2.4 Give context-free grammars that generate the following languages. In all parts the alphabet $\Sigma$ is \{0, 1\}.

(b) $\{w \mid w \text{ starts and ends with the same symbol}\}$

(c) $\{w \mid \text{the length of } w \text{ is odd}\}$

(e) $\{w \mid w = w^R, \text{ that is, } w \text{ is a palindrome}\}$

(f) The empty set

2.12 Convert the CFG $G$ given in Exercise 2.3 to an equivalent PDA, using the procedure given in Theorem 2.20.

2.3 Answer each part for the following context-free grammar $G$.

$$
R \to XR X \mid S \\
S \to aTb \mid bTa \\
T \to XTX \mid X \mid \varepsilon \\
X \to a \mid b
$$

2.14 Convert the following CFG into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

$$
A \to BAB \mid B \mid \varepsilon \\
B \to 00 \mid \varepsilon
$$

2.25 For any language $A$, let $\text{SUFFIX}(A) = \{v \mid uv \in A \text{ for some string } u\}$. Show that the class of context-free languages is closed under the $\text{SUFFIX}$ operation.
2.27 Let $G = (V, \Sigma, R, \langle \text{STMT} \rangle)$ be the following grammar.

\[
\langle \text{STMT} \rangle \rightarrow \langle \text{ASSIGN} \rangle | \langle \text{IF-THEN} \rangle | \langle \text{IF-THEN-ELSE} \rangle
\]

\[
\langle \text{IF-THEN} \rangle \rightarrow \text{if condition then } \langle \text{STMT} \rangle
\]

\[
\langle \text{IF-THEN-ELSE} \rangle \rightarrow \text{if condition then } \langle \text{STMT} \rangle \text{ else } \langle \text{STMT} \rangle
\]

\[
\langle \text{ASSIGN} \rangle \rightarrow a := 1
\]

$\Sigma = \{\text{if, condition, then, else, a:=1}\}$.

$V = \{\langle \text{STMT} \rangle, \langle \text{IF-THEN} \rangle, \langle \text{IF-THEN-ELSE} \rangle, \langle \text{ASSIGN} \rangle\}$

$G$ is a natural-looking grammar for a fragment of a programming language, but $G$ is ambiguous.

a. Show that $G$ is ambiguous.

b. Give a new unambiguous grammar for the same language.

(DON'T BE INTIMIDATED BY THE "**")

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2.32 Let $\Sigma = \{1, 2, 3, 4\}$ and $C = \{w \in \Sigma^* \mid \text{in } w, \text{ the number of 1s equals the number of 2s, and the number of 3s equals the number of 4s} \}$. Show that $C$ is not context free.

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2.36 Give an example of a language that is not context free but that acts like a CFL in the pumping lemma. Prove that your example works. (See the analogous example for regular languages in Problem 1.54.)

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3.9 Let a $k$-PDA be a pushdown automaton that has $k$ stacks. Thus a 0-PDA is an NFA and a 1-PDA is a conventional PDA. You already know that 1-PDAs are more powerful (recognize a larger class of languages) than 0-PDAs.

a. Show that 2-PDAs are more powerful than 1-PDAs.

b. Show that 3-PDAs are not more powerful than 2-PDAs.

(Hint: Simulate a Turing machine tape with two stacks.)

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3.12 A Turing machine with left reset is similar to an ordinary Turing machine, but the transition function has the form

\[
\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{R, \text{RESET}\}.
\]

If $\delta(q, a) = (r, b, \text{RESET})$, when the machine is in state $q$ reading an $a$, the machine's head jumps to the left-hand end of the tape after it writes $b$ on the tape and enters state $r$. Note that these machines do not have the usual ability to move the head one symbol left. Show that Turing machines with left reset recognize the class of Turing-recognizable languages.