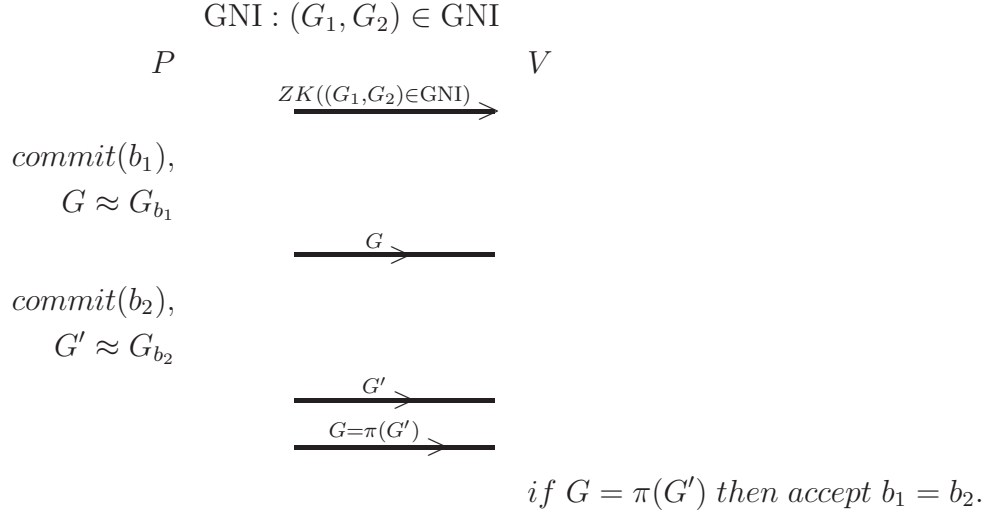


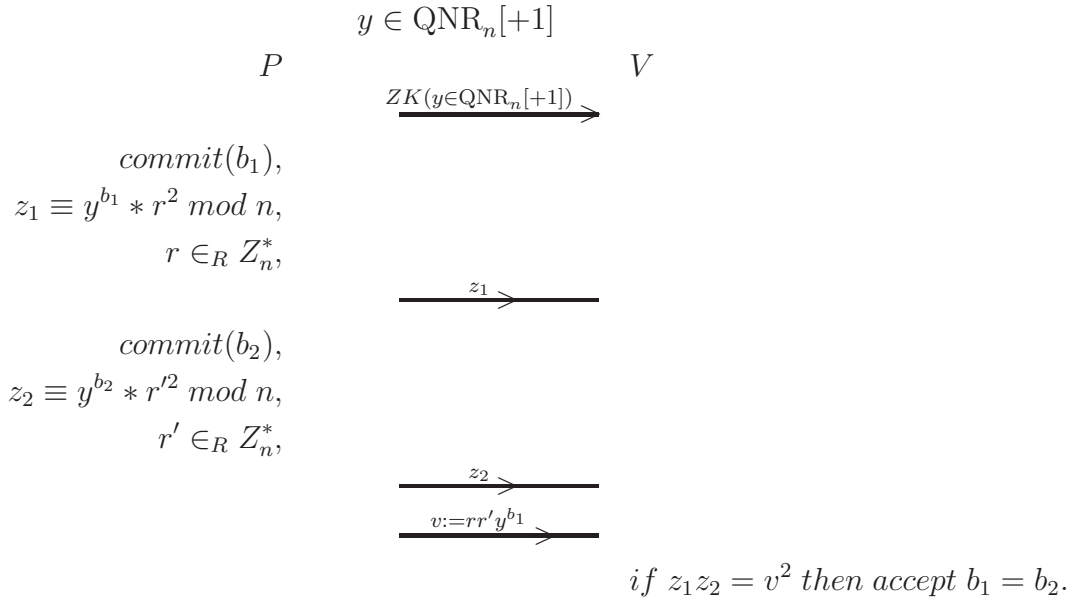
# 1 BC with equality

## 1.1 Comparing equality of two committed inputs

**Example 1.1** *bit commitment based on GNI.*



**Example 1.2** *bit commitment based on QNR.*



To prove  $b_1 \neq b_2$  P sends  $\sqrt{yz_1z_2}$ .

