Winter 2016
COMP-250: Introduction to Computer Science

Lecture 20, March 24, 2016

Public Announcement

GETTING YOUR DREAM TECH INTERNSHIP

MARCH 29TH 6PM-7:30PM

TROTTIER 0070

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Public Announcement

Course : COMP 250 Sect: 1	Title: Intro to Computer Science
Exam Date: 4/28/2016	**Exam Time:** 2:00:00 PM

Public Announcement

Mercury Course Evaluations

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Course evaluations matter. Evaluate your courses and instructors!

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March 21 - May 1

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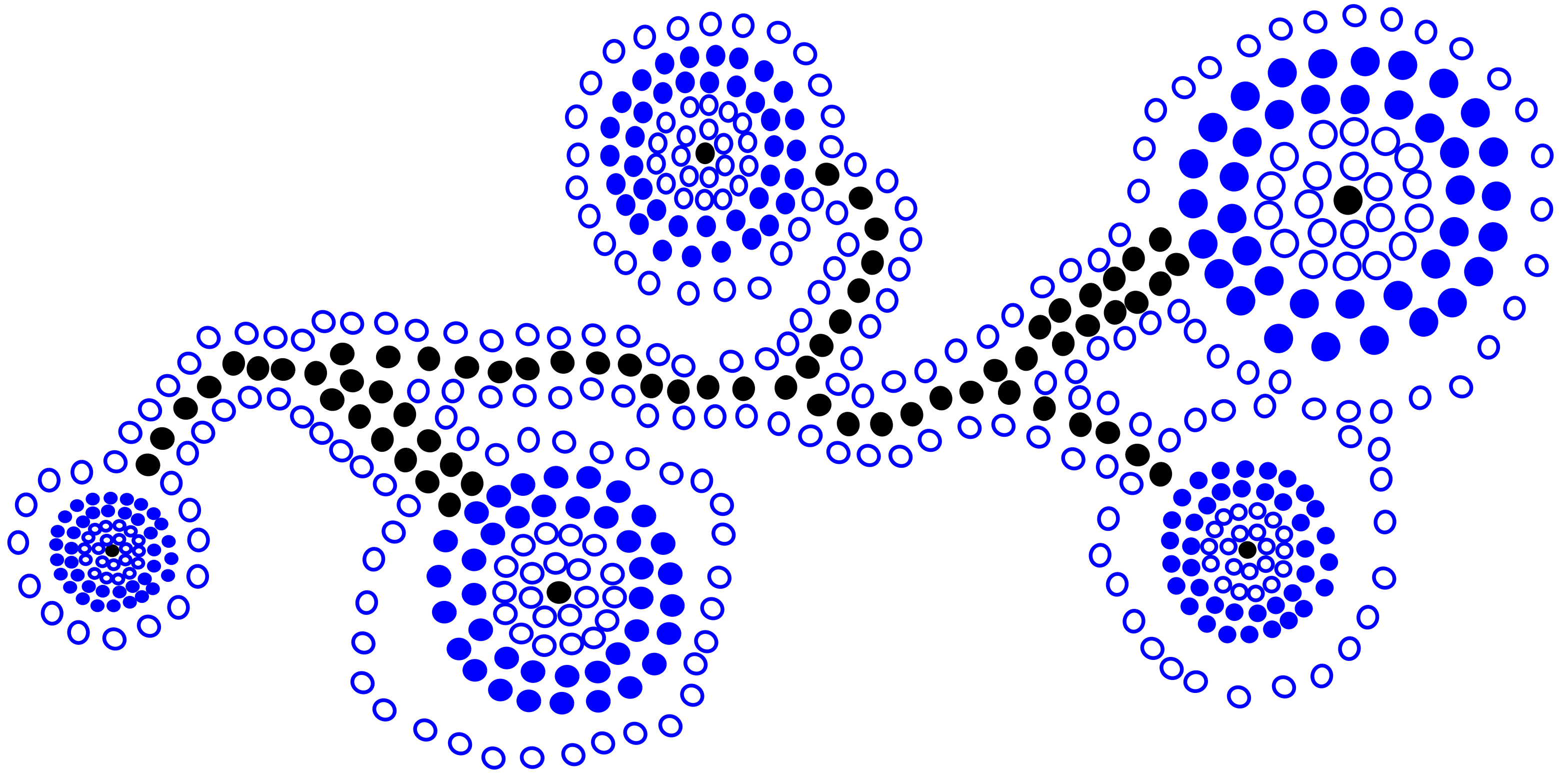
Click **HERE** to complete your course evaluations.

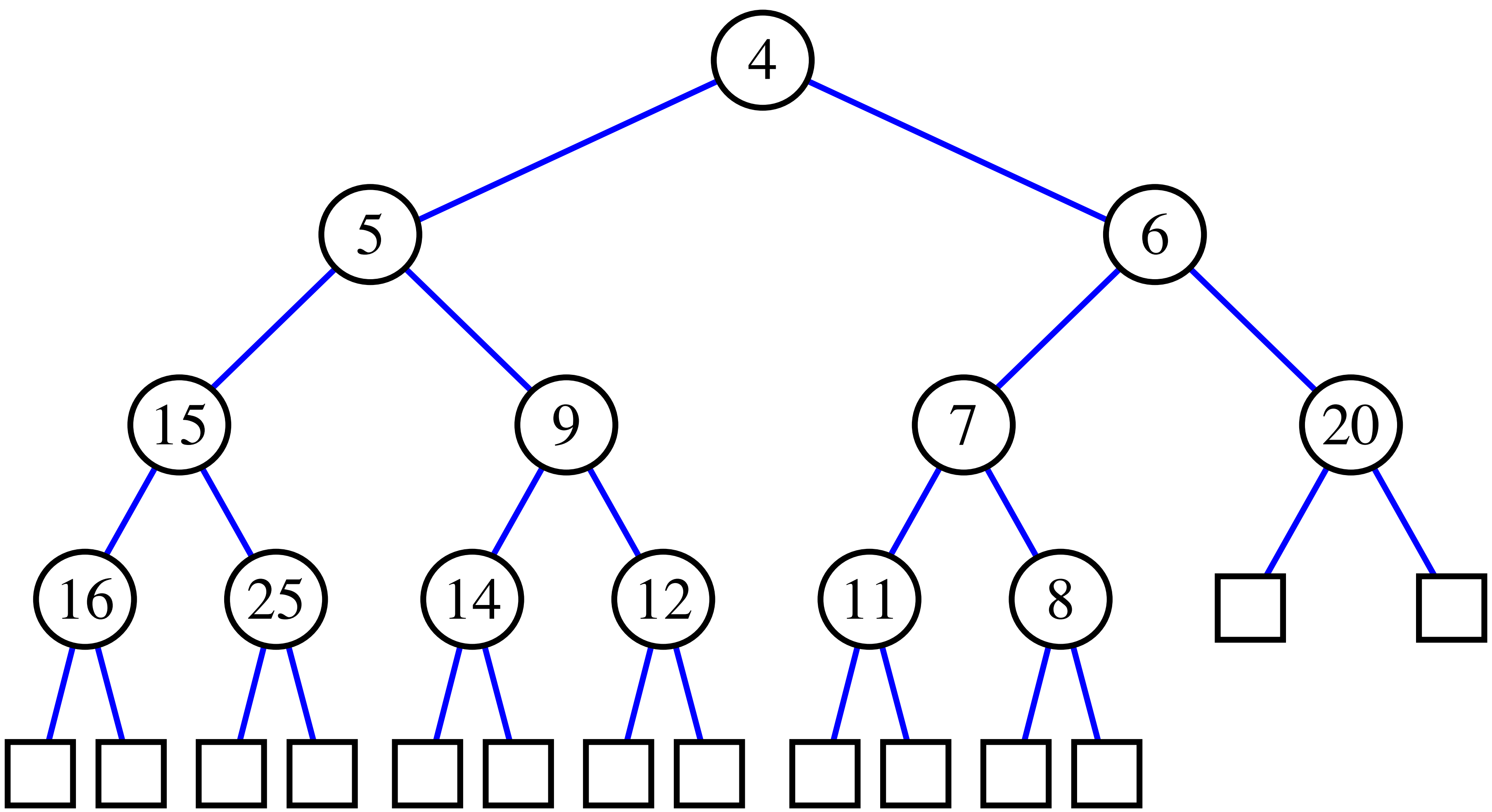
HEAPS I

- Heaps
- Properties
- Insertion and Deletion



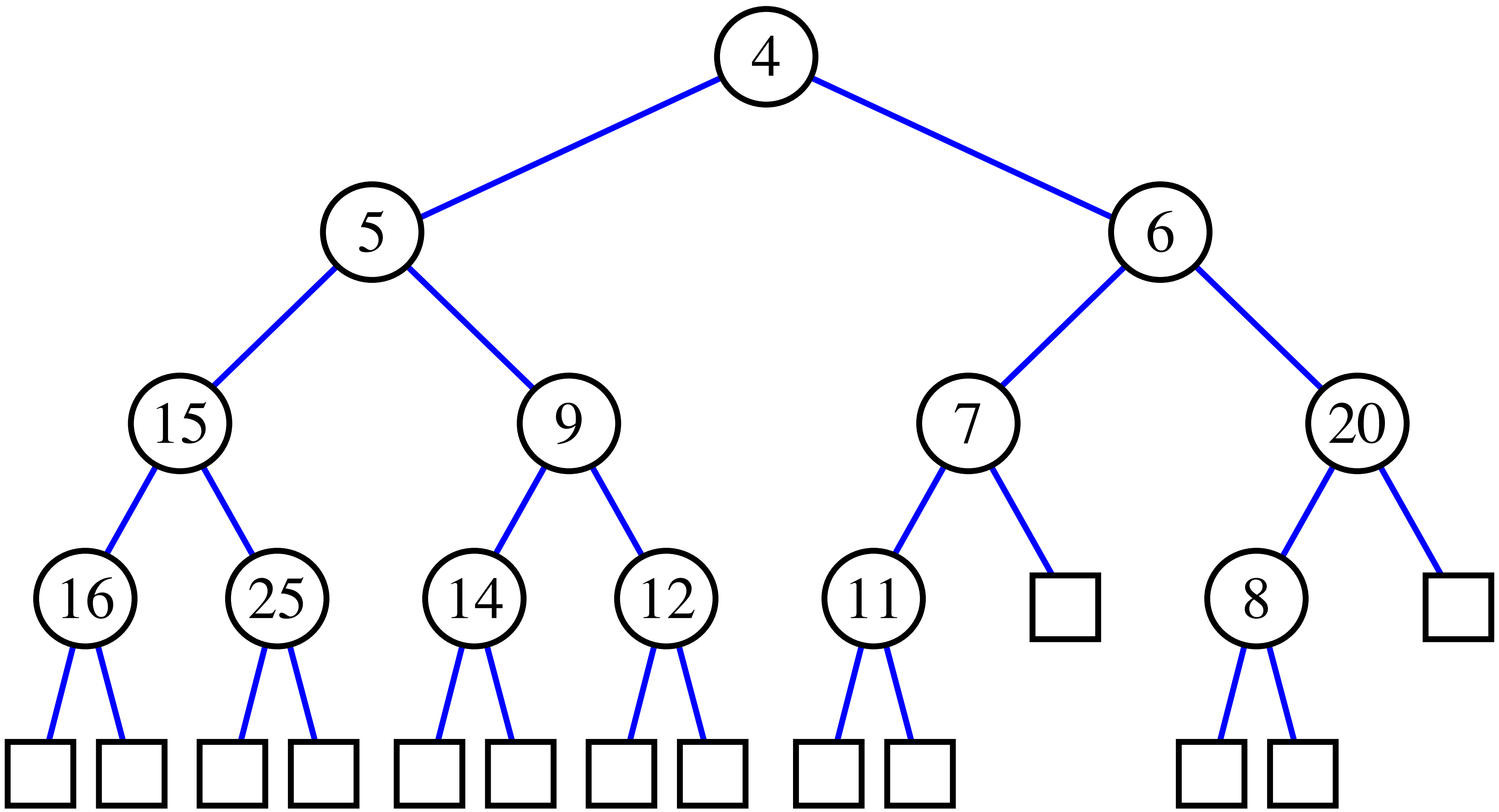
Heaps

- A ***heap*** is a binary tree *T* that stores a collection of keys (or key-element pairs) at its internal nodes and that satisfies two additional properties:
  - ***Order Property:*** key(parent) ≤ key(child)
  - ***Structural Property***: all levels are full, except the last one, which is left-filled (***complete binary tree***)



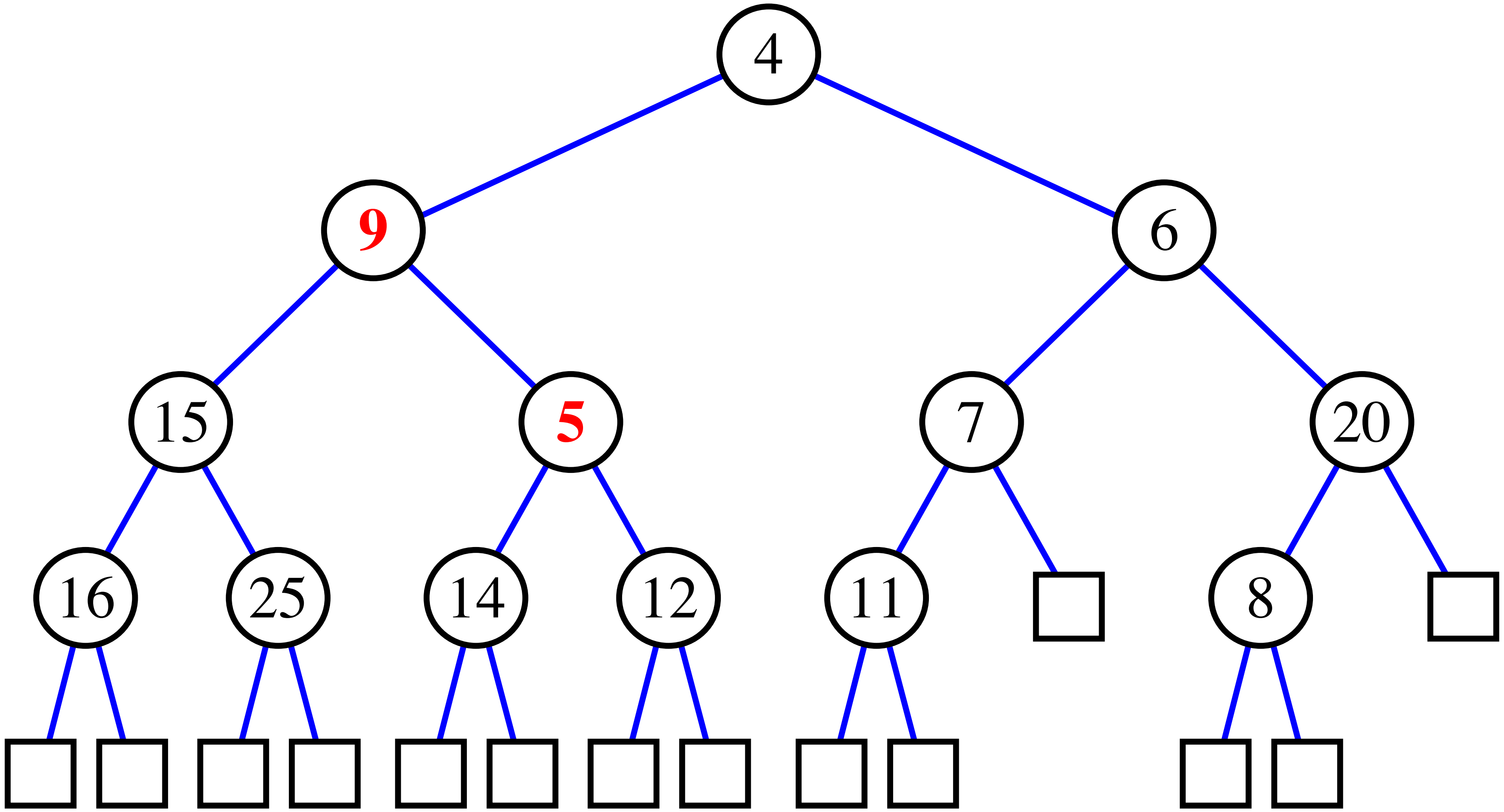
Not Heaps

- bottom level is not left-filled



Not Heaps

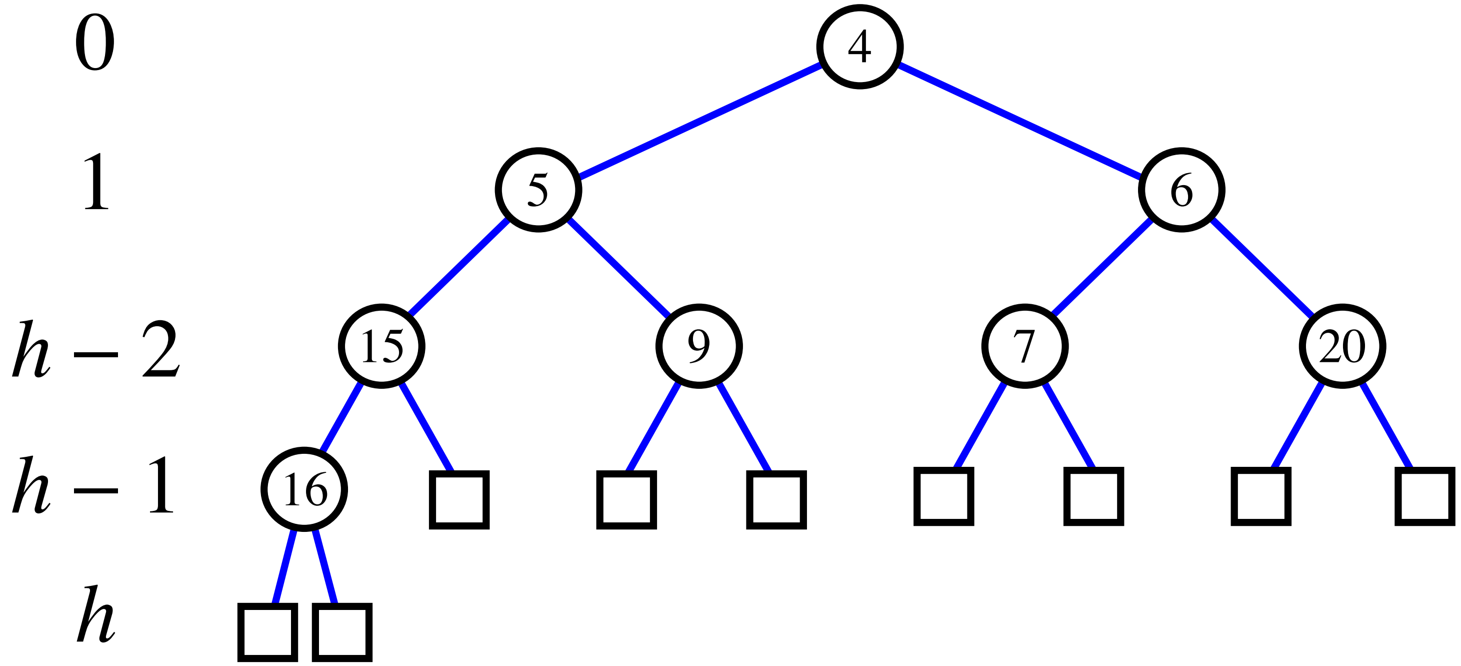
- key(parent)> key(child)



