1) a) Write a recursive algorithm such that the related time recurrence cannot be solved with the Master Theorem.

b) Write two recurrences such that their solutions are $\Theta(n^3)$ according to distinct cases of the Master Theorem.

2) Consider the 9x9 smiley below as the adjacency matrix of a graph (dark=0, light=1).

a) Is this graph directed or undirected?

b) Is this graph cyclic or not?

c) Draw the graph using vertices and edges.

D) Give this same graph in its adjacency list representation.
3) a) Construct a $O(\log n)$-time Divide-and-conquer algorithm to find in a sorted array $A[1..n]$ whether there is an index $i$ such that $A[i]=i$.

**Hint**: notice that $A[i]=i$ if and only if $A[i]-i = 0$.

b) Write the recurrence associated to the running time of your algorithm, and show its solution is $O(\log n)$.

4) a) Consider this map of Vietnamese provinces. Is it possible to assign one of 4 colors to each province so that no two touching provinces are of the same color?

Why?

b) How many topological ordering does this graph have?

c) Order the following Big-O functions using $=$ or $\subset$ (Justify): $O(9^{\lg n})$, $O(n^3)$, $O(3^3)$