Winter 2016
COMP-250: Introduction to Computer Science

Lecture 3, January 19, 2016

COMP-250 Weekly Schedule

Omar MC-102	Faizy TR-3110	DavidB-R TR-3110	**Thu** 10:00	Chris TR-3110
			Thu 10:30	
			Thu 11:00	
Mon 11:30	**Tue** 11:30	**Wed** 11:30	**Thu** 11:30	**Fri** 11:30
Faiz TR-3110	**Tue** 12:00	DoYeon TR-3110	**Thu** 12:00	Claude MC-110N
	Tue 12:30		**Thu** 12:30	
Mon 13:30	Claude ST-S1/4	DavidB TR-3110	Claude ST-S1/4	
Mon 14:00				
Mon 14:30	**Tue** 14:30		**Thu** 14:30	
Mon 15:00	**Tue** 15:00	**Wed** 15:00	Hardik TR-3110	**Fri** 15:00
Mon 15:30	**Tue** 15:30	**Wed** 15:30		**Fri** 15:30
Mon 16:00	**Tue** 16:00	**Wed** 16:00		**Fri** 16:00

MC=MCENG ST=STBIO TR=TRENG

Analysis of Algorithms

Analysis of Addition

$$carry = 0$$

for $i = 0$ to $N - 1$ **do**

$$r[i] \leftarrow (a[i] + b[i] + carry) \ \% \ 10$$

$$carry \leftarrow (a[i] + b[i] + carry)/10$$

end for

$$r[N] \leftarrow carry$$

Analysis of Addition

$$\texttt{cst}\ \{ \mathit{carry} = 0$$

$$\textbf{for } i = 0 \text{ to } N-1 \ \ \textbf{do}$$

$$r[i] \leftarrow (a[i] + b[i] + \mathit{carry})\ \%\ 10$$

$$\mathit{carry} \leftarrow (a[i] + b[i] + \mathit{carry})/10$$

$$\textbf{end for}$$

$$r[N] \leftarrow \mathit{carry}$$

Analysis of Addition

cst { $carry = 0$

for $i = 0$ to $N - 1$ **do**

cst { $r[i] \leftarrow (a[i] + b[i] + carry)\ \%\ 10$

$carry \leftarrow (a[i] + b[i] + carry)/10$ }

end for

$r[N] \leftarrow carry$

Analysis of Addition

cst { $carry = 0$

linear {

for $i = 0$ to $N - 1$ **do**

cst {

$r[i] \leftarrow (a[i] + b[i] + carry)\ \%\ 10$

$carry \leftarrow (a[i] + b[i] + carry)/10$

end for

$r[N] \leftarrow carry$

Analysis of Addition

cst $\{ carry = 0$

linear $\left\{ \begin{array}{l} \textbf{for } i = 0 \text{ to } N-1 \textbf{ do} \\ \text{cst} \left\{ \begin{array}{l} r[i] \leftarrow (a[i] + b[i] + carry) \;\%\; 10 \\ carry \leftarrow (a[i] + b[i] + carry)/10 \end{array} \right. \\ \textbf{end for} \end{array} \right.$

cst $\{ r[N] \leftarrow carry$

Analysis of Addition

cst $\{ carry = 0$

linear $\left\{ \begin{array}{l} \textbf{for } i = 0 \text{ to } N-1 \textbf{ do} \\ \text{cst} \left\{ \begin{array}{l} r[i] \leftarrow (a[i] + b[i] + carry) \ \% \ 10 \\ carry \leftarrow (a[i] + b[i] + carry)/10 \end{array} \right. \\ \textbf{end for} \end{array} \right.$

cst $\{ r[N] \leftarrow carry$

$$\text{Time}(N) = c_1 + c_2 \times N$$

Analysis of Multiplication

for $j = 0$ to $N - 1$ **do**
  $carry \leftarrow 0$
  for $i = 0$ to $N - 1$ **do**
    $prod \leftarrow (a[i] * b[j] + carry)$
    $tmp[j][i + j] \leftarrow prod\%10$
    $carry \leftarrow prod/10$
  end for
  $tmp[j][N + j] \leftarrow carry$
end for