Height of a Heap

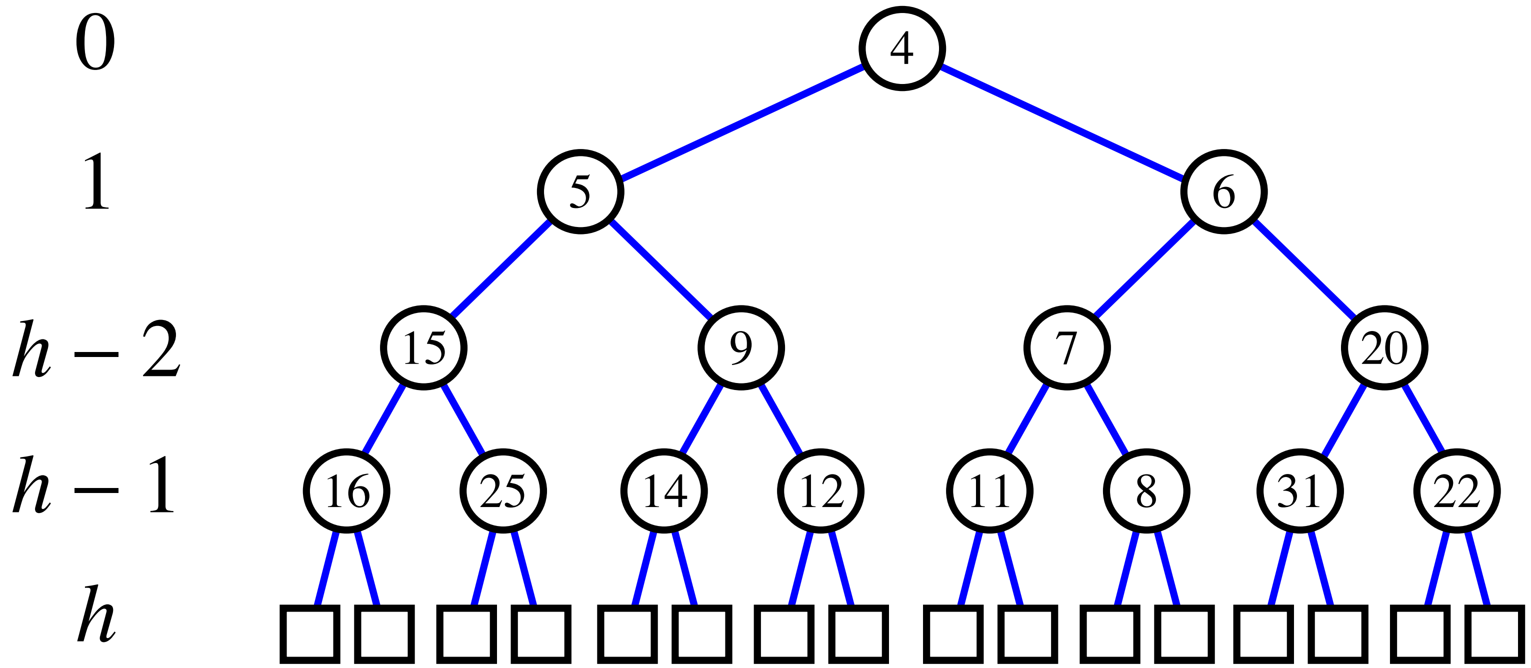
A heap T storing n keys has height $h = \lceil \log(n + 1) \rceil$, which is $O(\log n)$

- $n \geq 1 + 2 + 4 + \ldots + 2^{h-2} + 1 = 2^{h-1} - 1 + 1 = 2^{h-1}$



Height of a Heap

- $n \leq 1 + 2 + 4 + \ldots + 2^{h-1} = 2^h - 1$

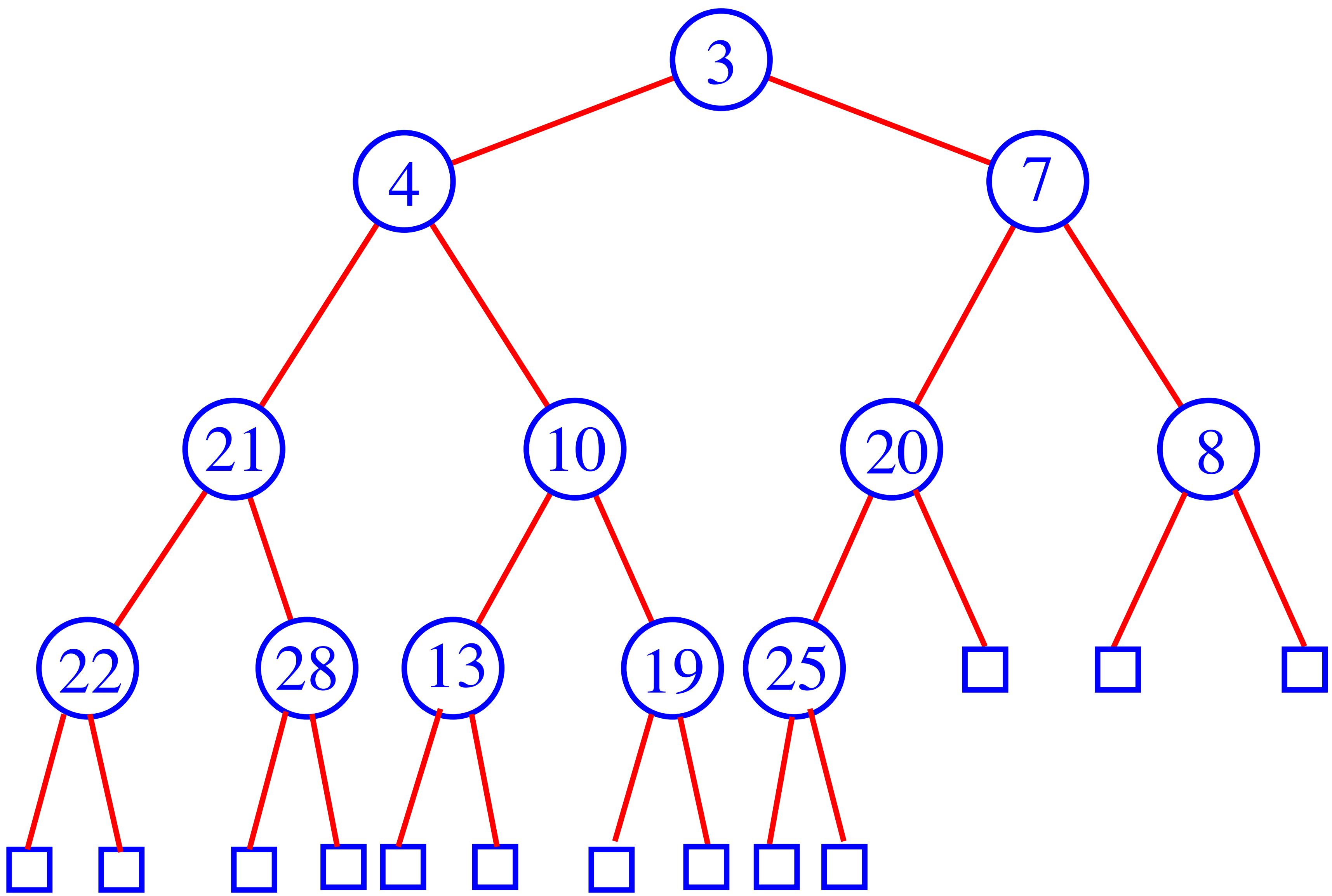


- Therefore $2^{h-1} \leq n \leq 2^h - 1$
- Taking logs, we get $\log(n+1) \leq h \leq \log n + 1$
- Which implies $\boldsymbol{h = \lceil \log(n+1) \rceil}$

Heap Insertion

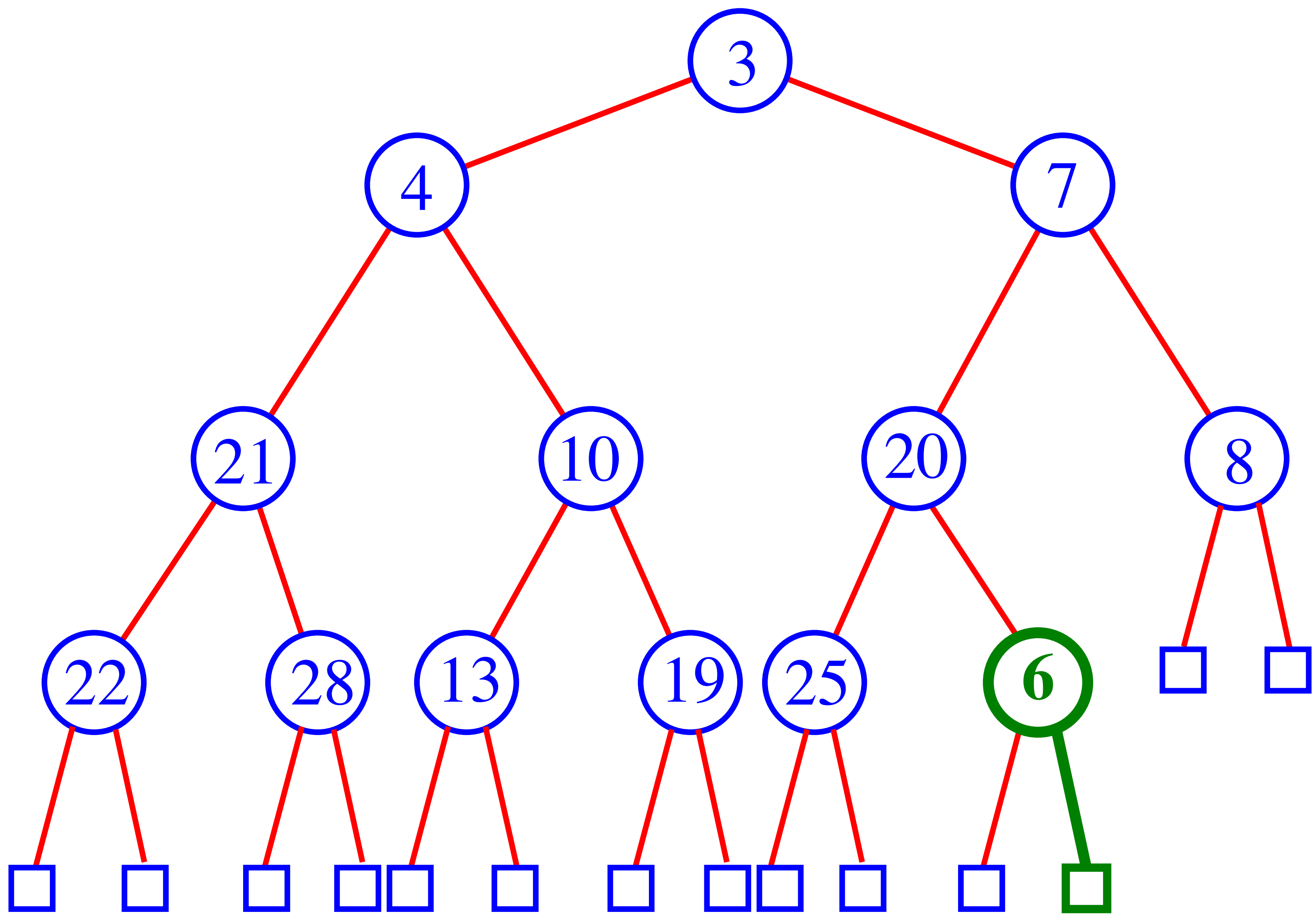
So here we go ...

The key to insert is **6**



Heap Insertion

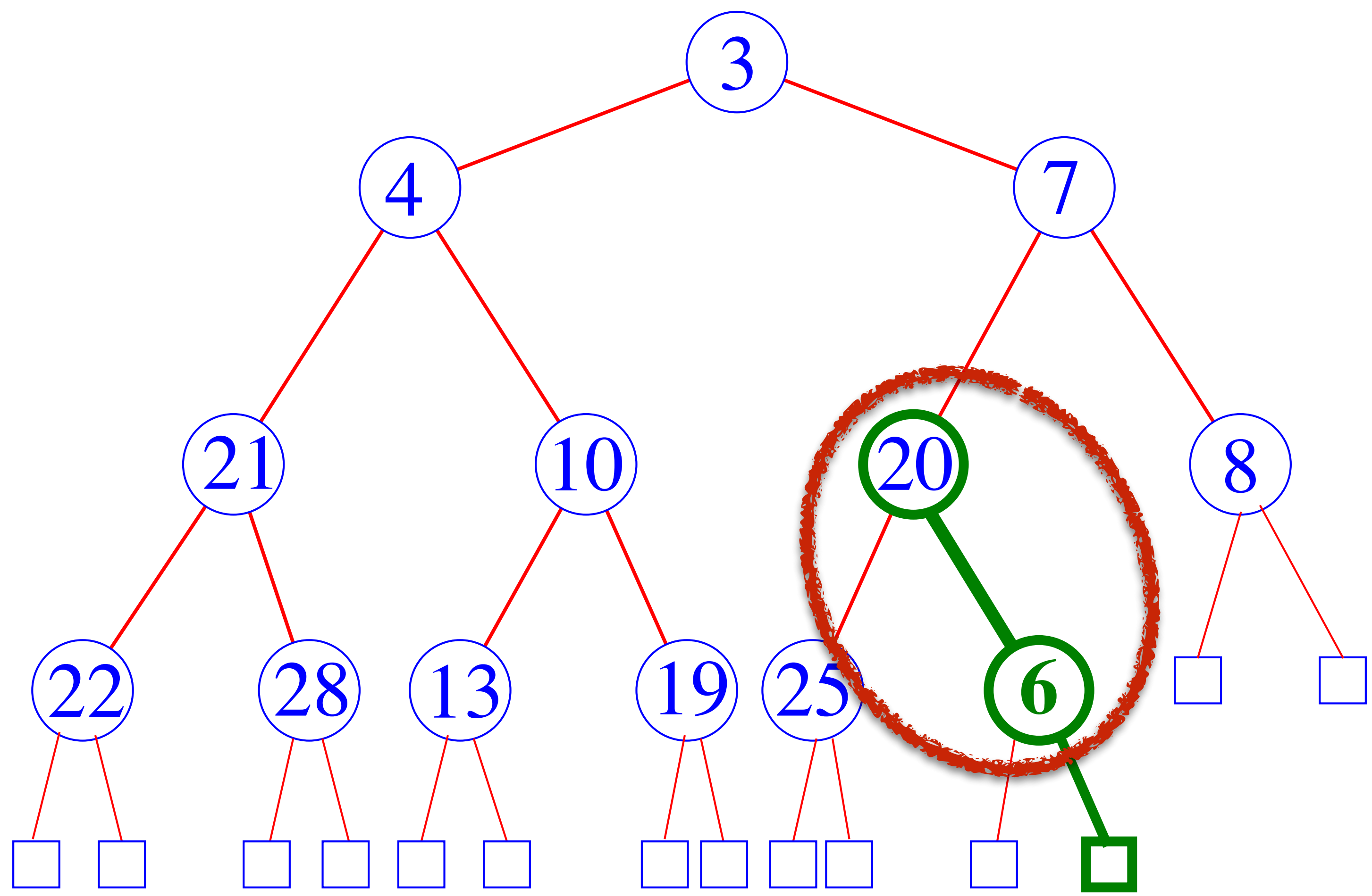
Add the key in the ***next available position*** in the heap.



Now begin ***Upheap***.

