Public Announcement

GETTING YOUR DREAM TECH INTERNSHIP

MARCH 29TH 6PM-7:30PM

TROTTIER 0070

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LIFE AFTER UNDERGRAD

ACADEMIA AND CAREERS IN SCIENCE

WEDNESDAY MARCH 23
7 – 9 PM AT LEACOCK 14

Unsure of your future with your BSc? Advice, anecdotes and guidance from professors, graduate students, and industry professionals who've "been there, done that."

SPEAKERS

Dr. Kenneth J. Ragan
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McGill Department of Physics

Dr. Daniel Bernard
Professor
McGill Department of Pharmacology

Bogdan Istrate
Full Stack Java Developer
TickSmith

Victoria Mallet
Product Manager
Ananda Microfluidics

Arjuna Rajakumar
Graduate Student
Abouheif Lab

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Comment about input size…

2)

Write *any* algorithm that runs in time $\Theta(n^2 \log^2 n)$ in worse case.
Explain why this is its running time. I don't care what it does.
I only care about its running time…

```
WhatEver(int n)

FOR i=1 TO n
  FOR j=1 TO n
      x=n; WHILE x>1 DO { x=x/2; y=n;
                          WHILE y>1 DO y=y/2 }
```

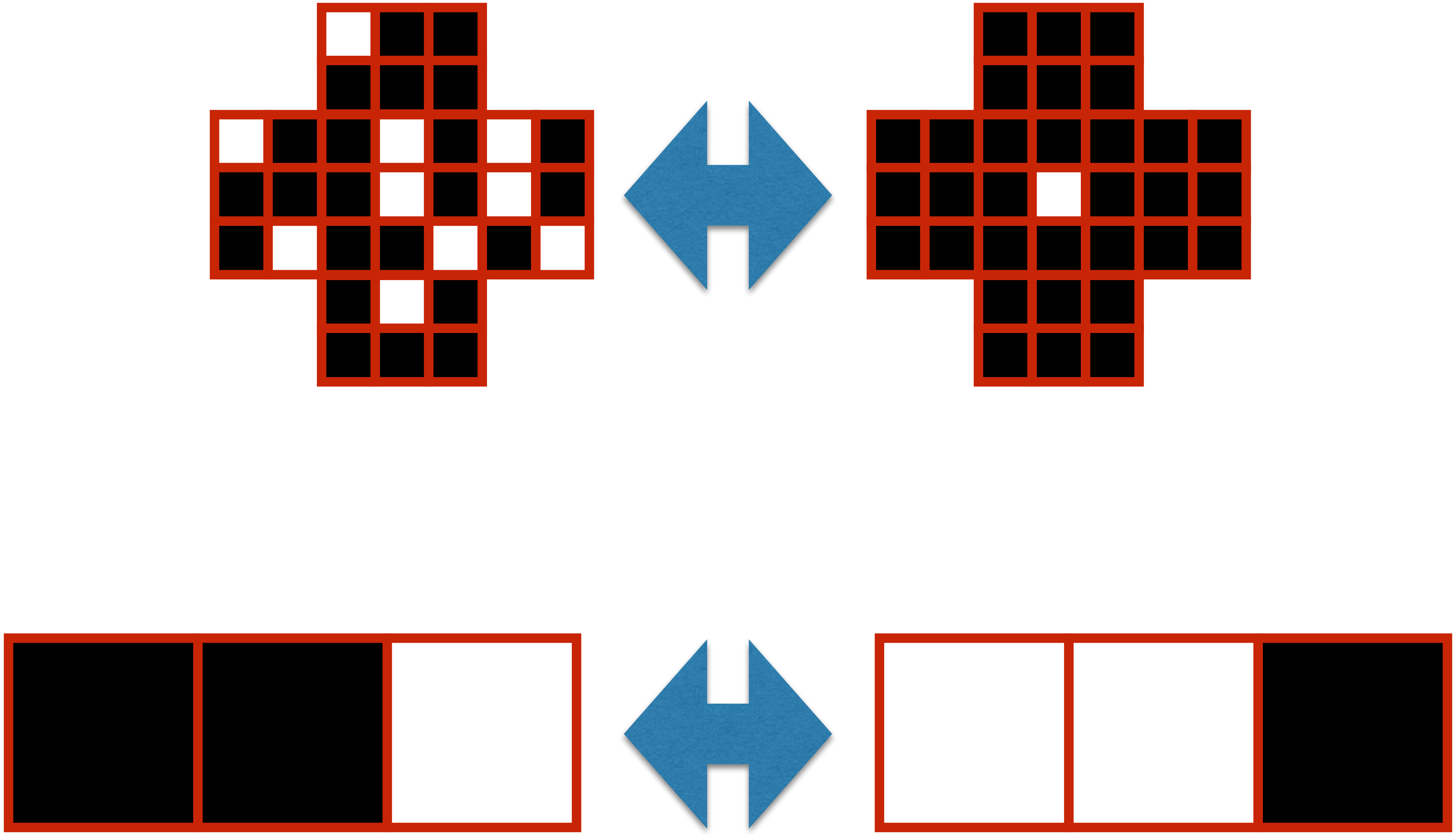
Comment about input size…

2)
Write *any* algorithm that runs in time $\Theta(n^2 \log^2 n)$ in worse case.
Explain why this is its running time. I don't care what it does.
I only care about its running time…

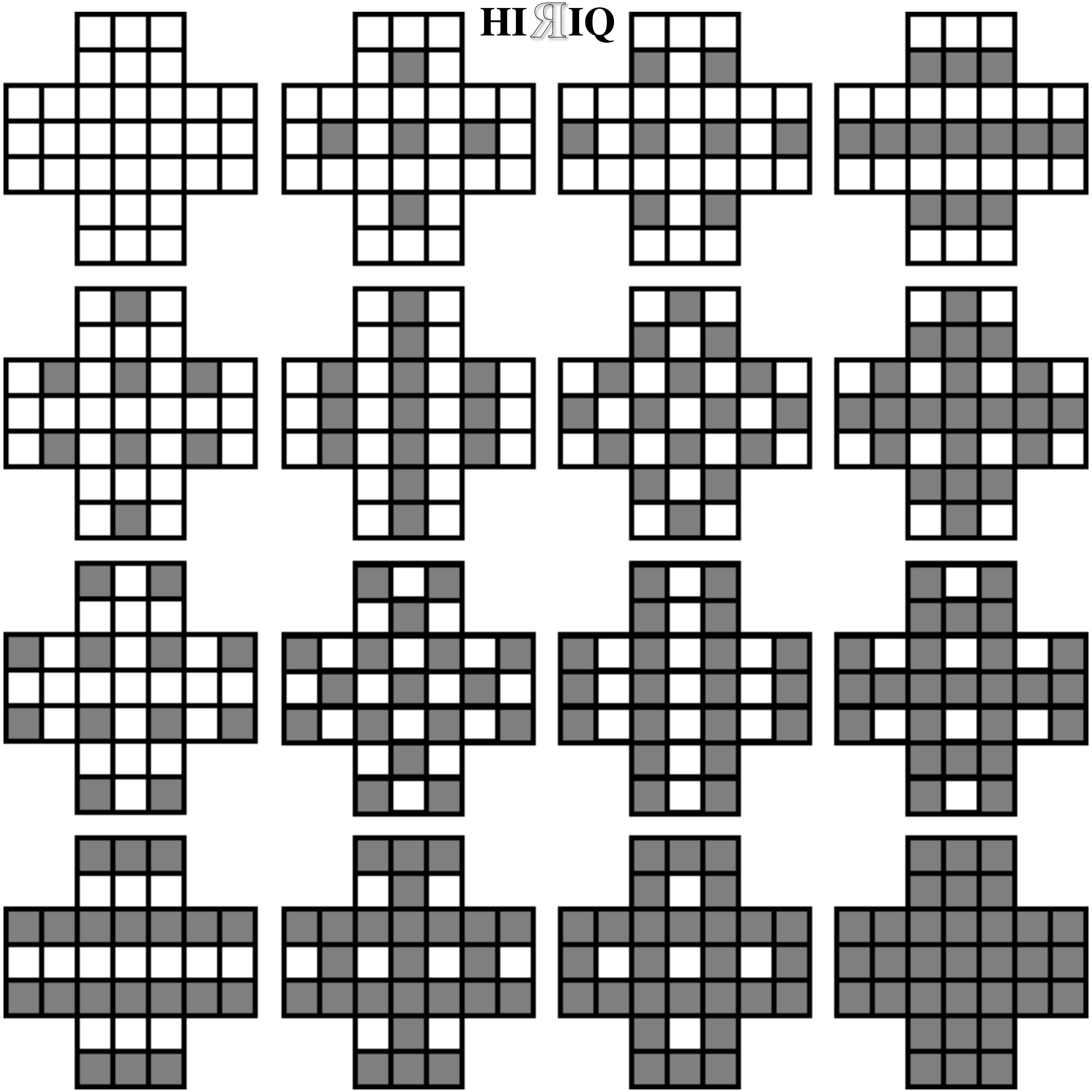
```
WhatEver(int[] A)

n = A.length;
FOR i=1 TO n
  FOR j=1 TO n
      x=n; WHILE x>1 DO { x=x/2; y=n;
                                   WHILE y>1 DO y=y/2 }
```

