COMP 330 2017, Assignment 3
Due Thursday, November 16th 2017 23:59

2.4 Give context-free grammars that generate the following languages. In all parts the alphabet \( \Sigma \) is \{0,1\}.

b. \( \{w | w \) starts and ends with the same symbol\}
c. \( \{w | \) the length of \( w \) is odd\}
e. \( \{w | w = w^R, \) that is, \( w \) is a palindrome\}
f. The empty set

2.12 Convert the CFG \( G \) given in Exercise 2.3 to an equivalent PDA, using the procedure given in Theorem 2.20.

2.3 Answer each part for the following context-free grammar \( G \).

\[
\begin{align*}
R & \rightarrow XRX \mid S \\
S & \rightarrow aTb \mid bTa \\
T & \rightarrow XTX \mid X \mid \varepsilon \\
X & \rightarrow a \mid b
\end{align*}
\]

2.14 Convert the following CFG into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

\[
\begin{align*}
A & \rightarrow BAB \mid B \mid \varepsilon \\
B & \rightarrow 00 \mid \varepsilon
\end{align*}
\]

2.25 For any language \( A \), let \( \text{SUFFIX}(A) = \{v | uv \in A \) for some string \( u\}\}. Show that the class of context-free languages is closed under the \( \text{SUFFIX} \) operation.
2.27 Let $G = (V, \Sigma, R, \langle \text{STMT} \rangle)$ be the following grammar.

$\langle \text{STMT} \rangle \rightarrow \langle \text{ASSIGN} \rangle \mid \langle \text{IF-THEN} \rangle \mid \langle \text{IF-THEN-ELSE} \rangle$

$\langle \text{IF-THEN} \rangle \rightarrow \text{if condition then } \langle \text{STMT} \rangle$

$\langle \text{IF-THEN-ELSE} \rangle \rightarrow \text{if condition then } \langle \text{STMT} \rangle \text{ else } \langle \text{STMT} \rangle$

$\langle \text{ASSIGN} \rangle \rightarrow a:=1$

$\Sigma = \{ \text{if, condition, then, else, a:=1} \}$

$V = \{ \langle \text{STMT} \rangle, \langle \text{IF-THEN} \rangle, \langle \text{IF-THEN-ELSE} \rangle, \langle \text{ASSIGN} \rangle \}$

$G$ is a natural-looking grammar for a fragment of a programming language, but $G$ is ambiguous.

a. Show that $G$ is ambiguous.

b. Give a new unambiguous grammar for the same language.

(\text{DON'T BE INTIMIDATED BY THE '*'})

2.32 Let $\Sigma = \{1, 2, 3, 4\}$ and $C = \{ w \in \Sigma^* \mid$ in $w$, the number of $1$s equals the number of $2$s, and the number of $3$s equals the number of $4$s $\}$. Show that $C$ is not context free.

2.36 Give an example of a language that is not context free but that acts like a CFL in the pumping lemma. Prove that your example works. (See the analogous example for regular languages in Problem 1.54.)

3.9 Let a $k$-PDA be a pushdown automaton that has $k$ stacks. Thus a 0-PDA is an NFA and a 1-PDA is a conventional PDA. You already know that 1-PDAs are more powerful (recognize a larger class of languages) than 0-PDAs.

a. Show that 2-PDAs are more powerful than 1-PDAs.

b. Show that 3-PDAs are not more powerful than 2-PDAs.

(Hint: Simulate a Turing machine tape with two stacks.)

3.12 A \textbf{Turing machine with left reset} is similar to an ordinary Turing machine, but the transition function has the form

$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{ R, \text{RESET} \}.$

If $\delta(q, a) = (r, b, \text{RESET})$, when the machine is in state $q$ reading an $a$, the machine's head jumps to the left-hand end of the tape after it writes $b$ on the tape and enters state $r$. Note that these machines do not have the usual ability to move the head one symbol left. Show that Turing machines with left reset recognize the class of Turing-recognizable languages.