## 1.2 Computation on boolean circuits using committed inputs

We shall use the previous two examples to do computations on boolean circuits. Prover commits to three bits:  $b_1, b_2, b_3 \in \{0, 1\}$ , s.t. B:  $b_1 \wedge b_2 = b_3$ . There are only four possible situations of B (see Table T), i.e. B (three bits) must belong to one of the following situation  $T_j$  (three bits).

**Table T**

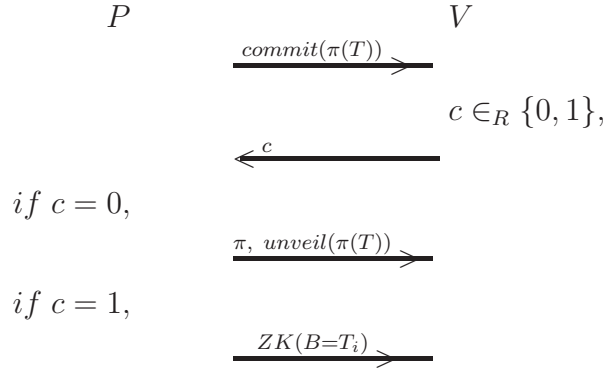
$$T_1 : 0 \wedge 0 = 0$$

$$T_2 : 0 \wedge 1 = 0$$

$$T_3 : 1 \wedge 0 = 0$$

$$T_4 : 1 \wedge 1 = 1$$

We design the protocol by using the "Cut and Choose" technique in order to prove that  $B \in T$ . P permutes the Table T then commit to  $(\pi(T))$  to V.



Note:

1.  $\pi$  is a permutation of Table T.
2. Using example 1.1, P can show to V that the three committed bits B:  $b_1, b_2$  and  $b_3$  are equal to the three committed bits of  $T_i$  respectively.

If  $b_1 \wedge b_2 = b_3$ ,  $\Pr[\text{accept}] = 1$ ,

If  $b_1 \wedge b_2 \neq b_3$ ,  $\Pr[\text{accept}] \leq 1/2^k$ , where  $k = \#$  of rounds.

We can use this method for any logical gate:  $\wedge, \vee, -, \oplus$ .

### 1.3 Rudich's Trick

Now we are going to talk about a general way to obtain a bit commitment where we can prove equality based on any bit commitment. Here “Rudich's Trick” is the way to show two committed bits are equal.

Suppose  $b = b'$ , where  $b, b' \in \{0, 1\}$ , let  $u_i, x_i$  be random bits and  $v_i, y_i$  be defined according to  $u_i \oplus v_i = b$ ,  $x_i \oplus y_i = b'$ ,  $i = 1, \dots, 2n$ . We shall use  $4n$  committed bits to commit one bit.

<i>Alice</i>	<i>Bob</i>
$commit(b) :$	$commit(b') :$
$\alpha_1 : C(u_1), C(v_1)$	$C(x_1), C(y_1) : \beta_1$
$\alpha_2 : C(u_2), C(v_2)$	$C(x_2), C(y_2) : \beta_2$
$\vdots$	$\vdots$
$\alpha_{2n} : C(u_{2n}), C(v_{2n})$	$C(x_{2n}), C(y_{2n}) : \beta_{2n}$

where  $\alpha_i$  and  $\beta_i$  are two committed bits and  $C$  denotes commit.

- Bob imposes two random permutations  $\pi_\alpha, \pi_\beta$  to Alice who permutes  $\alpha_i$  using  $\pi_\alpha$  and  $\beta_i$  using  $\pi_\beta$ .