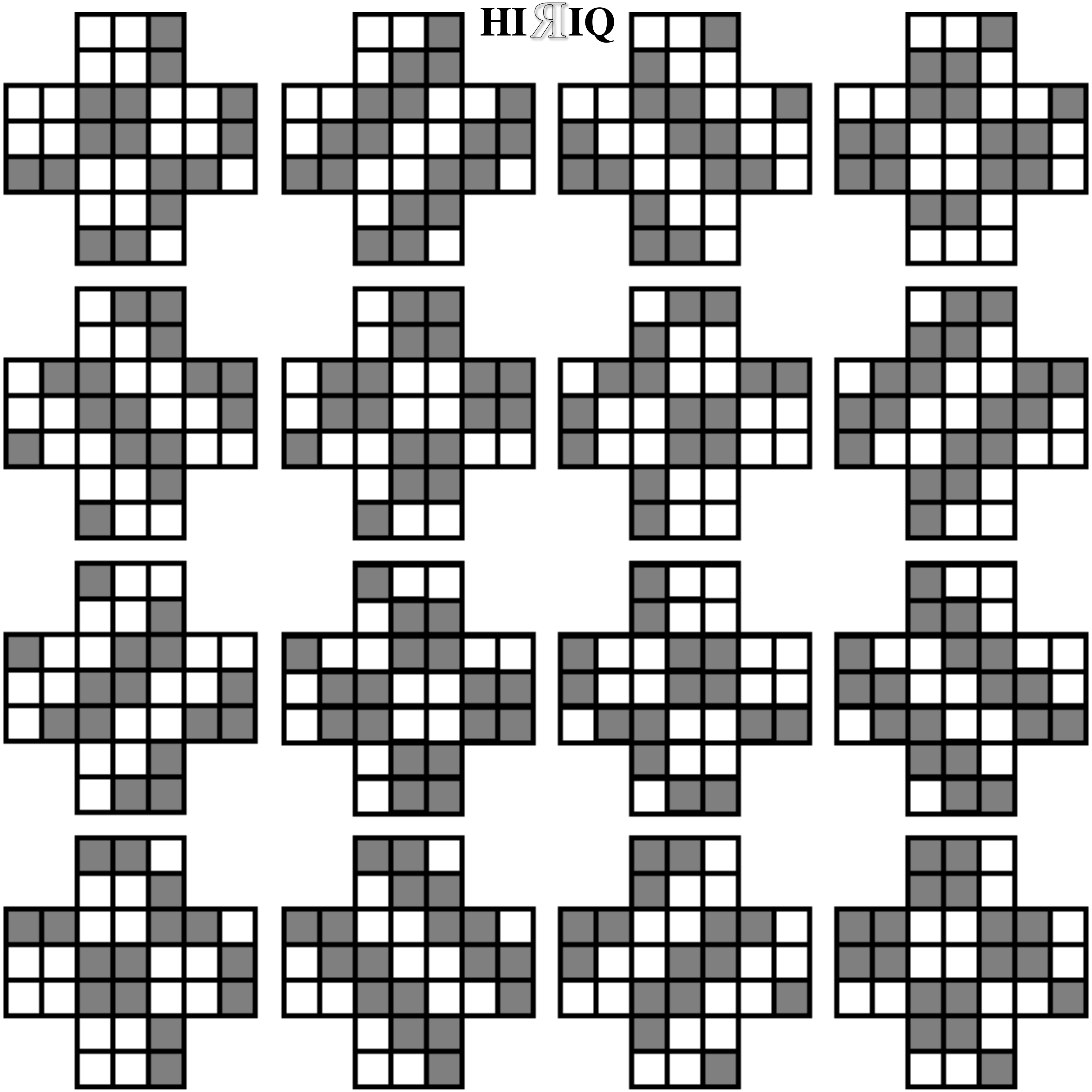
HIЯIQ



16 configurations with 0 neighbours



16 configurations with 38 neighbours



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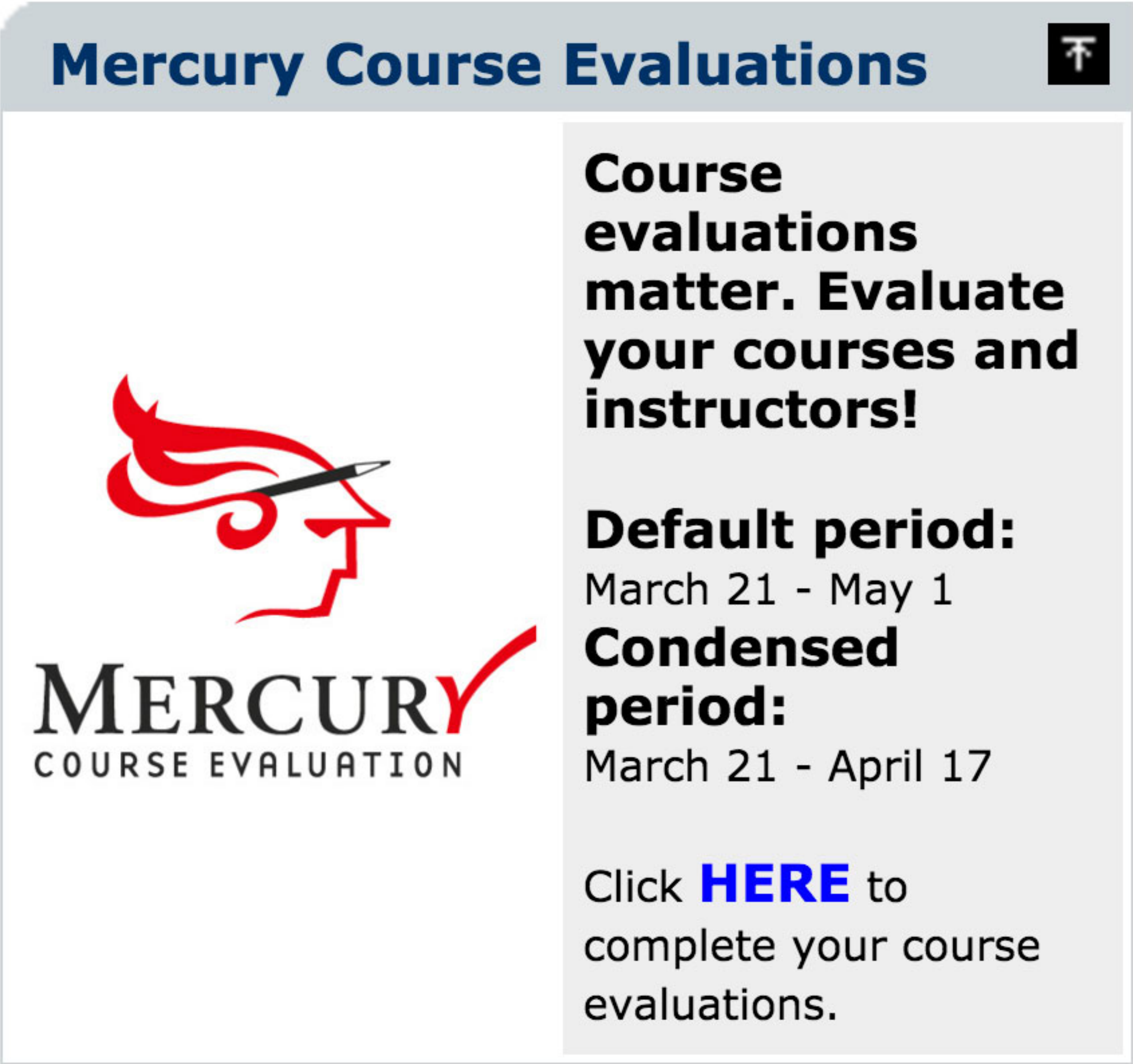