Analysis of Multiplication

for $j = 0$ to $N - 1$ **do**

cst $\{ carry \leftarrow 0$

  for $i = 0$ to $N - 1$ **do**

    $prod \leftarrow (a[i] * b[j] + carry)$

    $tmp[j][i + j] \leftarrow prod\%10$

    $carry \leftarrow prod/10$

  end for

  $tmp[j][N + j] \leftarrow carry$

end for

Analysis of Multiplication

for $j = 0$ to $N - 1$ **do**

cst $\{carry \leftarrow 0$

for $i = 0$ to $N - 1$ **do**

cst $\left\{\begin{array}{l} prod \leftarrow (a[i] * b[j] + carry) \\ tmp[j][i+j] \leftarrow prod\%10 \\ carry \leftarrow prod/10 \end{array}\right.$

end for

$tmp[j][N + j] \leftarrow carry$

end for

Analysis of Multiplication

for $j = 0$ to $N-1$ **do**

cst { $carry \leftarrow 0$

linear {

  for $i = 0$ to $N-1$ **do**

  cst {

    $prod \leftarrow (a[i] * b[j] + carry)$

    $tmp[j][i+j] \leftarrow prod\%10$

    $carry \leftarrow prod/10$

  end for

  $tmp[j][N+j] \leftarrow carry$

end for

Analysis of Multiplication

for $j = 0$ to $N-1$ **do**

cst { $carry \leftarrow 0$

linear {

for $i = 0$ to $N-1$ **do**

cst {

$prod \leftarrow (a[i] * b[j] + carry)$

$tmp[j][i+j] \leftarrow prod\%10$

$carry \leftarrow prod/10$

end for

cst { $tmp[j][N+j] \leftarrow carry$

end for

Analysis of Multiplication

quadratic {

for $j = 0$ to $N-1$ **do**

cst $\{carry \leftarrow 0$

linear {

for $i = 0$ to $N-1$ **do**

cst {

$prod \leftarrow (a[i] * b[j] + carry)$

$tmp[j][i+j] \leftarrow prod\%10$

$carry \leftarrow prod/10$

end for

cst $\{tmp[j][N+j] \leftarrow carry$

end for

Analysis of Multiplication

$carry \leftarrow 0$
for $i = 0$ to $2 * N - 1$ **do**
    $sum \leftarrow carry$
    for $j = 0$ to $N - 1$ **do**
        $sum \leftarrow sum + tmp[j][i]$
    end for
    $r[i] \leftarrow sum\%10$
    $carry \leftarrow sum/10$
end for
$r[2 * N] \leftarrow carry$

