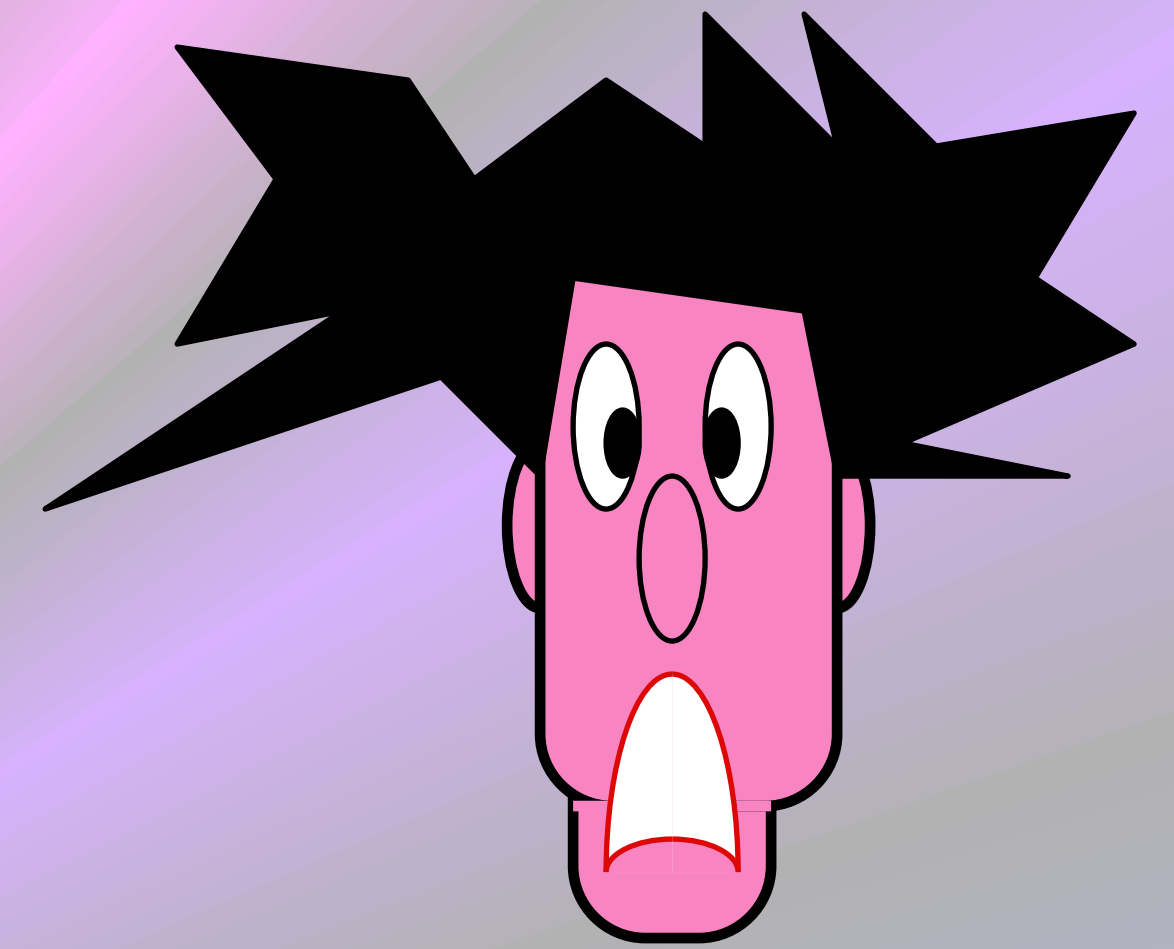
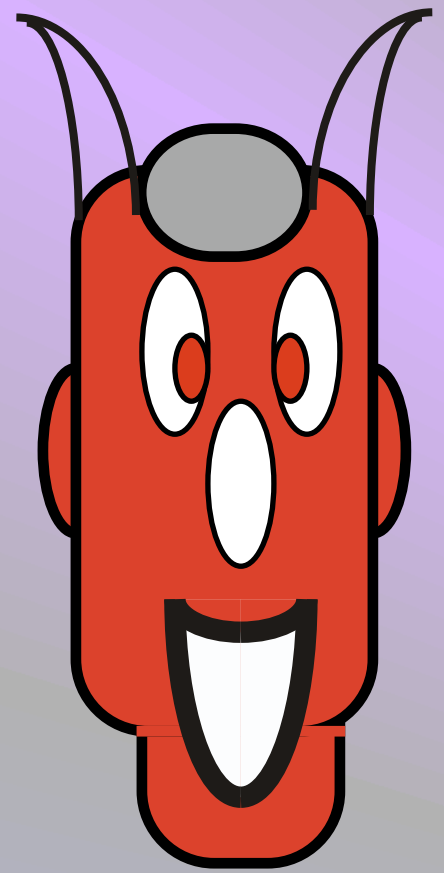


$$G_0 = \pi(G_1)$$

YES !

$$\forall x \in L \exists \pi \Pr( [\text{spiky black hair}] (x, \pi) = \text{YES} ) = 1$$

# Proofs



$x \notin L$

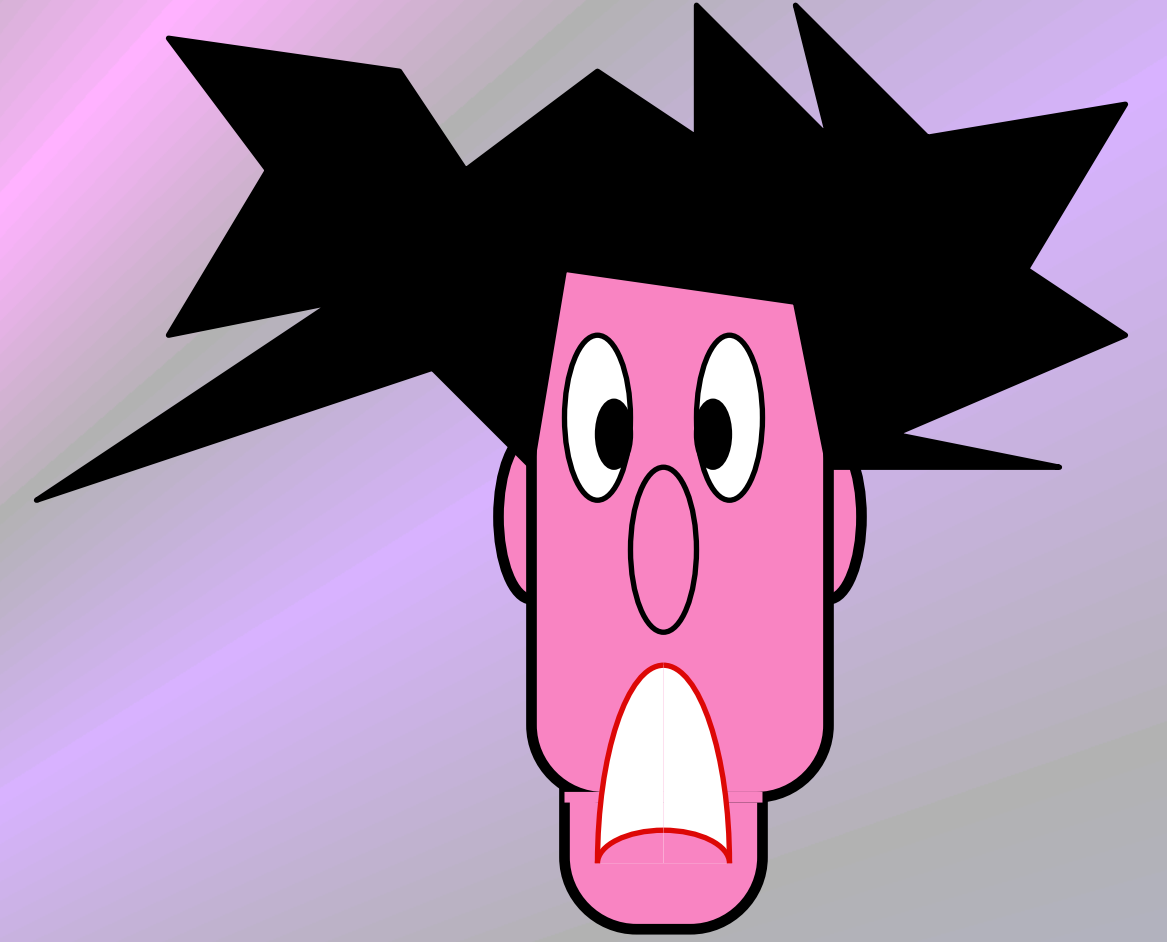
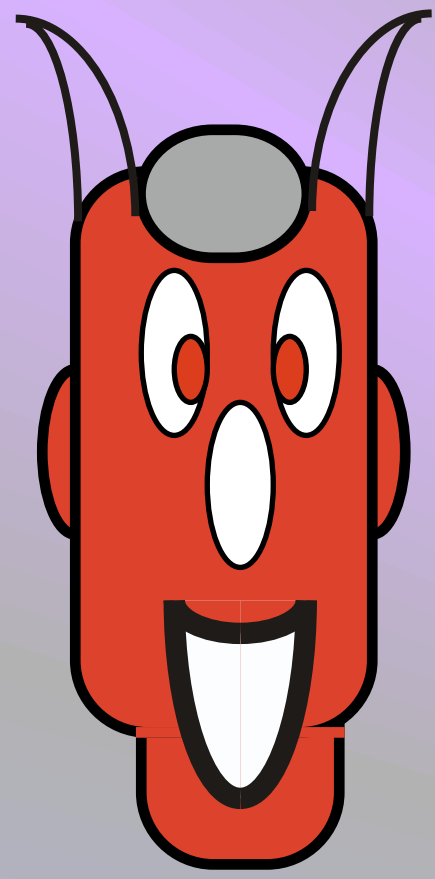
$w$



NO !

$$\forall x \notin L \forall w \Pr( \text{[surprised face]}(x, w) = \text{YES} ) = 0$$

# Proofs




$(G_0, G_1) \notin \text{ISO}$



$G_0 \neq \pi(G_1)$

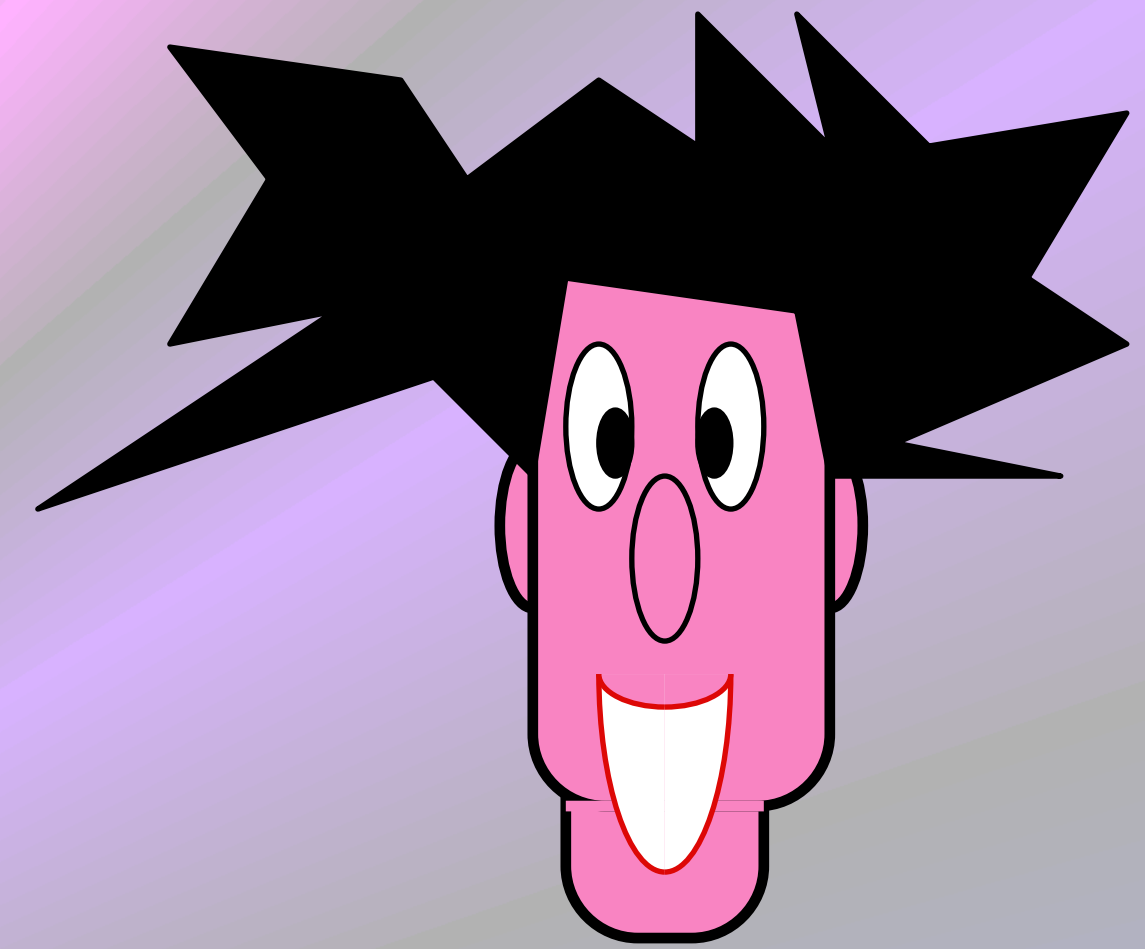
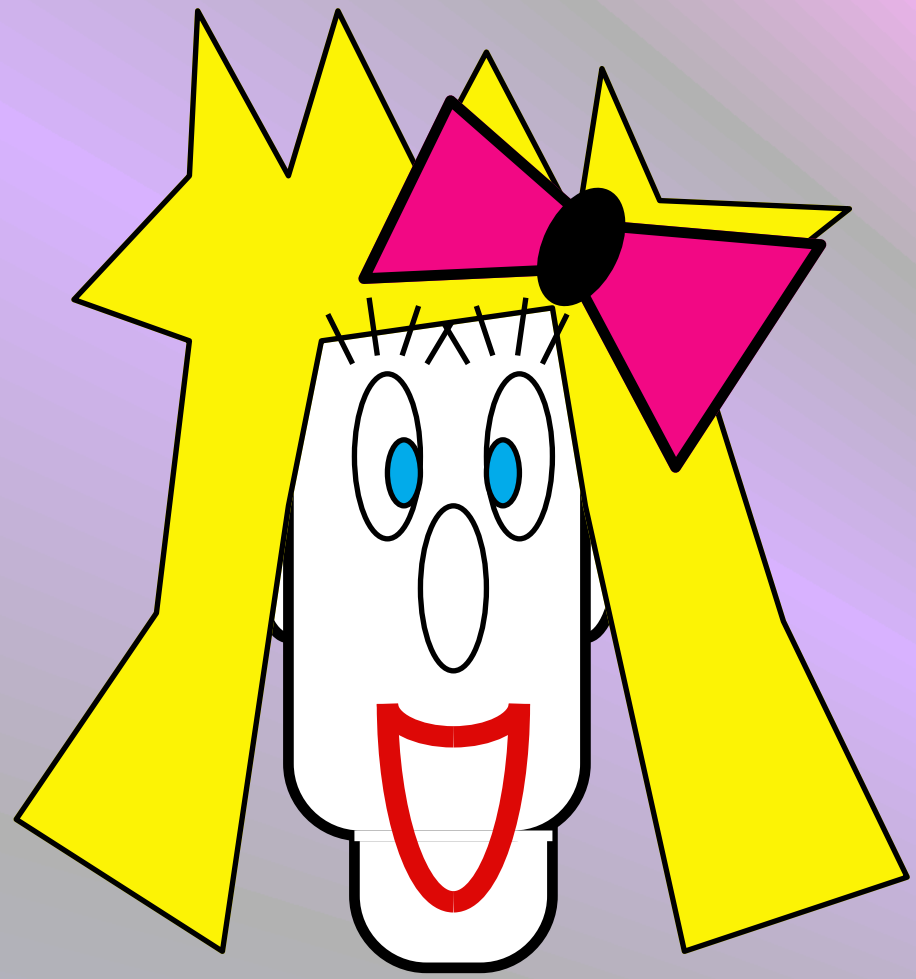
**NO !**

$$\forall x \notin L \quad \forall \pi \quad \Pr( [\text{shocked face}] (x, \pi) = \text{YES} ) = 0$$

