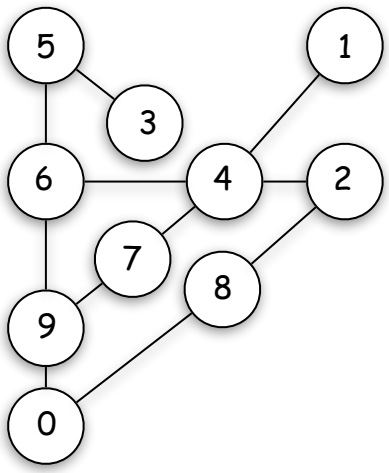
Computer Science COMP-251A
Midterm, Oct 20, 2015, 11:35-12:55.
O P E N • B O O K S •/• O P E N • N O T E S

[40%]

1) Consider the following graph:



a) Give this graph in its adjacency matrix representation.

b) Give this graph in its adjacency list representation.

c) How would you determine whether this graph is bipartite or not ?

d) Show as whether *this* graph is actually bipartite or not.

[40%]

2)

a) Provide an algorithm that runs in time $\Theta(n^2 \log n)$ in worse case. Explain why this is its running time. I don't care what it does. I only care about running time…

b) Provide a recursive algorithm that runs in time $T(n)$, defined recursively, but such that the solution cannot be found by the Master Method.

c) Let $f(n)=25n^4+n^2+124$ and $g(n)=13n^3+5n^2+n \log n$.
Prove that $f(n) \bullet g(n)$ **is** $\Theta(n^7)$.

d) We know that $\sum_{i=1}^{n} i = n(n+1)/2$, $\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$ and $\sum_{i=1}^{n} i^3 = (n(n+1)/2)^2$.
Show that in general $\sum_{i=1}^{n} i^k$ **is** $\Theta(n^{k+1})$ for any positive integer k.

[30%]

3)

Given a sorted array of n distinct integers A[1,…,n], you want to find out whether there is an index i for which A[i] = i.

a) Give an algorithm that runs in time $O(\log n)$ for this problem.

b) Set the recurrence to describe the worse case running time T(n) of your algorithm.

c) Show that T(n) **is** O(log n) using the Master Method.