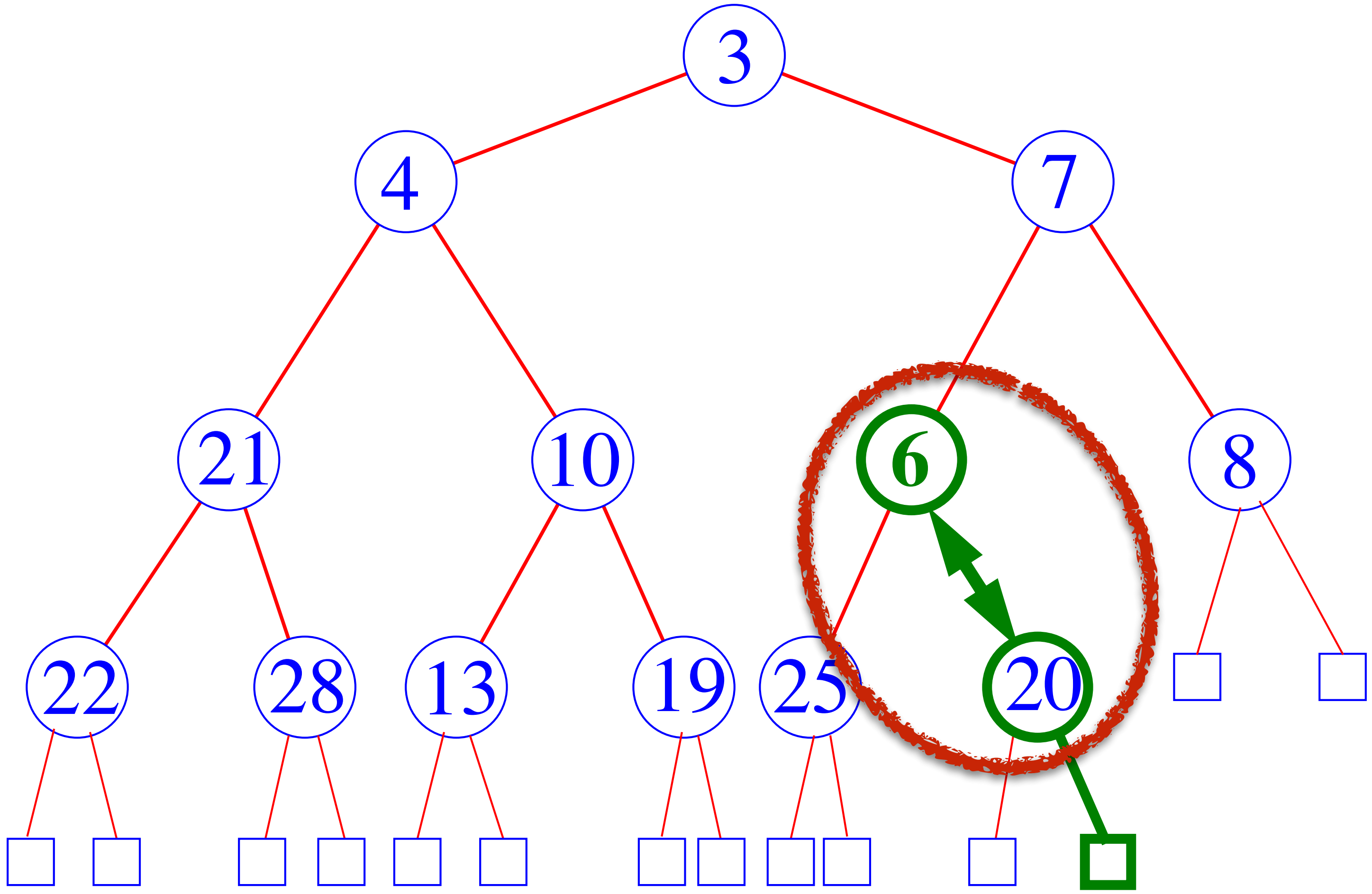
Upheap

- *Swap parent-child keys out of order*

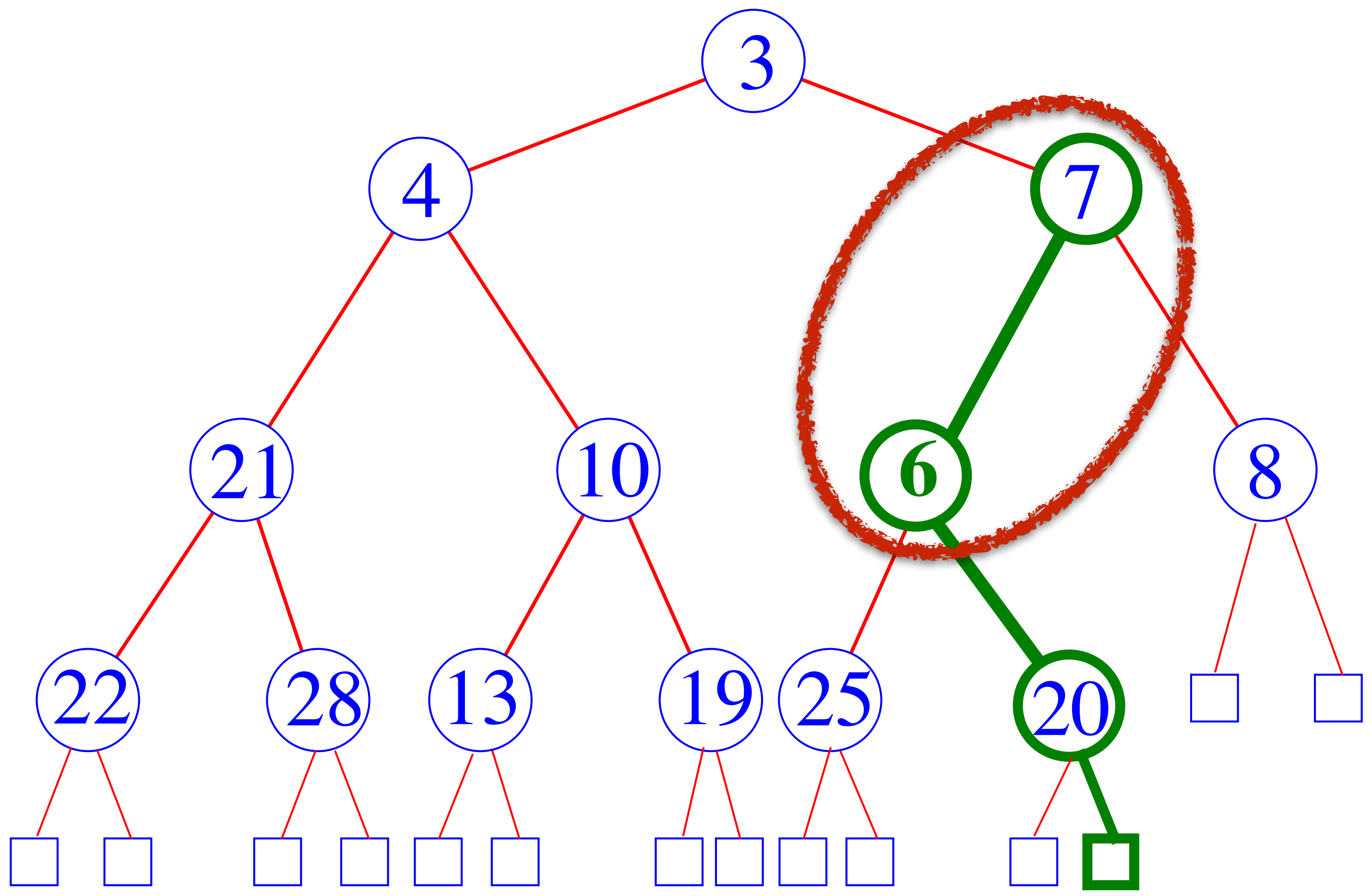


Upheap

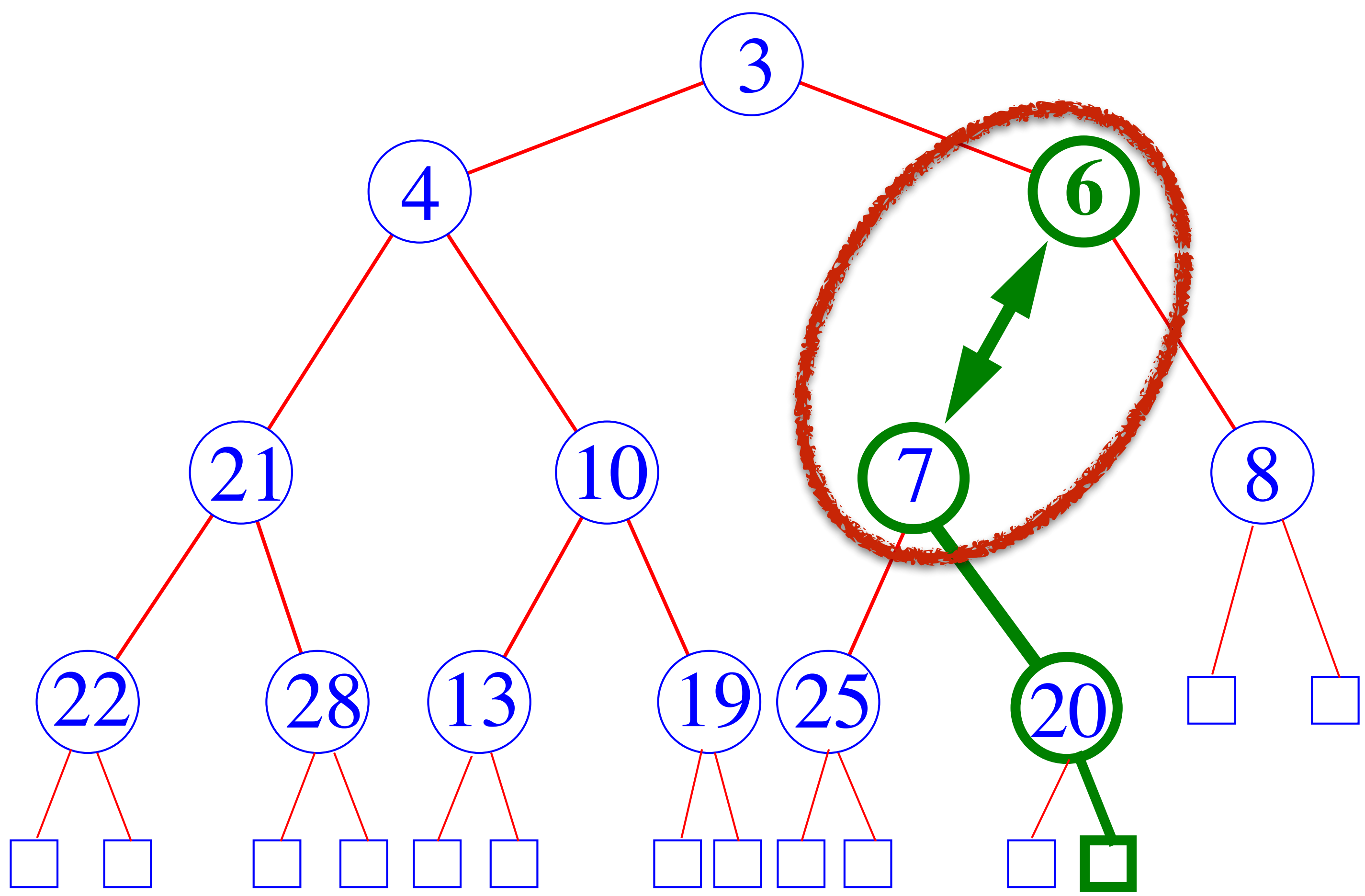
- ***Swap parent-child keys out of order***



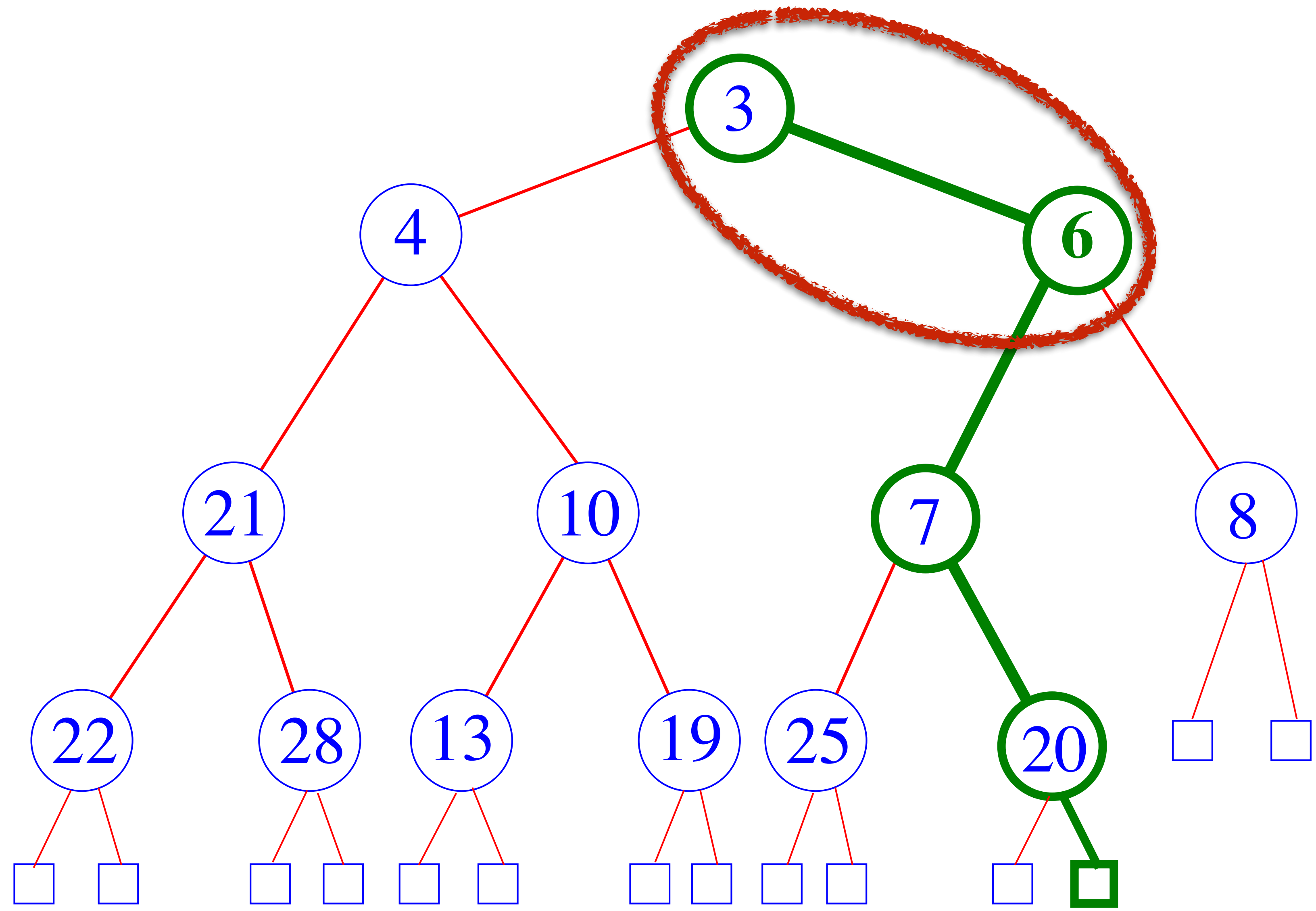
Upheap Continues



Upheap Continues



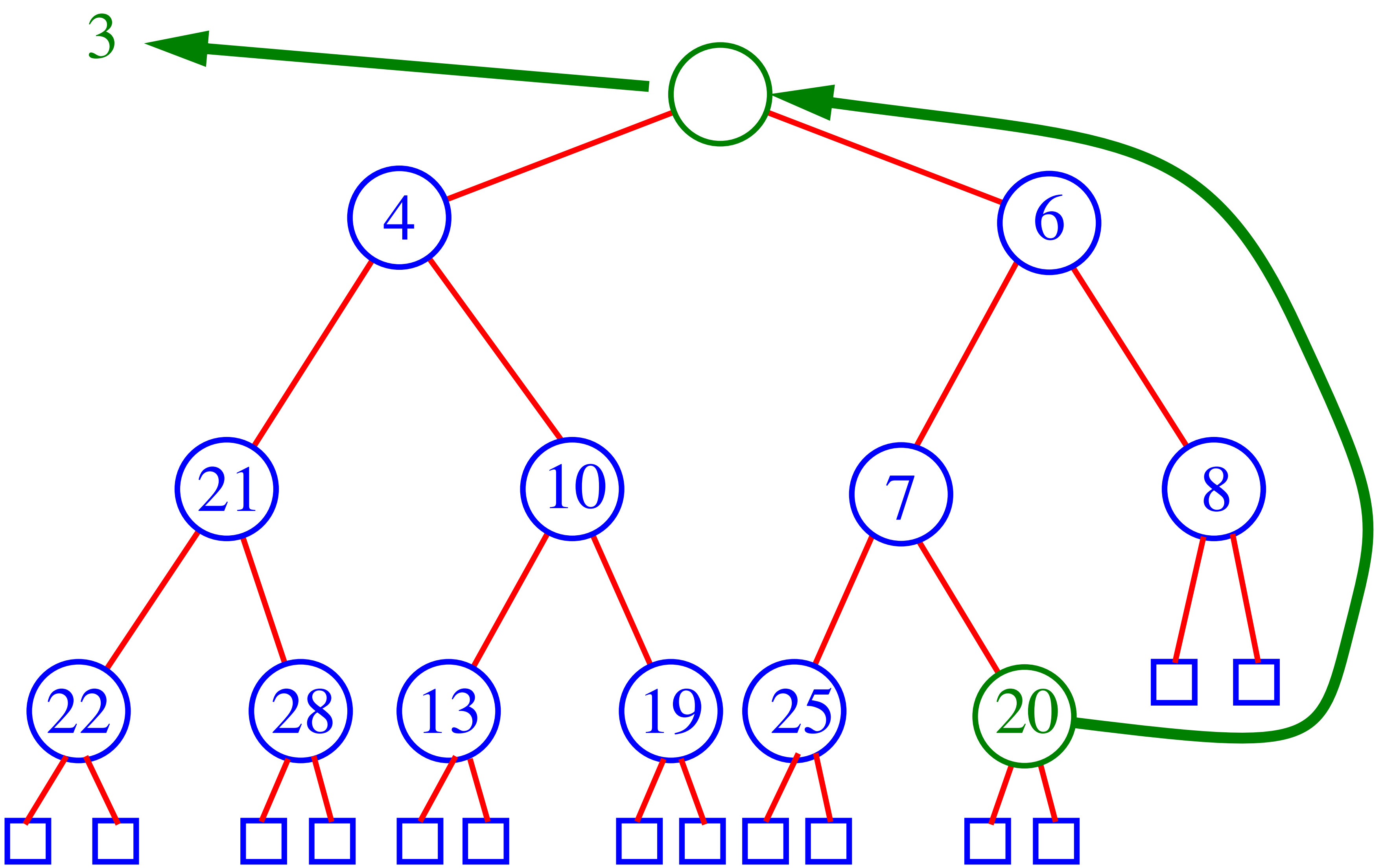
End of Upheap

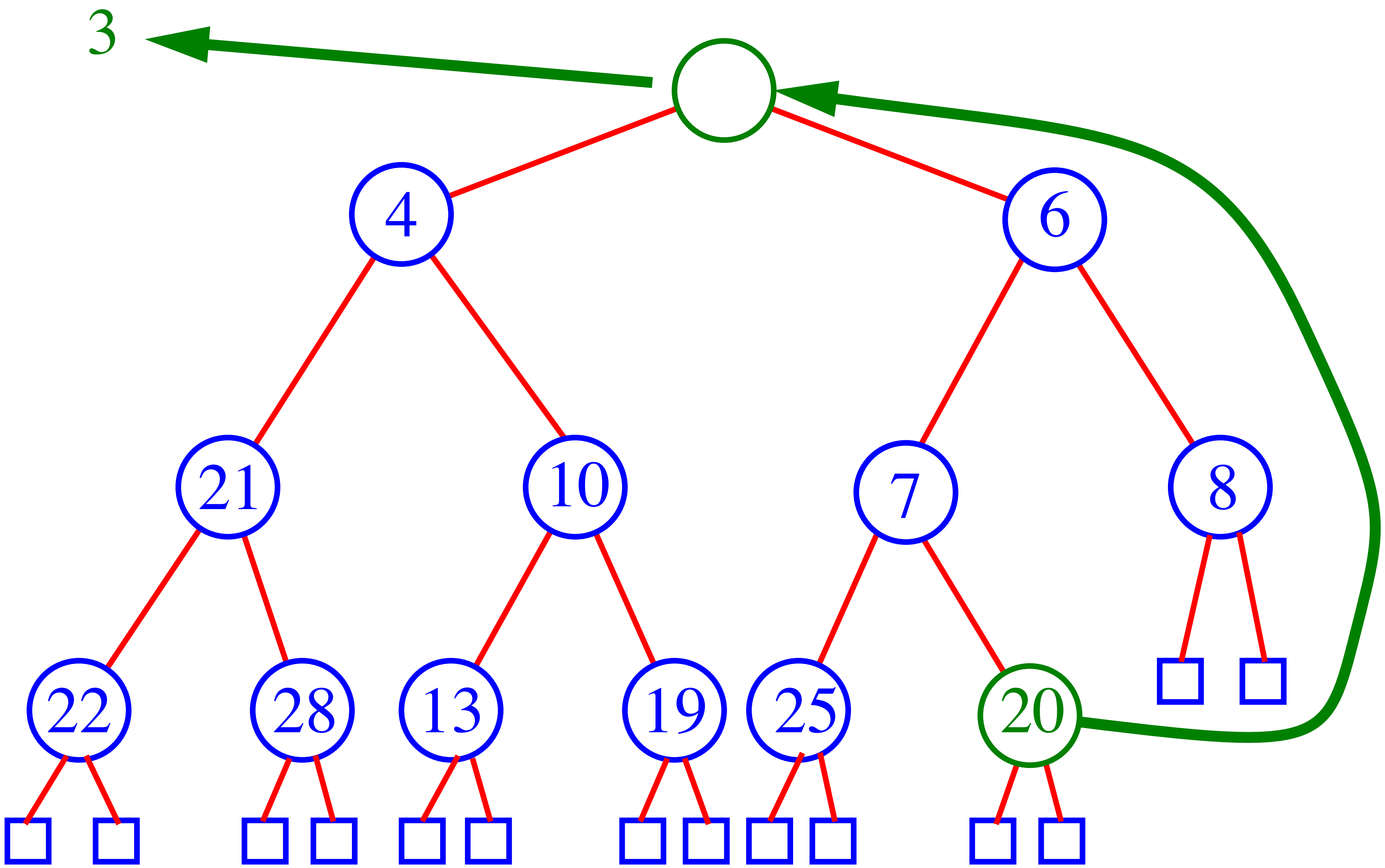


- ***Upheap*** terminates when new key is greater than the key of its parent **or** the top of the heap is reached
- (total #swaps) $\leq$ $(h - 1)$, which is $O(\log n)$

