Grade School Algorithms
Grade School Algorithms

Representation quite inefficient
"+" easy to describe
Grade School Algorithms

Representation quite inefficient
"X" easy to describe
Inefficient Representation

1 million kids meditate for world peace
@ Phra Shammakaya temple Thailand
Roman Grade School

Representation very efficient
Roman Grade School

MMCCCXXIV
+ MCMXXXVII

Representation very efficient
"+" complicated to describe
Roman Grade School

MMCCCXXIV
X MCMXXXVII

Representation very efficient
"X" complicated to describe
**Algorithm 1** Addition (base 10): Add two \( N \) digit numbers \( a \) and \( b \) which are represented as arrays of digits.
Algorithm 1 Addition (base 10): Add two $N$ digit numbers $a$ and $b$ which are represented as arrays of digits

\[
\begin{array}{c}
\begin{array}{c}
0 \\
0 \\
0 \\
0 \\
1 \\
\end{array}
\end{array}
\begin{array}{c}
2343 \\
+ \\
4519 \\
\end{array}
\begin{array}{c}
6862 \\
\end{array}
\]
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**Algorithm 1** Addition (base 10): Add two $N$ digit numbers $a$ and $b$ which are represented as arrays of digits

```
carry ← 0
for $i ← 0$ to $N-1$ do
    $r[i] ← R[a[i], b[i], carry]$
    carry ← $L[a[i], b[i], carry]$
end for
$r[N] ← carry$
```
Algorithm 1 Addition (base 10): Add two $N$ digit numbers $a$ and $b$ which are represented as arrays of digits

\[ L[i,j,0] \quad R[i,j,0] \]
Algorithm 1 Addition (base 10): Add two $N$ digit numbers $a$ and $b$ which are represented as arrays of digits

$L[i,j,0]$  

$R[i,j,0]$
**Algorithm 1** Addition (base 10): Add two $N$ digit numbers $a$ and $b$ which are represented as arrays of digits.

<table>
<thead>
<tr>
<th>$L[i,j,1]$</th>
<th>$R[i,j,1]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
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<tr>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
</tr>
</tbody>
</table>

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Algorithm 1: Addition (base 10): Add two $N$ digit numbers $a$ and $b$ which are represented as arrays of digits.

$L[i,j,1]$  $R[i,j,1]$
Algorithm 1 Addition (base 10): Add two $N$ digit numbers $a$ and $b$ which are represented as arrays of digits

$$(3+9+1) / 10 = 1 \quad 3 \quad 1 = (3+9+1) \mod 10$$
Algorithm 1 Addition (base 10): Add two $N$ digit numbers $a$ and $b$ which are represented as arrays of digits.

\[(A+B+C) / 10 = E \quad D = (A+B+C) \mod 10\]
Algorithm 1  Addition (base 10): Add two $N$ digit numbers $a$ and $b$ which are represented as arrays of digits

\[
\begin{align*}
carry & = 0 \\
\text{for } i = 0 \text{ to } N - 1 \text{ do} & \\
& r[i] \leftarrow (a[i] + b[i] + carry) \% 10 \\
& carry \leftarrow (a[i] + b[i] + carry) / 10 \\
\text{end for} & \\
& r[N] \leftarrow carry
\end{align*}
\]
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**Algorithm 1** Addition (base $\beta$): Add two $N \beta$-git numbers $a$ and $b$ which are represented as arrays of $\beta$-gits.

$$ (A+B+C) \mod \beta = E D = (A+B+C) \div \beta $$
Grade School Algorithms

Algorithm 1 Addition (base $\beta$): Add two $\beta$-git numbers $a$ and $b$ which are represented as arrays of $\beta$-gits.

\[
\begin{align*}
\text{carry} & = 0 \\
\text{for } i = 0 & \text{ to } N - 1 \text{ do} \\
& \quad r[i] \leftarrow (a[i] + b[i] + \text{carry}) \% \beta \\
& \quad \text{carry} \leftarrow (a[i] + b[i] + \text{carry}) / \beta \\
\text{end for} \\
r[N] & \leftarrow \text{carry}
\end{align*}
\]
Example: addition base 8

\[
\begin{array}{c}
  \underline{101} \\
  (1205)_8 \\
  + \ (736)_8 \\
  \hline \\
  (2143)_8
\end{array}
\]

