COMP 330A 2019, Assignment 1
Due Tuesday, October 1st 2019

[8%]

1.5 Each of the following languages is the complement of a simpler language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.

d. $\{w|\ w$ is any string not in $\mathtt{a}^*\mathtt{b}^*\}$

e. $\{w|\ w$ is any string not in $(\mathtt{ab}^+)^*\}$

f. $\{w|\ w$ is any string not in $\mathtt{a}^* \cup \mathtt{b}^*\}$

g. $\{w|\ w$ is any string that doesn't contain exactly two a's$\}$

[14%]

1.6 Give state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is $\{\mathtt{0},\mathtt{1}\}$.

a. $\{w|\ w$ begins with a 1 and ends with a 0$\}$

c. $\{w|\ w$ contains the substring 0101 (i.e., $w = x\mathtt{0101}y$ for some x and y)$\}$

d. $\{w|\ w$ has length at least 3 and its third symbol is a 0$\}$

i. $\{w|$ every odd position of w is a 1$\}$

j. $\{w|\ w$ contains at least two 0s and at most one 1$\}$

m. The empty set

n. All strings except the empty string

[16%]

1.14 **a.** Show that if M is a DFA that recognizes language B, swapping the accept and nonaccept states in M yields a new DFA recognizing the complement of B. Conclude that the class of regular languages is closed under complement.

b. Show by giving an example that if M is an NFA that recognizes language C, swapping the accept and nonaccept states in M doesn't necessarily yield a new NFA that recognizes the complement of C. Is the class of languages recognized by NFAs closed under complement? Explain your answer.

In question 1.20 below please provide strings of size at least 7. If no such string exists, just say "none of size ≥7" and give examples of any size.

[12%]

1.20 For each of the following languages, give two strings that are members and two strings that are *not* members—a total of four strings for each part. Assume the alphabet $\Sigma = \{\mathtt{a},\mathtt{b}\}$ in all parts.

a. $\mathtt{a}^*\mathtt{b}^*$

b. $\mathtt{a}(\mathtt{ba})^*\mathtt{b}$

c. $\mathtt{a}^* \cup \mathtt{b}^*$

e. $\Sigma^*\mathtt{a}\Sigma^*\mathtt{b}\Sigma^*\mathtt{a}\Sigma^*$

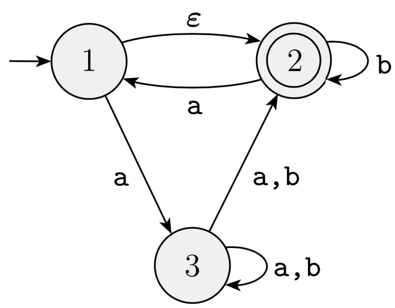
f. $\mathtt{aba} \cup \mathtt{bab}$

g. $(\varepsilon \cup \mathtt{a})\mathtt{b}$

[8%]

1.16 Use the construction given in Theorem 1.39 to convert the following **NFA** to an equivalent **DFA.**

Please notice the instance below is *NOT* the same as in the book…



[8%]

1.31 For any string $w = w_1w_2\cdots w_n$, the ***reverse*** of w, written $w^{\mathcal{R}}$, is the string w in reverse order, $w_n \cdots w_2w_1$. For any language A, let $A^{\mathcal{R}} = \{w^{\mathcal{R}}|\ w \in A\}$. Show that if A is regular, so is $A^{\mathcal{R}}$.

[10%]

1.46 Prove that the following languages are not regular. You may use the **MYHILL-NERODE THEOREM** and the closure of the class of regular languages under union, intersection, and complement.

a. $\{0^n1^m0^n|\ m,n \geq 0\}$

c. $\{w|\ w \in \{0,1\}^*$ is not a palindrome$\}$[8]

[12%] **1.98** Consider the (binary) language **L**={ w | **010** or **101** is a substring of w}.

a) Draw a DFA that recognizes exactly the regular language **L**.

b) Prove by mathematical induction that the DFA you provided in a) recognizes exactly the language **L**.

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[8]A ***palindrome*** is a string that reads the same forward and backward.

[12%] **1.99** Consider the following regular languages over binary alphabet $\{0,1\}$

$$\mathbf{L_1} = \{\text{ number w is a multiple of 3}\},$$

$$\mathbf{L_2} = \left\{ w = w_1 w_2 \ldots w_n \;\middle|\; \sum_{k=1}^{n} (-1)^k w_k \text{ is a multiple of 3} \right\}$$

$$\mathbf{L_3} = \left\{ w = w_n w_{n-1} \ldots w_1 \;\middle|\; \sum_{k=1}^{n} (-1)^k w_k \text{ is a multiple of 3} \right\}.$$

a) Identify manually which of these numbers are multiples of 3:
(Each line is a single number. I broke it down in blocks of 6 bits for readability.)

1) 101010 101011 010100 111010 110110 110101 010101 101111 101101
2) 111111 111111 111111 111011 110111 101111 110111 111110 111111
3) 100001 000000 100000 000001 000000 000010 000000 100000 010101
4) 110011 001100 110011 001100 110011 001100 110011 001100 110011

b) Prove that $\mathbf{L_1} = \mathbf{L_2} = \mathbf{L_3}$.

c) Explain the link with the automaton seen in class:

$M_{3,2}$