[10%]

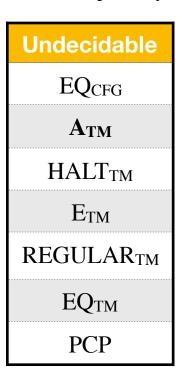
1) Let EQ_{CFG-TM} be the language of CFG and Turing machine descriptions with identical languages, i.e.

$$EQ_{CFG-TM} = \{ \langle G, M \rangle \mid L(G) = L(M) \}.$$

Show that **EQ**_{CFG-TM} is an undecidable language.

The table on the right is provided as a reminder of what we already showed in class.

You are allowed to use any if relevant...



[10%]

2) Using the Pumping Lemma show that

FIB =
$$\{1^{F_n} \mid n \ge 0\}$$
 is NON-REG.

$$F_0 = 0$$
, $F_1 = 1$

$$F_n = F_{n-1} + F_{n-2}$$
 $n \ge 2$

3) At some point in class we saw the grammar: G_2 : $R \rightarrow \epsilon \mid 0R \mid 1R$.

[5%]

a) Give a grammar C_2 in Chomsky normal form equivalent to G_2 using the general method learned in class to transform CFGs into CNF.

[10%]

b) Obtain a **PDA** accepting the language generated by **G**₂ using the general method learned in class to transform **CFGs** into **PDAs**.

[5%]

c) Provide the simplest possible PDA accepting L(G₂).

[+10%]

[10%]

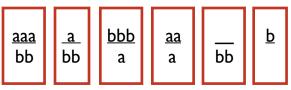
[10%]

4) In class (with prof. Panangaden) we have seen techniques to minimize DFAs and the Myhill-Nerode Theorem which states exactly the least number of states necessary to recognize a given regular language.

- a) Give some technique to minimize the number of variables used in a CFG.
- **b)** Argue that an analog of the *Myhill-Nerode* Theorem is not likely to be discovered.
- **c)** If you can, prove formally that the grammar with fewest variables equivalent to a given grammar **G** is uncomputable.

5) Short and sweet

(a) Consider the following tiles as an instance of MPCP. Give an equivalent instance of PCP which is solvable if and only if the instance below is solvable.



[10%]

- (b) Give a unary language (using only input alphabet $\Sigma = \{1\}$) that is not Turingrecognizable and prove that statement.
- 6) Consider the problem:

PARTITION

INSTANCE: A finite set A and a "size" $s(a) \in Z^+$ for each $a \in A$.

QUESTION: Is there a subset $A' \subseteq A$ such that

$$\sum_{a\in A'} s(a) = \sum_{a\in A-A'} s(a) ?$$

[1**0**%]

a) Prove that **PARTITION** \in **NP**.

For the next problem you may use without proof that **PARTITION** \in **NP-complete**. Consider the problem:

SUBSET SUM

INSTANCE: An integer K, a finite set A and a "size" $s(a) \in Z^+$ for each $a \in A$. QUESTION: Is there a non-empty subset A' ⊆ A such that

$$\sum_{a \in A'} s(a) = K?$$

b) Prove that SUBSET

SUM \in NP-complete.