1) Consider the following graph:

a) Give this graph in its adjacency matrix representation.
b) Give this graph in its adjacency list representation.
c) How would you determine whether this graph is bipartite or not?
d) Show as whether this graph is actually bipartite or not.

2) a) Provide an algorithm that runs in time $\theta(n^2 \log n)$ in worse case. Explain why this is its running time. I don’t care what it does. I only care about running time…

b) Provide a recursive algorithm that runs in time $T(n)$, defined recursively, but such that the solution cannot be found by the Master Method.

c) Let $f(n)=25n^4+n^2+124$ and $g(n)=13n^3+5n^2+n \log n$. Prove that $f(n)\cdot g(n)$ is $\theta(n^7)$.

d) We know that $\sum_{i=1}^{n} i = n(n+1)/2$, $\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$ and $\sum_{i=1}^{n} i^3 = (n(n+1)/2)^2$. Show that in general $\sum_{i=1}^{n} i^k$ is $\theta(n^{k+1})$ for any positive integer $k$. 
3) Given a sorted array of \( n \) distinct integers \( A[1, \ldots, n] \), you want to find out whether there is an index \( i \) for which \( A[i] = i \).

   a) Give an algorithm that runs in time \( O(\log n) \) for this problem.

   b) Set the recurrence to describe the worse case running time \( T(n) \) of your algorithm.

   c) Show that \( T(n) \) is \( O(\log n) \) using the Master Method.