[10%]

1) Let $EQ_{CFG\text{-}TM}$ be the language of CFG and Turing machine descriptions with identical languages, i.e.

$$EQ_{CFG\text{-}TM} = \{ \langle G,M \rangle \mid L(G)=L(M) \} .$$

Show that $\mathbf{EQ_{CFG\text{-}TM}}$ is an undecidable language.

The table on the right is provided as a reminder of what we already showed in class.

You are allowed to use any if relevant…

Undecidable
EQ_{CFG}
A_{TM}
$HALT_{TM}$
E_{TM}
$REGULAR_{TM}$
EQ_{TM}
PCP

[10%]

2) Using the Pumping Lemma show that

$$FIB = \{ 1^{F_n} \mid n \geq 0 \} \text{ is NON-REG.}$$

$$F_0 = 0,\ F_1 = 1$$

$$F_n = F_{n-1} + F_{n-2} \qquad n \geq 2$$

3) At some point in class we saw the grammar: G_2: $R \rightarrow \varepsilon \mid 0R \mid 1R$.

[5%] a) Give a grammar $\mathbf{C_2}$ in Chomsky normal form equivalent to $\mathbf{G_2}$ using the general method learned in class to transform **CFGs** into **CNF**.

[10%] b) Obtain a **PDA** accepting the language generated by $\mathbf{G_2}$ using the general method learned in class to transform **CFGs** into **PDAs.**

[5%] c) Provide the simplest possible **PDA** accepting $\mathbf{L(G_2)}$.

• • <u>OPEN • BOOKS •/• OPEN • NOTES</u> • •

4) In class (with prof. *Panangaden*) we have seen techniques to minimize DFAs and the *Myhill-Nerode* Theorem which states exactly the least number of states necessary to recognize a given regular language.

[10%] **a)** Give some technique to minimize the number of variables used in a **CFG**.

[10%] **b)** Argue that an analog of the *Myhill-Nerode* Theorem is not likely to be discovered,

[+10%] **c)** If you can, prove formally that the grammar with fewest variables equivalent to a given grammar **G** is uncomputable.

5) Short and sweet

[10%] (a) Consider the following tiles as an instance of MPCP. Give an equivalent instance of PCP which is solvable if and only if the instance below is solvable.

aaa	a	bbb	aa		b
bb	bb	a	a	bb	

[10%] (b) Give a unary language (using only input alphabet $\Sigma = \{ 1 \}$) that is not Turing-recognizable and prove that statement.

6) Consider the problem:

PARTITION
INSTANCE: A finite set A and a "size" $s(a) \in Z^+$ for each $a \in A$.
QUESTION: Is there a subset $A' \subseteq A$ such that

$$\sum_{a \in A'} s(a) = \sum_{a \in A - A'} s(a) \ ?$$

[10%] **a)** Prove that **PARTITION** $\in$ **NP**.

For the next problem you may use without proof that **PARTITION** $\in$ **NP-complete**.
Consider the problem:

SUBSET SUM
INSTANCE: An integer K, a finite set A and a "size" $s(a) \in Z^+$ for each $a \in A$.
QUESTION: Is there a non-empty subset $A' \subseteq A$ such that

$$\sum_{a \in A'} s(a) = K\,?$$

[10%] **b)** Prove that **SUBSET SUM** $\in$ **NP-complete**.