Removal From a Heap

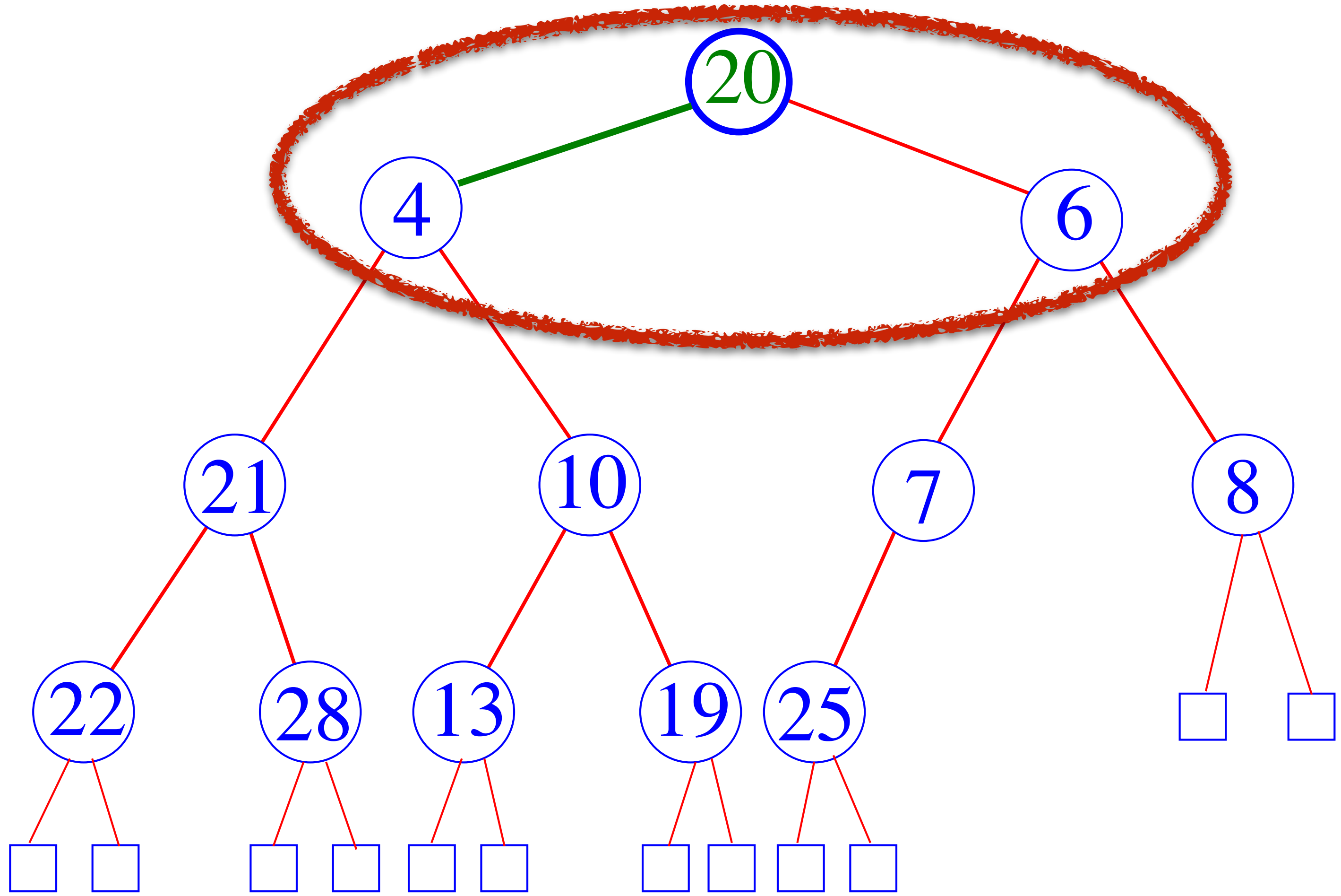
RemoveMin()



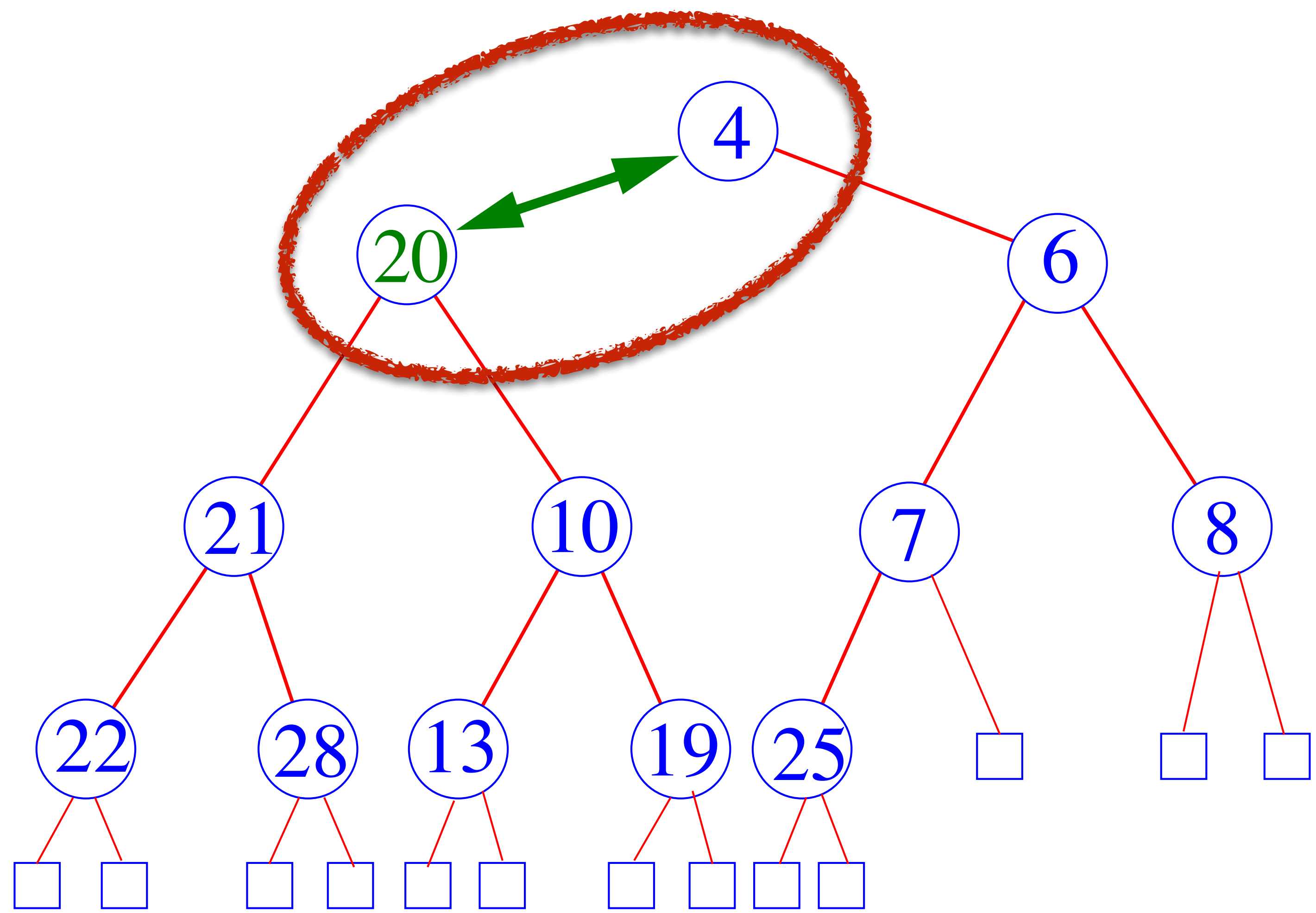


- The removal of the top key leaves a hole
- We need to fix the heap
- First, replace the hole with the last key in the heap
- Then, begin ***Downheap***

Downheap

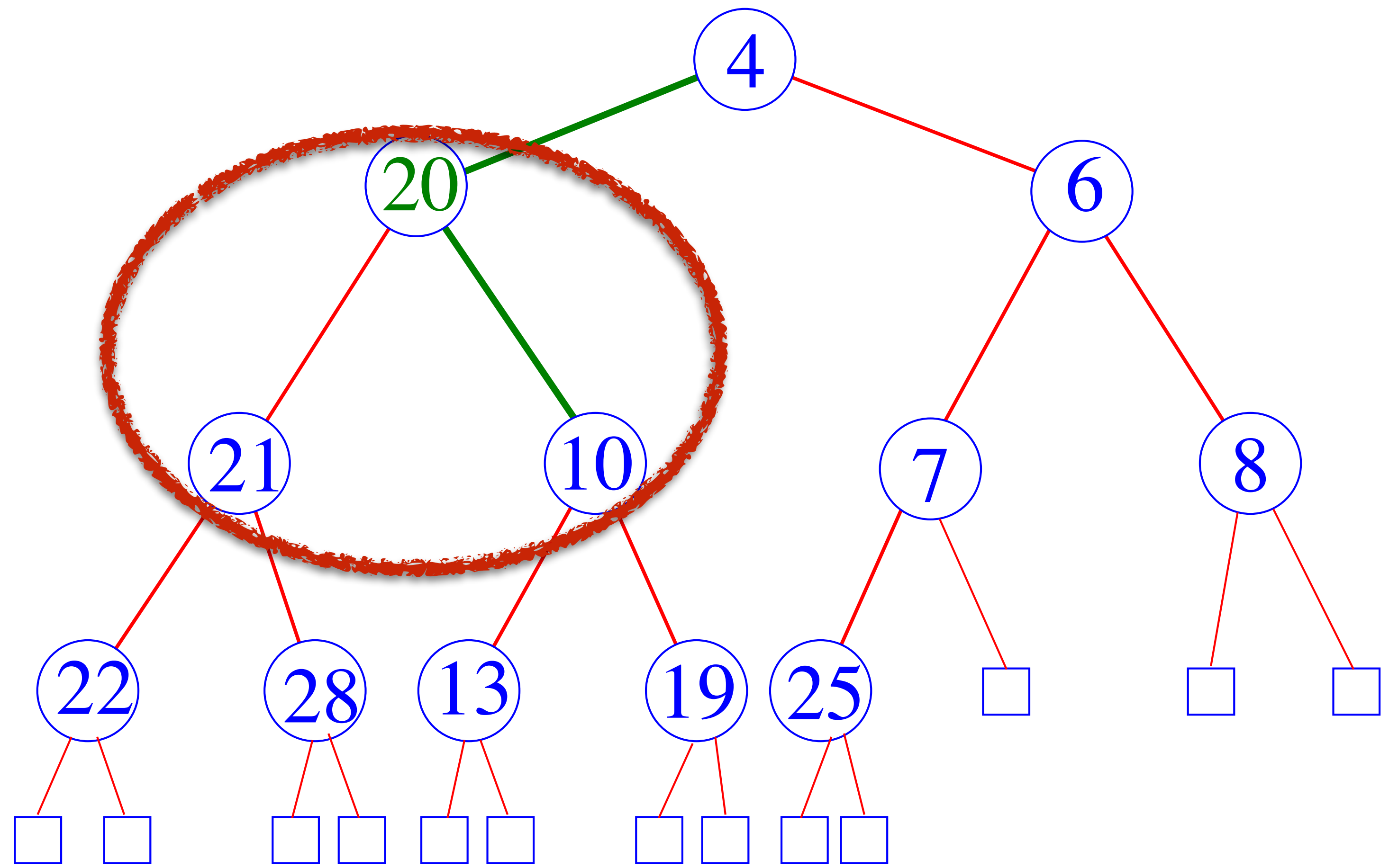


Downheap

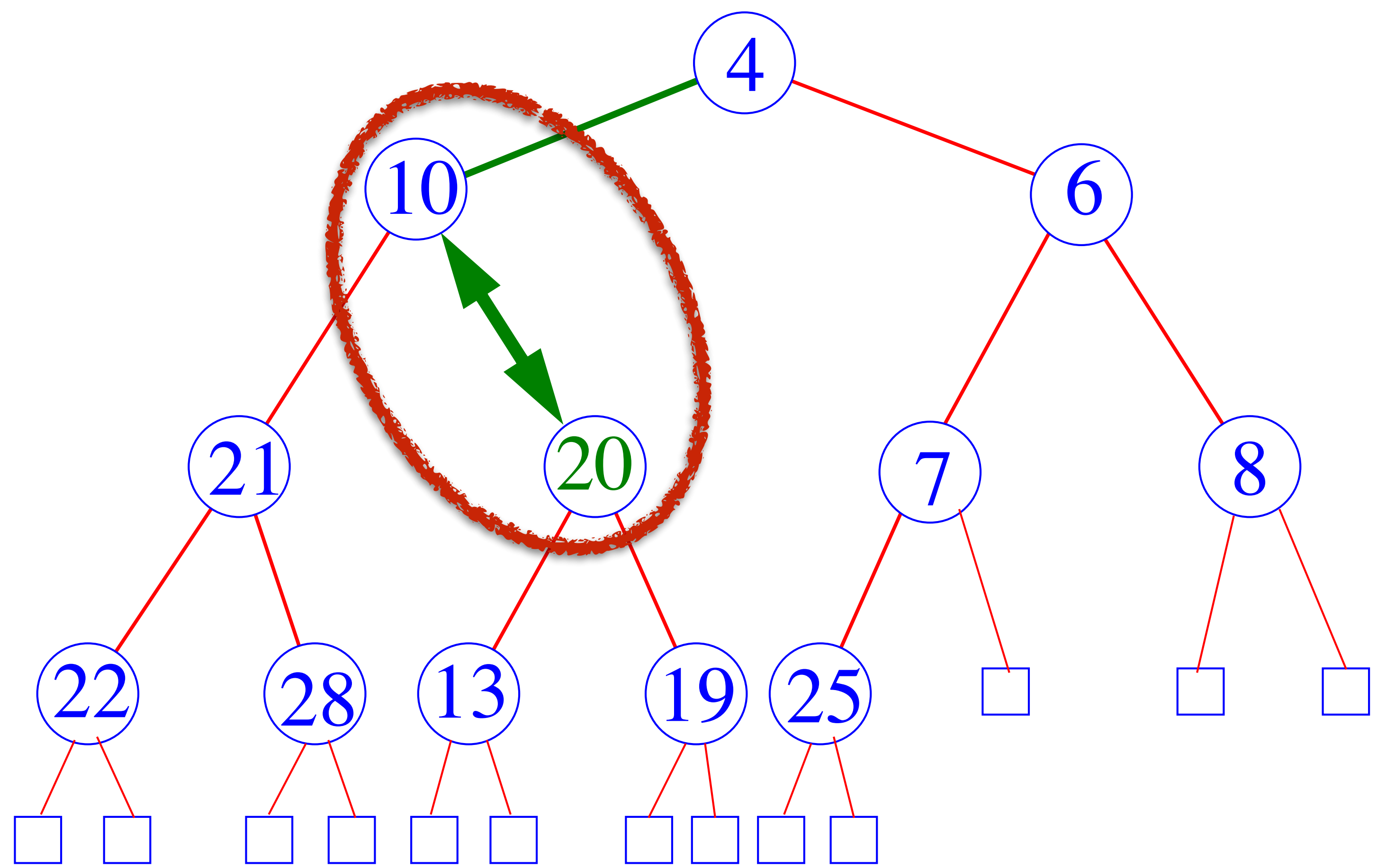


Downheap compares the parent with the smallest child. If the child is smaller, it switches the two.