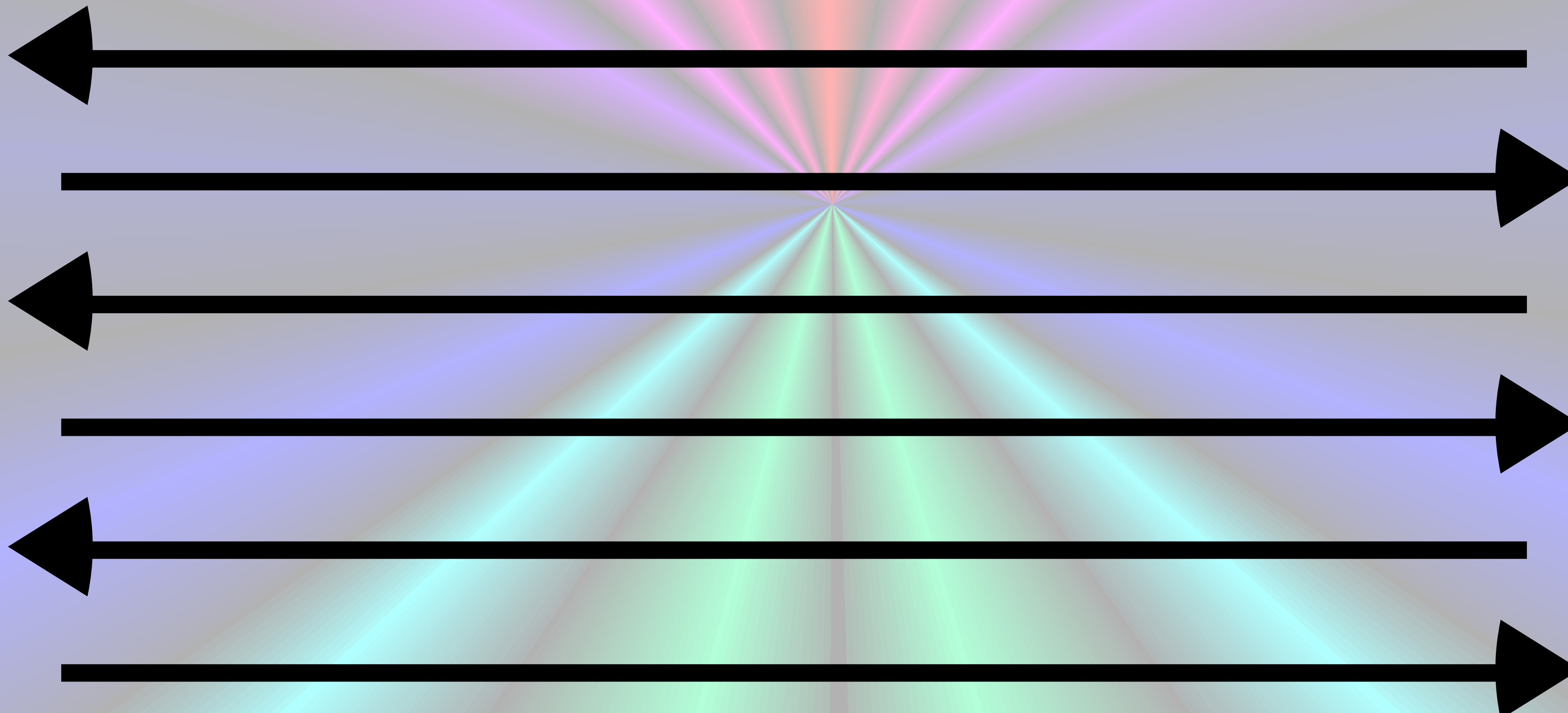


# Interactive Proofs

# Interactive Proofs and Zero-Knowledge



$x \in L$

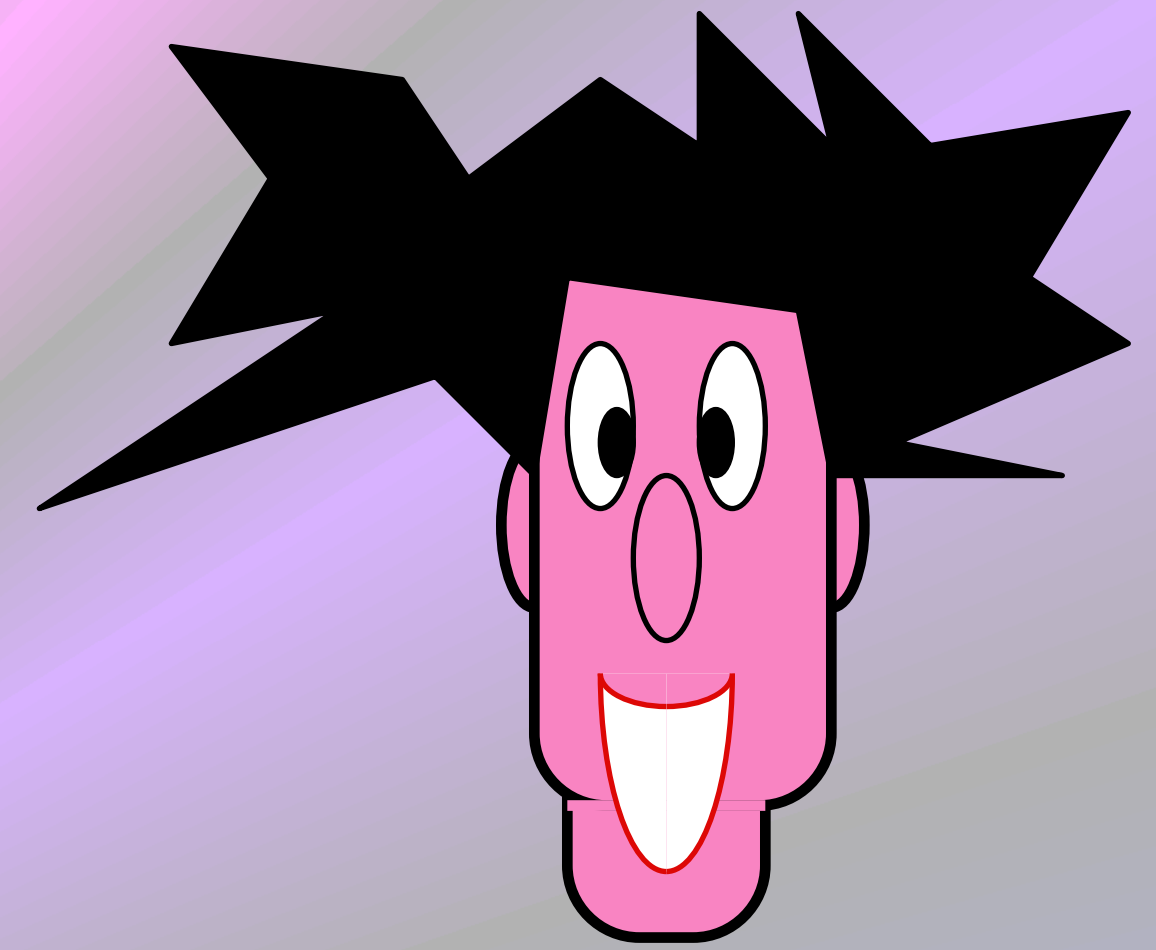
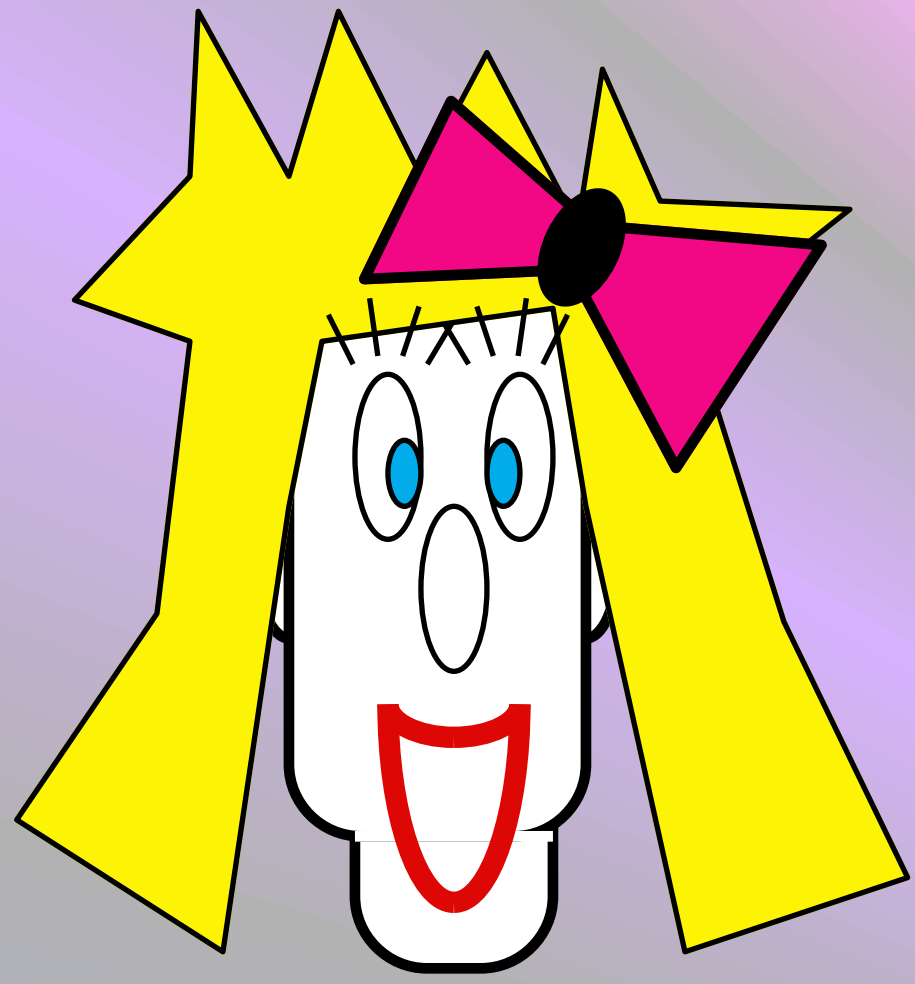


YES !

$$\forall x \in L \Pr( [\text{Character 1}, \text{Character 2}](x) = \text{YES} ) \approx 1$$



# Interactive Proofs and Zero-Knowledge

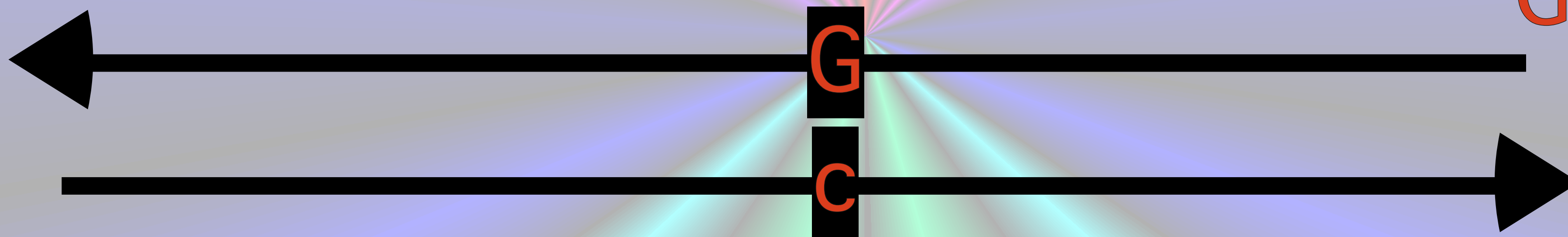


$(G_0, G_1) \notin \text{ISO}$

$G \neq G_0$  or  $G \neq G_1$

$G \approx G_c$

random  $b, \pi$   
 $G = \pi(G_b)$

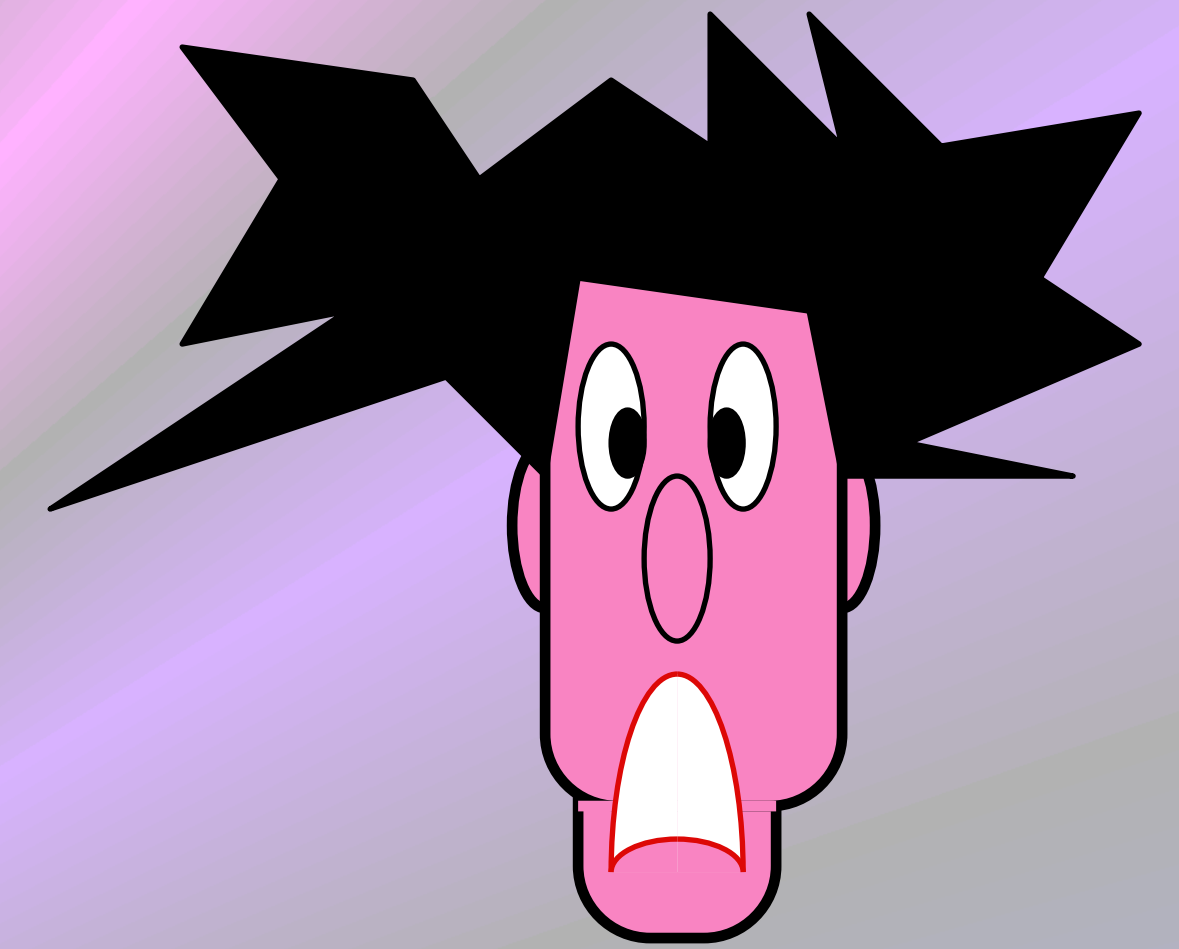
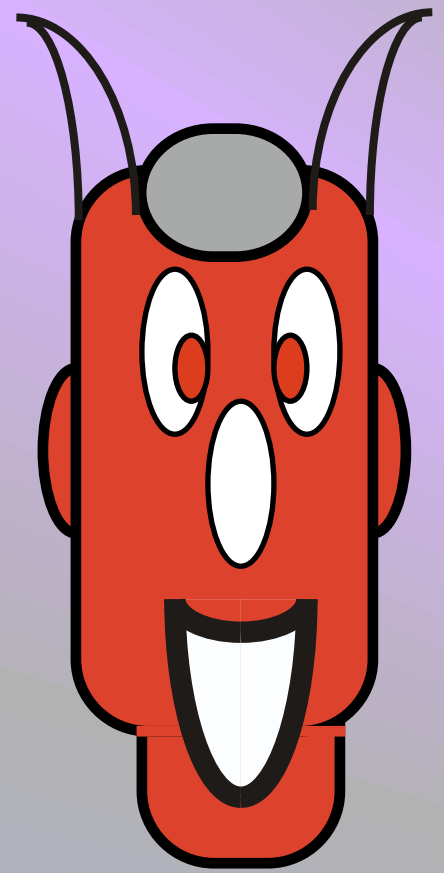


$b = c?$

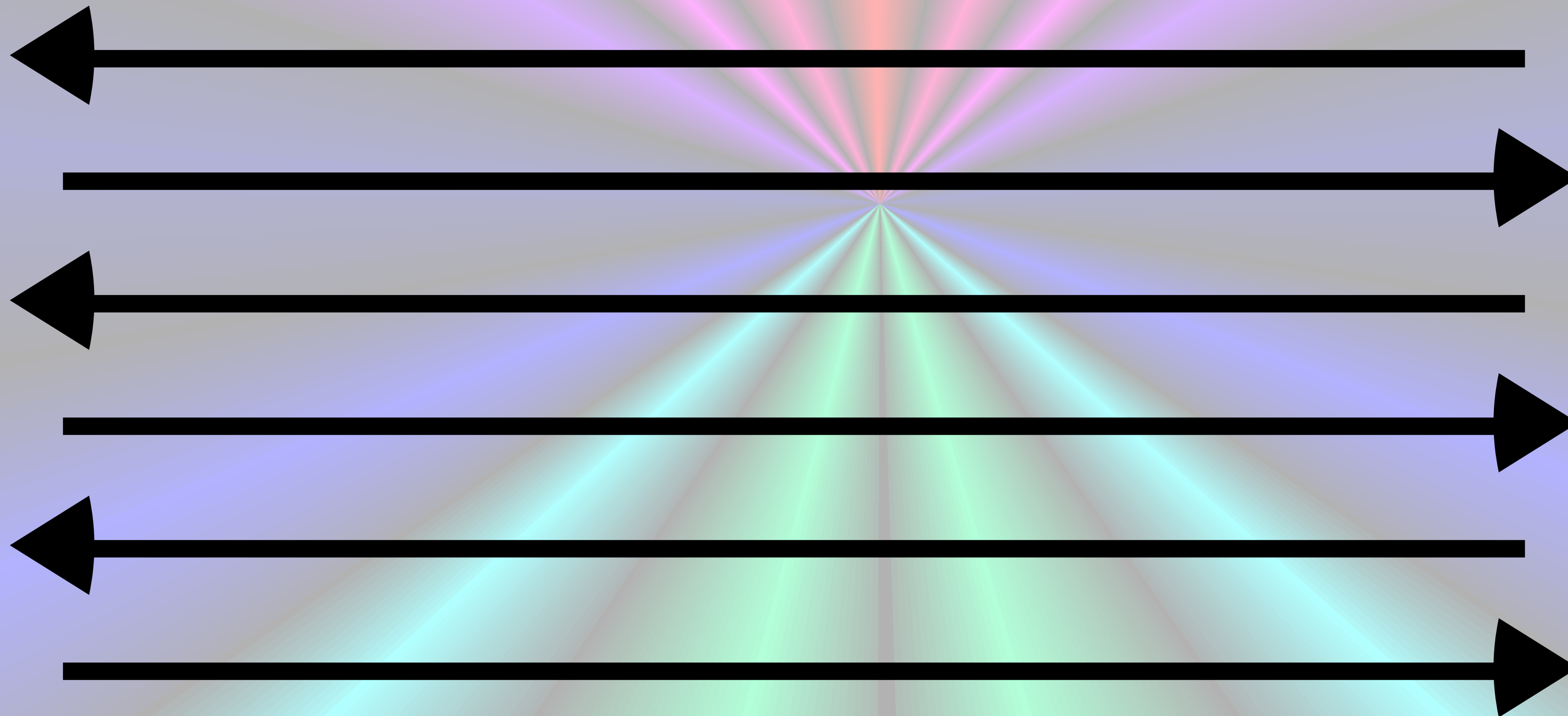
YES !

