

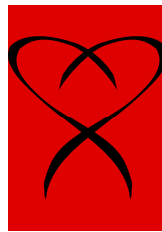
Computer Science COMP-251B
Midterm, Feb 14, 2008, 14:35-15:55.
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[25%]

1) Let

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ T(\lceil n/6 \rceil) + 2T(\lceil n/3 \rceil) + O(n) & \text{if } n>1 \end{cases}$$

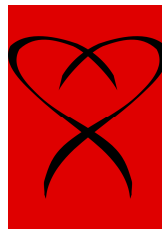
Prove by constructive induction that $T(n)$ is $O(n)$.



[25%]

2) **Exercises 9.3-5**

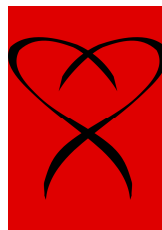
Suppose that you have a “black-box” (you don’t know how it solves the problem), worst-case linear-time, median subroutine. Give a simple, linear-time algorithm that solves the selection problem for an arbitrary order statistic.



[25%]

3) **Exercises 5.2-4**

Use indicator random variables to solve the following problem, which is known as the **hat-check problem**. Each of n customers gives a hat to a hat-check person at a restaurant. The hat-check person gives the hats back to the customers in a random order. What is the expected number of customers that get back their own hat?



[25%]

4) **Exercises 7.4-2**

Show that quicksort’s best-case running time is $\Omega(n \lg n)$.