Analysis of Multiplication

cst {

$carry \leftarrow 0$

for $i = 0$ to $2 * N - 1$ **do**

$sum \leftarrow carry$

for $j = 0$ to $N - 1$ **do**

$sum \leftarrow sum + tmp[j][i]$

end for

$r[i] \leftarrow sum\%10$

$carry \leftarrow sum/10$

end for

$r[2 * N] \leftarrow carry$

Analysis of Multiplication

cst { $carry \leftarrow 0$

for $i = 0$ to $2 * N - 1$ **do**

cst { $sum \leftarrow carry$

for $j = 0$ to $N - 1$ **do**

$sum \leftarrow sum + tmp[j][i]$

end for

$r[i] \leftarrow sum\%10$

$carry \leftarrow sum/10$

end for

$r[2 * N] \leftarrow carry$

Analysis of Multiplication

cst { $carry \leftarrow 0$

for $i = 0$ to $2 * N - 1$ **do**

cst { $sum \leftarrow carry$

for $j = 0$ to $N - 1$ **do**

cst { $sum \leftarrow sum + tmp[j][i]$

end for

$r[i] \leftarrow sum\%10$

$carry \leftarrow sum/10$

end for

$r[2 * N] \leftarrow carry$

Analysis of Multiplication

cst { $carry \leftarrow 0$

for $i = 0$ to $2 * N - 1$ **do**

cst { $sum \leftarrow carry$

linear {

for $j = 0$ to $N - 1$ **do**

cst { $sum \leftarrow sum + tmp[j][i]$

end for

}

$r[i] \leftarrow sum\%10$

$carry \leftarrow sum/10$

end for

$r[2 * N] \leftarrow carry$

Analysis of Multiplication

cst { $carry \leftarrow 0$

for $i = 0$ to $2 * N - 1$ **do**

cst { $sum \leftarrow carry$

linear {

for $j = 0$ to $N - 1$ **do**

cst { $sum \leftarrow sum + tmp[j][i]$

end for

}

cst {

$r[i] \leftarrow sum\%10$

$carry \leftarrow sum/10$

}

end for

$r[2 * N] \leftarrow carry$

Analysis of Multiplication

cst { $carry \leftarrow 0$

quadratic {

for $i = 0$ to $2 * N - 1$ **do**

  cst { $sum \leftarrow carry$

  linear {

  for $j = 0$ to $N - 1$ **do**

    cst { $sum \leftarrow sum + tmp[j][i]$

  end for

  cst {

  $r[i] \leftarrow sum\%10$

  $carry \leftarrow sum/10$

end for

$r[2 * N] \leftarrow carry$

Analysis of Multiplication

[cst] $carry \leftarrow 0$

[quadratic] **for** $i = 0$ to $2 * N - 1$ **do**

[cst] $sum \leftarrow carry$

[linear] **for** $j = 0$ to $N - 1$ **do**

[cst] $sum \leftarrow sum + tmp[j][i]$

end for

[cst] $r[i] \leftarrow sum\%10$

$carry \leftarrow sum/10$

end for

[cst] $r[2 * N] \leftarrow carry$

Analysis of Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

for $j = 0$ to $N - 1$ **do**
  $carry \leftarrow 0$
  for $i = 0$ to $N - 1$ **do**
    $prod \leftarrow (a[i] * b[j] + carry)$
    $tmp[j][i + j] \leftarrow prod\%10$
    $carry \leftarrow prod/10$
  end for
  $tmp[j][N + j] \leftarrow carry$
end for

$carry \leftarrow 0$
for $i = 0$ to $2 * N - 1$ **do**
  $sum \leftarrow carry$
  for $j = 0$ to $N - 1$ **do**
    $sum \leftarrow sum + tmp[j][i]$
  end for
  $r[i] \leftarrow sum\%10$
  $carry \leftarrow sum/10$
end for
$r[2 * N] \leftarrow carry$

Analysis of Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

```
for j = 0 to N − 1 do
    carry ← 0
    for i = 0 to N − 1 do
        prod ← (a[i] * b[j] + carry)
        tmp[j][i + j] ← prod%10
        carry ← prod/10
    end for
    tmp[j][N + j] ← carry
end for
```

```
carry ← 0
for i = 0 to 2 * N − 1 do
    sum ← carry
    for j = 0 to N − 1 do
        sum ← sum + tmp[j][i]
    end for
    r[i] ← sum%10
    carry ← sum/10
end for
r[2 * N] ← carry
```