$$\forall x \in L \Pr( [\text{Character 1}, \text{Character 2}](x) = \text{YES} ) = 1$$

# Interactive Proofs and Zero-Knowledge



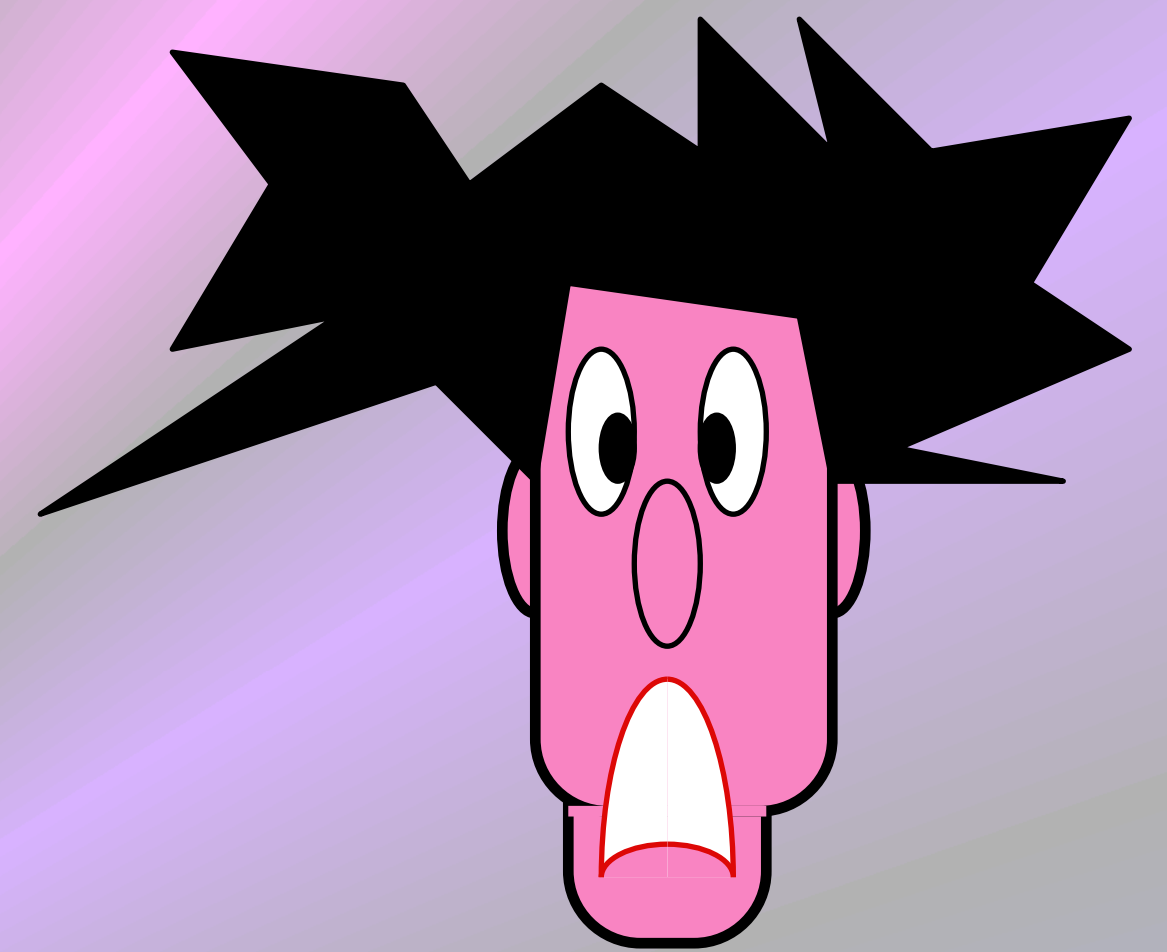
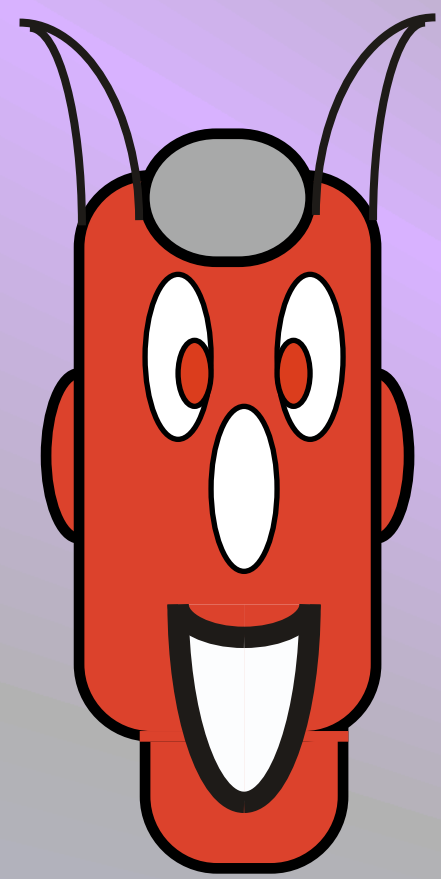
$x \notin L$



NO !

$$\forall x \notin L \forall \text{Red} \Pr( [\text{Red}, \text{Pink}](x) = \text{YES} ) \approx 0$$

# Interactive Proofs and Zero-Knowledge

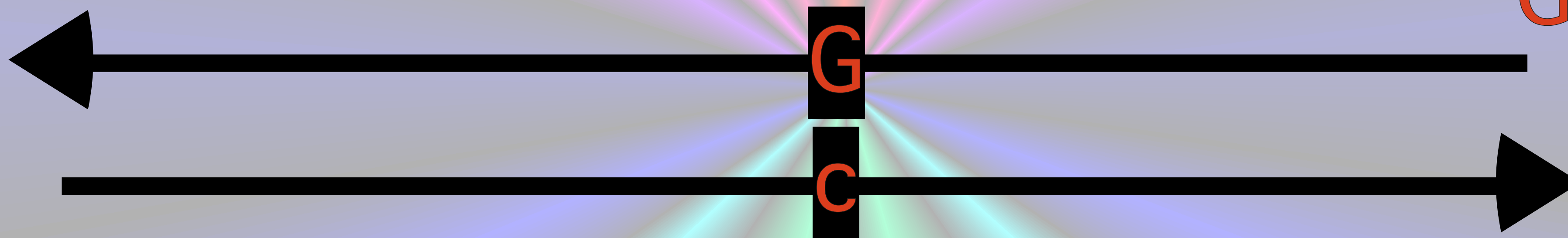


$$(G_0, G_1) \in \text{ISO}$$

$$G \approx G_0 \text{ and } G \approx G_1$$

$$\text{random } b, \pi$$
$$G = \pi(G_b)$$

$$G \approx G_c$$

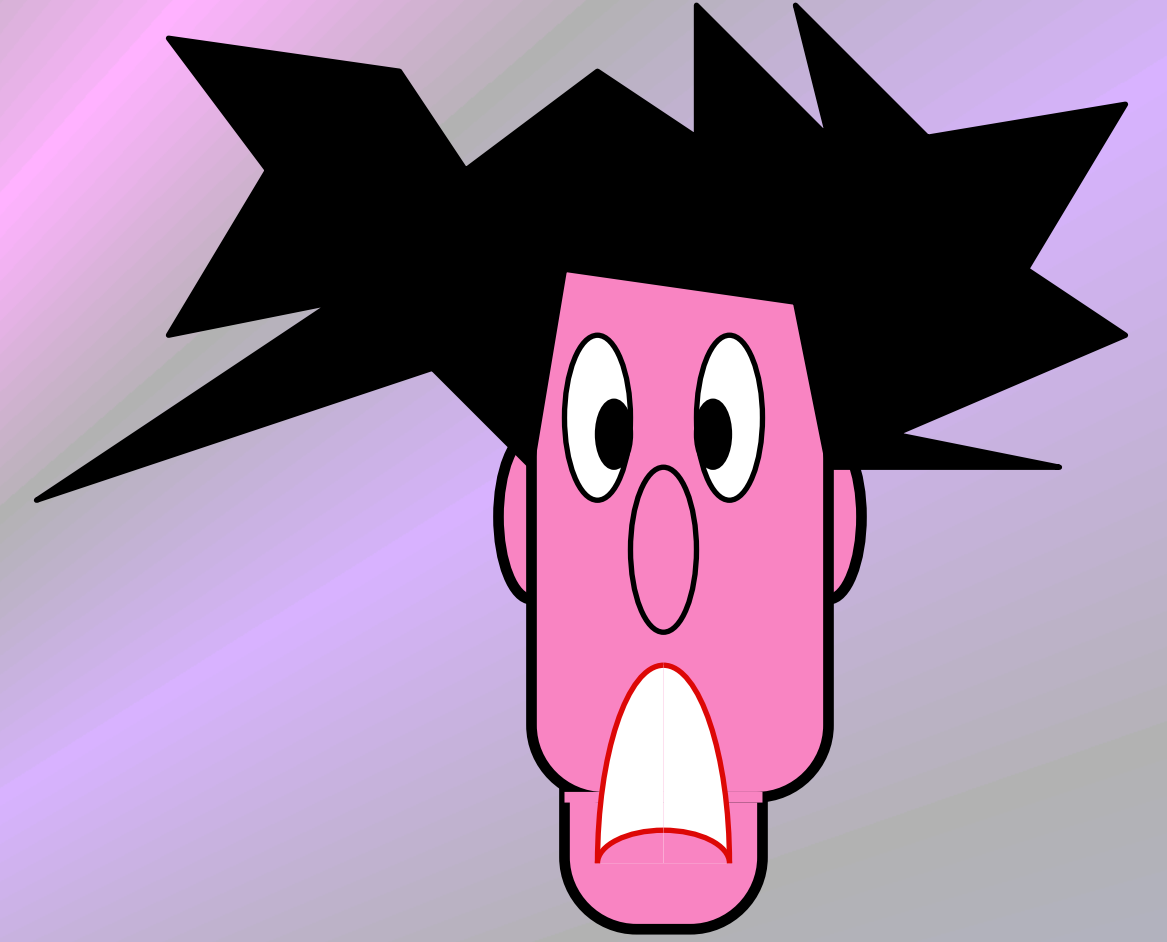
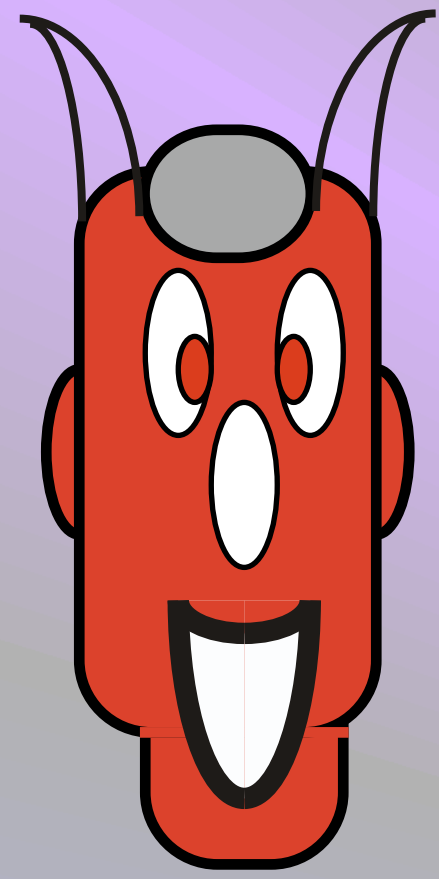


$$b = c?$$

NO !

$$\forall x \notin L \quad \forall \text{ [red alien, pink alien]} \quad \Pr( [\text{red alien}, \text{pink alien}](x) = \text{YES} ) \leq 1/2$$

# Interactive Proofs and Zero-Knowledge

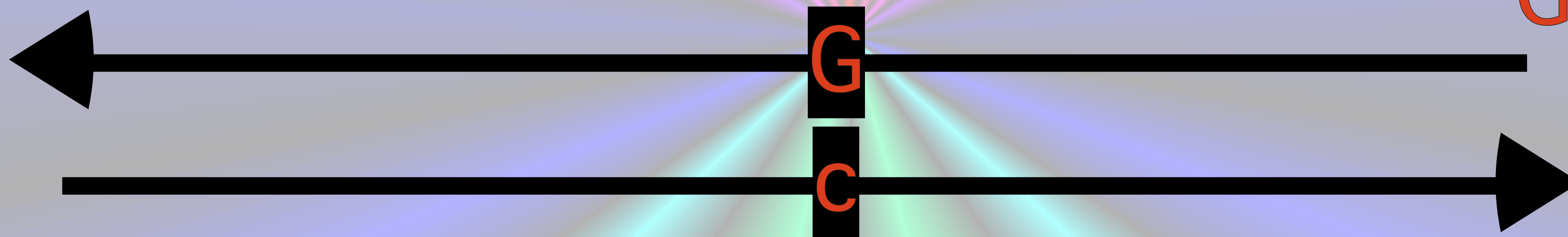


$$(G_0, G_1) \in \text{ISO}$$

random  $b, \pi$   
 $G = \pi(G_b)$

$G \approx G_0$  and  $G \approx G_1$

$G \approx G_c$



$b = c?$

**REPEAT  $k$  TIMES**  
**and say "YES" only if all "YES"**

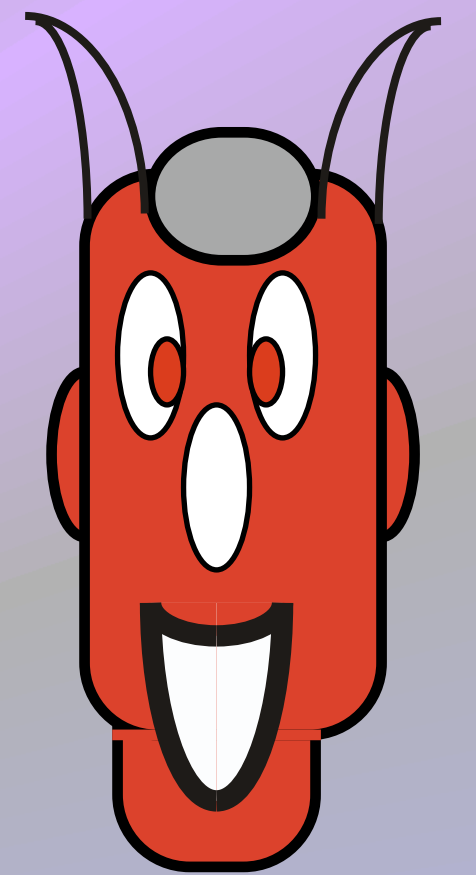
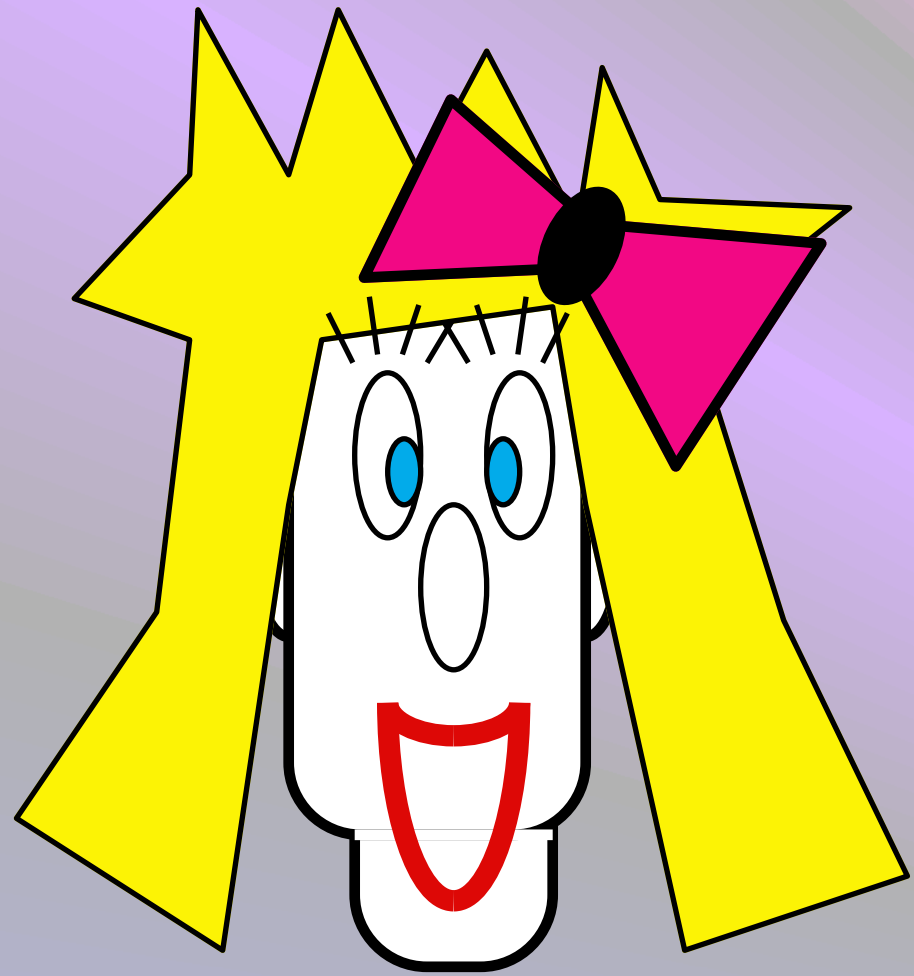
**NO !**

$$\forall x \notin L \quad \forall \text{ [red alien, pink alien] } \Pr( \text{ [red alien, pink alien] } (x) = \text{YES} ) \leq 1/2^k$$

