1) Let $\text{EQ}_{\text{CFG-TM}}$ be the language of CFG and Turing machine descriptions with identical languages, i.e.

$$\text{EQ}_{\text{CFG-TM}} = \{ \langle G, M \rangle \mid L(G) = L(M) \}.$$ 

Show that $\text{EQ}_{\text{CFG-TM}}$ is an undecidable language.

The table on the right is provided as a reminder of what we already showed in class.

You are allowed to use any if relevant…

2) Using the Pumping Lemma show that

$\text{FIB} = \{ 1^n \mid n \geq 0 \}$ is NON-REG.

$$F_0 = 0, \quad F_1 = 1$$

$$F_n = F_{n-1} + F_{n-2} \quad n \geq 2$$

3) At some point in class we saw the grammar: $G_2: R \rightarrow \varepsilon | 0R | 1R$.

a) Give a grammar $C_2$ in Chomsky normal form equivalent to $G_2$ using the general method learned in class to transform CFGs into CNF.

b) Obtain a PDA accepting the language generated by $G_2$ using the general method learned in class to transform CFGs into PDAs.

c) Provide the simplest possible PDA accepting $L(G_2)$.
4) In class (with prof. Panangaden) we have seen techniques to minimize DFAs and the *Myhill-Nerode* Theorem which states exactly the least number of states necessary to recognize a given regular language.

a) Give some technique to minimize the number of variables used in a *CFG*.

b) Argue that an analog of the *Myhill-Nerode* Theorem is not likely to be discovered.

c) If you can, prove formally that the grammar with fewest variables equivalent to a given grammar *G* is uncomputable.

5) Short and sweet

(a) Consider the following tiles as an instance of MPCP. Give an equivalent instance of PCP which is solvable if and only if the instance below is solvable.

```
aaa
bb
```
```
 a
bb
```
```
bbb
a
```
```
a
bb
```
```
b
```

(b) Give a unary language (using only input alphabet \( \Sigma = \{1\} \)) that is not Turing-recognizable and prove that statement.

6) Consider the problem:

**PARTITION**

INSTANCE: A finite set \( A \) and a "size" \( s(a) \in \mathbb{Z}^+ \) for each \( a \in A \).

QUESTION: Is there a subset \( A' \subseteq A \) such that

\[
\sum_{a \in A'} s(a) = \sum_{a \in A - A'} s(a) ?
\]

a) Prove that **PARTITION** \( \in \text{NP} \).

For the next problem you may use without proof that **PARTITION** \( \in \text{NP-complete} \).

Consider the problem:

**SUBSET SUM**

INSTANCE: An integer \( K \), a finite set \( A \) and a "size" \( s(a) \in \mathbb{Z}^+ \) for each \( a \in A \).

QUESTION: Is there a non-empty subset \( A' \subseteq A \) such that

\[
\sum_{a \in A'} s(a) = K ?
\]

b) Prove that **SUBSET SUM** \( \in \text{NP-complete} \).