$$\text{Time}(N) = c_1 + c_2 \times N + c_3 \times N^2$$

Analysis of Algorithms

Addition

$$\texttt{Time(N)} = c_1 + c_2 \times N$$

Multiplication

$$\texttt{Time(N)} = c_1 + c_2 \times N + c_3 \times N^2$$

Analysis of Algorithms

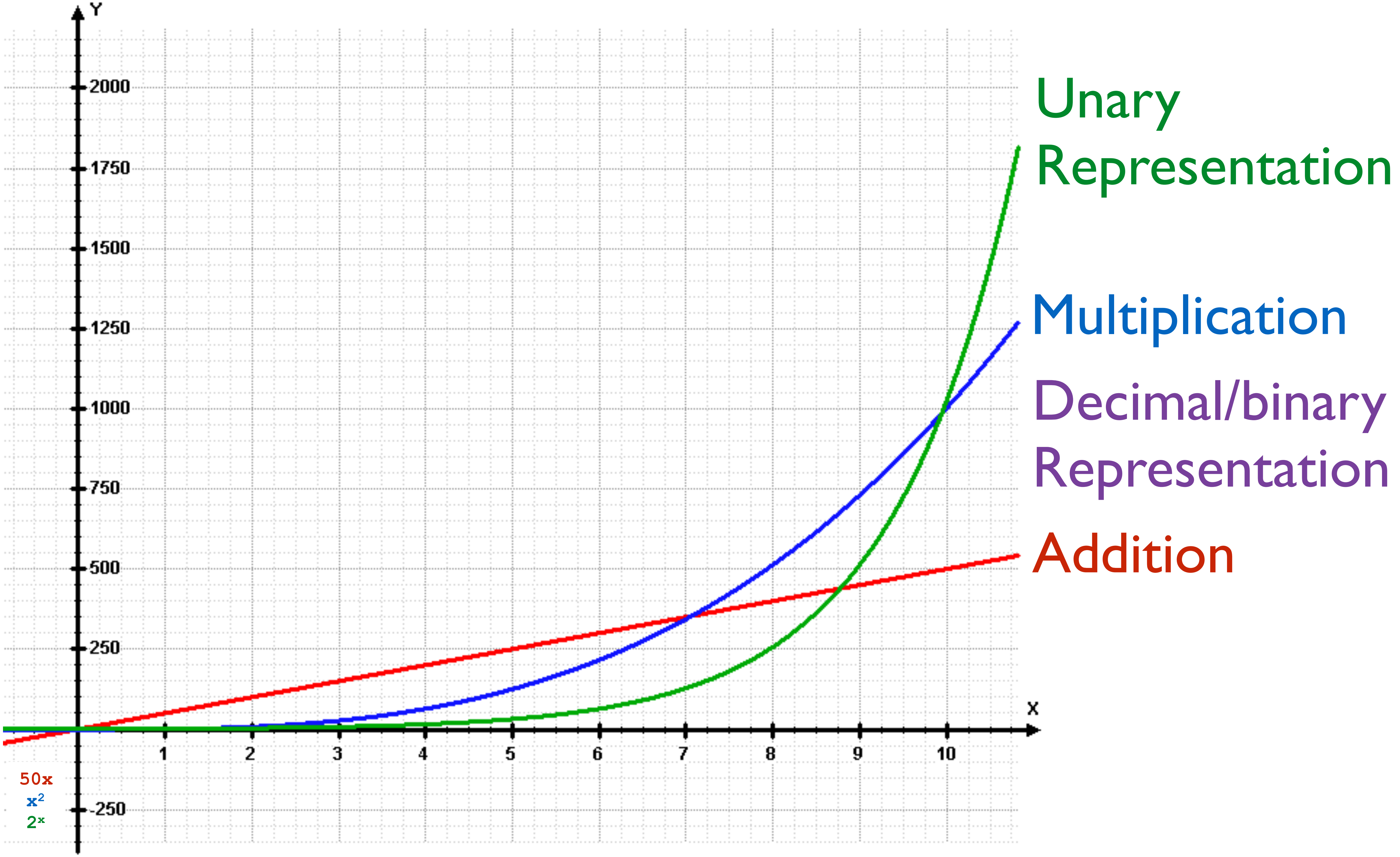
Addition

`Time(N)` **`is`** `O(N)`

Multiplication

`Time(N)` **`is`** $O(N^2)$

Analysis of Algorithms



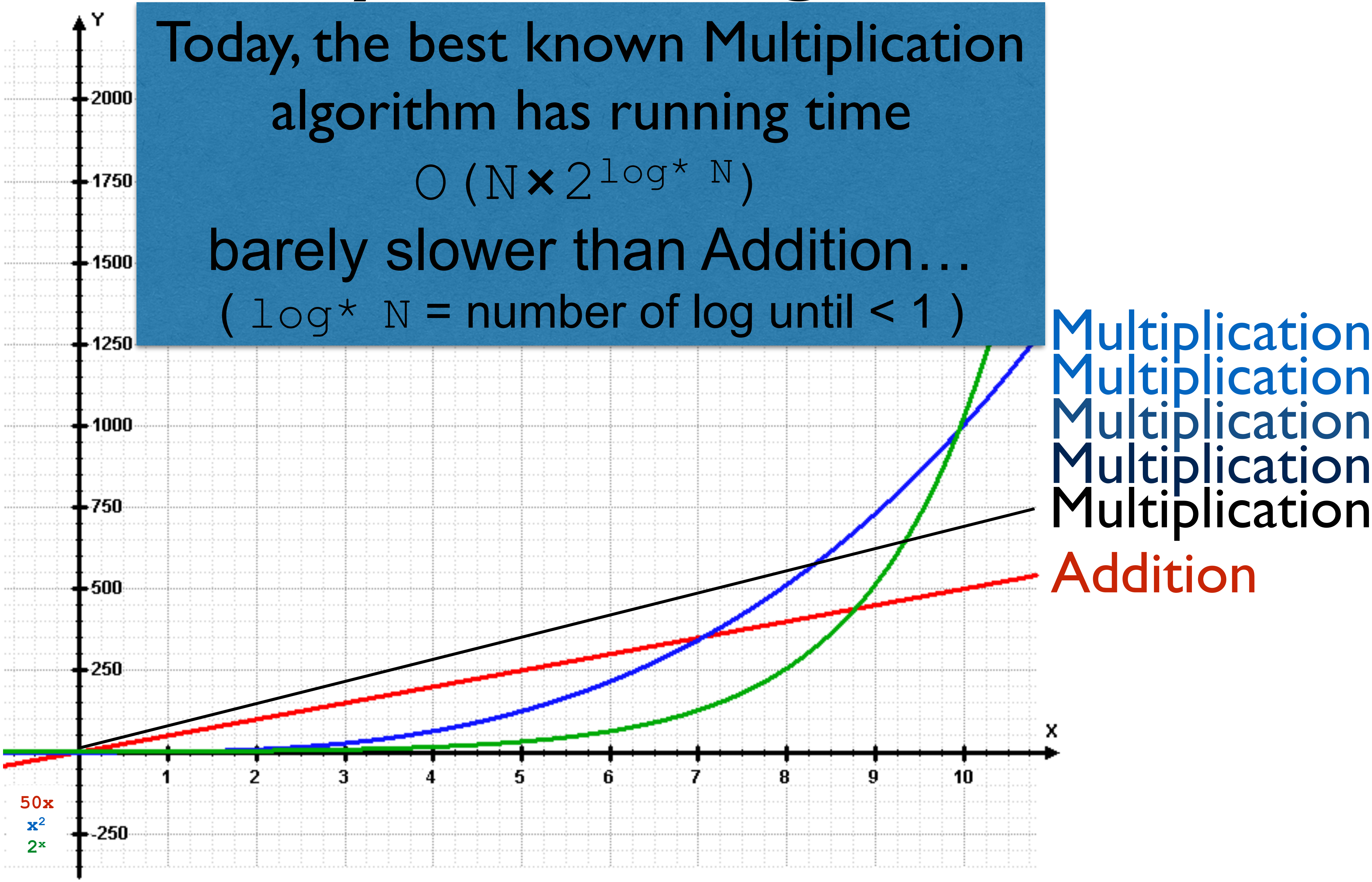
Analysis of Algorithms

Today, the best known Multiplication algorithm has running time

$O(N \times 2^{\log^* N})$

barely slower than Addition…

($\log^* N$ = number of log until < 1)



Bases and Binary Representation

Base 8 vs Base 2

$$(2143)_8$$

$$=(????)_2$$

Base 8 vs Base 2

$$(2143)_8$$

$$=(\ 010\ \ 001\ \ 100\ \ 101\ )_2$$

$$=(10001100101)_2$$