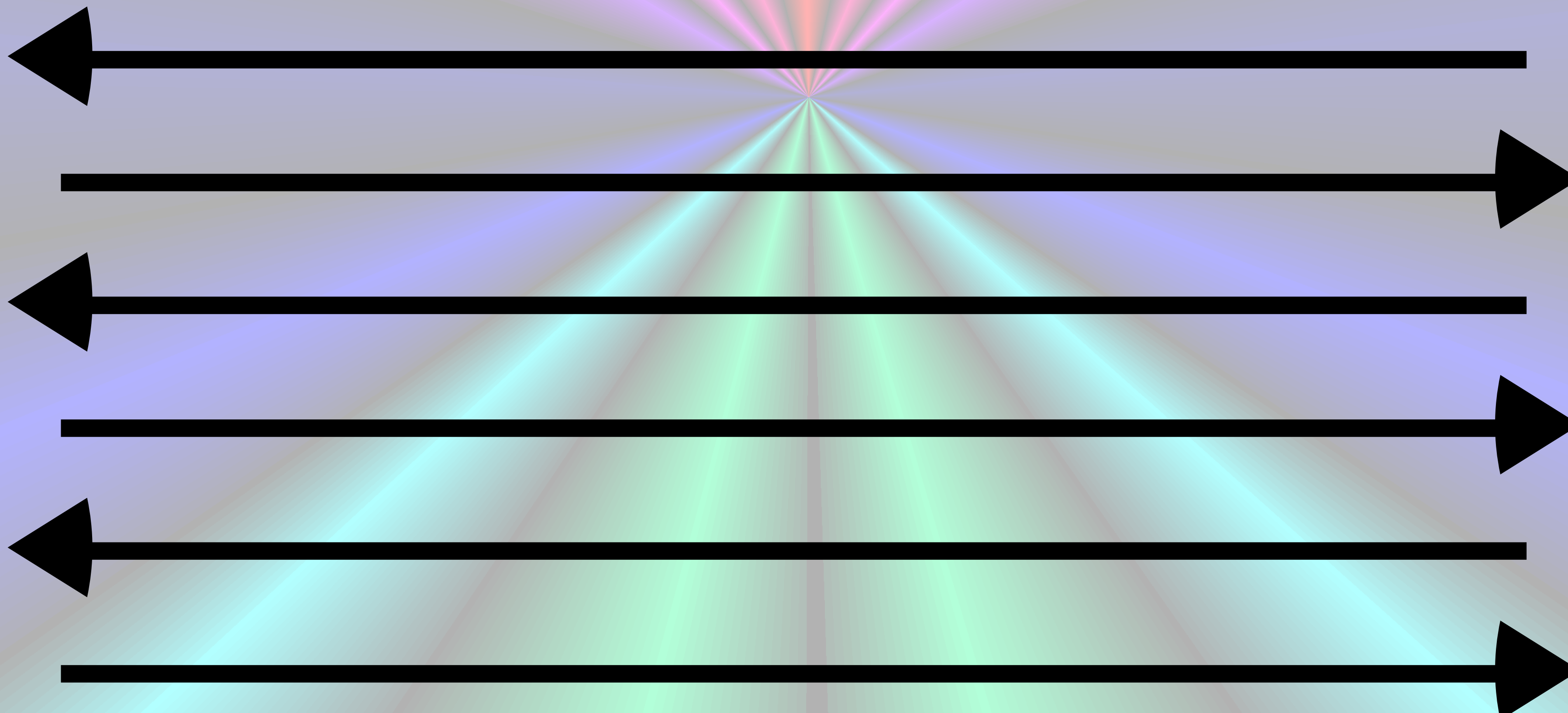


# Zero-Knowledge

# Interactive Proofs and Zero-Knowledge

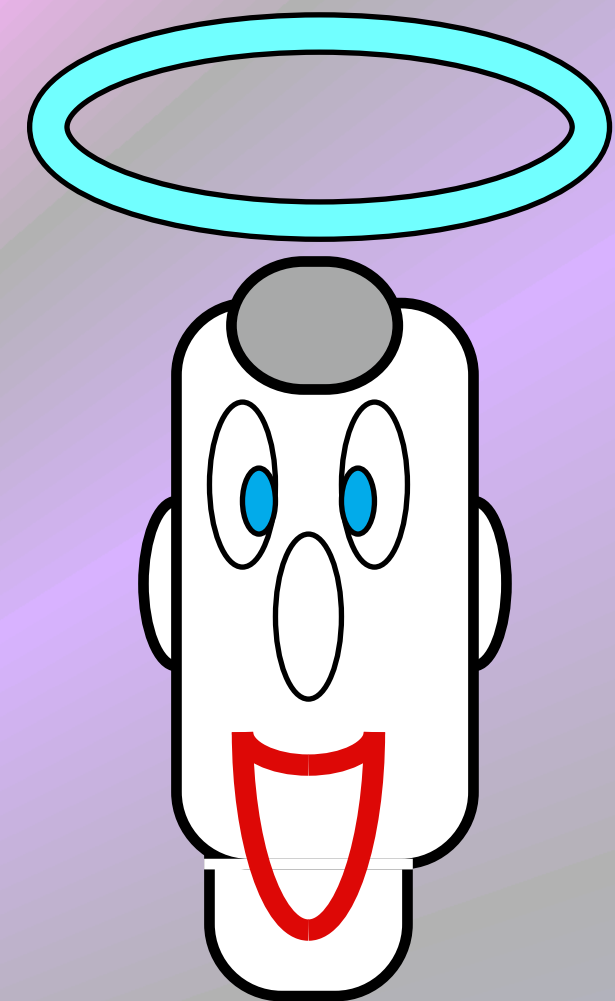
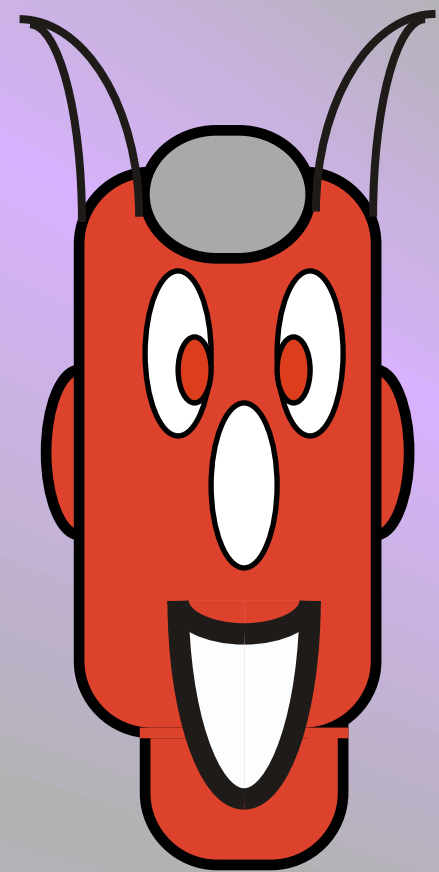


$x \in L$

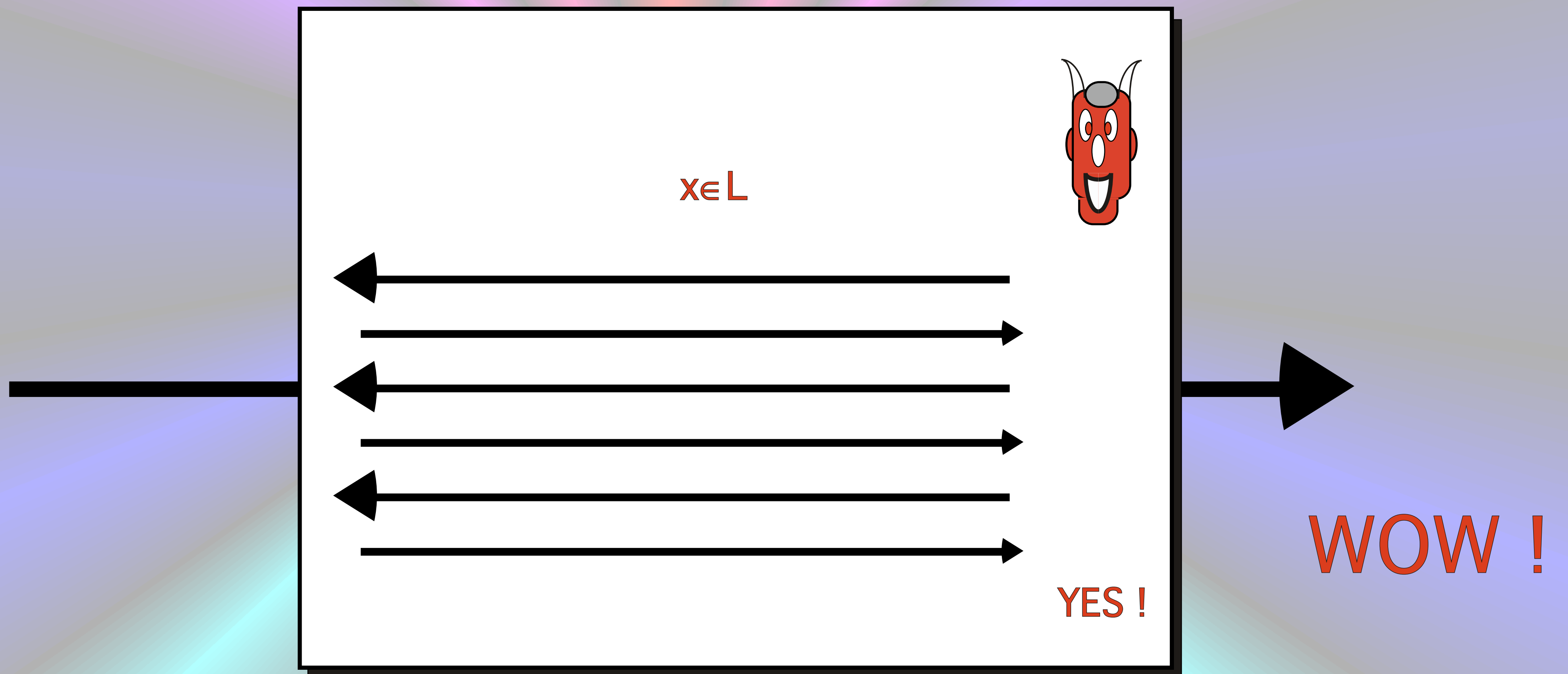


YES !

# Interactive Proofs and Zero-Knowledge



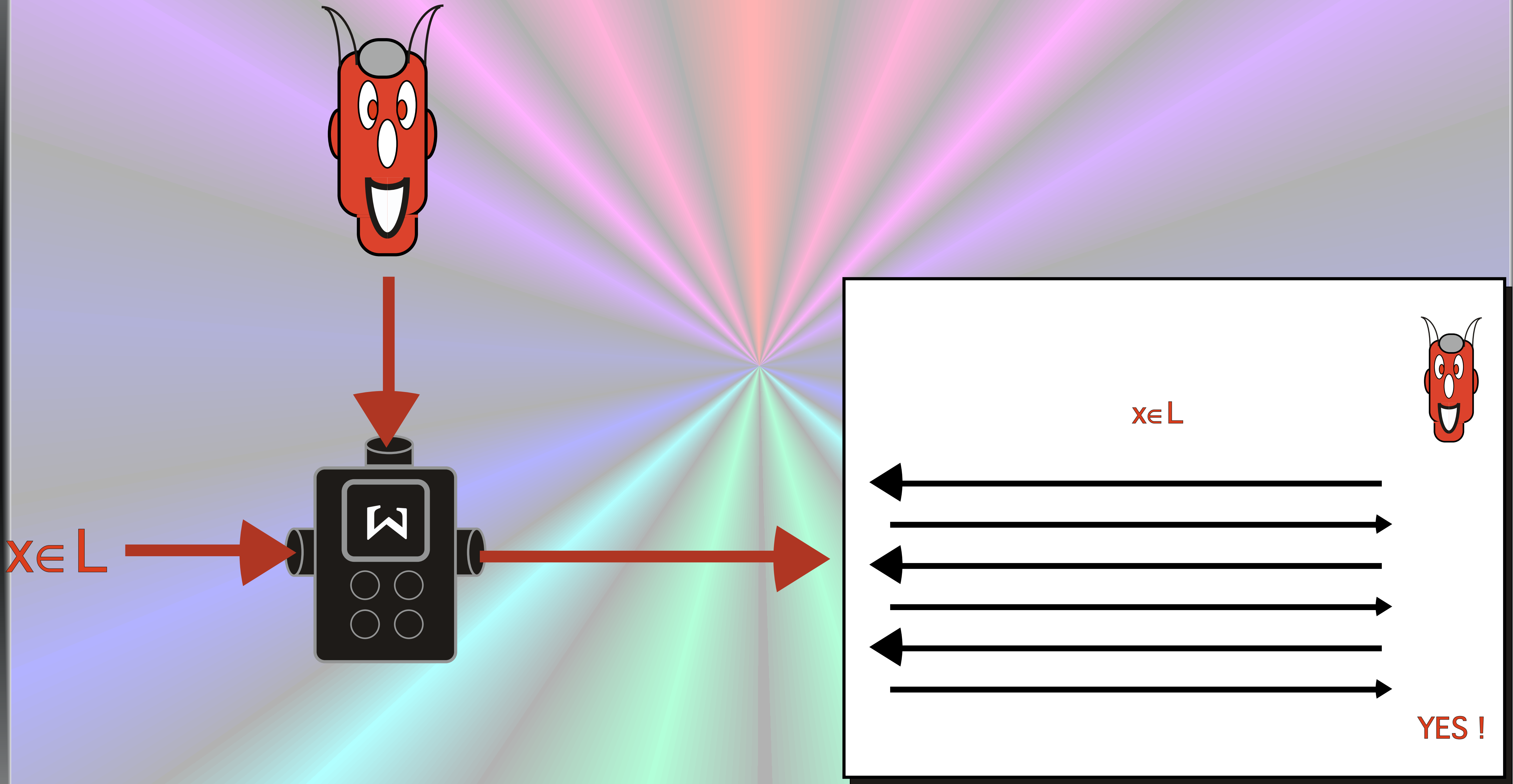
$x \in L$



WOW !

YES !

# Zero-Knowledge and Simulator

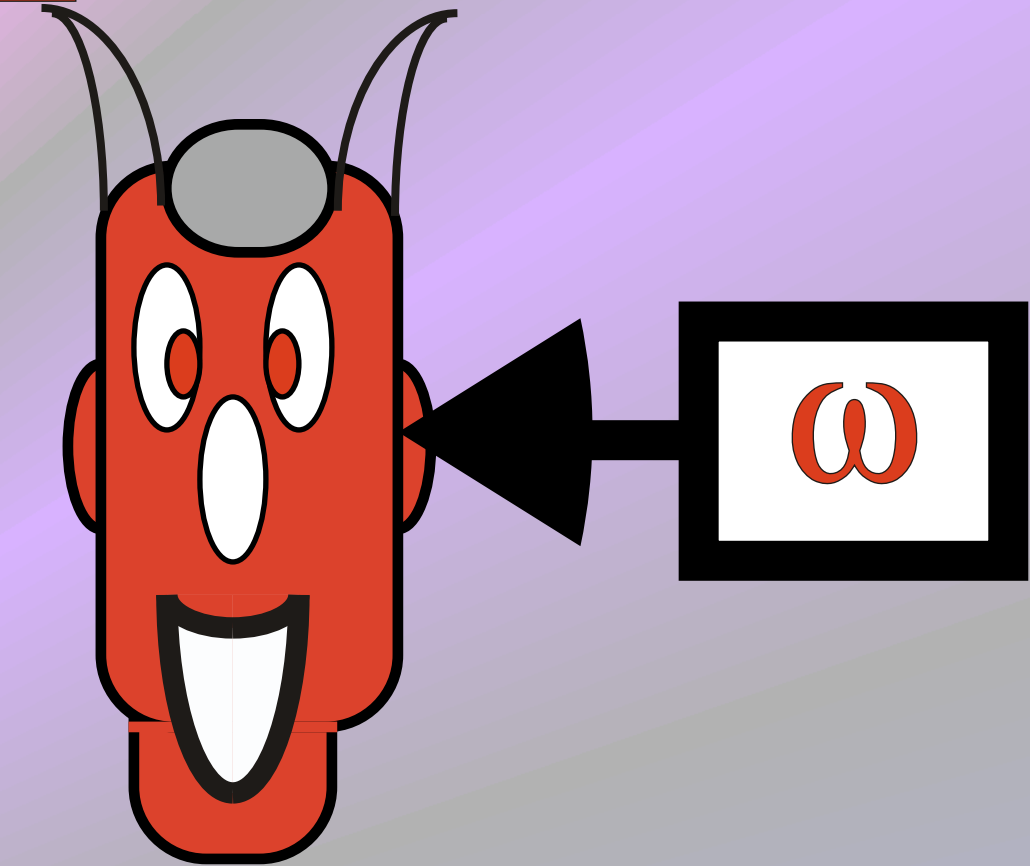
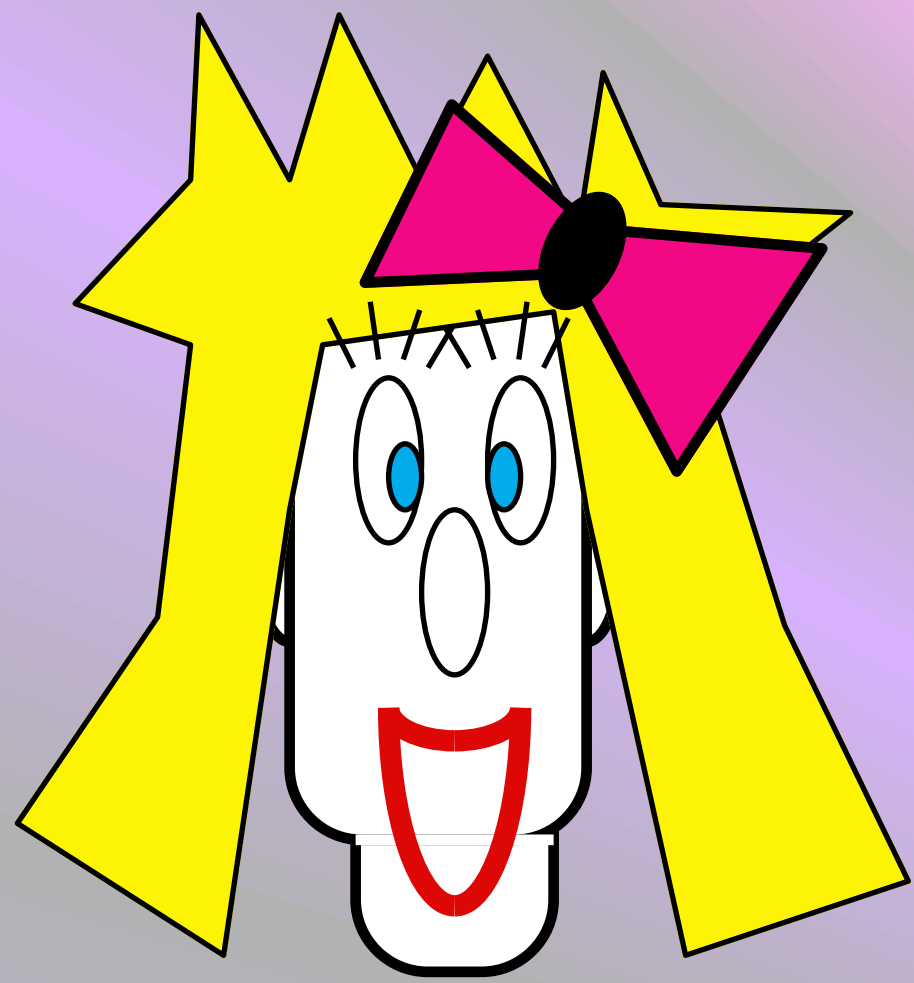