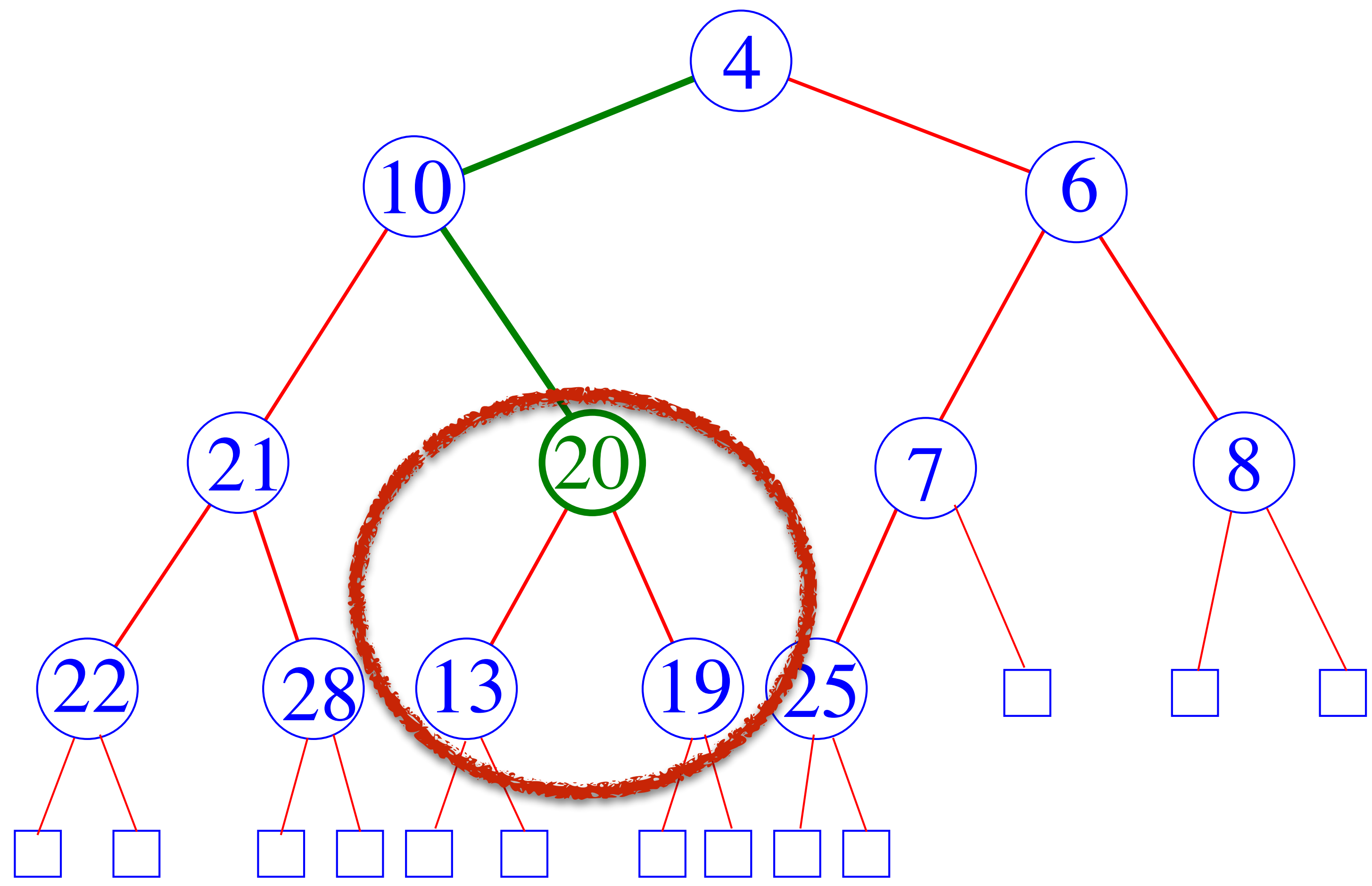
Downheap Continues



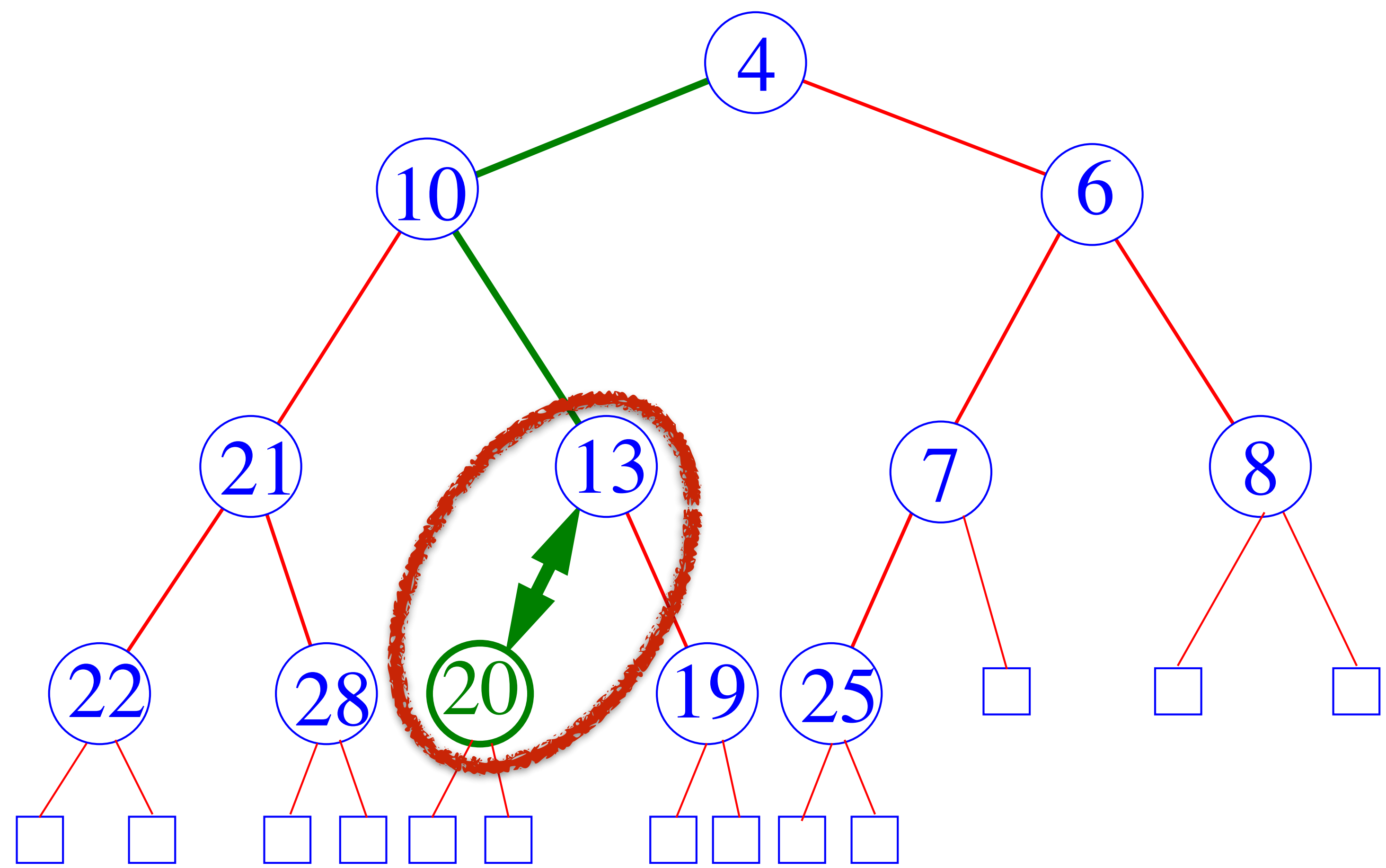
Downheap Continues



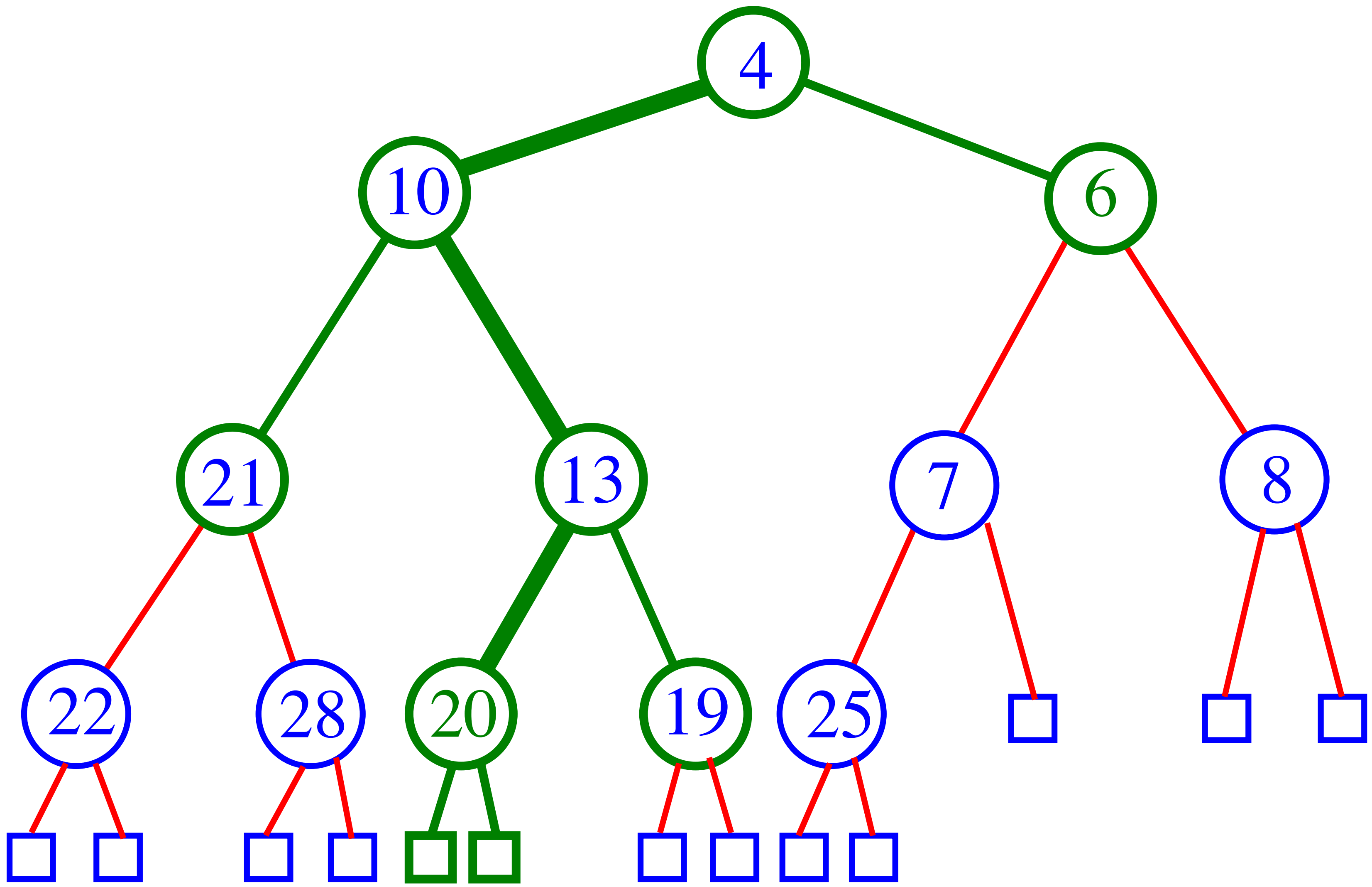
Downheap Continues



Downheap Continues



End of Downheap

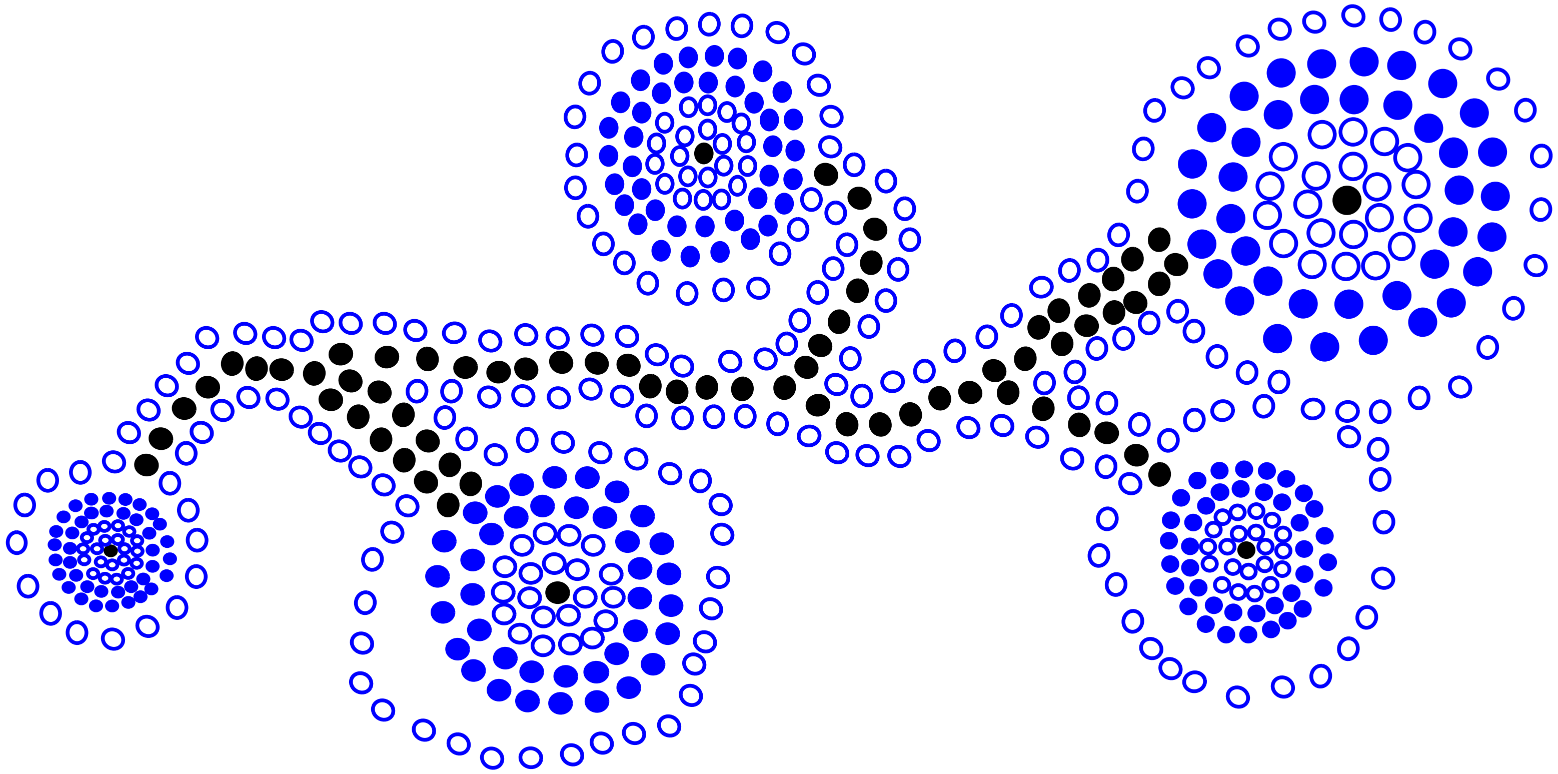


- ***Downheap*** terminates when the key is greater than the keys of both its children **or** the bottom of the heap is reached.

- (total #swaps) $\leq$ $(h - 1)$, which is $O(\log n)$

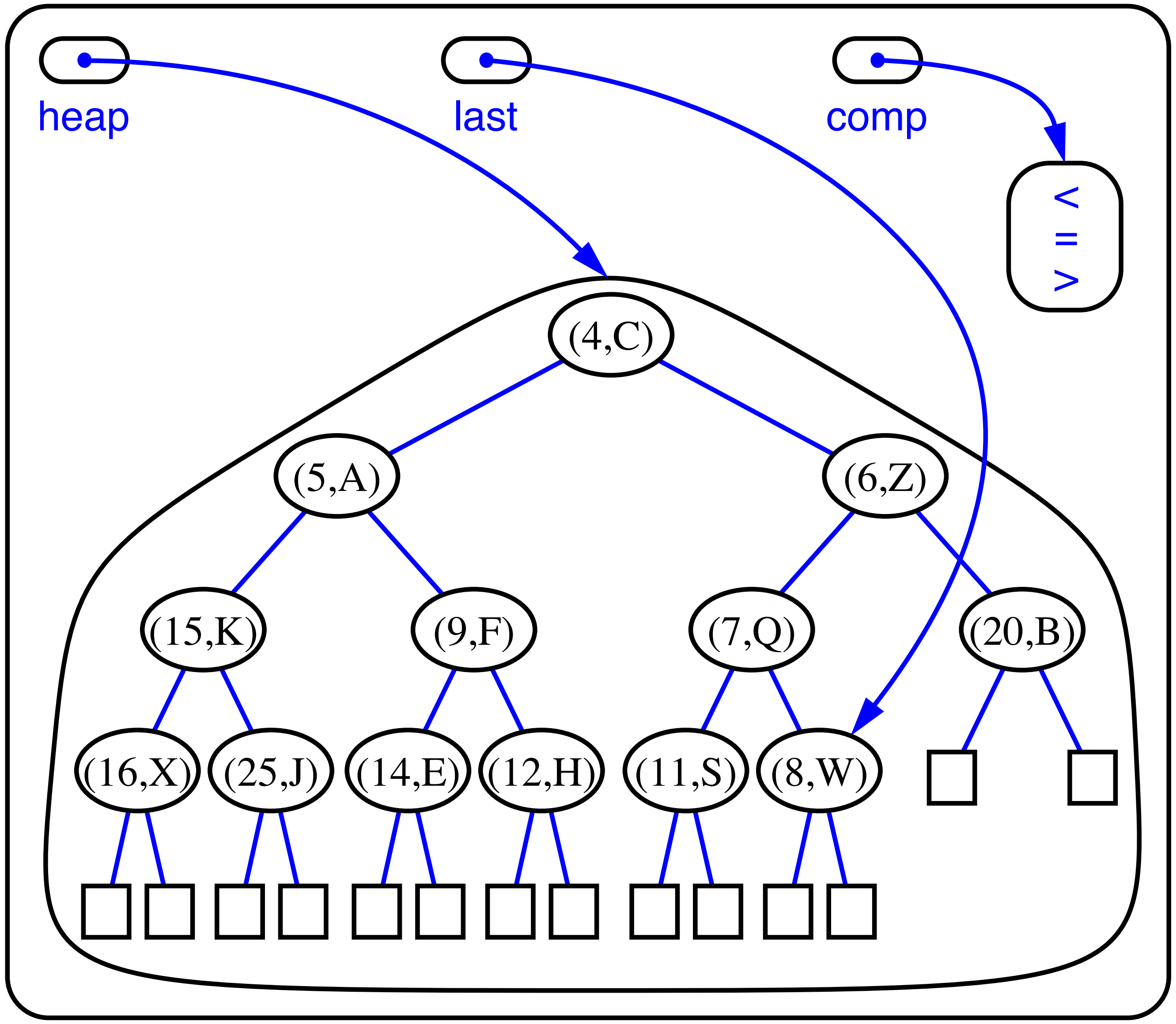
HEAPS II

- Implementation
- HeapSort
- Bottom-Up Heap Construction
- Locators



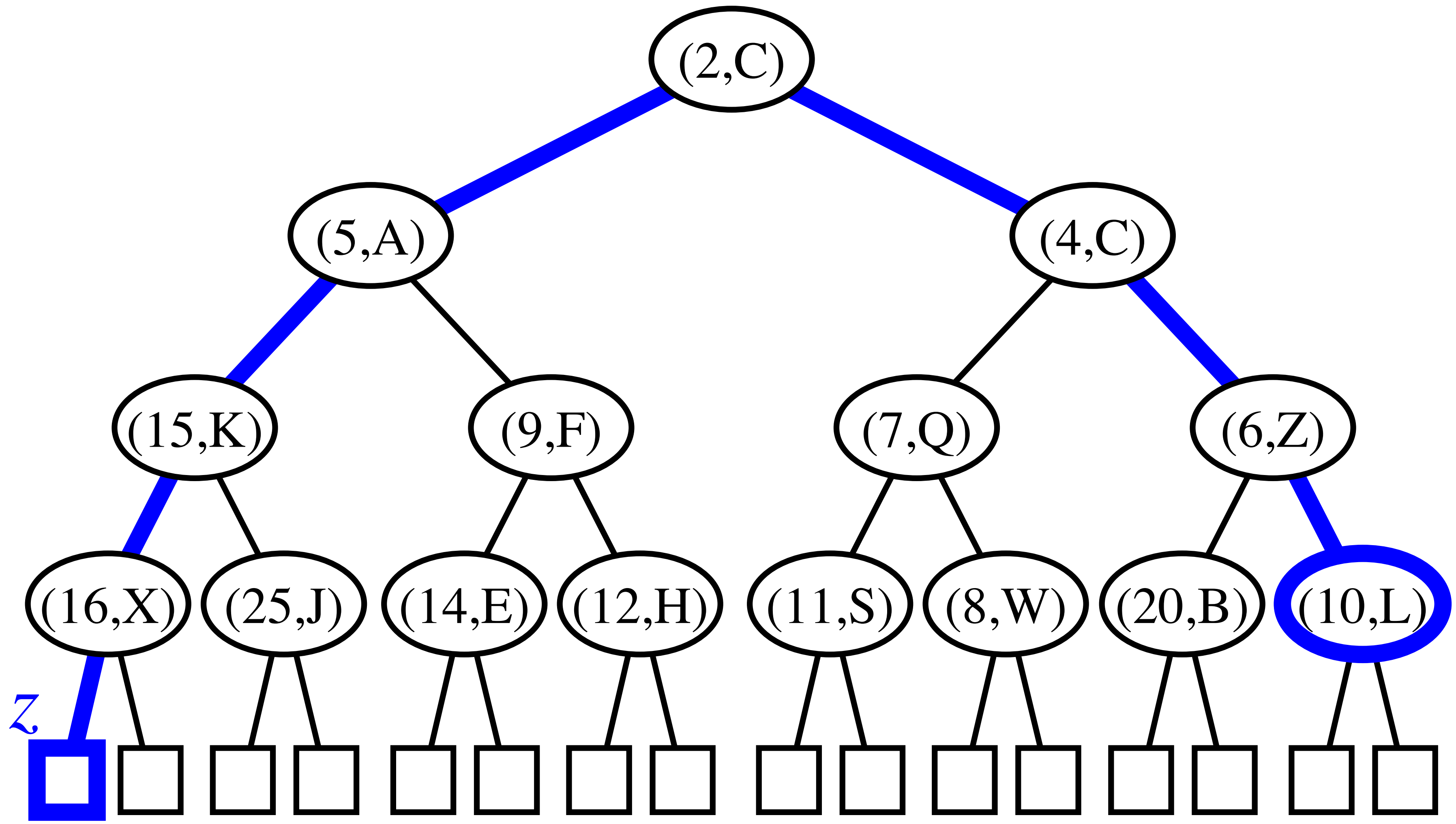
Implementation of a Heap

```
public class HeapPriorityQueue implements PriorityQueue
{
   BinaryTree T;
   Position last;
   Comparator comparator;
   ...
}
```