Mercury Course Evaluations

Course evaluations matter. Evaluate your courses and instructors!

Default period:
March 21 - May 1
Condensed period:
March 21 - April 17

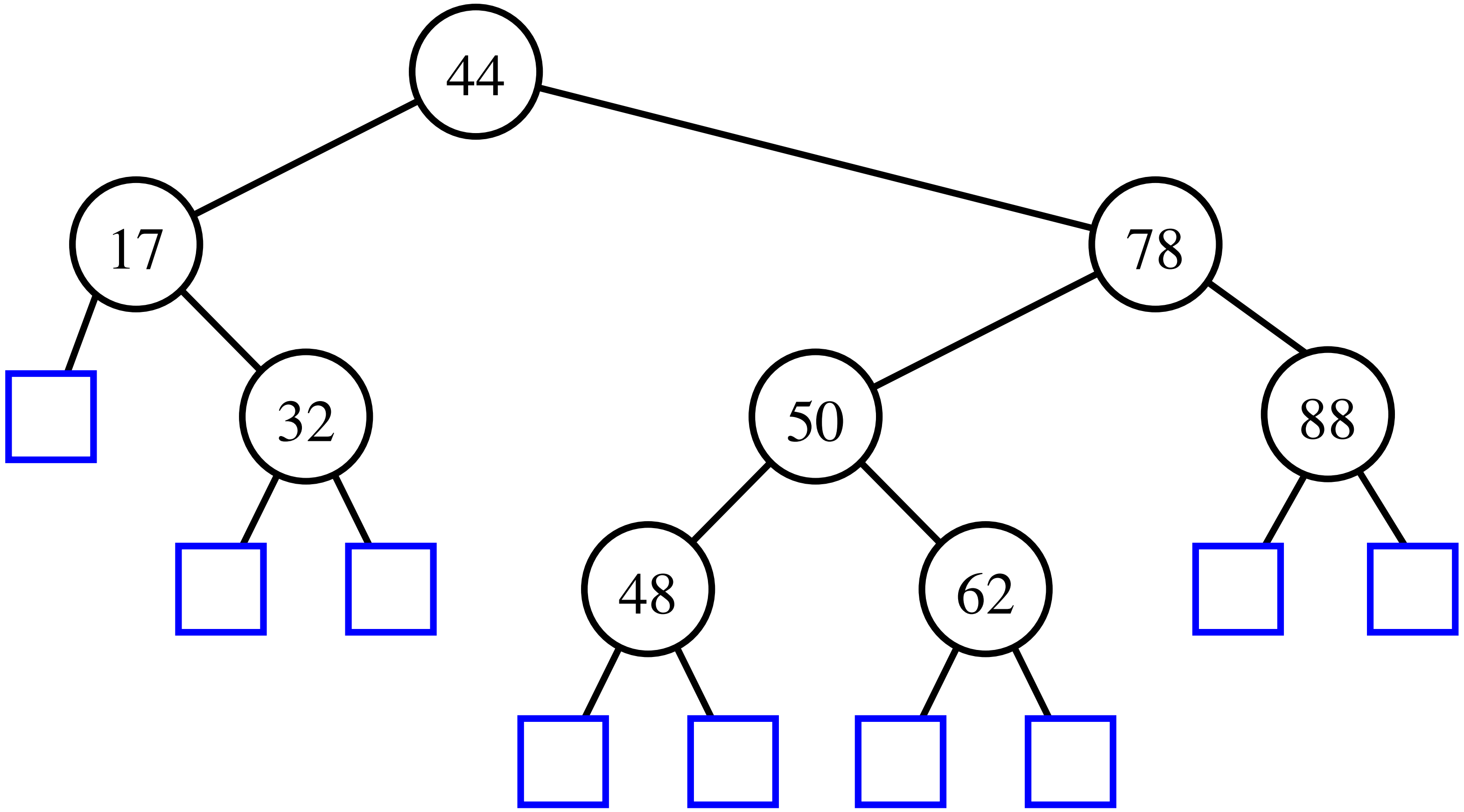
Click **HERE** to complete your course evaluations.

Winter 2016
COMP-250: Introduction to Computer Science

Lecture 19, March 22, 2016

SEARCHING

- the dictionary ADT
- binary search trees



The Dictionary ADT

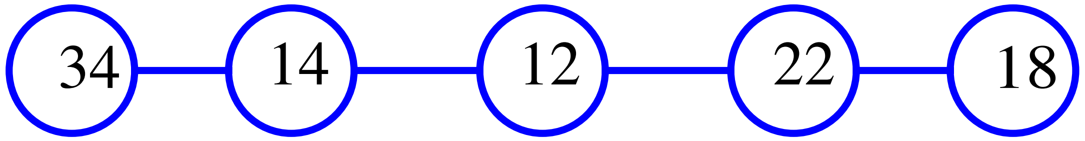
- a dictionary is an abstract model of a database
- like a priority queue, a dictionary stores key-element pairs
- the main operation supported by a dictionary is searching by key
- simple container methods:
  - size()
  - isEmpty()
  - elements()

The Dictionary ADT

- query methods:
  - findElement(k)
  - findAllElements(k)
- update methods:
  - insertItem(k, e)
  - removeElement(k)
  - removeAllElements(k)
- special element
  - NO_SUCH_KEY, returned by an unsuccessful search

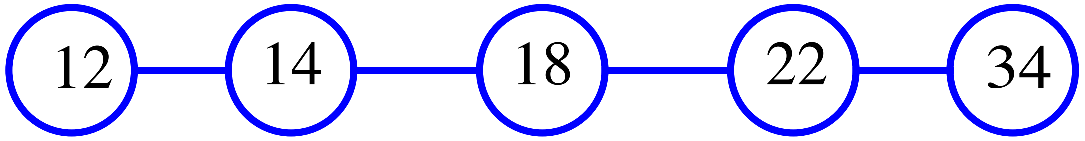
Implementing a Dictionary with a Sequence

