1.21 Use the procedure described in Lemma 1.60 to convert the following finite automata to regular expressions.

(a)  

(b)  

1.22 In certain programming languages, comments appear between delimiters such as /#/ and #/. Let $C$ be the language of all valid delimited comment strings. A member of $C$ must begin with /#/ and end with #/ but have no intervening #/. For simplicity, we'll say that the comments themselves are written with only the symbols a and b; hence the alphabet of $C$ is $\Sigma = \{a, b, /, #\}$.

a. Give a DFA that recognizes $C$.

b. Give a regular expression that generates $C$.

1.47 Let $\Sigma = \{1, #\}$ and let

$$Y = \{w \mid w = x_1#x_2#\cdots#x_k \text{ for } k \geq 0, \text{ each } x_i \in 1^*, \text{ and } x_i \neq x_j \text{ for } i \neq j\}.$$  

Prove that $Y$ is not regular.

1.53 Let $\Sigma = \{0, 1, +, =\}$ and

$$ADD = \{x=y+z \mid x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}.$$  

Show that $ADD$ is not regular.
1.999 Consider the (decimal) languages defined below. For each one, either give a regular expression for its elements or prove the language is non-regular:

In all examples, a number cannot start with a 0 (unless it is 0 itself)

a) \( L_a = \{ w \mid \text{as an integer } w \text{ is a multiple of 25}\} \).
b) \( L_b = \{ w \mid \text{as an integer } w \text{ is a multiple of 10}\} \).
c) \( L_c = \{ w \mid \text{as an integer } w \text{ is a power of 10}\} \).
d) \( L_d = \{ w \mid \text{as an integer } w \text{ is a multiple of 2}\} \).
e) \( L_e = \{ w \mid \text{as an integer } w \text{ is such that the sum of its digits is a multiple of 2}\} \).
f) \( L_f = \{ w \mid \text{as an integer } w \text{ is a power of 2}\} \).
g) \( L_g = \{ w \mid w \text{ is an integer}\} (\text{with } \sum = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, - \} ) \).
h) \( L_h = \{ w \mid w \text{ is the decimal representation of a rational number}\} (\text{with } \sum = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, - , . , [ , ] \} ) \)

Examples of such strings are 0.[3] representing the number 0.3333… = 1/3 and -23.15[24] representing the number -23.1524242424… = -76403/3300.