$$\forall \text{ (robot head) } \exists \text{ (black device) } \forall x \in L \text{ view}[\text{ (yellow robot head), (robot head) }](x) = \text{ (black device) } (x)$$

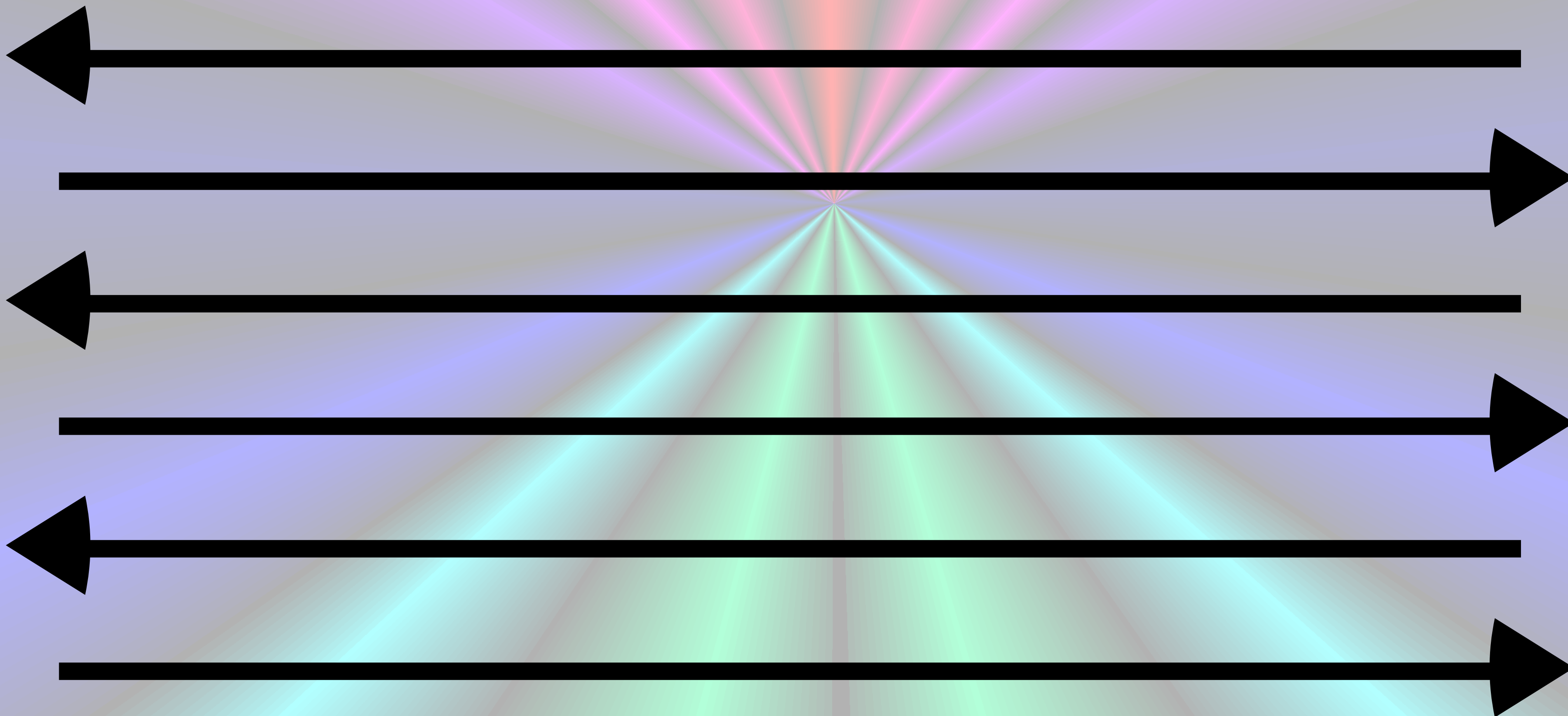


# Auxiliary Input Zero-Knowledge

$\omega$  = existing knowledge about  $x \in L$

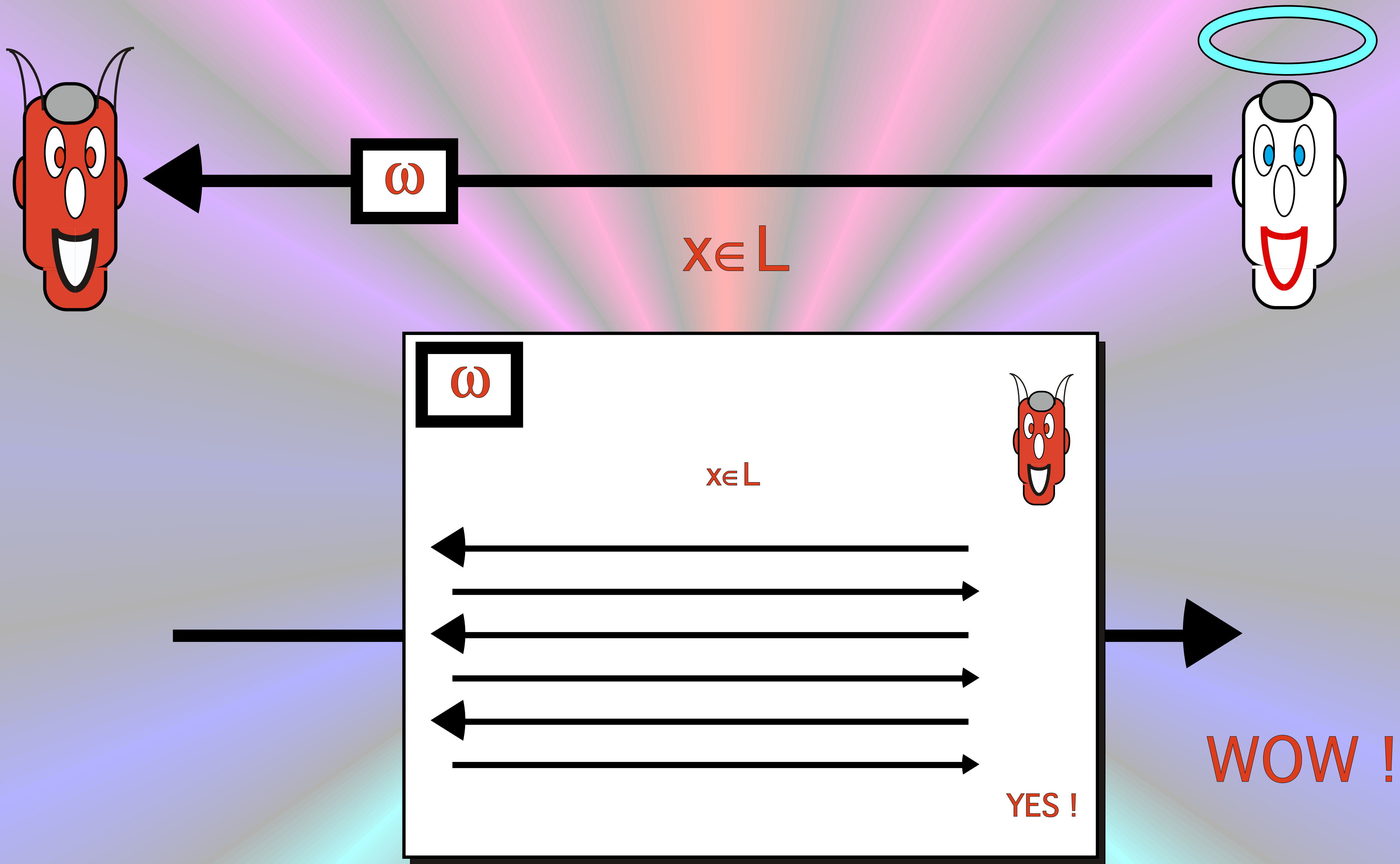


$x \in L$

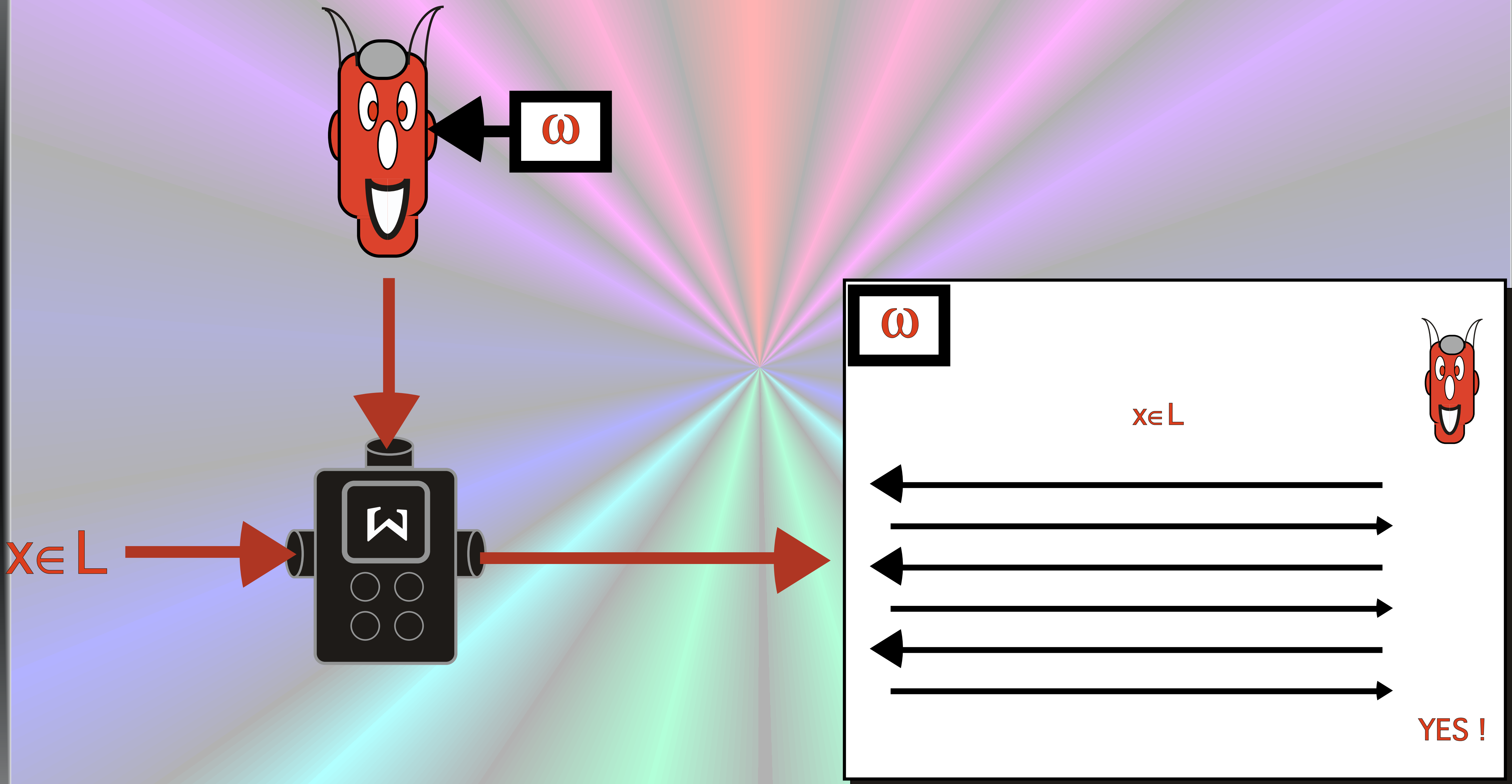


YES !

# Auxiliary Input Zero-Knowledge



# Auxiliary Input Zero-Knowledge

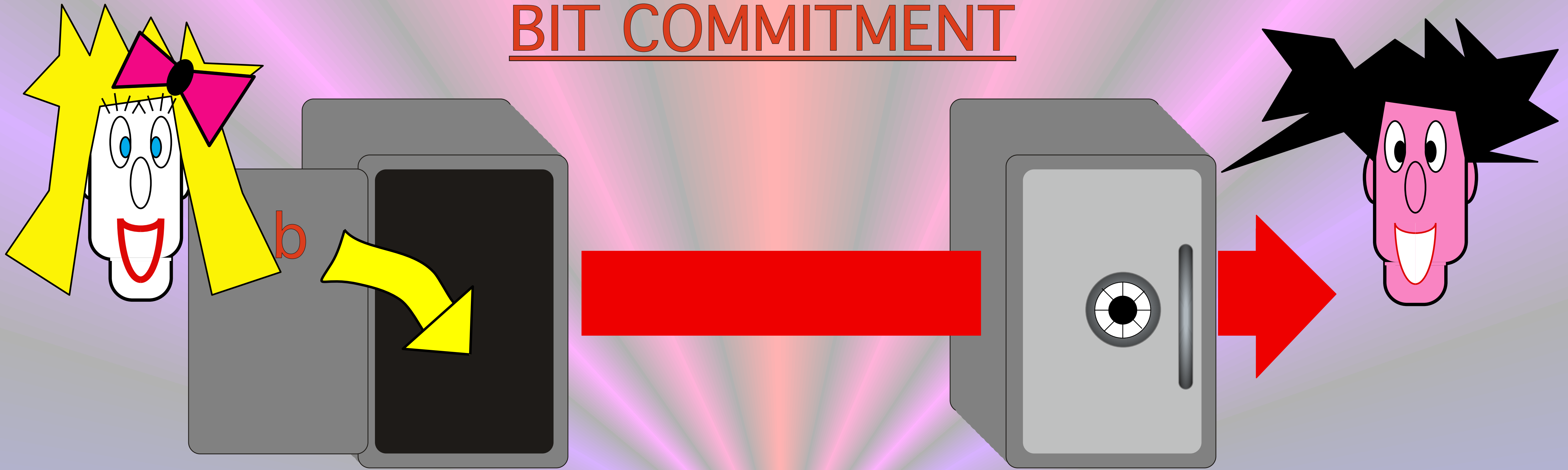


$$\forall \text{ (Red Alien)} \exists \text{ (Black Device)} \forall \omega \forall x \in L \text{ view}[\text{ (Yellow Alien)}, \text{ (Red Alien)}](\omega, x) = \text{ (Black Device)}(\omega, x)$$

# Bit Commitment

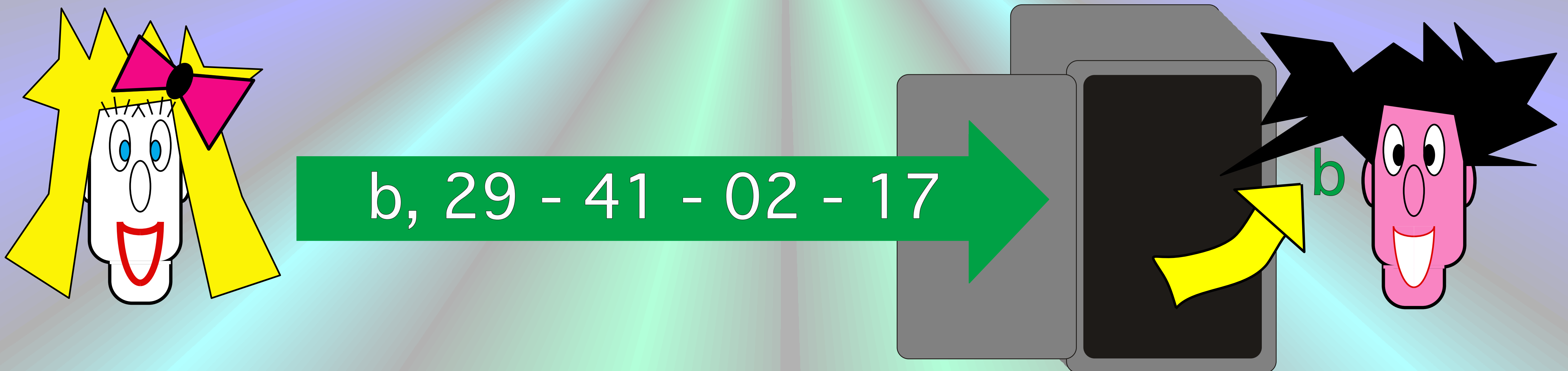
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# BIT COMMITMENT