- ***unordered sequence***



  - searching and removing takes O(*n*) time
  - inserting takes O(1) time
  - applications to log files (frequent insertions, rare searches and removals)

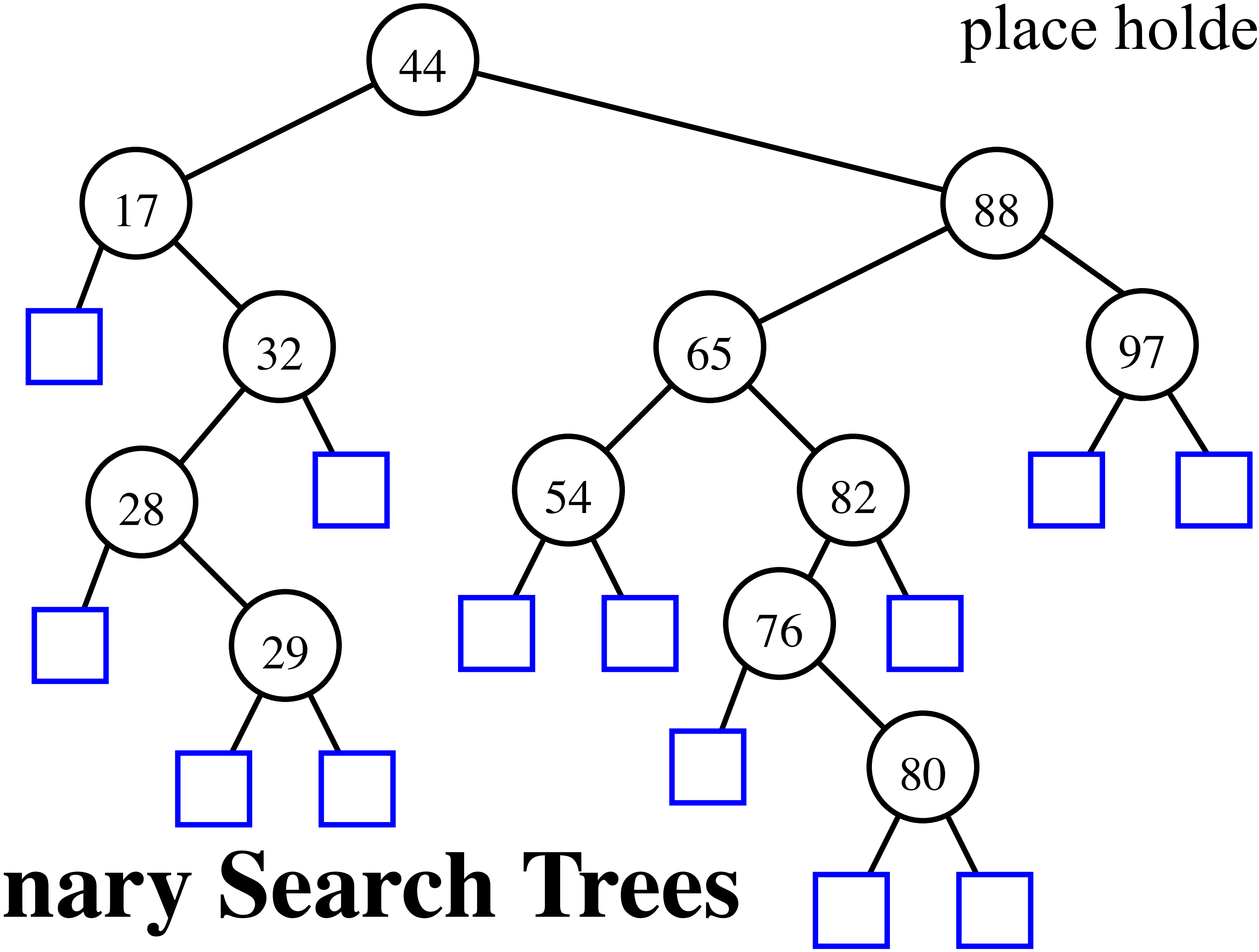
- ***array-based ordered sequence*** (assumes keys can be ordered)



  - searching takes O(log *n*) time (***binary search***)
  - inserting and removing takes O(*n*) time
  - application to look-up tables (frequent searches, rare insertions and removals)

Binary Search Trees

- A binary search tree is a binary tree T such that
  - each internal node stores an item (k, e) of a dictionary.
  - keys stored at nodes in the left subtree of v are less than or equal to k.
  - keys stored at nodes in the right subtree of v are greater than or equal to k.
  - external nodes do not hold elements but serve as place holders.



