1.21 Use the procedure described in Lemma 1.60 to convert the following finite automata to regular expressions.

(a) 

(b) 

1.22 In certain programming languages, comments appear between delimiters such as /# and #/. Let $C$ be the language of all valid delimited comment strings. A member of $C$ must begin with /# and end with #/ but have no intervening #/. For simplicity, we’ll say that the comments themselves are written with only the symbols a and b; hence the alphabet of $C$ is $\Sigma = \{a, b, /, #\}$.

a. Give a DFA that recognizes $C$.

b. Give a regular expression that generates $C$.

1.39 The construction in Theorem 1.54 shows that every GNFA is equivalent to a GNFA with only two states. We can show that an opposite phenomenon occurs for DFAs. Prove that for every $k > 1$ a language $A_k \subseteq \{0, 1\}^*$ exists that is recognized by a DFA with $k$ states but not by one with only $k - 1$ states.

1.47 Let $\Sigma = \{1, \#\}$ and let

$$Y = \{w \mid w = x_1\#x_2\# \cdots \#x_k \text{ for } k \geq 0, \text{ each } x_i \in 1^*, \text{ and } x_i \neq x_j \text{ for } i \neq j\}.$$ 

Prove that $Y$ is not regular.
1.53 Let \( \Sigma = \{0, 1, +, =\} \) and
\[
ADD = \{x = y + z \mid x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}.
\]
Show that \( ADD \) is not regular.

1.60 Let \( \Sigma = \{a, b\} \). For each \( k \geq 1 \), let \( C_k \) be the language consisting of all strings that contain an \( a \) exactly \( k \) places from the right-hand end. Thus \( C_k = \Sigma^* a \Sigma^{k-1} \).
Describe an NFA with \( k + 1 \) states that recognizes \( C_k \), both in terms of a state diagram and a formal description.

1.61 Consider the languages \( C_k \) defined in Problem 1.60. Prove that for each \( k \), no DFA can recognize \( C_k \) with fewer than \( 2^k \) states.

2.6 Give context-free grammars generating the following languages.

[b. The complement of the language \( \{a^n b^n \mid n \geq 0\} \)

d. \( \{x_1 \# x_2 \# \cdots \# x_k \mid k \geq 1, \text{each } x_i \in \{a, b\}^+, \text{ and for some } i \text{ and } j, x_i = x_j^R\} \)

1.64 Let \( N \) be an NFA with \( k \) states that recognizes some language \( A \).

[a. Show that, if \( A \) is nonempty, \( A \) contains some string of length at most \( k \).
[b. Show that, by giving an example, that part (a) is not necessarily true if you replace both \( A \)’s by \( \overline{A} \).
[c. Show that, if \( \overline{A} \) is nonempty, \( \overline{A} \) contains some string of length at most \( 2^k \).
[d. Show that the bound given in part (c) is nearly tight; that is, for each \( k \), demonstrate an NFA recognizing a language \( A_k \) where \( \overline{A_k} \) is nonempty and where \( \overline{A_k} \)’s shortest member strings are of length exponential in \( k \). Come as close to the bound in (c) as you can.