COMMIT

UNVEIL



b, 29 - 41 - 02 - 17

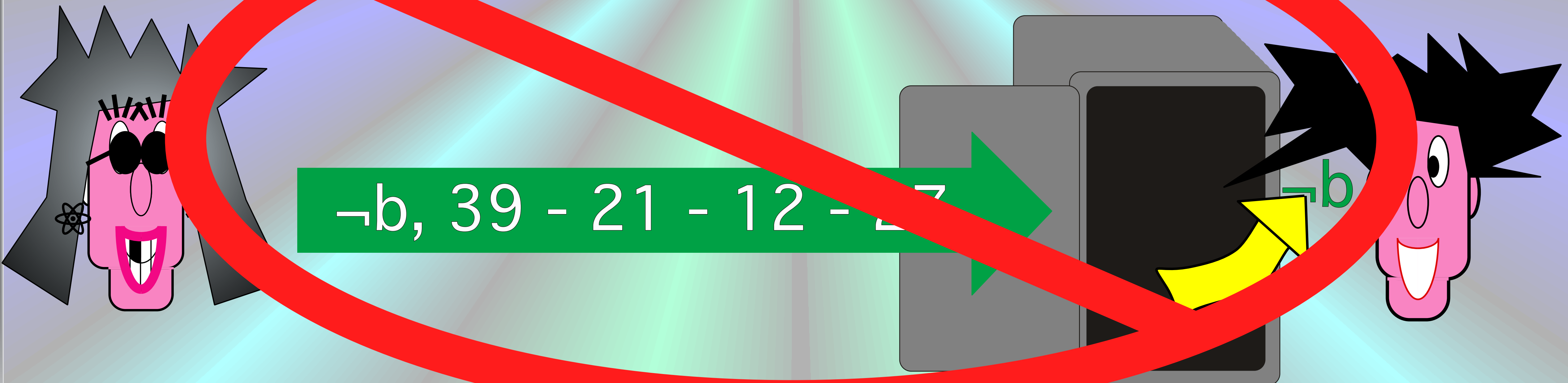
b

# BIT COMMITMENT



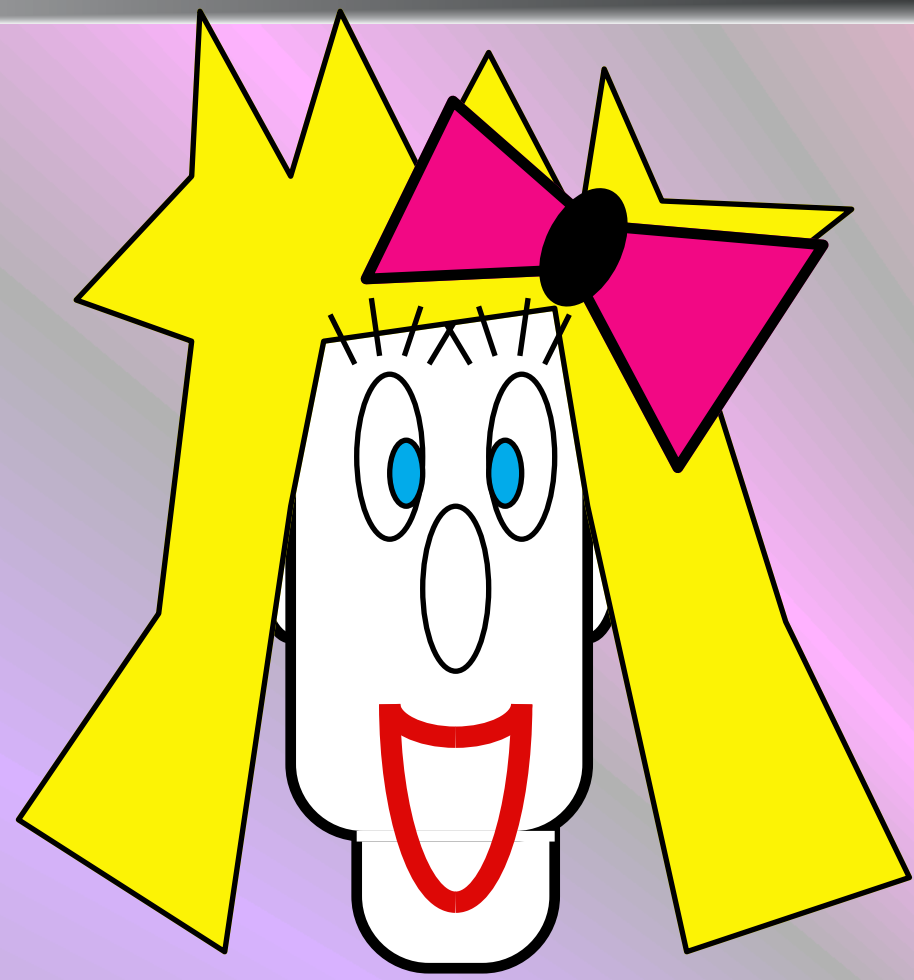
CONCEALING

# BINDING

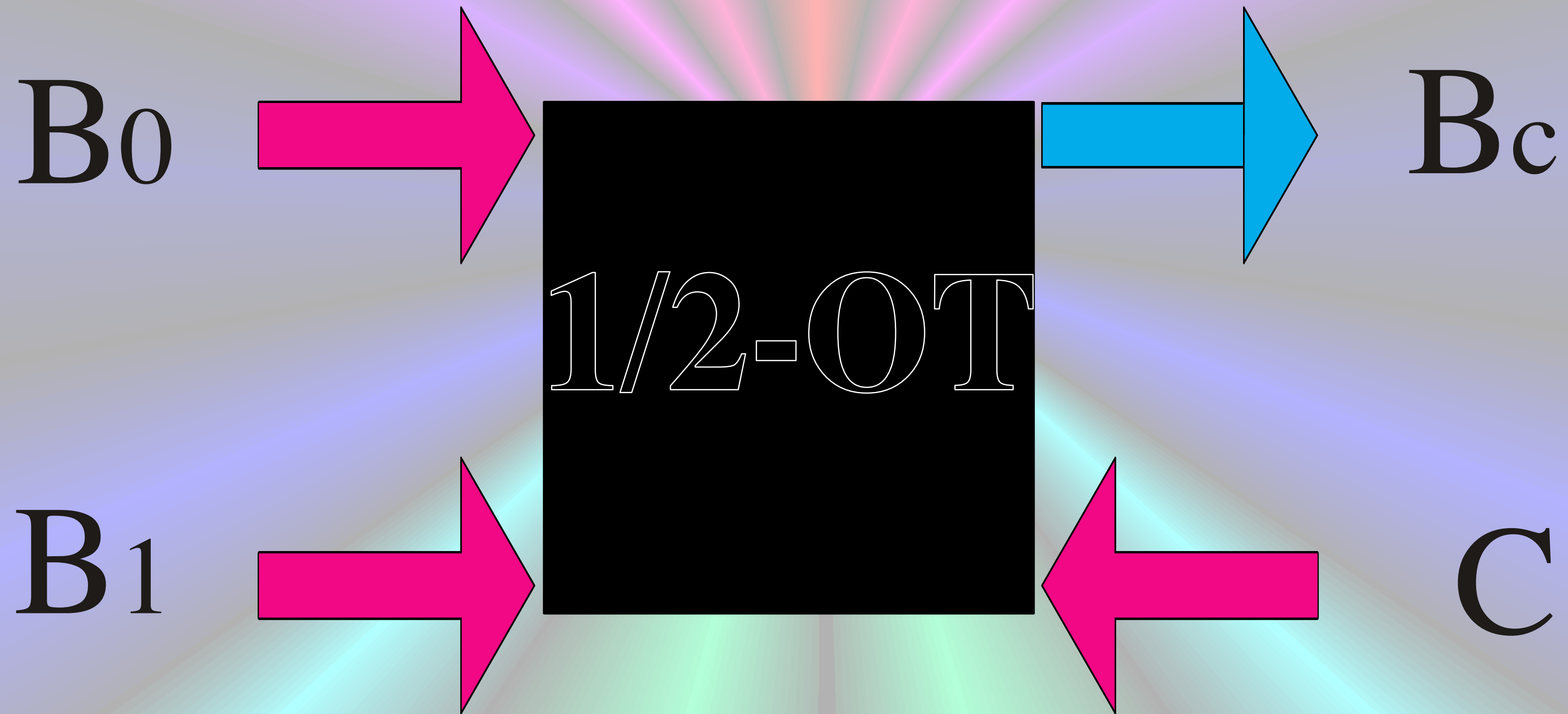
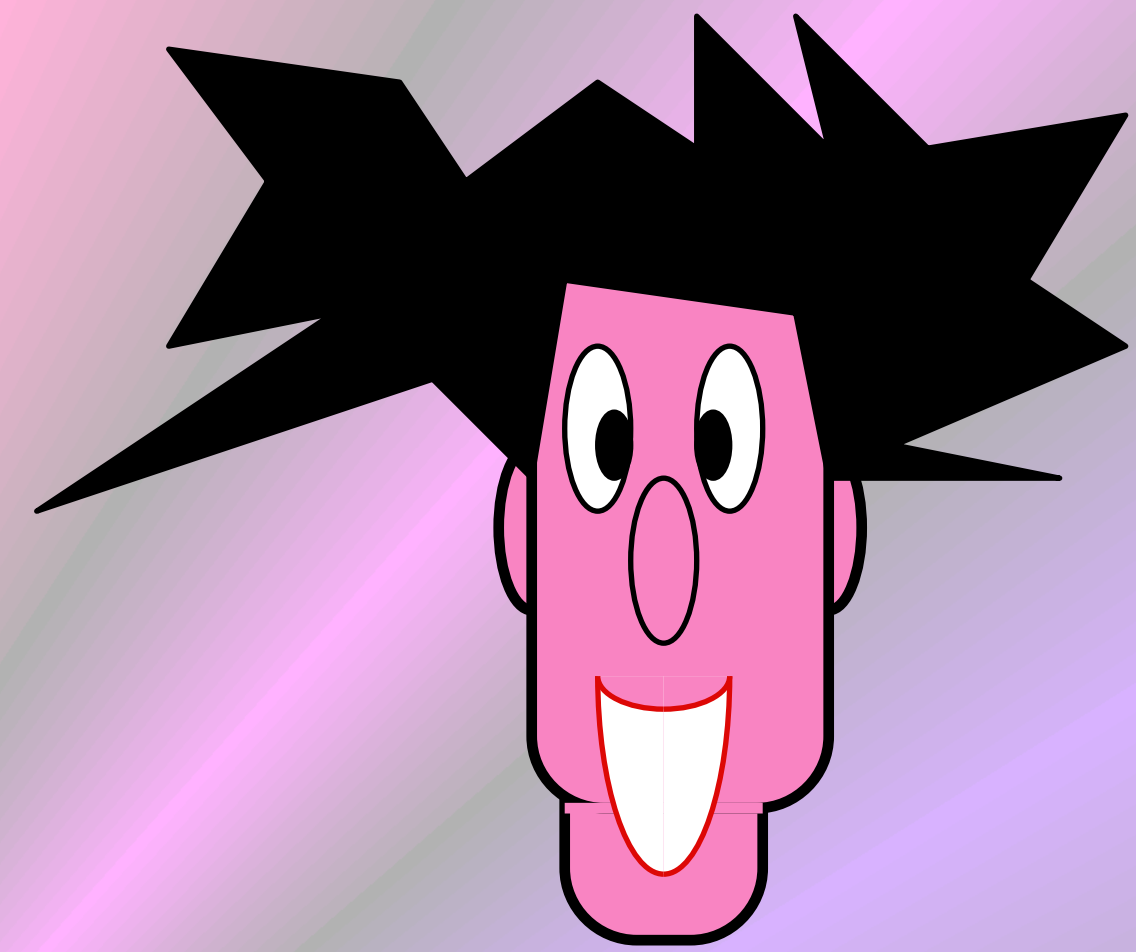