Base 2 vs Base 8

$$(1001001010101010101)_2$$
$$=(10\ 010\ 010\ 101\ 010\ 101)_2$$
$$=(2\ 2\ 2\ 5\ 2\ 5)_8$$
$$=(222525)_8$$

Base 2 vs Base 10

$$(10010010101010101)_2$$
$$=(65536+8192+1024+256+64+16+4+1)_{10}$$
$$=(75093)_{10}$$

Powers of 2 in Base 10

$2^0=1$ $2^1=2$ $2^2=4$

$2^3=8$ $2^4=16$ $2^5=32$

$2^6=64$ $2^7=128$ $2^8=256$

$2^9=512$ $2^{10}=1024$ $2^{11}=2048$

$2^{12}=4096$ $2^{13}=8192$ $2^{14}=16384$

$2^{15}=32768$ $2^{16}=65536$

$2^{32} = 4\,294\,967\,296$

Powers of 10 in Base 2

10^0=1

10^1=1010

10^2=1100110

10^3=1111101000 $\approx 2^{10}$

10^4=10011100010000

Base 10 vs Base 2

$$
\begin{aligned}
&(75093)_{10} \\
&= (111*10011100010000 \\
&\quad +101*1111101000 \\
&\quad +1001*1010 \\
&\quad +101)_{2} \\
&= (10010010101010101)_{2}
\end{aligned}
$$