(1123 in base ten)
Example: addition base 8

\[
\begin{align*}
\quad & \quad \quad \quad 1 \quad 0 \quad 1 \\
\hline
(1205)_8 & + \quad (736)_8 \\
\hline
(2143)_8 & = (1123)_x
\end{align*}
\]
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\[
\begin{array}{c}
51 \\
6343 \\
- \quad 4519 \\
1824
\end{array}
\]
Algorithm 2: Multiplication (base 10) of two numbers $a$ and $b$
Algorithm 2 Multiplication (base 10) of two numbers $a$ and $b$

\[ \begin{array}{c}
20 \\
\times 4 \\
\hline \\
352 \\
\hline \\
1408
\end{array} \]
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Algorithm 2 Multiplication (base 10) of two numbers $a$ and $b$

$$352 \times 964$$

_______

1408
Algorithm 2 Multiplication (base 10) of two numbers $a$ and $b$

\[
\begin{array}{c}
352 \\
\times \quad 964 \\
\hline
1408 \\
21120
\end{array}
\]
Algorithm 2 Multiplication (base 10) of two numbers $a$ and $b$

$$
352 \\
\times \ 964 \\
\hline
1408 \\
21120 \\
316800
$$
Algorithm 2 Multiplication (base 10) of two numbers $a$ and $b$

$$
\begin{array}{c}
352 \\
\times 964 \\
\hline
1408 \\
21120 \\
316800 \\
\hline
339328
\end{array}
$$
Algorithm 2 Multiplication (base 10) of two numbers \( a \) and \( b \)

```plaintext
for \( j = 0 \) to \( N - 1 \) do
  carry \( \leftarrow 0 \)
  for \( i = 0 \) to \( N - 1 \) do
    prod \( \leftarrow (a[i] \times b[j] + \text{carry}) \)
    \( tmp[j][i+j] \leftarrow \text{prod} \mod 10 \)
    carry \( \leftarrow \text{prod} / 10 \)
  end for
  \( tmp[j][N+j] \leftarrow \text{carry} \)
end for

carry \( \leftarrow 0 \)
for \( i = 0 \) to \( 2 \times N - 1 \) do
  sum \( \leftarrow \text{carry} \)
  for \( j = 0 \) to \( N - 1 \) do
    sum \( \leftarrow \text{sum} + \text{tmp}[j][i] \)
  end for
  \( r[i] \leftarrow \text{sum} \mod 10 \)
  carry \( \leftarrow \text{sum} / 10 \)
end for
\( r[2 \times N] \leftarrow \text{carry} \)
```
Multiplication

\[
\text{for } j = 0 \text{ to } N - 1 \text{ do}
\]
\[
\text{carry } \leftarrow 0
\]
\[
\text{for } i = 0 \text{ to } N - 1 \text{ do}
\]
\[
\text{prod } \leftarrow (a[i] \ast b[j] + \text{carry})
\]
\[
\text{tmp}[j][i + j] \leftarrow \text{prod} \mod 10
\]
\[
\text{carry } \leftarrow \text{prod} / 10
\]
\[
\text{end for}
\]
\[
\text{tmp}[j][N + j] \leftarrow \text{carry}
\]
\[
\text{end for}
\]
Multiplication

\[
\text{carry} \leftarrow 0 \\
\text{for } i = 0 \text{ to } 2 \times N - 1 \text{ do} \\
\quad \text{sum} \leftarrow \text{carry} \\
\quad \text{for } j = 0 \text{ to } N - 1 \text{ do} \\
\quad\quad \text{sum} \leftarrow \text{sum} + \text{tmp}[j][i] \\
\quad \text{end for} \\
\text{end for} \\
\text{r}[i] \leftarrow \text{sum}\%10 \\
\text{carry} \leftarrow \text{sum}/10 \\
\text{end for} \\
\text{r}[2 \times N] \leftarrow \text{carry}
Algorithm 2 Multiplication (base $\beta$) of two numbers $a$ and $b$

for $j = 0$ to $N - 1$
    $carry \leftarrow 0$
    for $i = 0$ to $N - 1$
        $prod \leftarrow (a[i] \times b[j] + carry)$
        $tmp[j][i + j] \leftarrow prod \mod \beta$
        $carry \leftarrow prod / \beta$
    end for
    $tmp[j][N + j] \leftarrow carry$
end for

carry $\leftarrow 0$
for $i = 0$ to $2 \times N - 1$
    $sum \leftarrow carry$
    for $j = 0$ to $N - 1$
        $sum \leftarrow sum + tmp[j][i]$  
        $r[i] \leftarrow sum \mod \beta$
        $carry \leftarrow sum / \beta$
    end for
end for
$r[2 \times N] \leftarrow carry$
Multiplication base 8

\[(1205)_8 \times (736)_8 = (308310)_{10}\]
Long Division

\[
\begin{array}{cccccccccccccccc}
723 & | & 41672542996 \\
\hline
\end{array}
\]
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\[
\begin{array}{c|c}
723 & 50 \\
\hline
\end{array}
\]

\[
41672542996 \div 723 = 57638372
\]

\[
41672542996 \% 723 = 50
\]