$\neg b, 39 - 21 - 12 - 7$

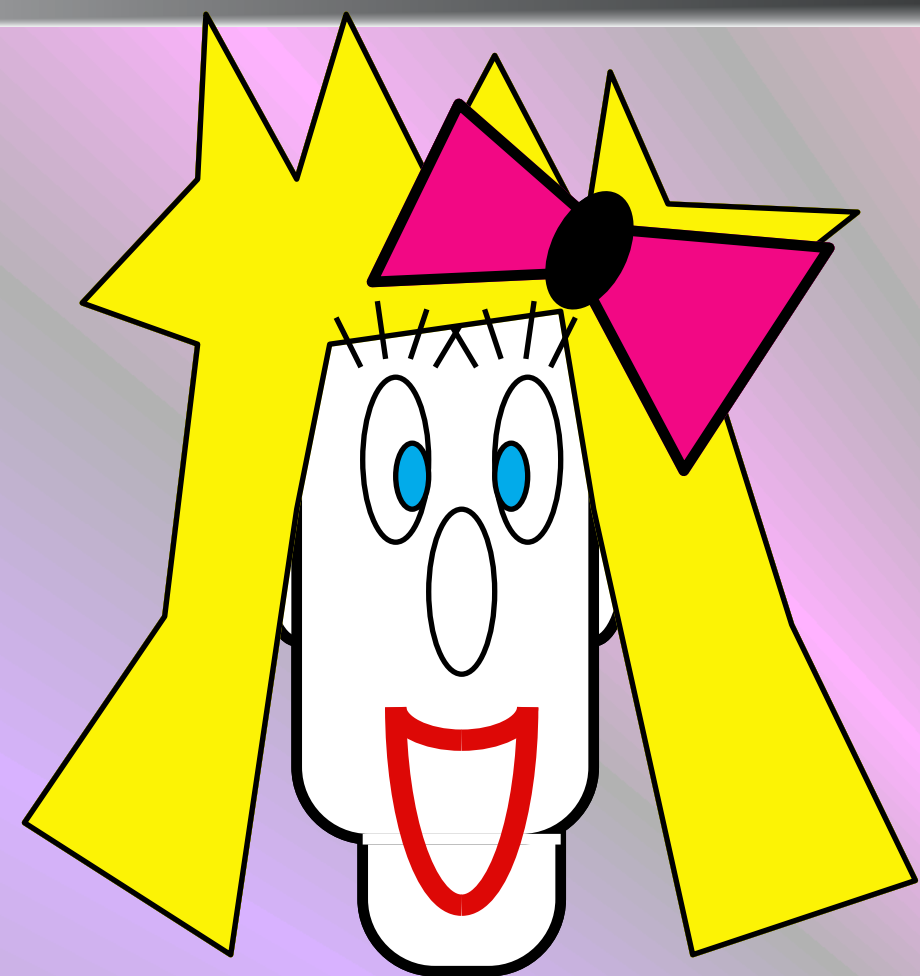
# Oblivious Transfer



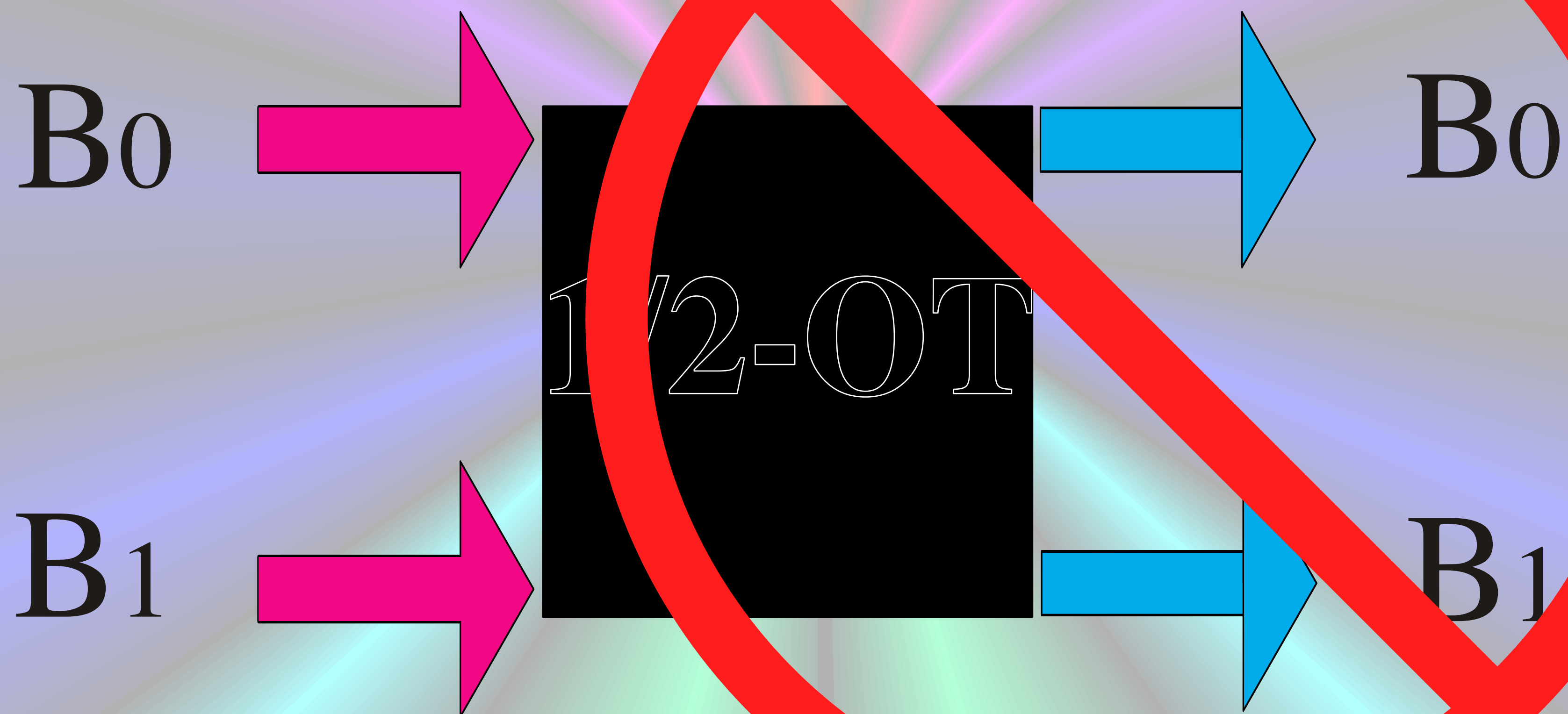
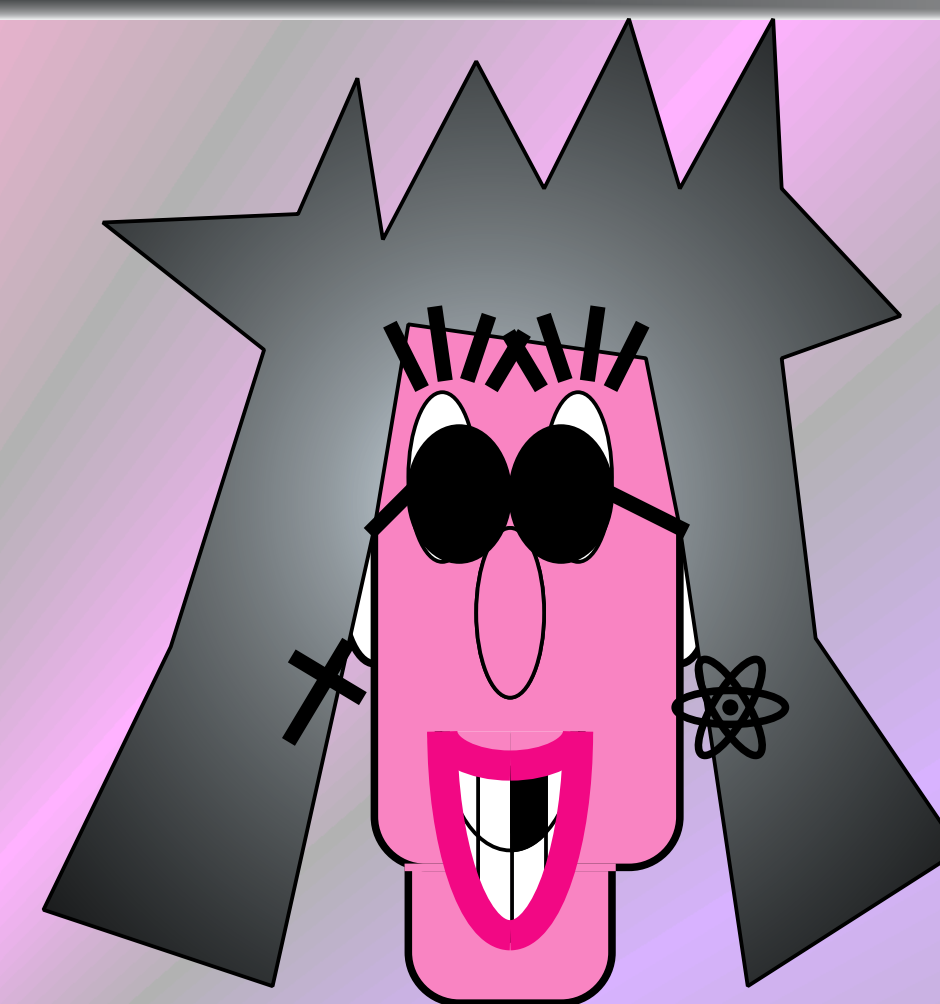
# Oblivious Transfer

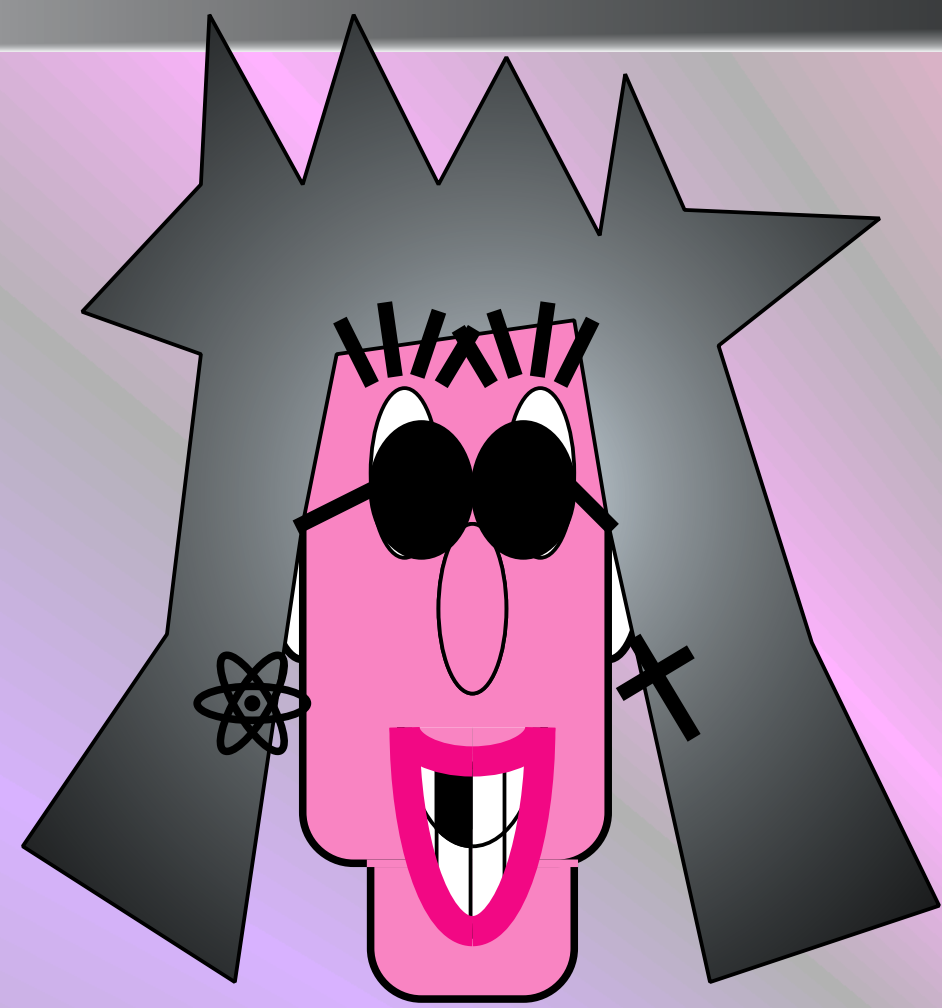




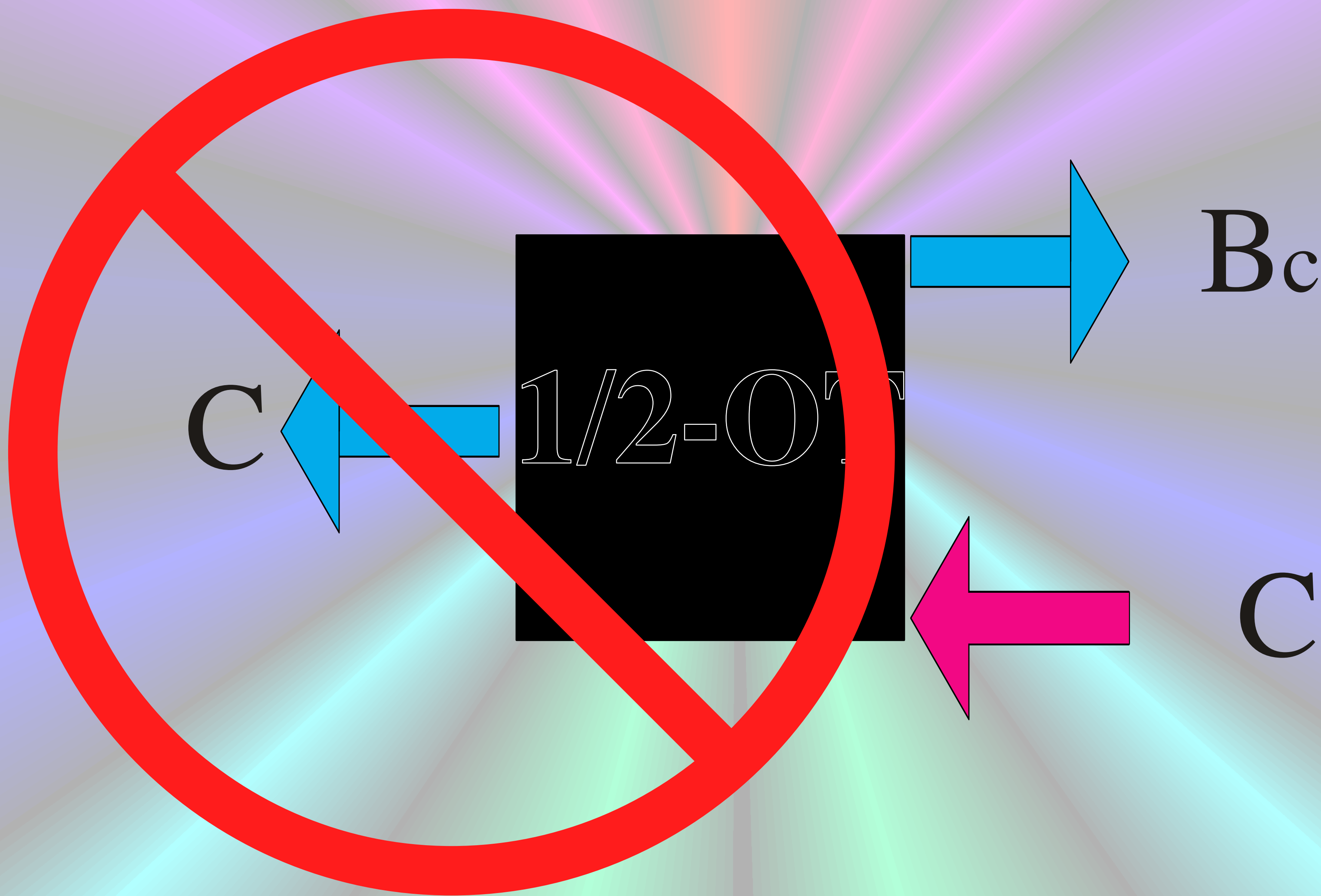
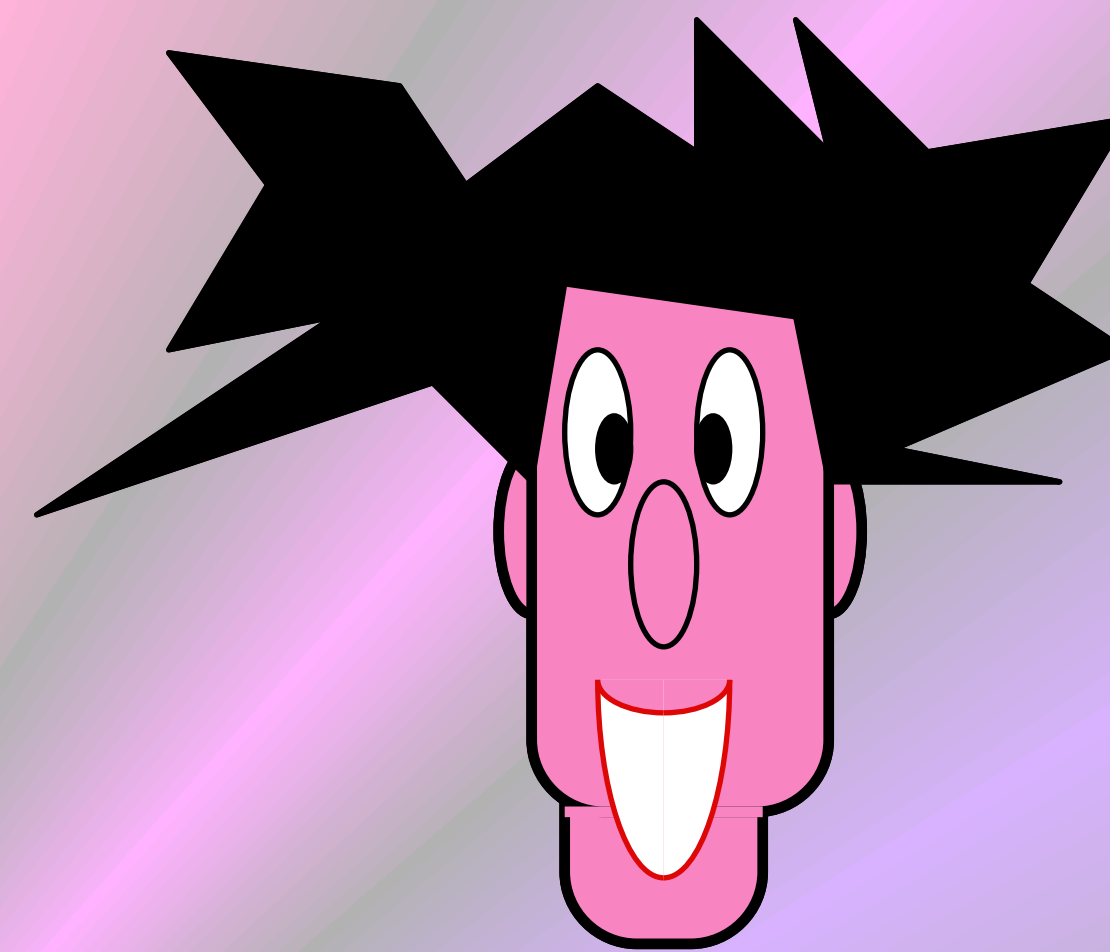