$C(u_9), C(v_9)$	$C(x_2), C(y_2)$
$C(u_n), C(v_n)$	$C(x_7), C(y_7)$
$\vdots$	$\vdots$
$C(u_3), C(v_3)$	$C(x_6), C(y_6)$

- Regardless of  $b$ ,  $u_i, x_i$  and  $v_i, y_i$  are either identical or opposite. Alice will claim for the first half of the lines whether they are “=” or “ $\neq$ ”.

$$\begin{array}{ccc}
C(u_9), C(v_9) & = & C(x_2), C(y_2) \\
C(u_n), C(v_n) & \neq & C(x_7), C(y_7) \\
& \vdots & \vdots \\
C(u_{41}), C(v_{41}) & = & C(x_{63}), C(y_{63}) \\
\hline
C(u_{99}), C(v_{99}) & & C(x_{n-9}), C(y_{n-9}) \\
C(u_{2n}), C(v_{2n}) & & C(x_{55}), C(y_{55}) \\
& \vdots & \vdots \\
C(u_3), C(v_3) & & C(x_6), C(y_6)
\end{array}$$

for example:

$$\begin{array}{ll}
b = b' = 0 & b = b' = 1 \\
0 \oplus 0 = 0 \oplus 0 : u_i = x_i, v_i = y_i, & 0 \oplus 1 = 0 \oplus 1 : u_i = x_i, v_i = y_i, \\
0 \oplus 0 = 1 \oplus 1 : u_i \neq x_i, v_i \neq y_i, & 1 \oplus 0 = 0 \oplus 1 : u_i \neq x_i, v_i \neq y_i.
\end{array}$$

- For each line Bob randomly chooses to see the both left sides or both right sides, but not both sides, then Alice unveils them to Bob.

$$\begin{array}{ccc}
C(u_9), U(v_9) & = & C(x_2), U(y_2) \\
U(u_n), C(v_n) & \neq & U(x_7), C(y_7) \\
& \vdots & \vdots \\
C(u_{41}), U(v_{41}) & = & C(x_{63}), U(y_{63}) \\
\hline
C(u_{99}), C(v_{99}) & & C(x_{n-9}), C(y_{n-9}) \\
C(u_{2n}), C(v_{2n}) & & C(x_{55}), C(y_{55}) \\
& \vdots & \vdots \\
C(u_3), C(v_3) & & C(x_6), C(y_6)
\end{array}$$

where  $U$  denotes unveil.

If Alice wants to cheat, suppose  $b=1$ ,  $b'=0$  and Alice claims that  $\alpha_i = \beta_i$ , for example:

$$\begin{array}{ccc} b = 1 & & b' = 0 \\ \alpha_i : 0 \oplus 1 & = & 1 \oplus 1 : \beta_i \end{array}$$

With  $1/2$  probability, Bob will request to see the left sides and Alice unveils to him that  $(u_i = 0) \neq (x_i = 1)$ , then Bob rejects.

With  $1/2$  probability, Bob will request to see the right sides and Alice unveils to him that  $v_i = 1 = y_i$ , then Bob accepts. So if  $b \neq b'$ ,  $1/2$  probability Bob will be cheated.

Therefore,

If  $b=b'$ ,  $\Pr[\text{accept}]=1$ ,

If  $b \neq b'$ ,  $\Pr[\text{accept}] \leq (1/2)^n$ , at each line, if  $b \neq b'$ , regardless of Alice's answer, the probability Bob finds out that Alice is cheating is  $1/2$ .

After a test is conclusive, Alice can construct a new valid commitment to represent both  $b$  and  $b'$  using the untouched commitments:

$$\begin{array}{l} \text{commit}(b, b') : \\ C(u_{99}), C(v_{99}) \\ C(u_{2n}), C(v_{2n}) \\ \vdots \\ C(u_3), C(v_3) \\ C(x_{n-9}), C(y_{n-9}) \\ C(x_{55}), C(y_{55}) \\ \vdots \\ C(x_6), C(y_6) \end{array}$$