to Base 2

Algorithm 3 Convert integer to binary

INPUT: a number m

OUTPUT: the number m **expressed in base 2 using a bit array** $b[\,]$

$i \leftarrow 0$

while $m > 0$ **do**

  $b[i] \leftarrow m\%2$

  $m \leftarrow m/2$

  $i \leftarrow i + 1$

end while

75093

75093

/2 %2

37546 1

75093	
/2	%2
37546	1
18773	0

75093	
/2	%2
37546	1
18773	0
9386	1

75093

/2	%2
37546	1
18773	0
9386	1
4693	0

75093	
/2	%2
37546	1
18773	0
9386	1
4693	0
2346	1

75093

/2	%2
37546	1
18773	0
9386	1
4693	0
2346	1
1173	0

75093

/2	%2
37546	1
18773	0
9386	1
4693	0
2346	1
1173	0
586	1

75093

/2	%2
37546	1
18773	0
9386	1
4693	0
2346	1
1173	0
586	1
293	0

75093

/2	%2
37546	1
18773	0
9386	1
4693	0
2346	1
1173	0
586	1
293	0
146	1

75093

/2	%2
37546	1
18773	0
9386	1
4693	0
2346	1
1173	0
586	1
293	0
146	1

/2	%2
73	0

75093

/2	%2
37546	1
18773	0
9386	1
4693	0
2346	1
1173	0
586	1
293	0
146	1

/2	%2
73	0
36	1

75093

/2	%2
37546	1
18773	0
9386	1
4693	0
2346	1
1173	0
586	1
293	0
146	1

/2	%2
73	0
36	1
18	0

75093

/2	%2
37546	1
18773	0
9386	1
4693	0
2346	1
1173	0
586	1
293	0
146	1

/2	%2
73	0
36	1
18	0
9	0

75093

/2	%2
37546	1
18773	0
9386	1
4693	0
2346	1
1173	0
586	1
293	0
146	1

/2	%2
73	0
36	1
18	0
9	0
4	1

75093

/2	%2
37546	1
18773	0
9386	1
4693	0
2346	1
1173	0
586	1
293	0
146	1

/2	%2
73	0
36	1
18	0
9	0
4	1
2	0

75093

/2	%2
37546	1
18773	0
9386	1
4693	0
2346	1
1173	0
586	1
293	0
146	1

/2	%2
73	0
36	1
18	0
9	0
4	1
2	0
1	0

75093

/2	%2
37546	1
18773	0
9386	1
4693	0
2346	1
1173	0
586	1
293	0
146	1

/2	%2
73	0
36	1
18	0
9	0
4	1
2	0
1	0
0	1

75093

/2	%2
37546	1
18773	0
9386	1
4693	0
2346	1
1173	0
586	1
293	0
146	1

/2	%2
73	0
36	1
18	0
9	0
4	1
2	0
1	0
0	1

75093 $= (10010010101010101)_2$

/2	%2	/2	%2
37546	1	73	0
18773	0	36	1
9386	1	18	0
4693	0	9	0
2346	1	4	1
1173	0	2	0
586	1	1	0
293	0	0	1
146	1		

why does it work ?

```
m = 2* (m/2) + m%2
```

```
    10010010101010101
= 2*1001001010101010 + 1
```

```
m = β* (m/β) + m%β
```

why does it work ?

why does it work ?

$$m = \sum_{i=0}^{n-1} b_i \beta^i$$

why does it work ?

$$\mathbf{m} = \sum_{i=0}^{n-1} b_i \beta^i$$

$$= (b_{n-1} b_{n-2} \ldots b_1 b_0)_\beta$$

why does it work ?

$$m = \sum_{i=0}^{n-1} b_i \beta^i$$

$$= (b_{n-1}b_{n-2}...b_1b_0)_\beta$$

$$= (b_{n-1}b_{n-2}...b_10)_\beta + b_0$$

why does it work ?