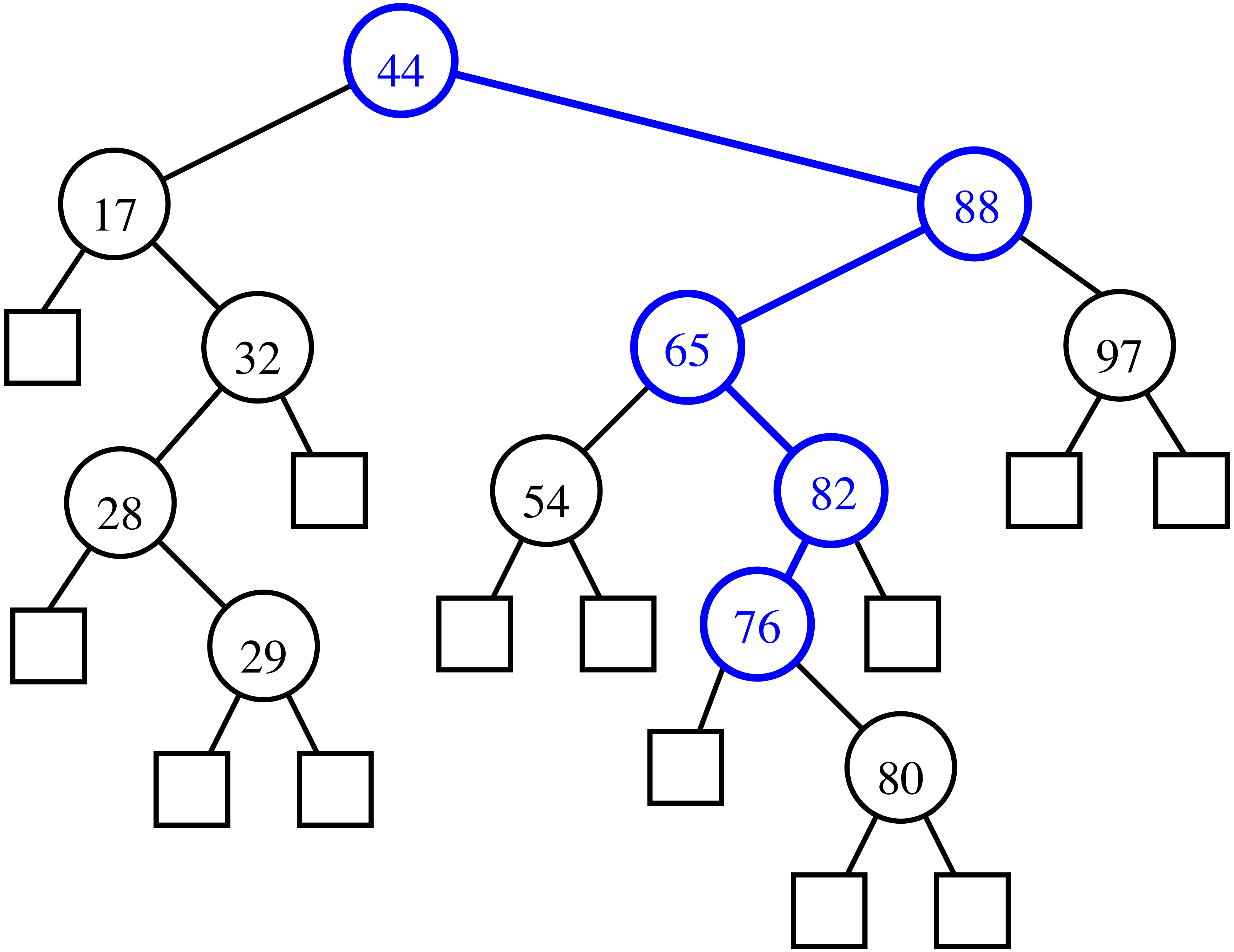
Search

- A binary search tree T is a ***decision tree***, where the question asked at an internal node v is whether the search key k is less than, equal to, or greater than the key stored at v.

```
Algorithm TreeSearch(k, v):
  Input: A search key k and a node v of a binary search
         tree T.
  Ouput: A node w of the subtree T(v) of T rooted at v,
  if v is an external node then
     return v
  if k = key(v) then
     return v
  else if k < key(v) then
     return TreeSearch(k, T.leftChild(v))
  else
     { k > key(v) }
     return TreeSearch(k, T.rightChild(v))
```

Search Example I

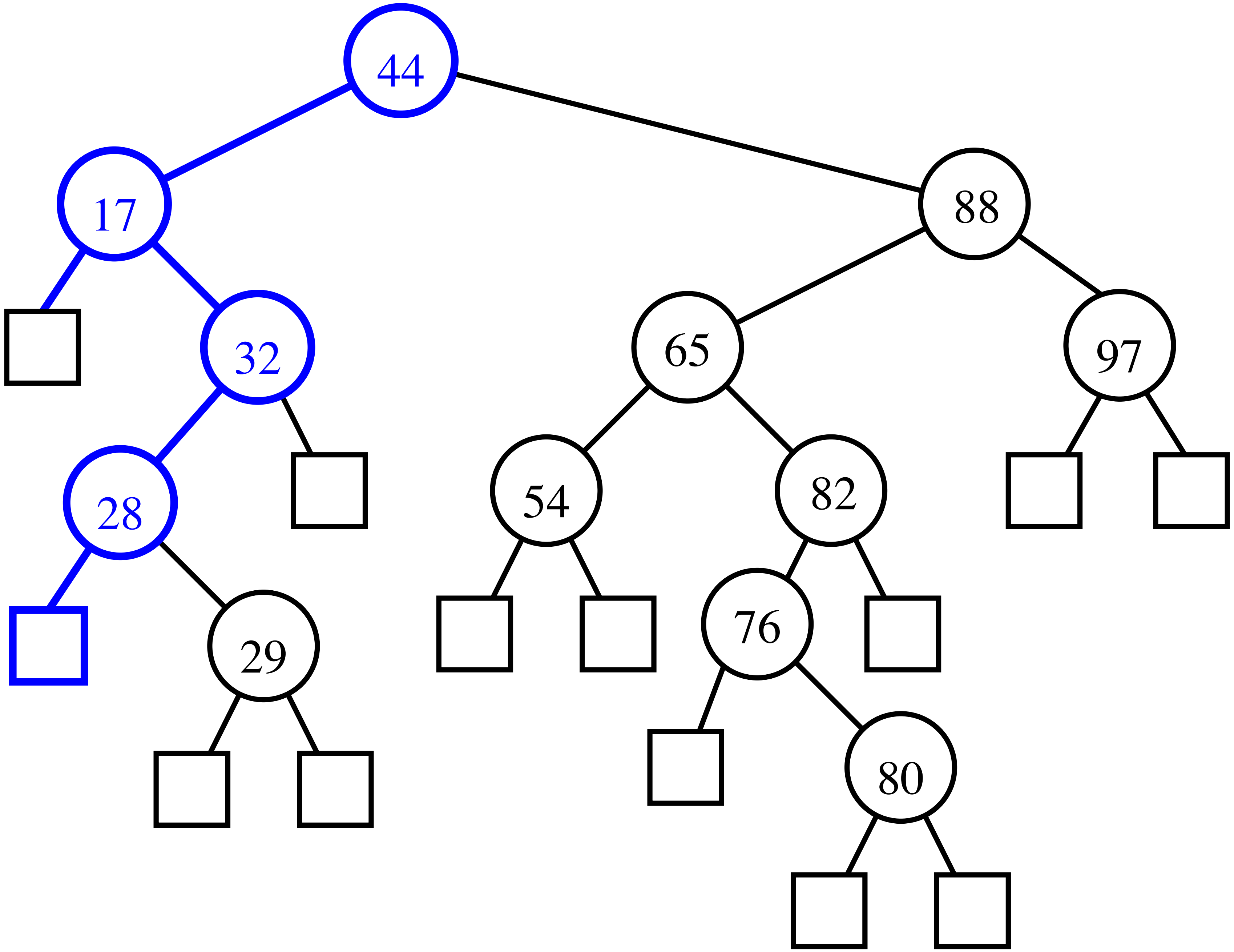
- Successful findElement(76)



- A successful search traverses a path starting at the root and ending at an internal node

Search Example II

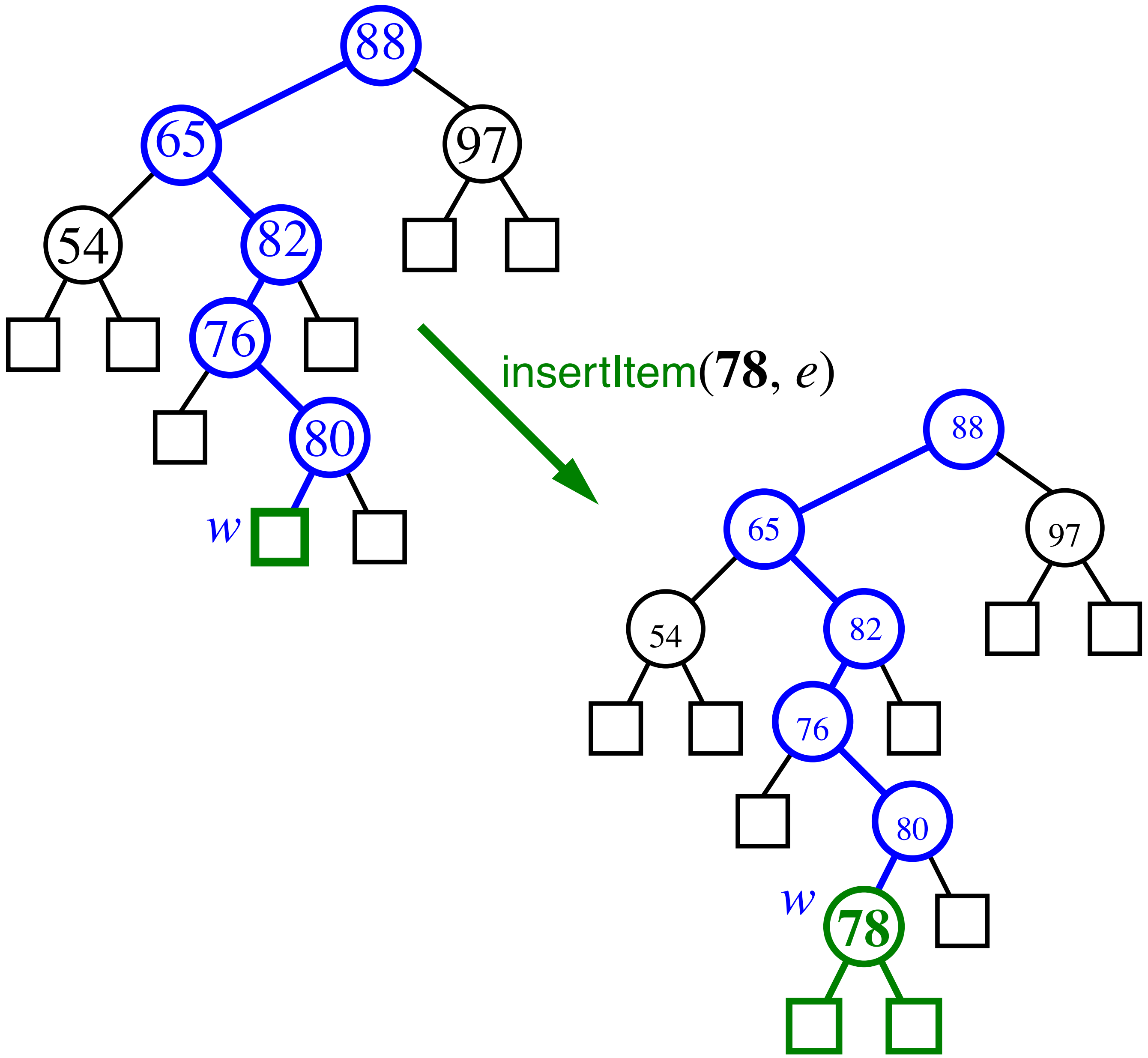
- Unsuccessful findElement(25)



