Computer Science COMP-251B

Midterm, Feb 14, 2008, 14:35-15:55.

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[25%] **1)**
Let

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ T(\lceil n/6 \rceil) + 2T(\lceil n/3 \rceil) + O(n) & \text{if } n>1 \end{cases}$$

Prove by constructive induction that T(n) is O(n).

[25%] **2) Exercises 9.3-5**
Suppose that you have a "black-box" (you don't know how it solves the problem), worst-case linear-time, median subroutine. Give a simple, linear-time algorithm that solves the selection problem for an arbitrary order statistic.

[25%] **3) Exercises 5.2-4**
Use indicator random variables to solve the following problem, which is known as the ***hat-check problem***. Each of *n* customers gives a hat to a hat-check person at a restaurant. The hat-check person gives the hats back to the customers in a random order. What is the expected number of customers that get back their own hat?

[25%] **4) Exercises 7.4-2**
Show that quicksort's best-case running time is $\Omega(n \lg n)$.