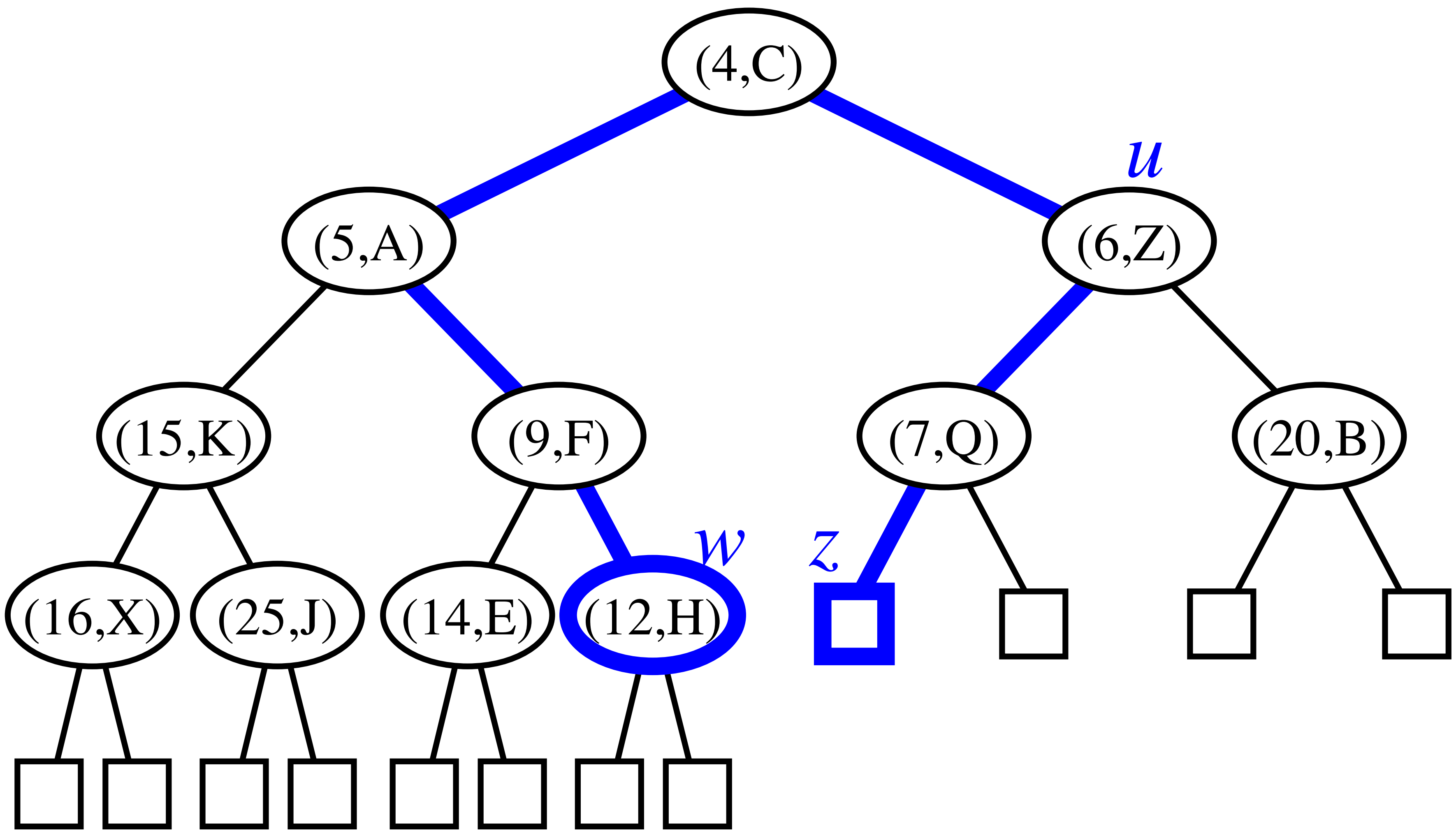
Implementation of a Heap(cont.)

- Two ways to find the insertion position z in a heap:



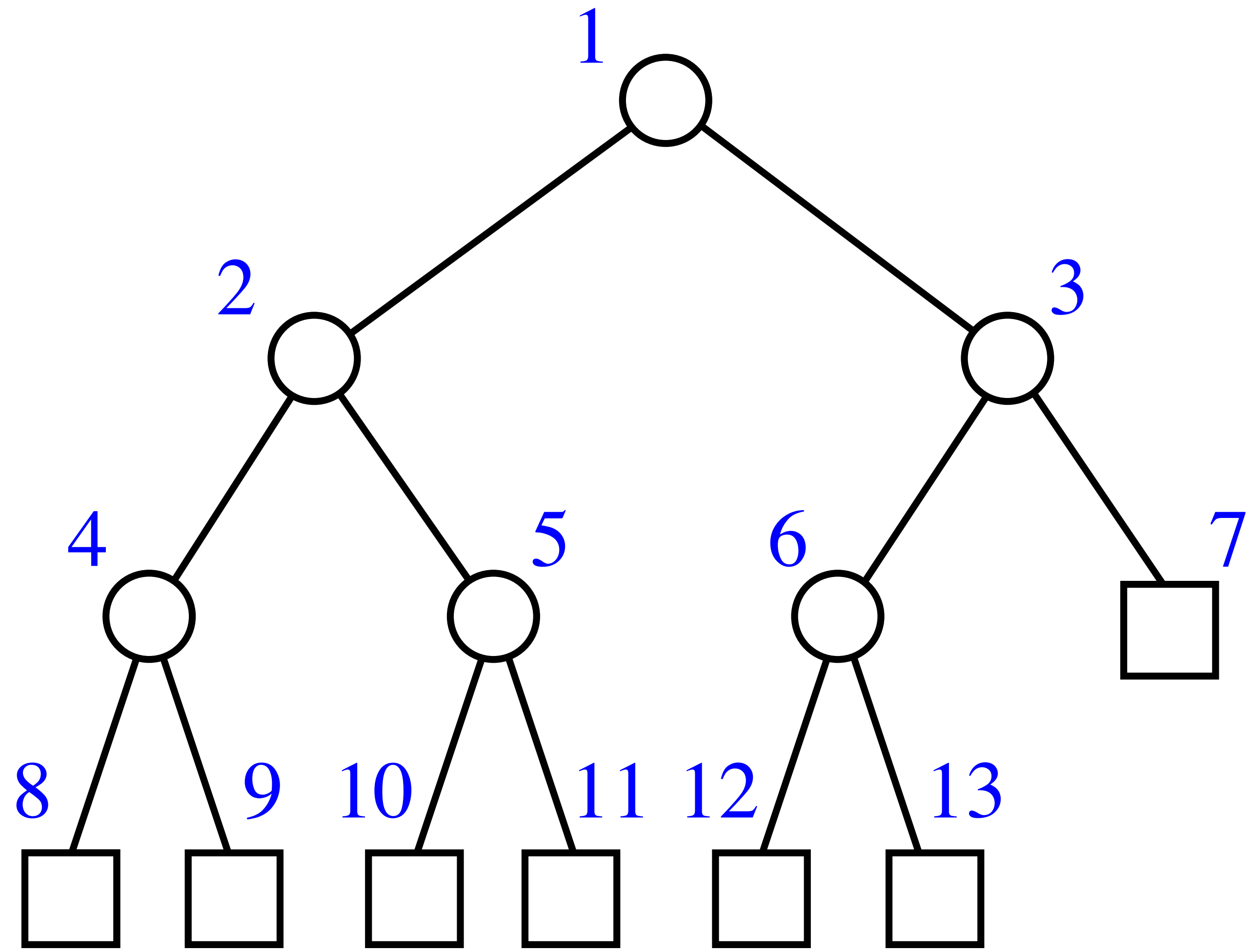
Implementation of a Heap(cont.)

- Two ways to find the insertion position z in a heap:

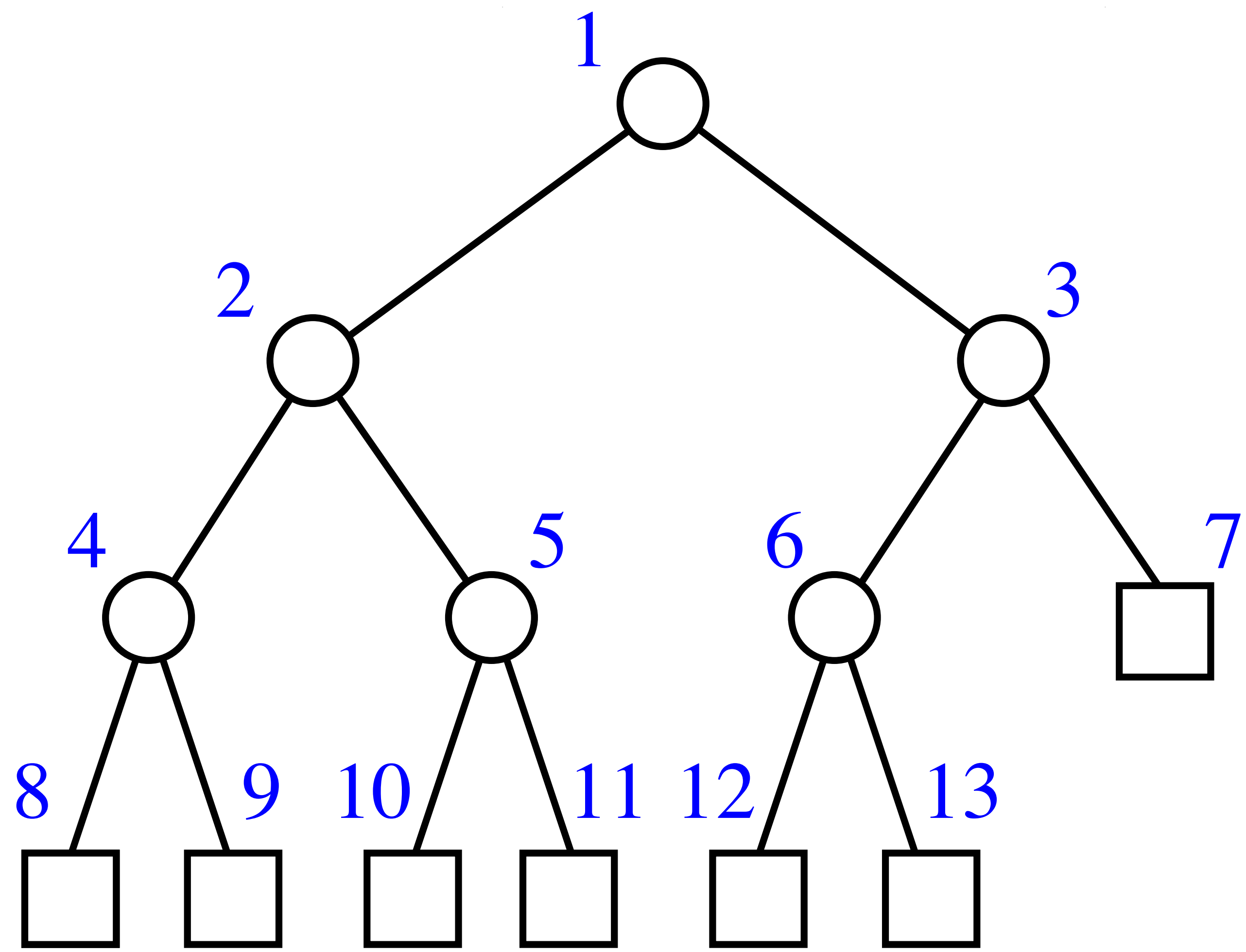


Vector Based Implementation

- Updates in the underlying tree occur only at the "last element"
- A heap can be represented by a vector, where the node at rank i has
  - left child at rank $2i$ and
  - right child at rank $2i + 1$



Vector Based Implementation



- The leaves do no need to be explicitly stored
- Insertion and removals into/from the heap correspond to insertLast and removeLast on the vector, respectively

Heap Sort

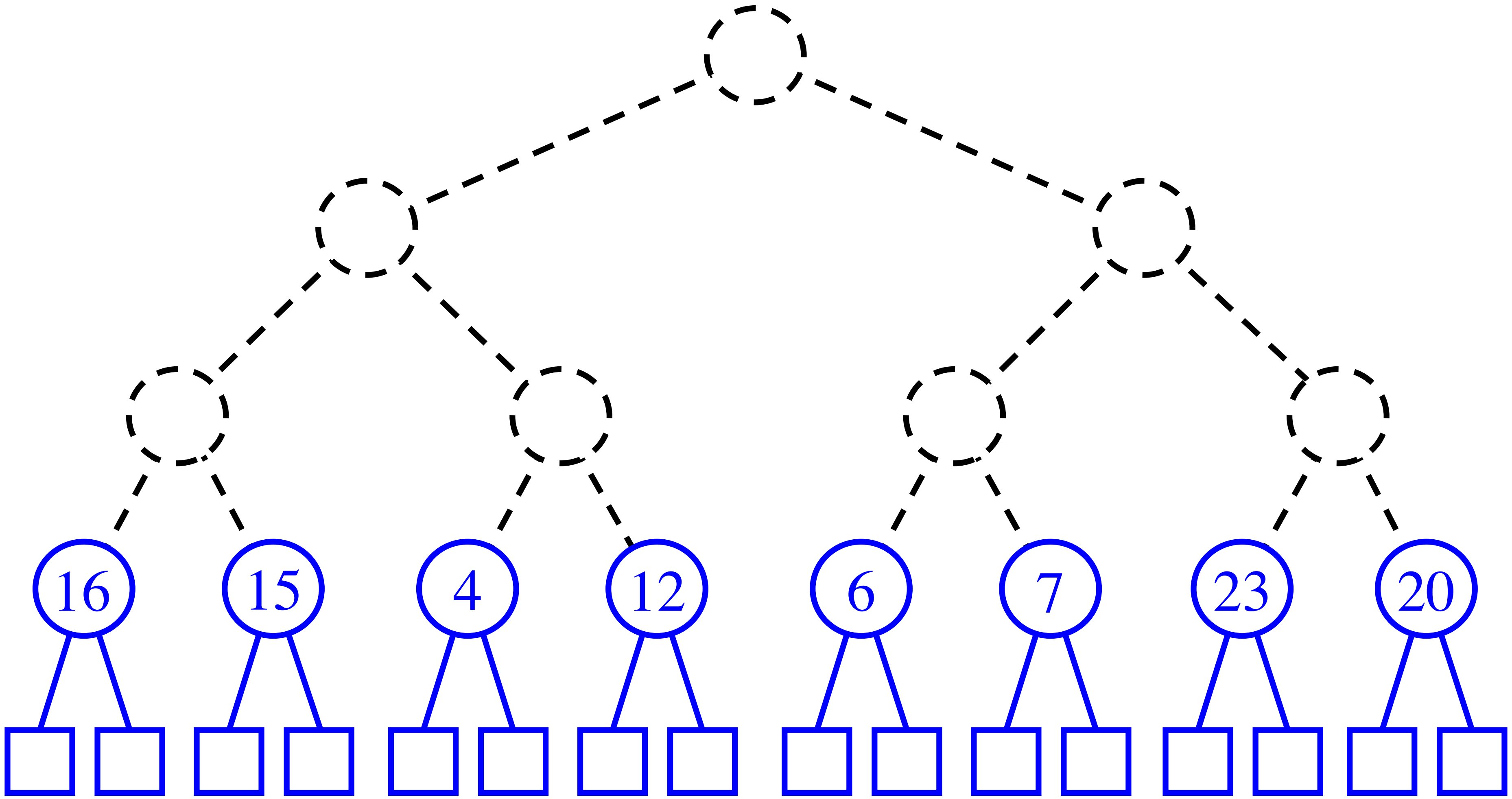
- All heap methods run in logarithmic time or better
- If we implement PriorityQueueSort using a heap for our priority queue, insertItem and removeMin each take O(log k), k being the number of elements in the heap at a given time.
- We always have at most n elements in the heap, so the worst case time complexity of these methods is O(log n).
- Thus each phase takes O(n log n) time, so the algorithm runs in O(n log n) time also.
- This sort is known as ***heap-sort***.
- The O(n log n) run time of heap-sort is much better than the O(n^2) run time of selection and insertion sort.

