COMP547B Homework set #3

Due Wednesday March 8th, 2017, 23:59:59

Exercises (from Katz and Lindell's book)

[10%]

3.2 Prove that Definition 3.8 cannot be satisfied if Π can encrypt arbitrary-length messages and the adversary is *not* restricted to output equal-length messages in experiment $\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi}$.

> **Hint:** Let $q(n)$ be a polynomial upper-bound on the length of the ciphertext when Π is used to encrypt a single bit. Then consider an adversary who outputs $m_0 \in \{0,1\}$ and a uniform $m_1 \in \{0,1\}^{q(n)+2}$.

[5%]

3.3 Say $\Pi = (\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ is such that for $k \in \{0,1\}^n$, algorithm Enc_k is only defined for messages of length at most $\ell(n)$ (for some polynomial ℓ). Construct a scheme satisfying Definition 3.8 even when the adversary is *not* restricted to outputting equal-length messages in $\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi}$.

[15%]

3.6 Let G be a pseudorandom generator with expansion factor $\ell(n) > 2n$. In each of the following cases, say whether G' is necessarily a pseudorandom generator. If yes, give a proof; if not, show a counterexample.

(a) Define $G'(s) \stackrel{\text{def}}{=} G(s_1 \cdots s_{\lfloor n/2 \rfloor})$, where $s = s_1 \cdots s_n$.

(b) Define $G'(s) \stackrel{\text{def}}{=} G\left(0^{|s|} \| s\right)$.

(c) Define $G'(s) \stackrel{\text{def}}{=} G(s) \,\|\, G(s+1)$.

[10%]

3.13 Consider the following keyed function F: For security parameter n, the key is an $n \times n$ boolean matrix A and an n-bit boolean vector b. Define $F_{A,b} : \{0,1\}^n \to \{0,1\}^n$ by $F_{A,b}(x) \stackrel{\text{def}}{=} Ax + b$, where all operations are done modulo 2. Show that F is not a pseudorandom function.

More on back…

[5%]

3.17 Assume pseudorandom permutations exist. Show that there exists a function F' that is a pseudorandom permutation but is *not* a strong pseudorandom permutation.

Hint: Construct F' such that $F'_k(k) = 0^{|k|}$.

[10%]

3.18 Let F be a pseudorandom permutation, and define a fixed-length encryption scheme $(\mathsf{Enc}, \mathsf{Dec})$ as follows: On input $m \in \{0,1\}^{n/2}$ and key $k \in \{0,1\}^n$, algorithm Enc chooses a uniform string $r \in \{0,1\}^{n/2}$ of length $n/2$ and computes $c := F_k(r \| m)$.

Show how to decrypt, and prove that this scheme is CPA-secure for messages of length $n/2$. (If you are looking for a real challenge, prove that this scheme is CCA-secure if F is a *strong* pseudorandom permutation.)

[+10% bonus]

[15%]

3.19 Let F be a pseudorandom function and G be a pseudorandom generator with expansion factor $\ell(n) = n+1$. For each of the following encryption schemes, state whether the scheme has indistinguishable encryptions in the presence of an eavesdropper and whether it is CPA-secure. (In each case, the shared key is a uniform $k \in \{0,1\}^n$.) Explain your answer.

(a) To encrypt $m \in \{0,1\}^{n+1}$, choose uniform $r \in \{0,1\}^n$ and output the ciphertext $\langle r, G(r) \oplus m \rangle$.

(b) To encrypt $m \in \{0,1\}^n$, output the ciphertext $m \oplus F_k(0^n)$.

(c) To encrypt $m \in \{0,1\}^{2n}$, parse m as $m_1 \| m_2$ with $|m_1| = |m_2|$, then choose uniform $r \in \{0,1\}^n$ and send $\langle r, m_1 \oplus F_k(r), m_2 \oplus F_k(r+1) \rangle$.

[10%]