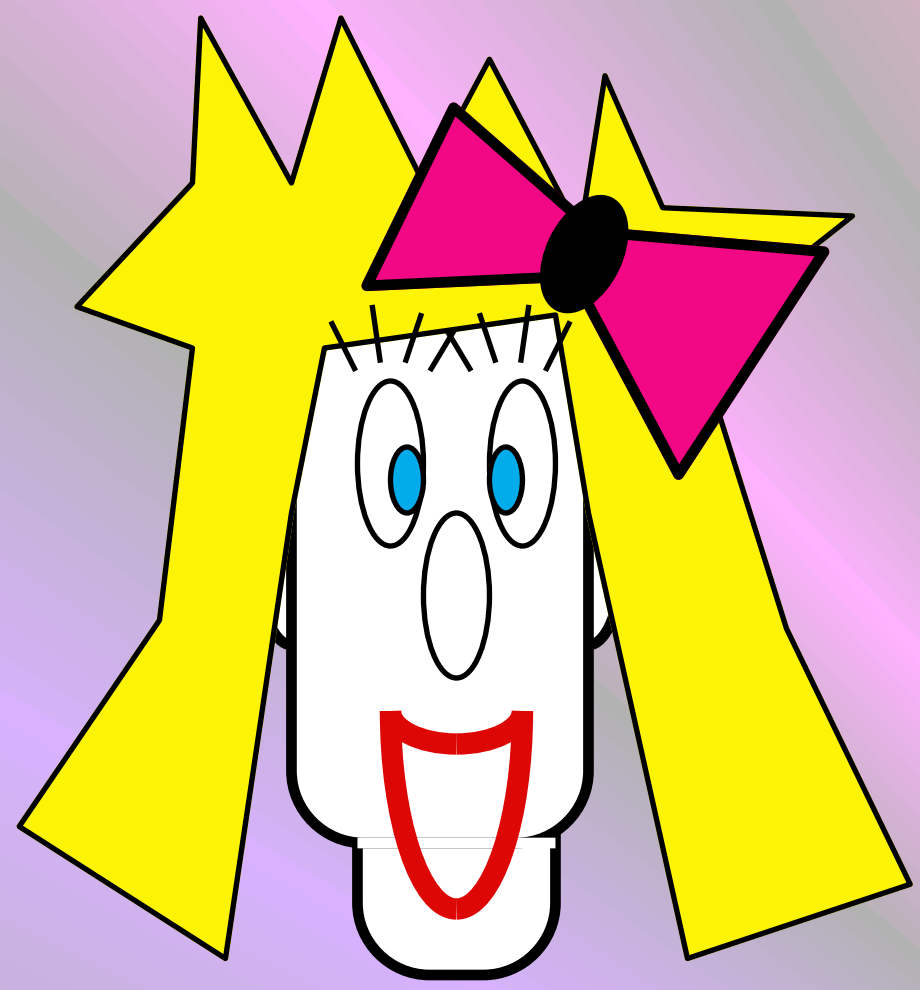
# Oblivious Transfer



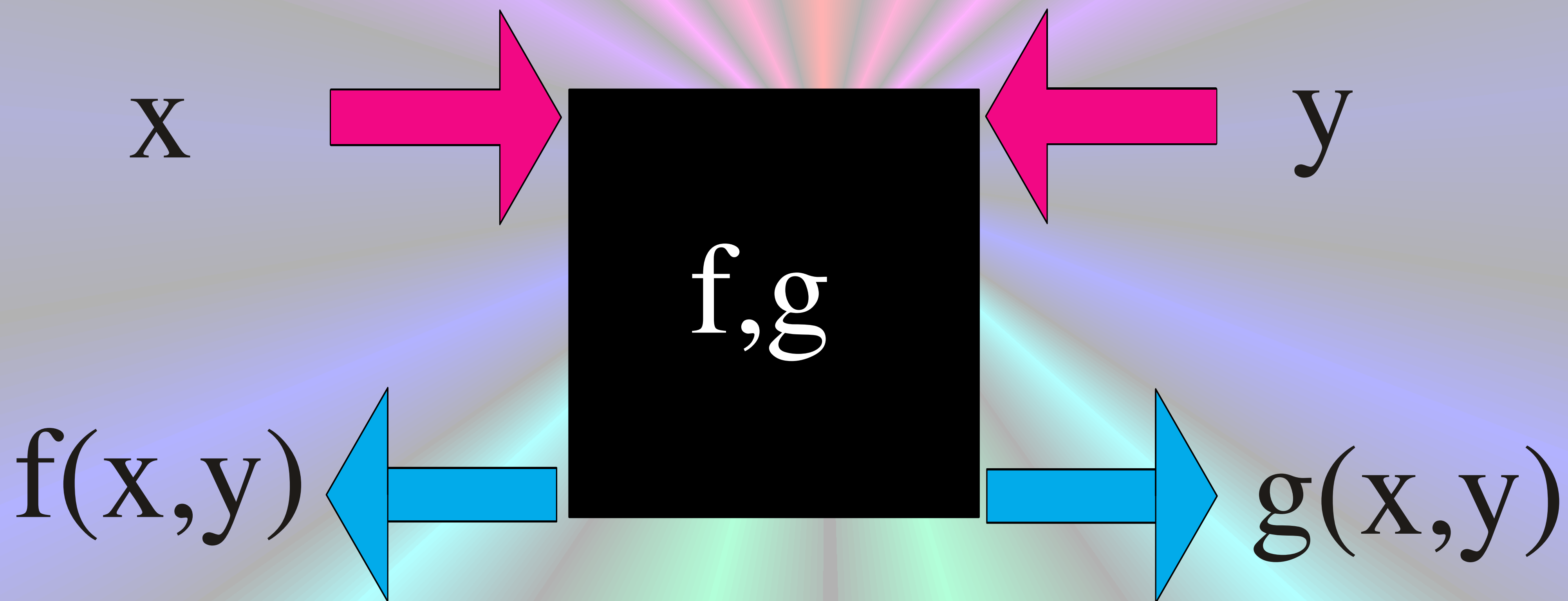
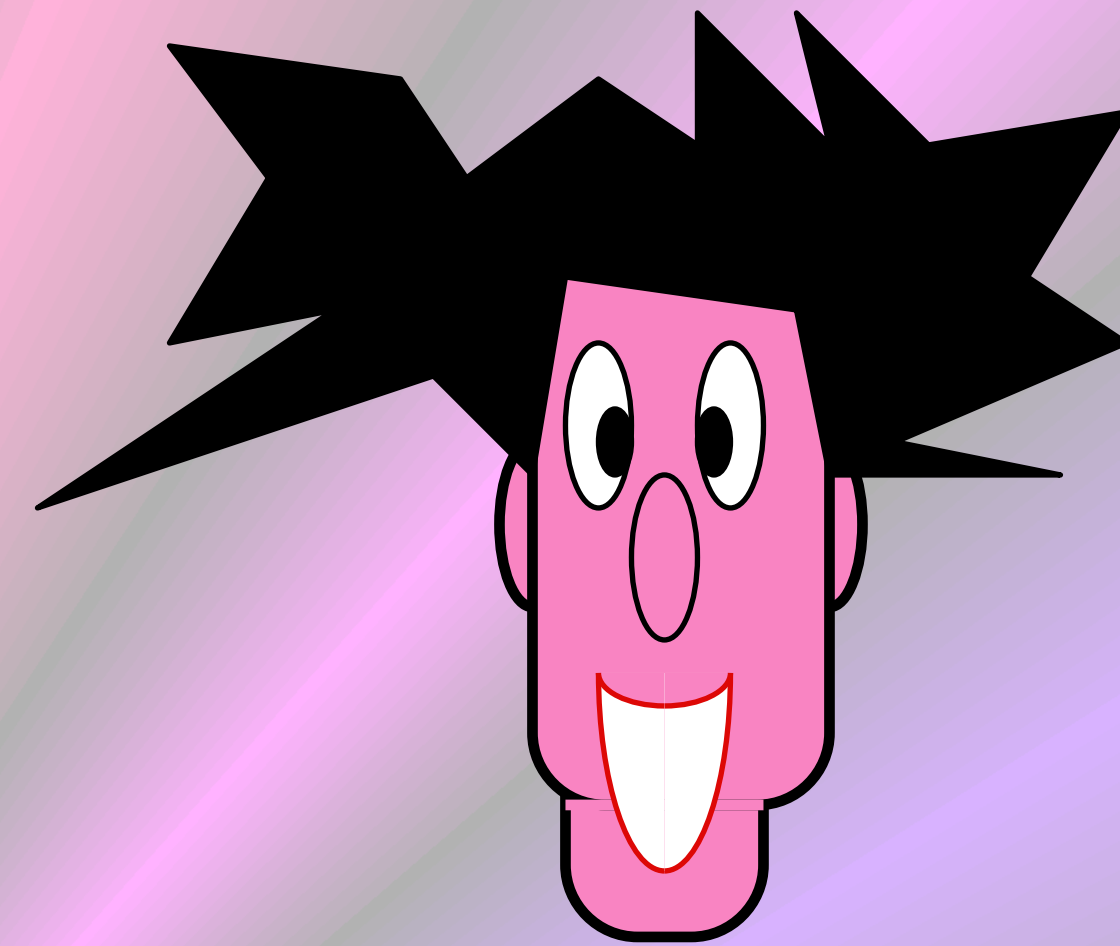


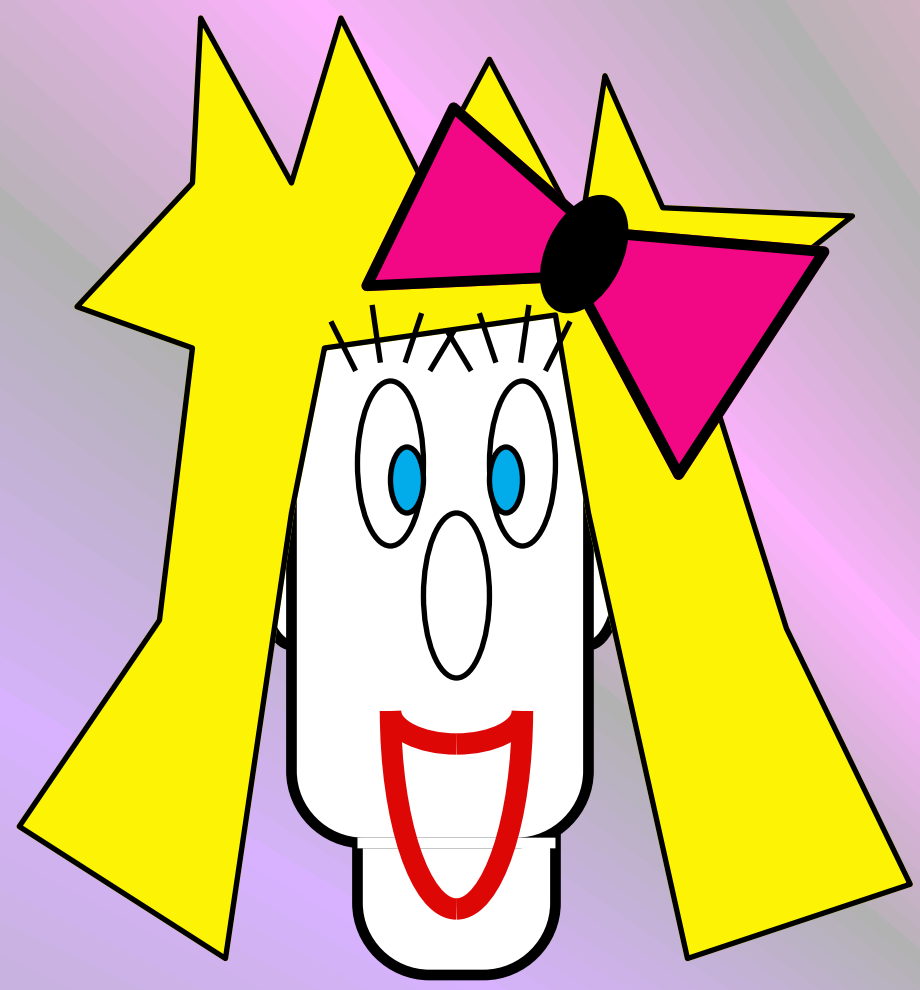
# Oblivious Transfer



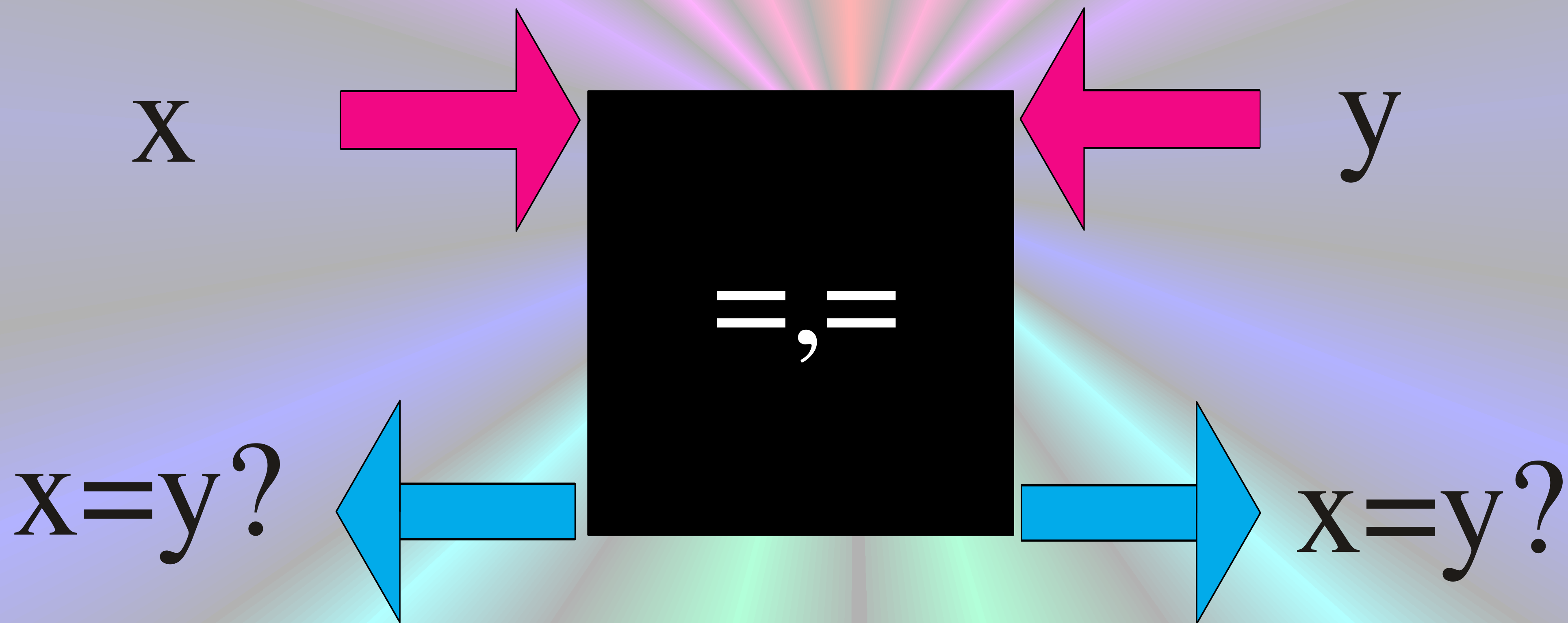
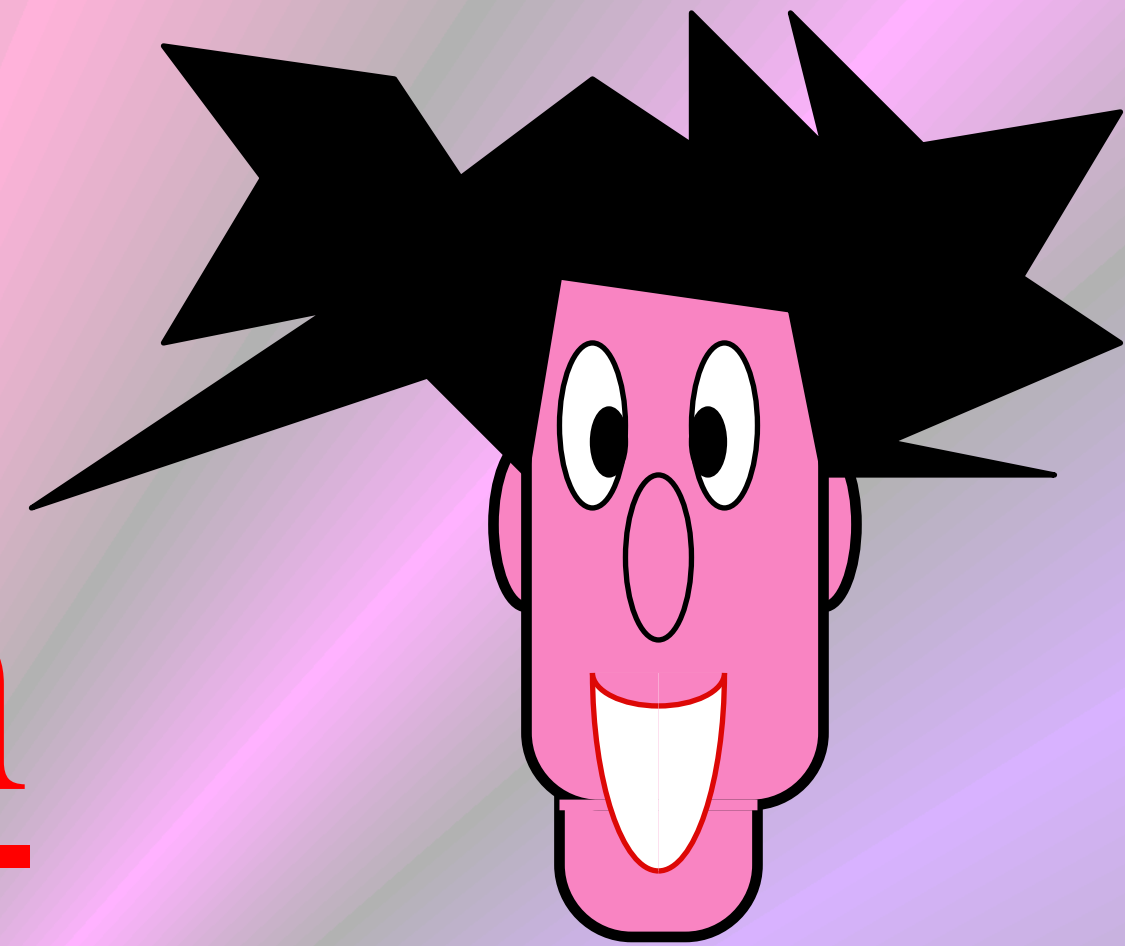


# Oblivious Function Evaluation

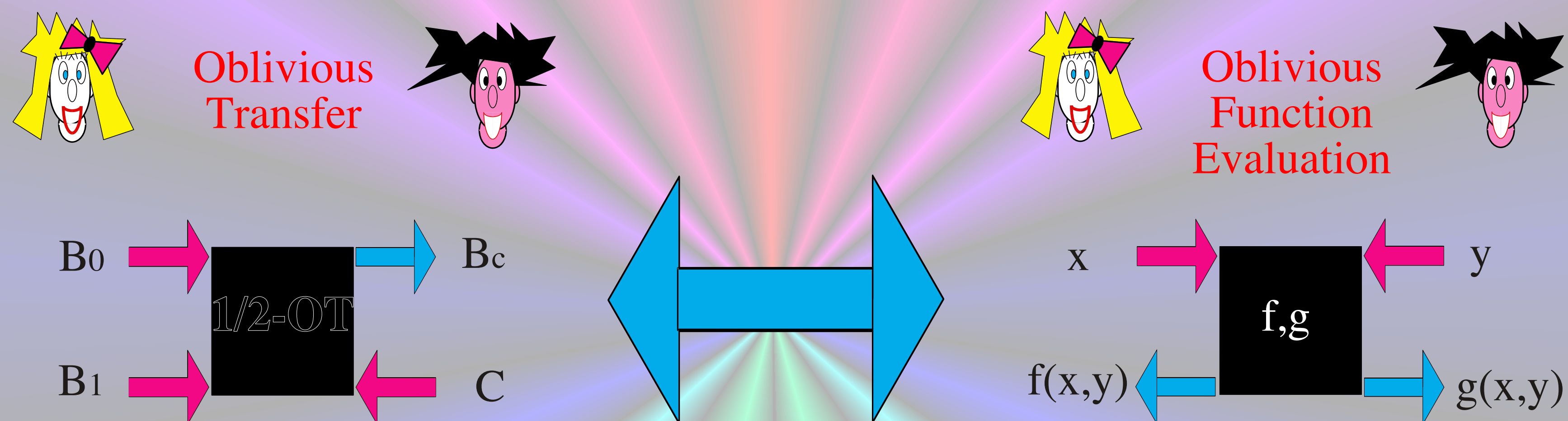




# Mutual Identification



# Cryptographic Reduction



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and other Two-Party  
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