INSTRUCTIONS:
• This examination is worth 10% of your final grade.
• The total of all questions is 100 points.
• Each question is assigned a value found in brackets next to it.
• OPEN BOOKS / OPEN NOTES
• Faculty standard calculator permitted only.
• This examination consists of 3 pages including title page.
• This examination consists of 4 questions.

SUGGESTION: read all the questions and their values before you start answering.

• OPEN BOOKS / OPEN NOTES • •
1) Prove that (using the Pumping Lemma and the Myhill-Nerode Theorem)

\[ L = \{ 1^n \mid n \text{ is a prime number} \} \] is NON-Regular.

2) Problem 1.31 was solved in HW-1.

1.31 For any string \( w = w_1 w_2 \cdots w_n \), the reverse of \( w \), written \( w^R \), is the string \( w \) in reverse order, \( w_n \cdots w_2 w_1 \). For any language \( A \), let \( A^R = \{ w^R \mid w \in A \} \).

Show that if \( A \) is regular, so is \( A^R \).

Use it to solve the following problem: verification of a binary addition is regular.

1.32 Let

\[ \Sigma_3 = \{ [0], [1], [0], \ldots, [1] \} \]

\( \Sigma_3 \) contains all size 3 columns of 0s and 1s. A string of symbols in \( \Sigma_3 \) gives three rows of 0s and 1s. Consider each row to be a binary number and let

\[ B = \{ w \in \Sigma_3^n \mid \text{the bottom row of } w \text{ is the sum of the top two rows} \} \]

For example,

\[ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \in B, \quad \text{but} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \notin B. \]

Show that \( B \) is regular. (Hint: Working with \( B^R \) is easier. You may assume the result claimed in Problem 1.31.)

3) Let \( \Sigma = \{ 0, 1, \cup, \circ, *, (,), \emptyset, \€ \} \) (I used the euro sign "€" for the empty string to distinguish it from the empty string itself "\( \€ \)". i.e. \( |\€|=0 \) while \( |\emptyset|=1 \).

Define \( L_{\text{REG}} = \{ w \in \Sigma^* \mid w \text{ is a valid REGULAR EXPRESSION}\} \).

A. Give examples of strings \( w \in L_{\text{REG}} \) and \( w' \notin L_{\text{REG}} \) such that \( |w| = |w'|=10 \).

Use strictly the definition below for valid REGULAR EXPRESSIONs.

B. Show that \( L_{\text{REG}} \) is a NON-regular language.

\( ^\dagger \) (see reminder (DEFINITION 1.52) about REGULAR EXPRESSIONs on next page)
4) You are given the state diagrams of two DFAs below.

Let \( L_1 = L(M_1) \) and \( L_2 = L(M_2) \).

a) Give a DFA for \( L_1 \cap L_2 \) based on the general construction of Theorem 1.25 and the footnote (explained in class) about intersection. Hint: the result should have exactly 12 states.

b) If instead you used the fact that \( L_1 \cap L_2 = (L_1^c \cup L_2^c)^c \), the construction of Theorem 1.45, and the construction of Theorem 1.39, how many states would result in such a DFA for \( L_1 \cap L_2 \)? Explain your calculation.