In-Place Heap-Sort

- Do not use an external heap
- Embed the heap into the sequence, using the vector representation

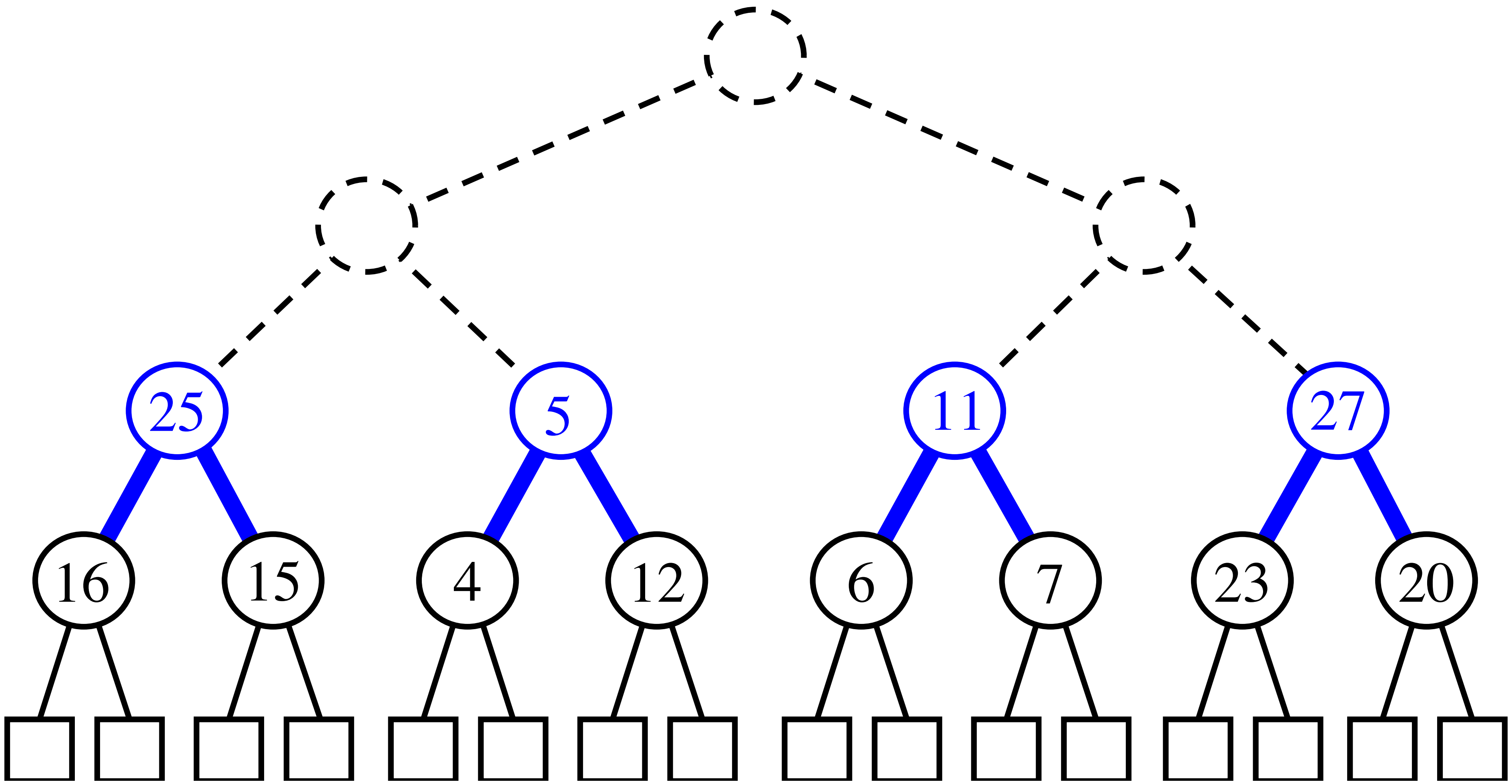
Bottom-Up Heap Construction

- build $(n + 1)/2$ trivial one-element heaps



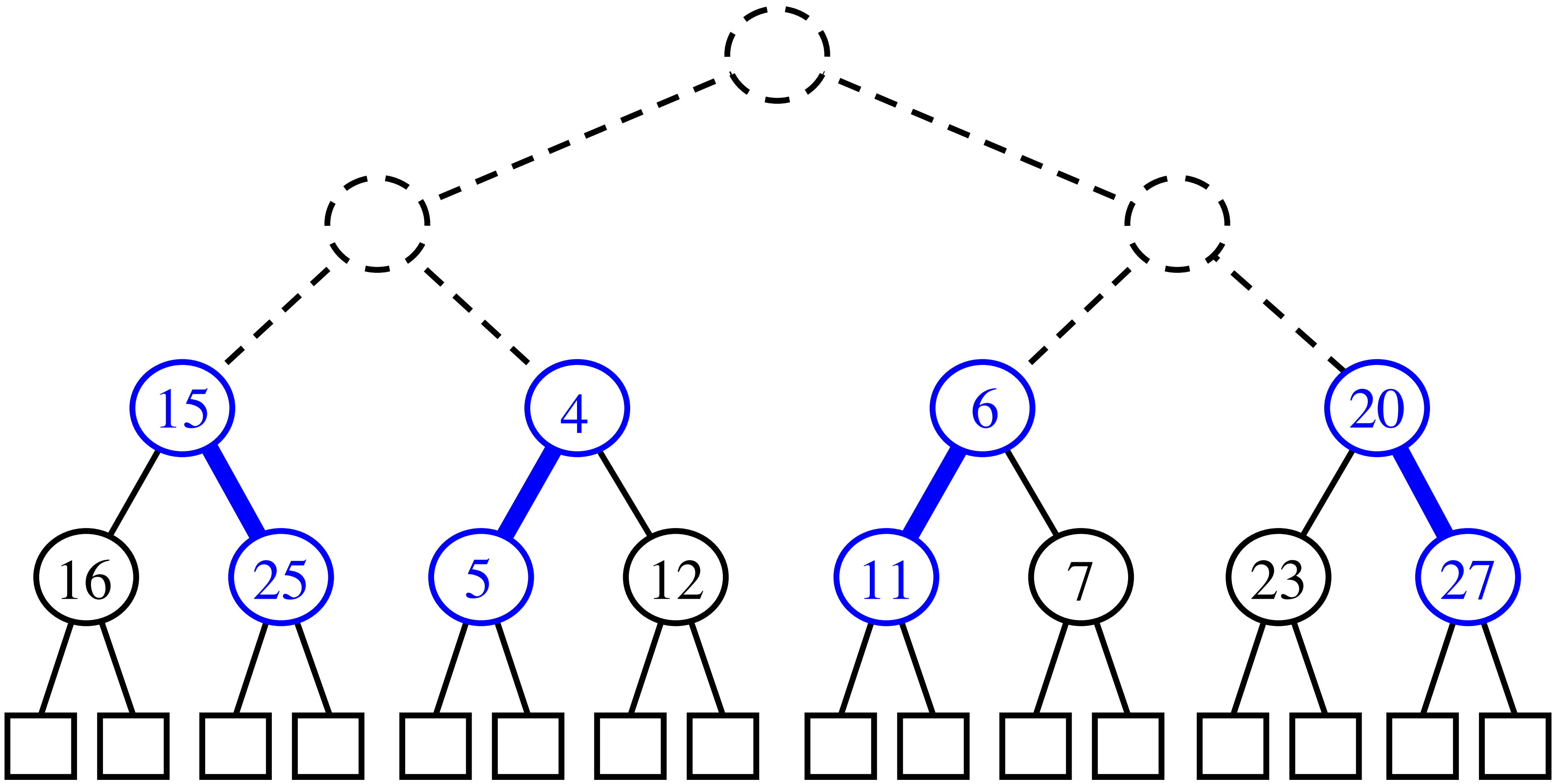
Bottom-Up Heap Construction

- now build three-element heaps on top of them



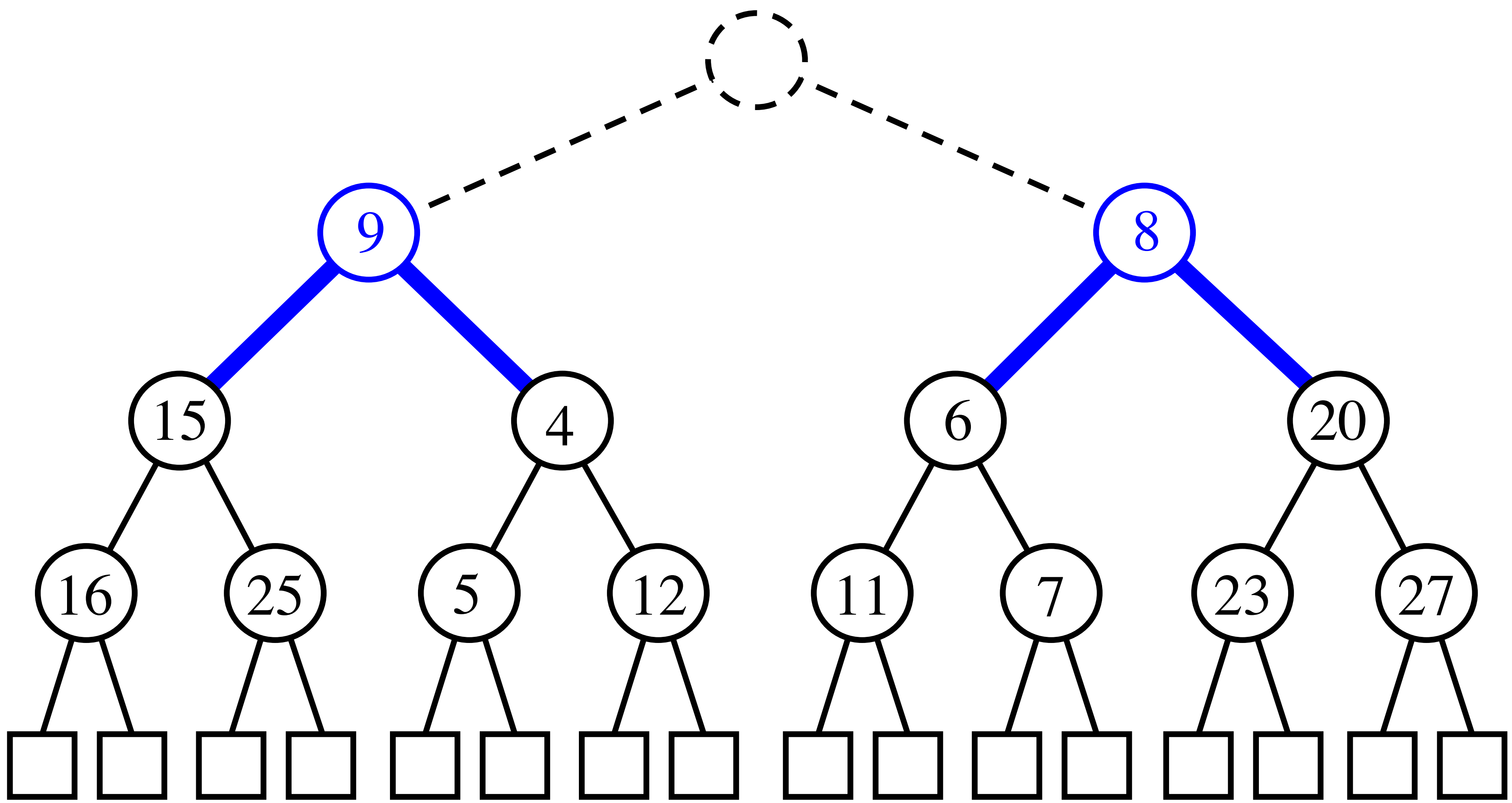
Bottom-Up Heap Construction

- ***downheap*** to preserve the order property

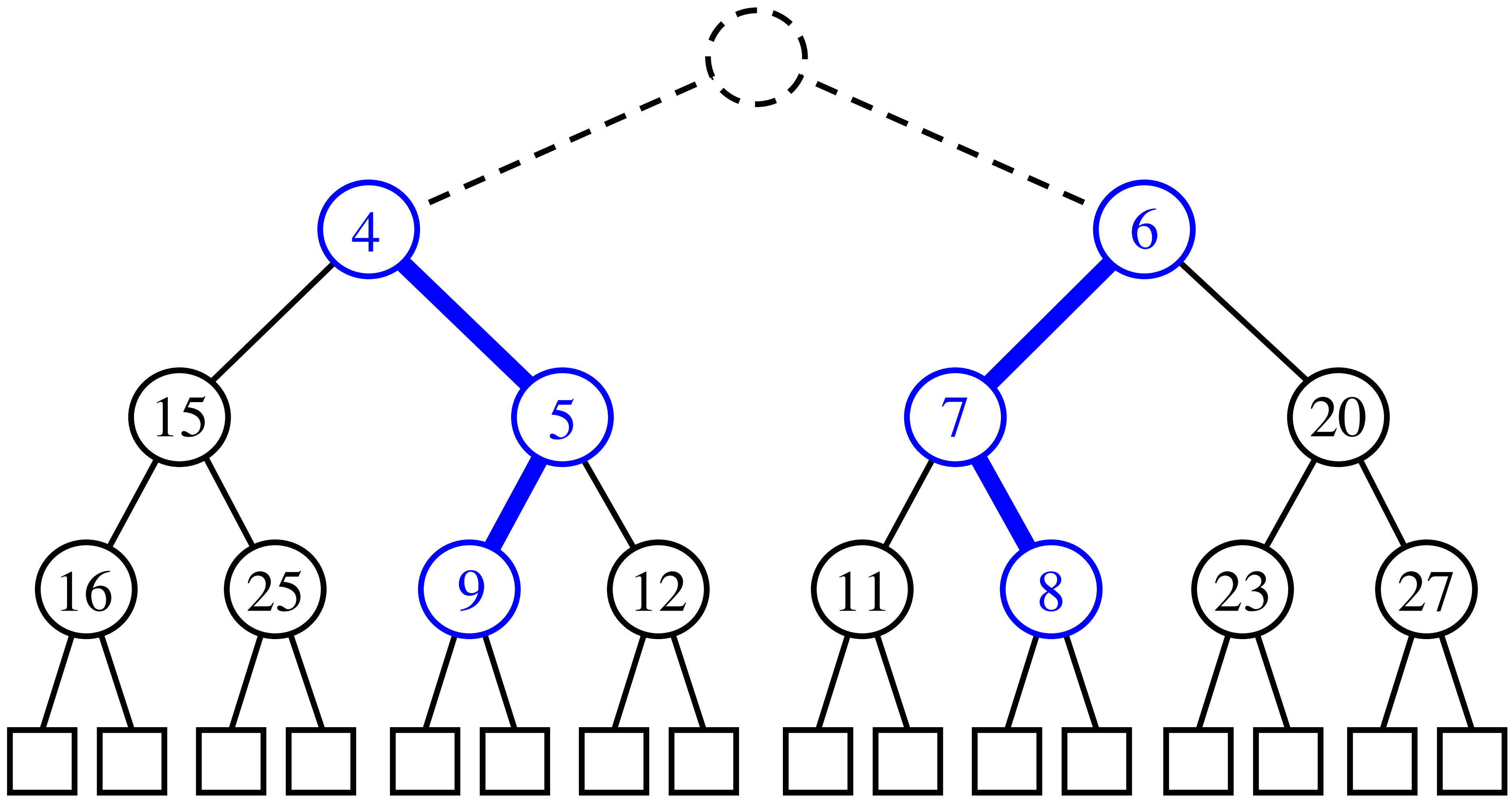


Bottom-Up Heap Construction

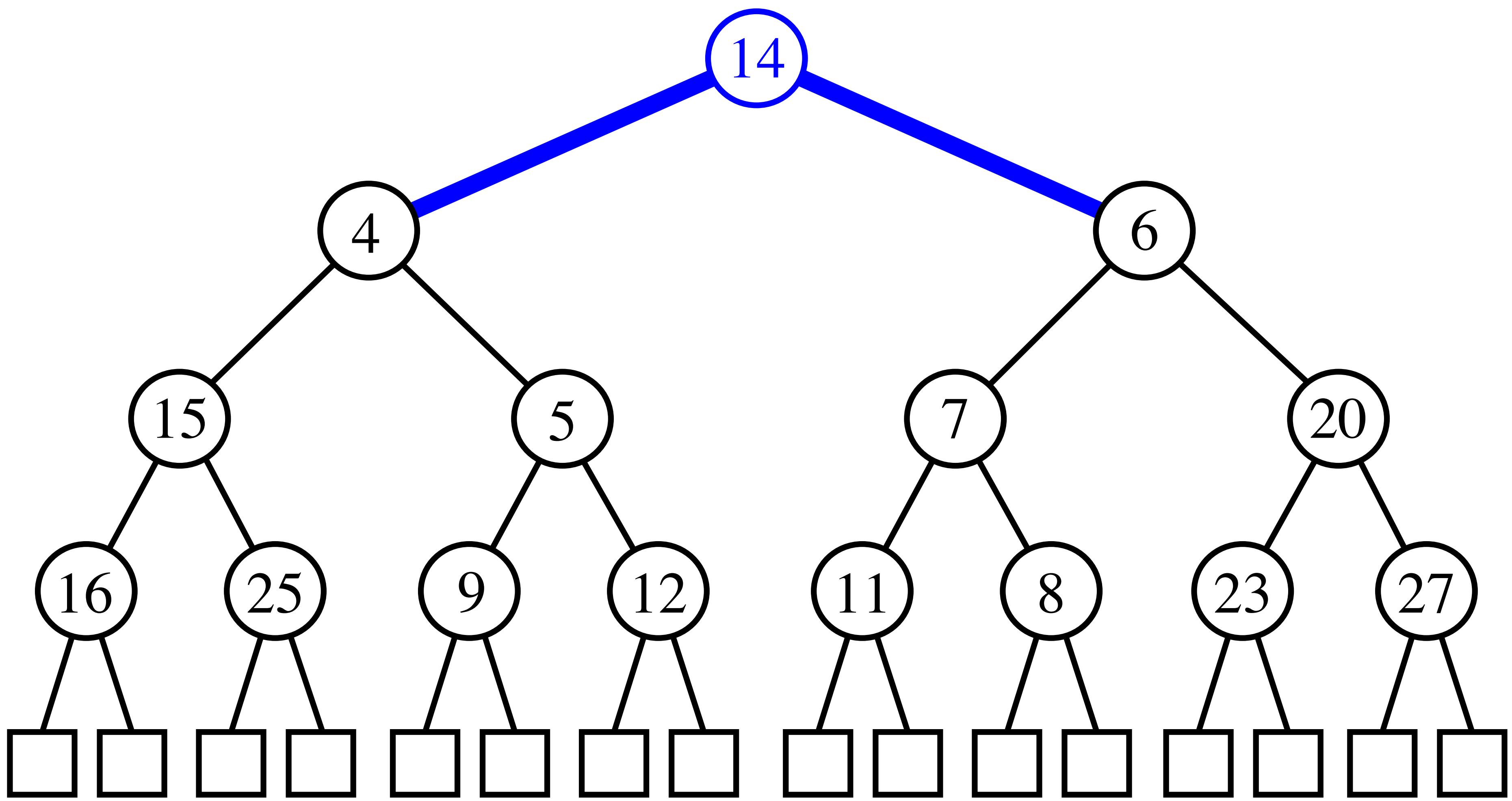
- now form seven-element heaps



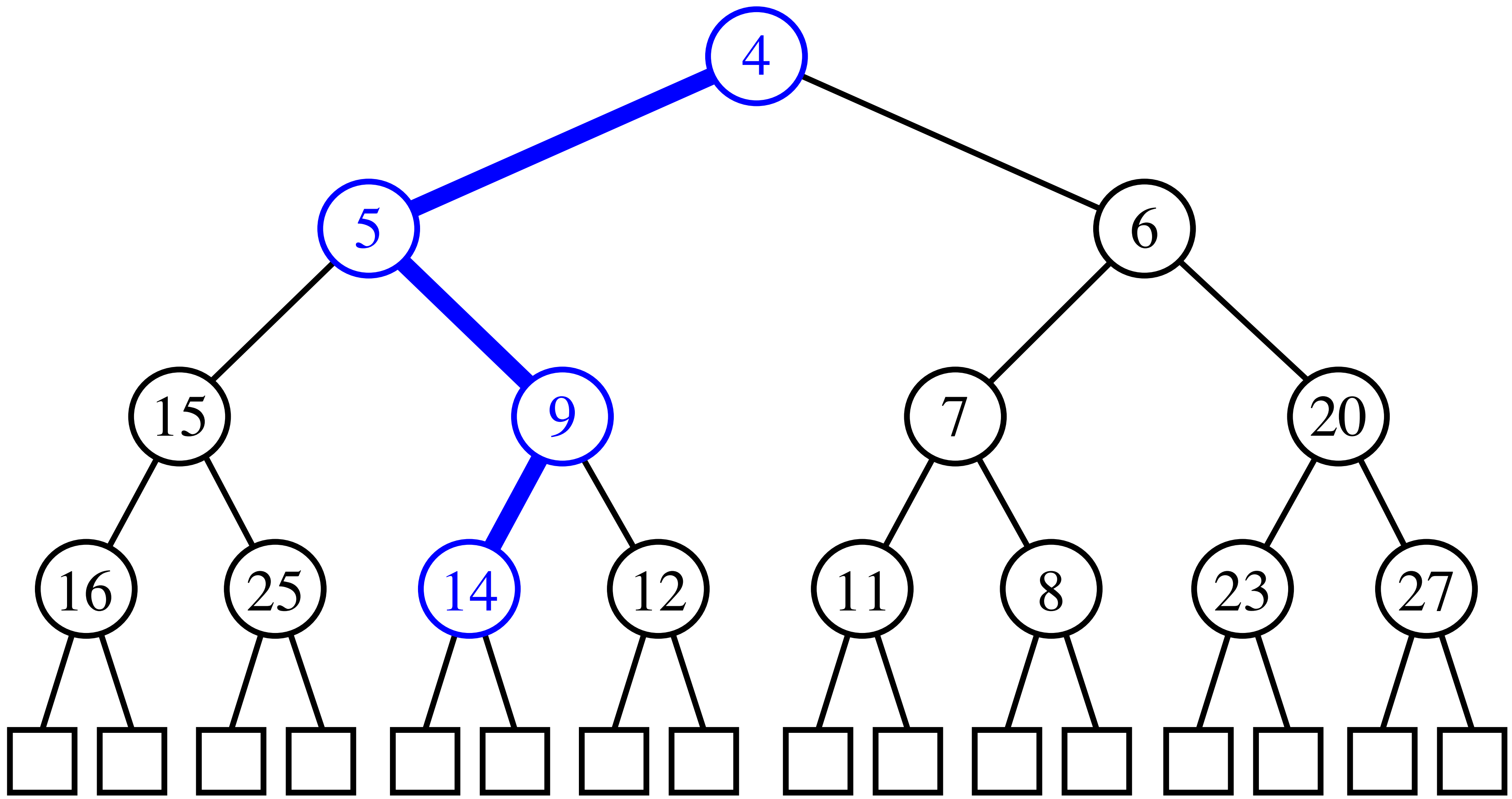
Bottom-Up Heap Construction (cont.)