- An unsuccessful search traverses a path starting at the root and ending at an external node

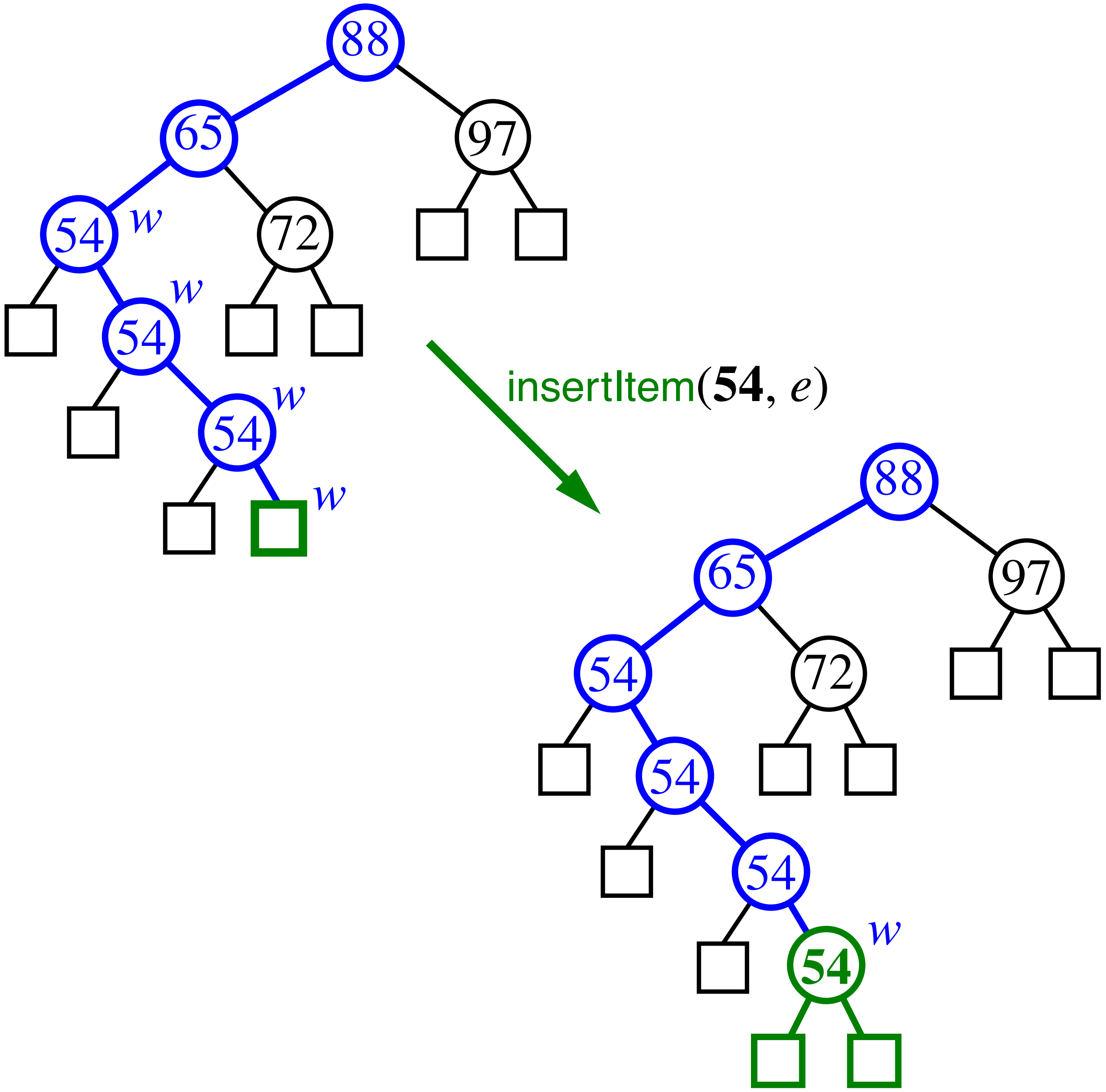
Insertion

- To perform insertItem(*k*, *e*), let *w* be the node returned by TreeSearch(*k*, *T*.root())
- If *w* is external, we know that *k* is not stored in *T*. We call expandExternal(*w*) on *T* and store (*k*, *e*) in *w*