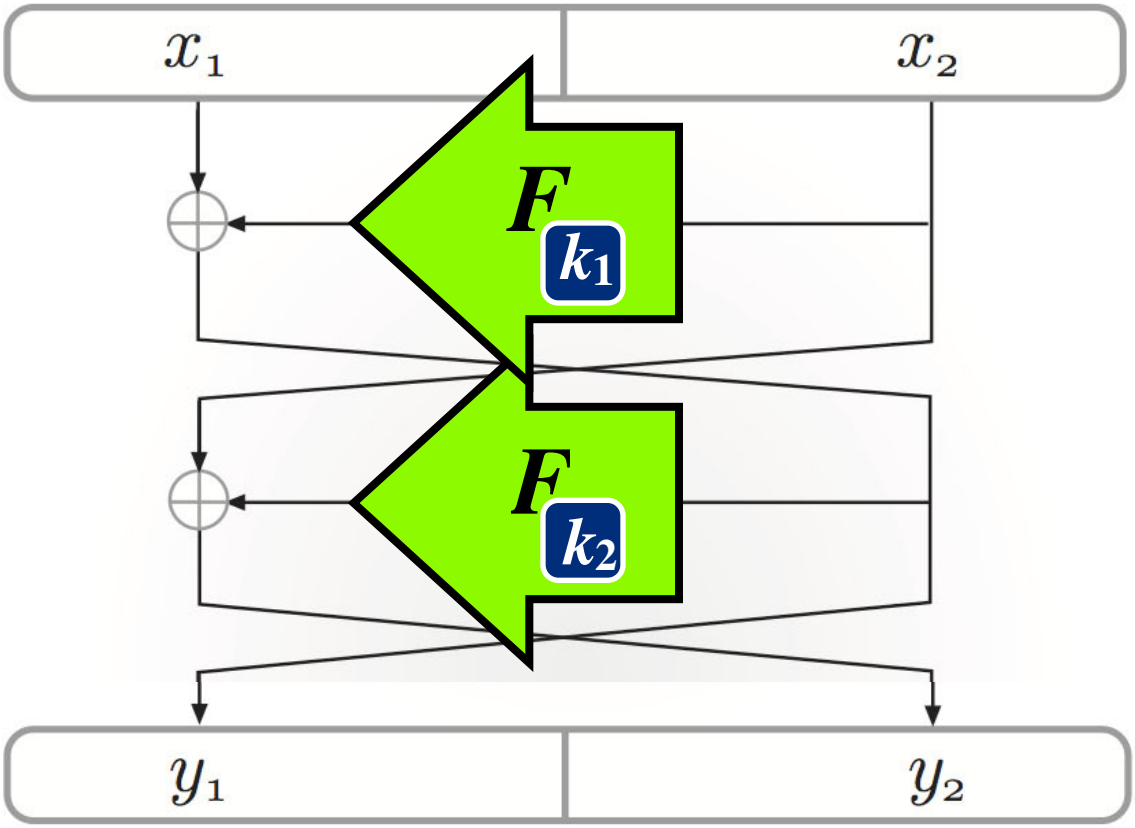
3.29 Let $\Pi_1 = (\mathsf{Enc}_1, \mathsf{Dec}_1)$ and $\Pi_2 = (\mathsf{Enc}_2, \mathsf{Dec}_2)$ be two encryption schemes for which it is known that at least one is CPA-secure (but you don't know which one). Show how to construct an encryption scheme Π that is guaranteed to be CPA-secure as long as at least one of Π_1 or Π_2 is CPA-secure. Provide a full proof of your solution.

Hint: Generate two plaintext messages from the original plaintext so that knowledge of either one reveals nothing about the original plaintext, but knowledge of both enables the original plaintext to be computed.

More on back…

Homemade Exercise: non Pseudo-Random Permutation

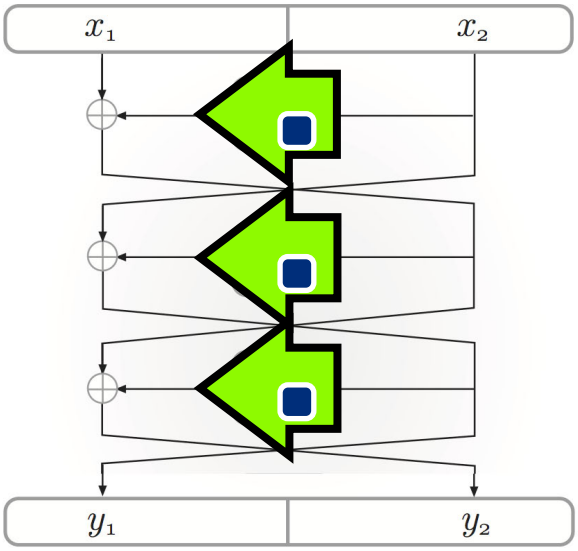
[10%] Let F be a pseudo-random family of functions. Let k_1 and k_2 be two independent random keys. Prove that $\pi_{k_1k_2}(x_1,x_2) := (x_1 \oplus F_{k_1}(x_2) , x_2 \oplus F_{k_2}(x_1 \oplus F_{k_1}(x_2)))$ is not a pseudo-random permutation family.



[10%] Let F be a pseudo-random family of functions. Let k_1, k_2 and k_3 be three independent random keys. Prove that

$\pi_{k_1k_2k_3}(x_1,x_2) := (x_2 \oplus F_{k_2}(x_1 \oplus F_{k_1}(x_2)) , x_1 \oplus F_{k_1}(x_2) \oplus F_{k_3}(x_2 \oplus F_{k_2}(x_1 \oplus F_{k_1}(x_2))))$

is not a **strong** pseudo-random permutation family.



More on back…