$$\mathbf{m} = \sum_{i=0}^{n-1} b_i \beta^i$$

$$= (b_{n-1}b_{n-2}\ldots b_1 b_0)_\beta$$

$$= (b_{n-1}b_{n-2}\ldots b_1 0)_\beta + b_0$$

$$= \beta * (b_{n-1}b_{n-2}\ldots b_1)_\beta + b_0$$

why does it work ?

$$m = \sum_{i=0}^{n-1} b_i \beta^i$$

$$= (b_{n-1} b_{n-2} \ldots b_1 b_0)_\beta$$

$$= (b_{n-1} b_{n-2} \ldots b_1 0)_\beta + b_0$$

$$= \beta * (b_{n-1} b_{n-2} \ldots b_1)_\beta + b_0$$

$$= \beta * (b_{n-1} b_{n-2} \ldots b_1 b_0 / \beta) + b_0$$

why does it work ?

$$m = \sum_{i=0}^{n-1} b_i \beta^i$$

$$= (b_{n-1} b_{n-2} \ldots b_1 b_0)_\beta$$

$$= (b_{n-1} b_{n-2} \ldots b_1 0)_\beta + b_0$$

$$= \beta * (b_{n-1} b_{n-2} \ldots b_1)_\beta + b_0$$

$$= \beta * (b_{n-1} b_{n-2} \ldots b_1 b_0 / \beta) + b_0$$

$$m = \beta * (m/\beta) + m \% \beta$$

why does it work ?

$$m = \sum_{i=0}^{n-1} b_i \beta^i$$

$$= (b_{n-1} b_{n-2} \ldots b_1 b_0)_\beta$$

$$= (b_{n-1} b_{n-2} \ldots b_1 0)_\beta + b_0$$

$$= \beta * (b_{n-1} b_{n-2} \ldots b_1)_\beta + b_0$$

$$= \beta * (b_{n-1} b_{n-2} \ldots b_1 b_0 / \beta) + b_0$$

$$m = \beta * (m/\beta) + m \% \beta$$

why does it work ?

$$m = \sum_{i=0}^{n-1} b_i\beta^i$$

$$= (b_{n-1}b_{n-2}...b_1b_0)_\beta$$

$$= (b_{n-1}b_{n-2}...b_1 0)_\beta + b_0$$

$$= \beta * (b_{n-1}b_{n-2}...b_1)_\beta + b_0$$

$$= \beta * (b_{n-1}b_{n-2}...b_1b_0/\beta) + b_0$$

$$m = \beta * (m/\beta) + m\%\beta$$

why does it work ?

$$m = \sum_{i=0}^{n-1} b_i \beta^i$$

$$= (b_{n-1} b_{n-2} \ldots b_1 b_0)_\beta$$

$$= (b_{n-1} b_{n-2} \ldots b_1 0)_\beta + b_0$$

$$= \beta * (b_{n-1} b_{n-2} \ldots b_1)_\beta + b_0$$

$$= \beta * (b_{n-1} b_{n-2} \ldots b_1 b_0 / \beta) + b_0$$

$$m = \beta * (m/\beta) + m\%\beta$$

to Base β

Algorithm 3 Convert integer to binary

INPUT: a number m

OUTPUT: the number m **expressed in base β using a bit array** $b[\,]$

$i \leftarrow 0$

while $m > 0$ **do**

$b[i] \leftarrow m\%\beta$

$m \leftarrow m/\beta$

$i \leftarrow i + 1$

end while

75093

/5 %5

15018 3

```
75093
/5     %5
15018  3
3003   3
```

75093

/5	%5
15018	3
3003	3
600	3

```
75093
/5      %5
15018 3
3003  3
600   3
120   0
```

75093	
/5	%5
15018	3
3003	3
600	3
120	0
24	0

```
75093
/5     %5
15018  3
3003   3
600    3
120    0
24     0
4      4
```

75093	
/5	%5
15018	3
3003	3
600	3
120	0
24	0
4	4
0	4

```
75093
/5     %5
15018  3
3003   3
600    3
120    0
24     0
4      4
0      4
```

```
75093
/5     %5
15018  3
3003   3
600    3
120    0
24     0
4      4
0      4
```

75093 $= (4400333)_5$

/5	%5
15018	3
3003	3
600	3
120	0
24	0
4	4
0	4

Winter 2016
COMP-250: Introduction to Computer Science

Lecture 3, January 19, 2016