Bottom-Up Heap Construction (cont.)



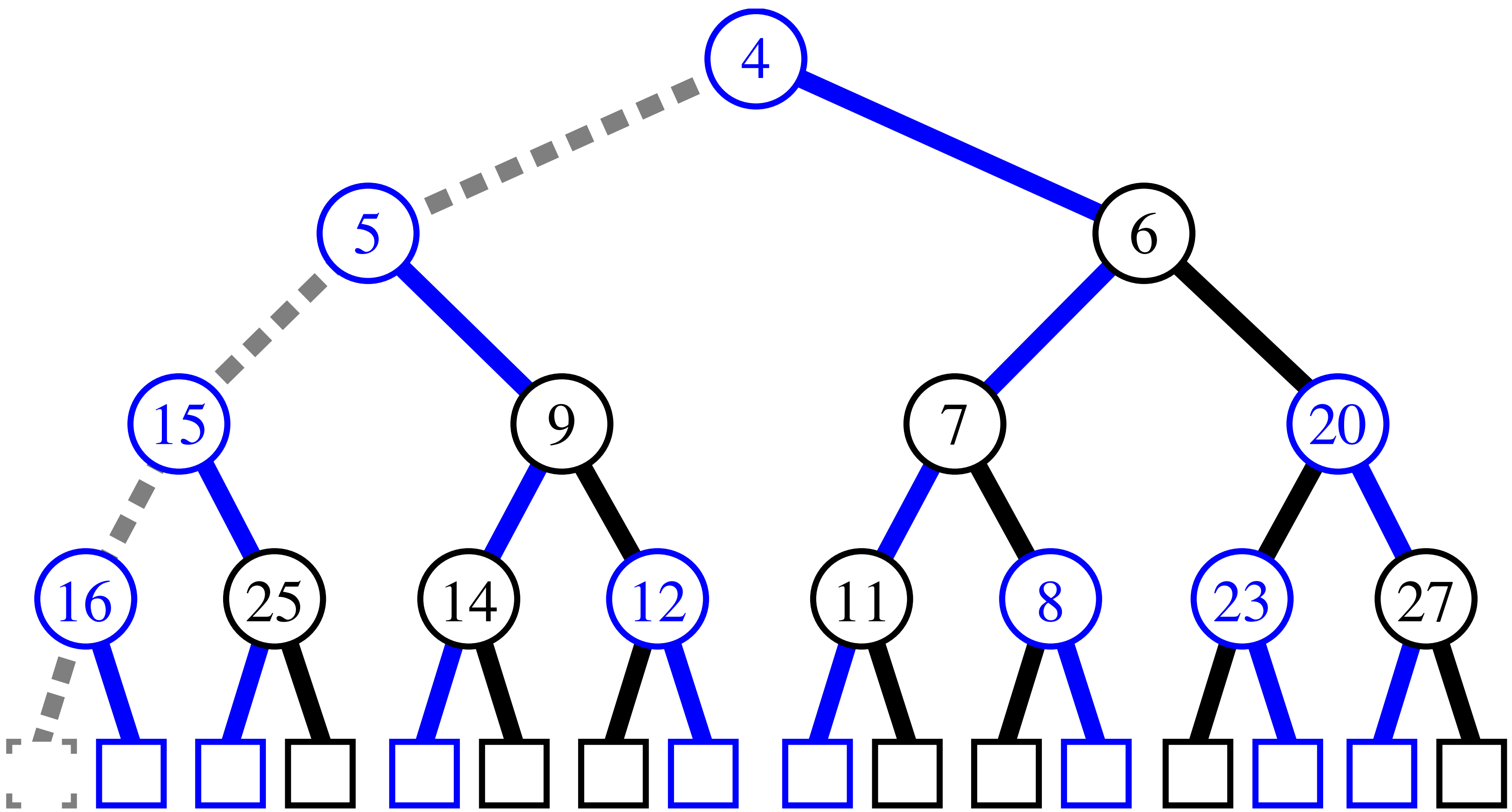
Bottom-Up Heap Construction (cont.)



The End

Analysis of Bottom-Up Heap Construction

- Proposition: Bottom-up heap construction with n keys takes $O(n)$ time.
  - Insert $(n + 1)/2$ nodes
  - Insert $(n + 1)/4$ nodes and downheap them
  - Insert $(n + 1)/8$ nodes and downheap them
  - ...



- n inserts, $n/2$ upheaps with total $O(n)$ running time

Chapter 8

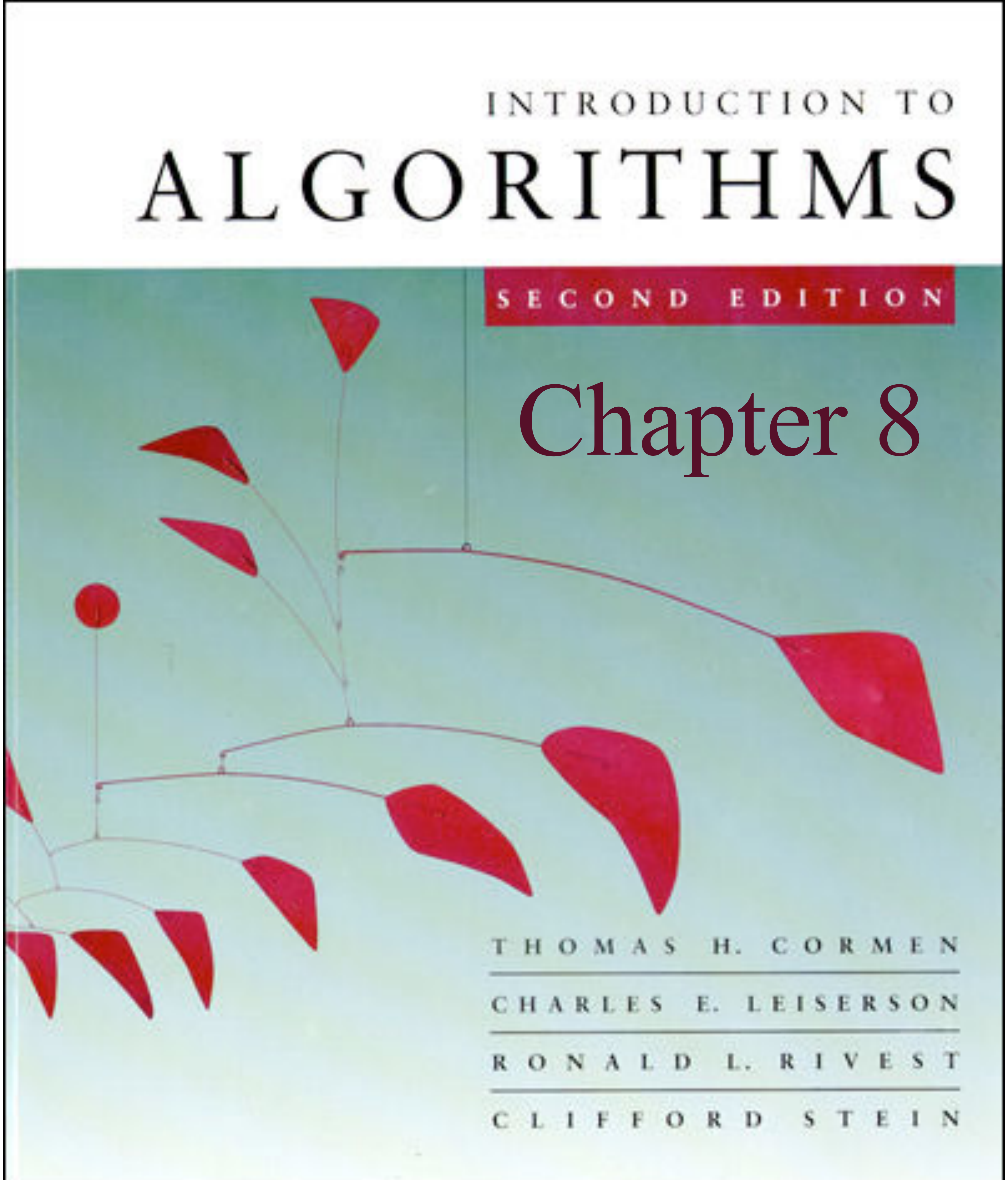

INTRODUCTION TO
ALGORITHMS
SECOND EDITION
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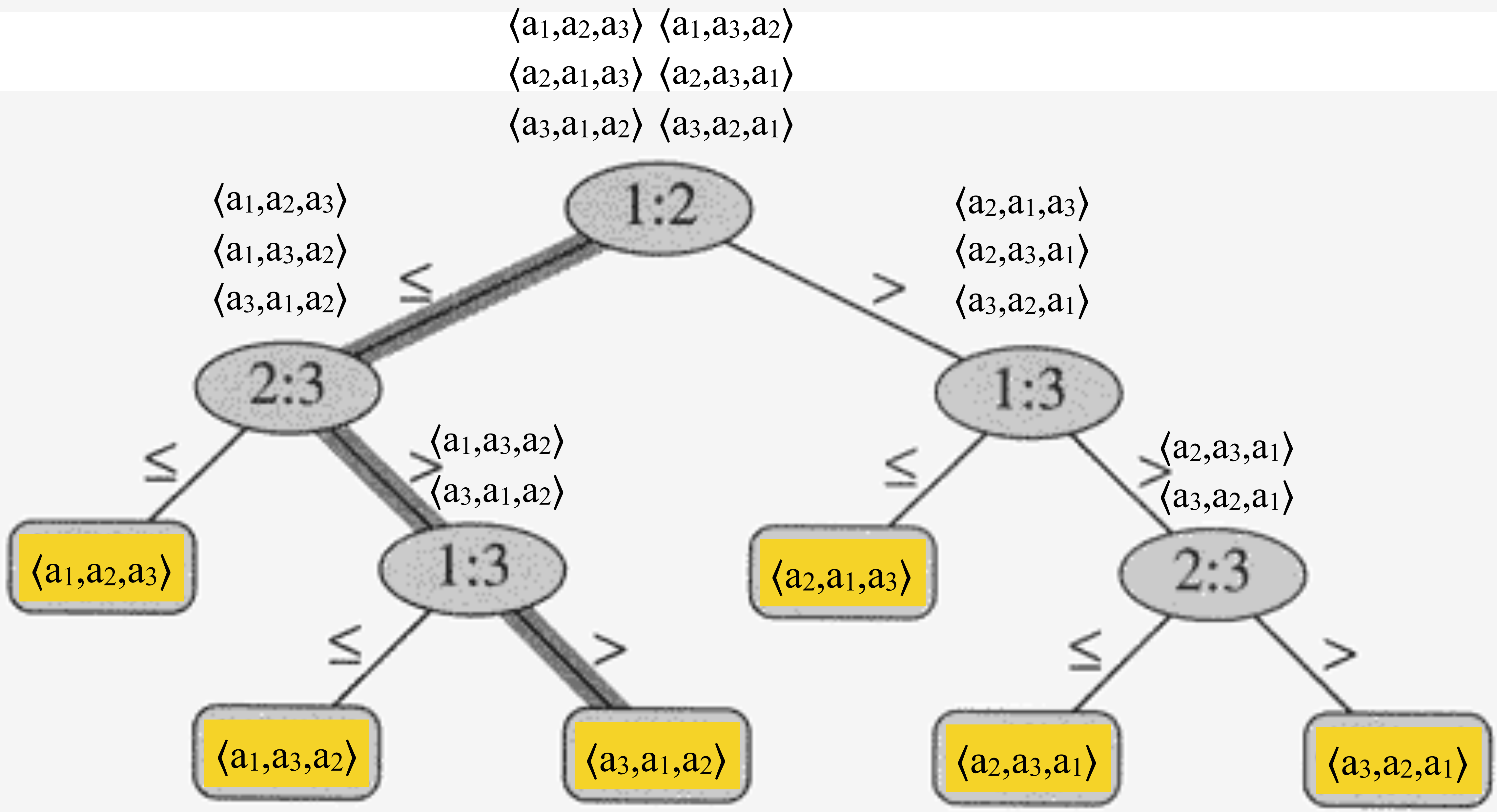
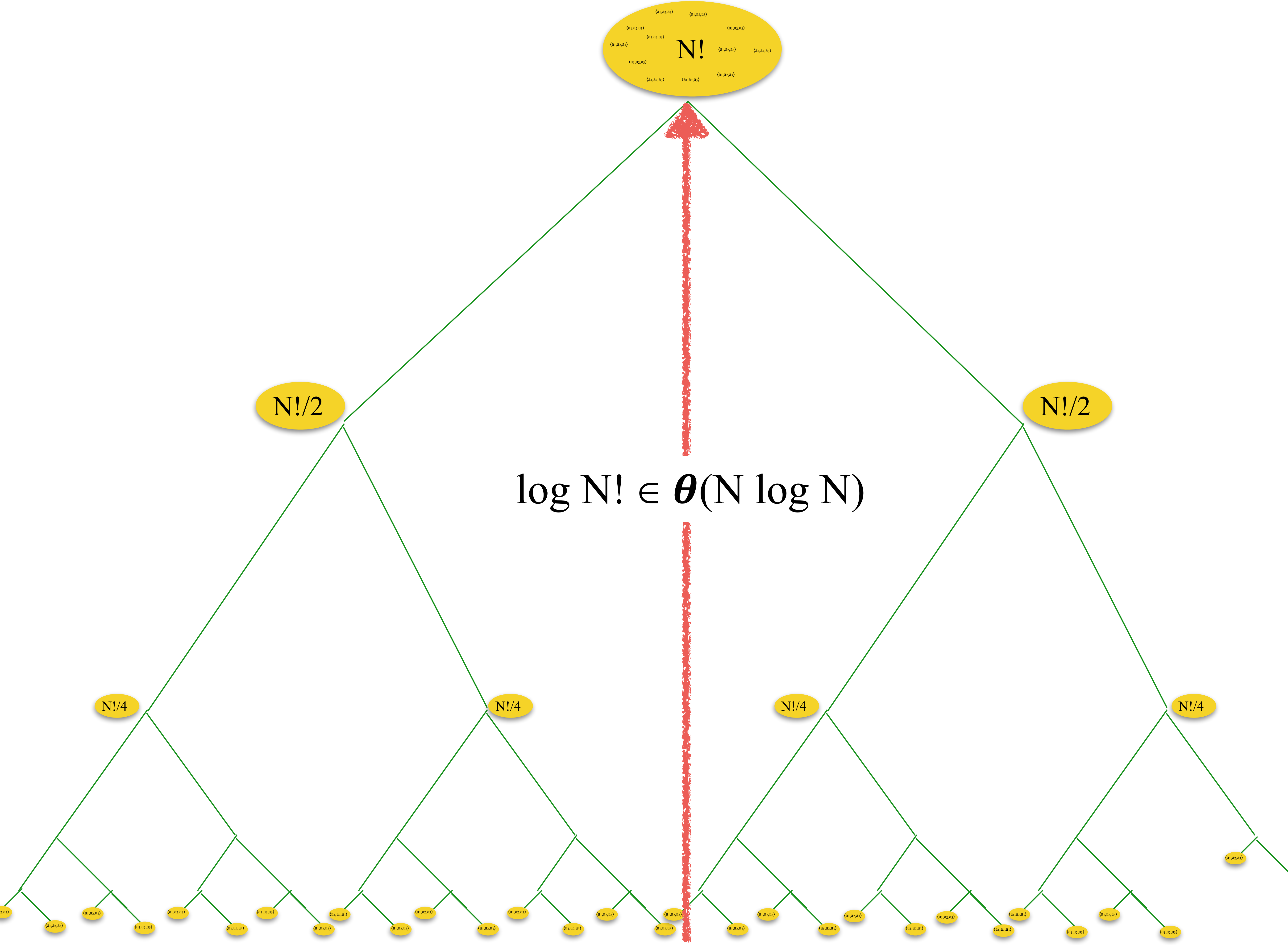
Figure 8.1 The decision tree for insertion sort operating on three elements. An internal node annotated by $i{:}j$ indicates a comparison between a_i and a_j. A leaf annotated by the permutation $\langle \pi(1), \pi(2), \ldots, \pi(n) \rangle$ indicates the ordering $a_{\pi(1)} \leq a_{\pi(2)} \leq \cdots \leq a_{\pi(n)}$. The shaded path indicates the decisions made when sorting the input sequence $\langle a_1 = 6, a_2 = 8, a_3 = 5 \rangle$; the permutation $\langle 3, 1, 2 \rangle$ at the leaf indicates that the sorted ordering is $a_3 = 5 \leq a_1 = 6 \leq a_2 = 8$. There are $3! = 6$ possible permutations of the input elements, so the decision tree must have at least 6 leaves.




N!
N!/2
N!/2
log N! ∈ θ(N log N)
N!/4
N!/4
N!/4
N!/4

Winter 2016
COMP-250: Introduction to Computer Science

Lecture 20, March 24, 2016

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