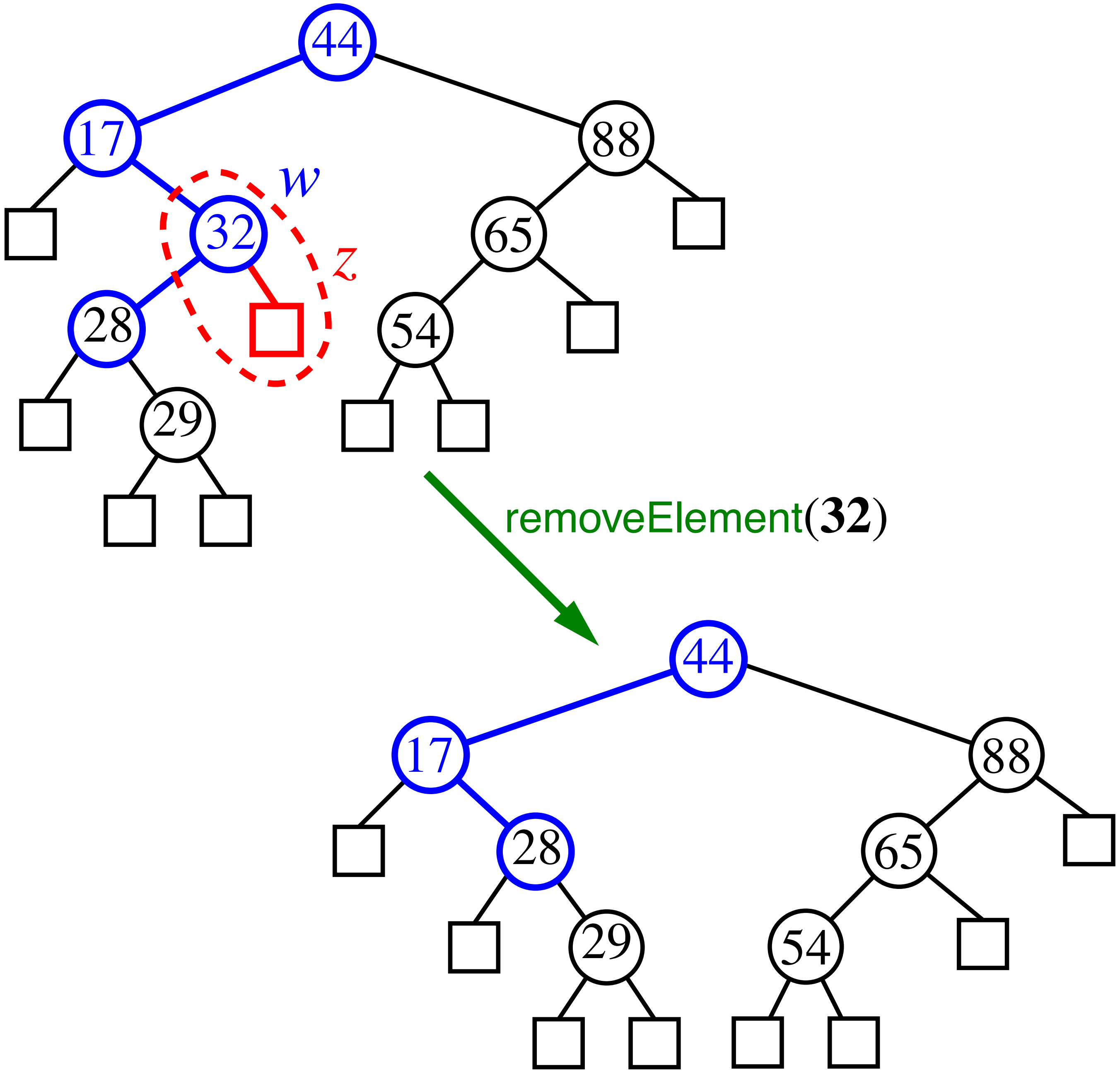
Insertion II

- If *w* is internal, we know another item with key *k* is stored at *w*. We call the algorithm recursively starting at *T*.rightChild(*w*) or *T*.leftChild(*w*)



Removal I

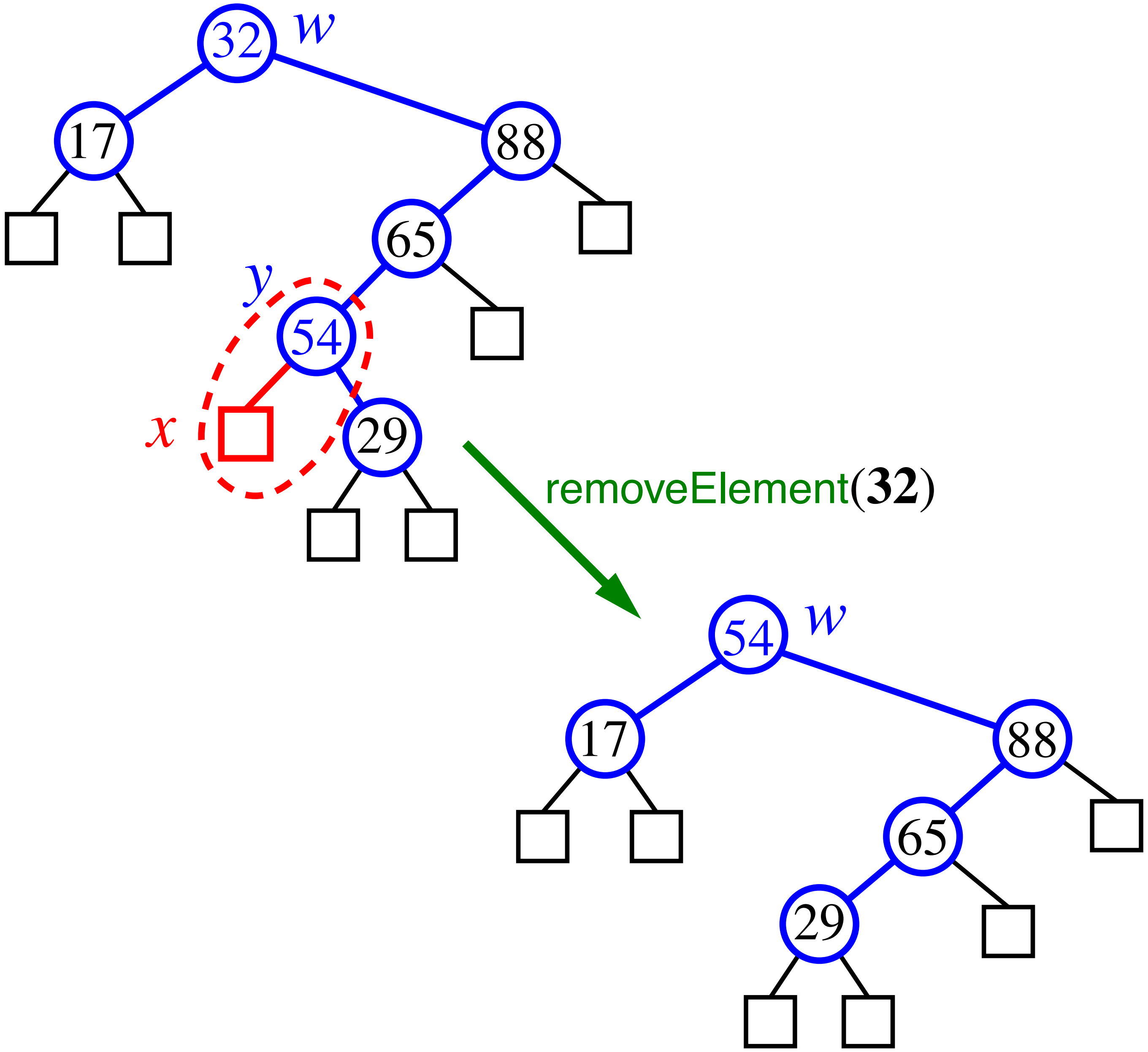
- We locate the node *w* where the key is stored with algorithm TreeSearch
- If *w* ***has an external child*** *z*, we remove *w* and *z* with removeAboveExternal(*z*)



removeElement(**32**)

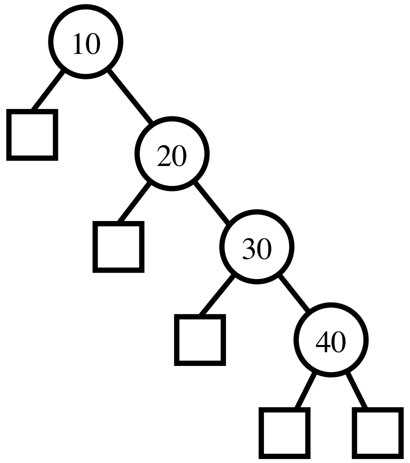
Removal II

- If *w* ***has an no external children***:
  - find the internal node *y* following *w* in inorder
  - move the item at *y* into *w*
  - perform removeAboveExternal(*x*), where *x* is the left child of *y* (guaranteed to be external)



