Computer Science COMP-251B
Midterm, Feb 17, 2005, 14:35-15:55.
O P E N • B O O K S •/• O P E N • N O T E S

[20%] **1)**
Let

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ T(\lceil n/5 \rceil) + T(\lceil n/4 \rceil) + T(\lceil n/3 \rceil) + O(n) & \text{if } n>1 \end{cases}$$

Prove by constructive induction that T(n) is O(n).

[20%] **2)**
Explain how to make quick-sort run in worse case O(n log n) time.
Explain why I named this modified algorithm "slow-sort" in class.

[20%] **3)**
Let n be a power of 2. Give an exact lower bound on the number of comparisons that merge-sort uses to sort n elements, in the worst case.
(**HINT**: Assume that the solution to problem 8-6 (2n-1) is an exact lower bound).

[20%] **4) exercise 8.4-2**
What is the worst-case running time for the bucket-sort algorithm? What simple change to the algorithm preserves its linear expected running time and makes its worst-case running time $O(n \lg n)$?

[20%] **5)**
Prove that $x=\text{median}(x_1,x_2,\ldots,x_n)$ (where $x_i \le x_{i+1}$) is <u>a</u> value that minimizes

$$SAD(x) = |x_1-x| + |x_2-x| + \ldots + |x_n-x|. \qquad \text{(SAD=Sum of Absolute Differences)}$$

HINT: rewrite

$$SAD(x) = (2i-n)x - (x_1 + x_2 + \ldots + x_i) + (x_{i+1} + x_{i+2} + \ldots + x_n) \text{ when } x_i \le x \le x_{i+1}.$$

Why cannot we say "$x=\text{median}(x_1,x_2,\ldots,x_n)$ is <u>the</u> value that minimizes SAD(x)" ?