# Computer Science 308-250B
## Midterm, Feb 20, 2004, 13:30-14:30.
## O P E N • B O O K S •/• O P E N • N O T E S

[30%]

1) Show the following (and justify your steps)

   a) $n^{\log n}$ **is** $O(n^{\sqrt{n}})$.

   b) for any positive constant **a** we have, n! **is** $\Omega(a^n)$.

[40%]

2)
**(a)** Solve the following recurrence T(n) and express your answer using Big-Θ

$$T(n) = \begin{cases} 12 & \text{if } n<3 \\ 5T(n/2) + n \log n + n^2 & \text{if } n>2 \end{cases}$$

**(b)** Let **A** and **B** be algorithms solving the same problem and let $\mathbf{T_A(n)}$ and $\mathbf{T_B(n)}$ be the worst case running times of **A** and **B** on inputs of size **n**. Consider a situation in which $\mathbf{T_A(n)}$ **is** $\mathbf{\Theta(n^2)}$, while $\mathbf{T_B(n)}$ **is** $\mathbf{\Theta(n^3)}$.

Enumerate 3 distinct cases where algorithm **B** may still be better than algorithm **A**.

[30%]

3) Consider the following algorithms for computing the Greatest Common Divisor (`GCD`) of two integers `a,b`.

```
GCD(a,b)

IF a=0 THEN RETURN b
 ELSE RETURN GCD((b mod a),a)
```

(a) Write an iterative version of this algorithm.

(b) Prove by mathematical induction that for all **a,b≥0** this algorithm always terminates.
*Suggestion:* prove by induction on **a** that for all **b≥0** this algorithm always terminates.