Time Complexity

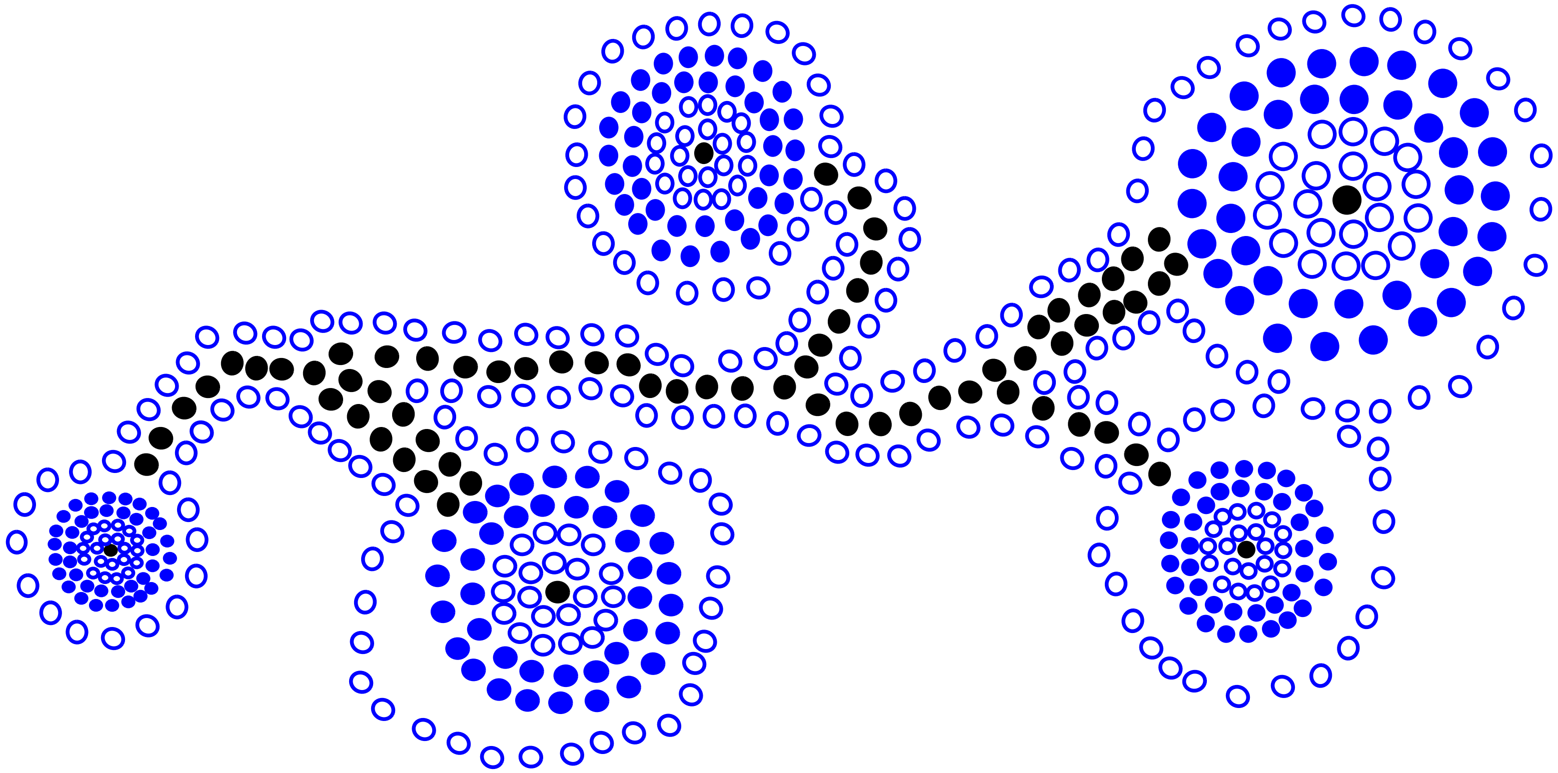
- A search, insertion, or removal, visits the nodes along a ***root-to leaf path***, plus possibly the ***siblings*** of such nodes
- Time O(1) is spent at each node
- The running time of each operation is O(h), where h is the height of the tree
- The height of binary serch tree is in n in the worst case, where a binary search tree looks like a sorted sequence



- To achive good running time, we need to keep the tree ***balanced***, i.e., with O(log n) height

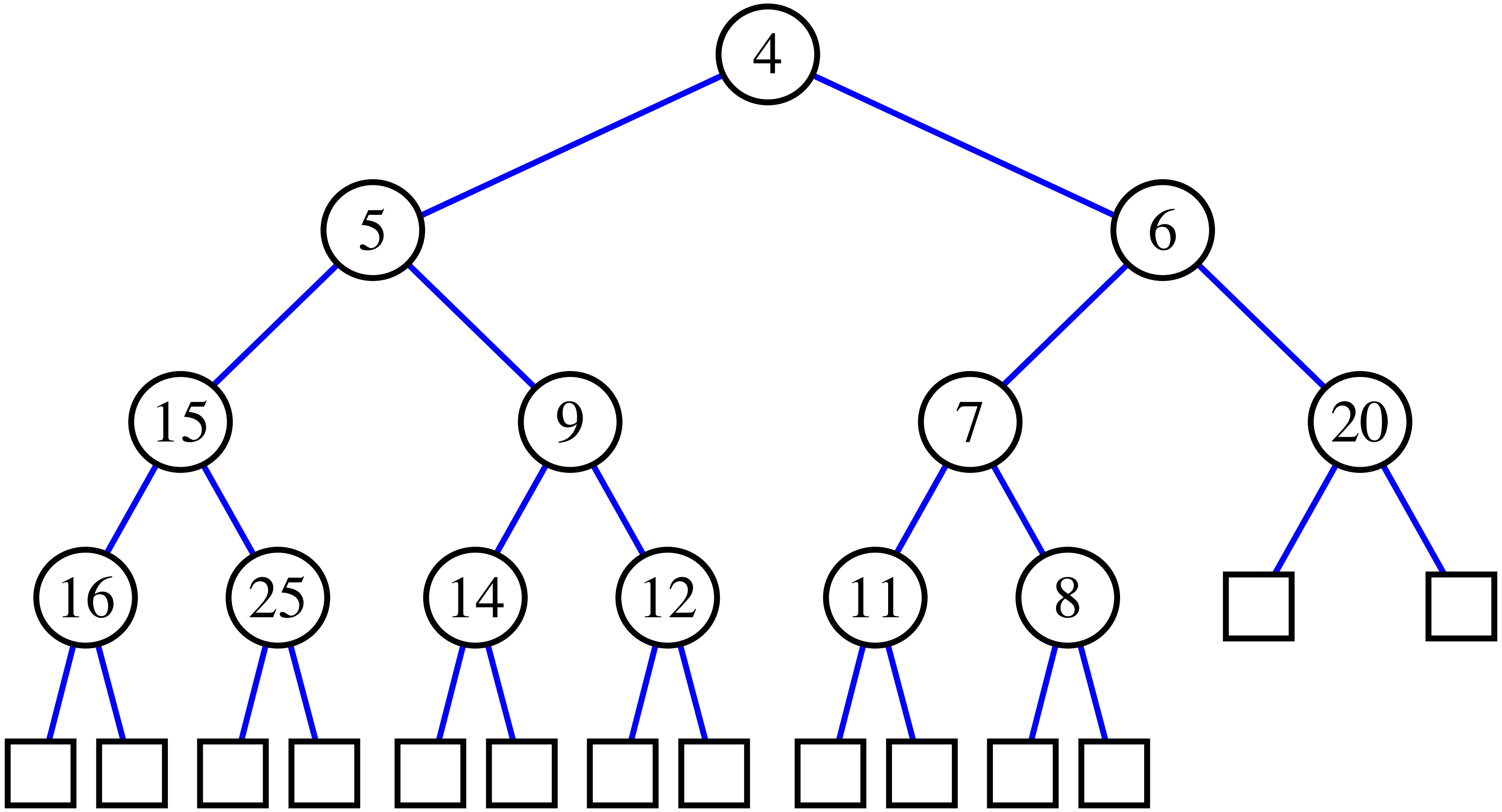
HEAPS I

- Heaps
- Properties
- Insertion and Deletion



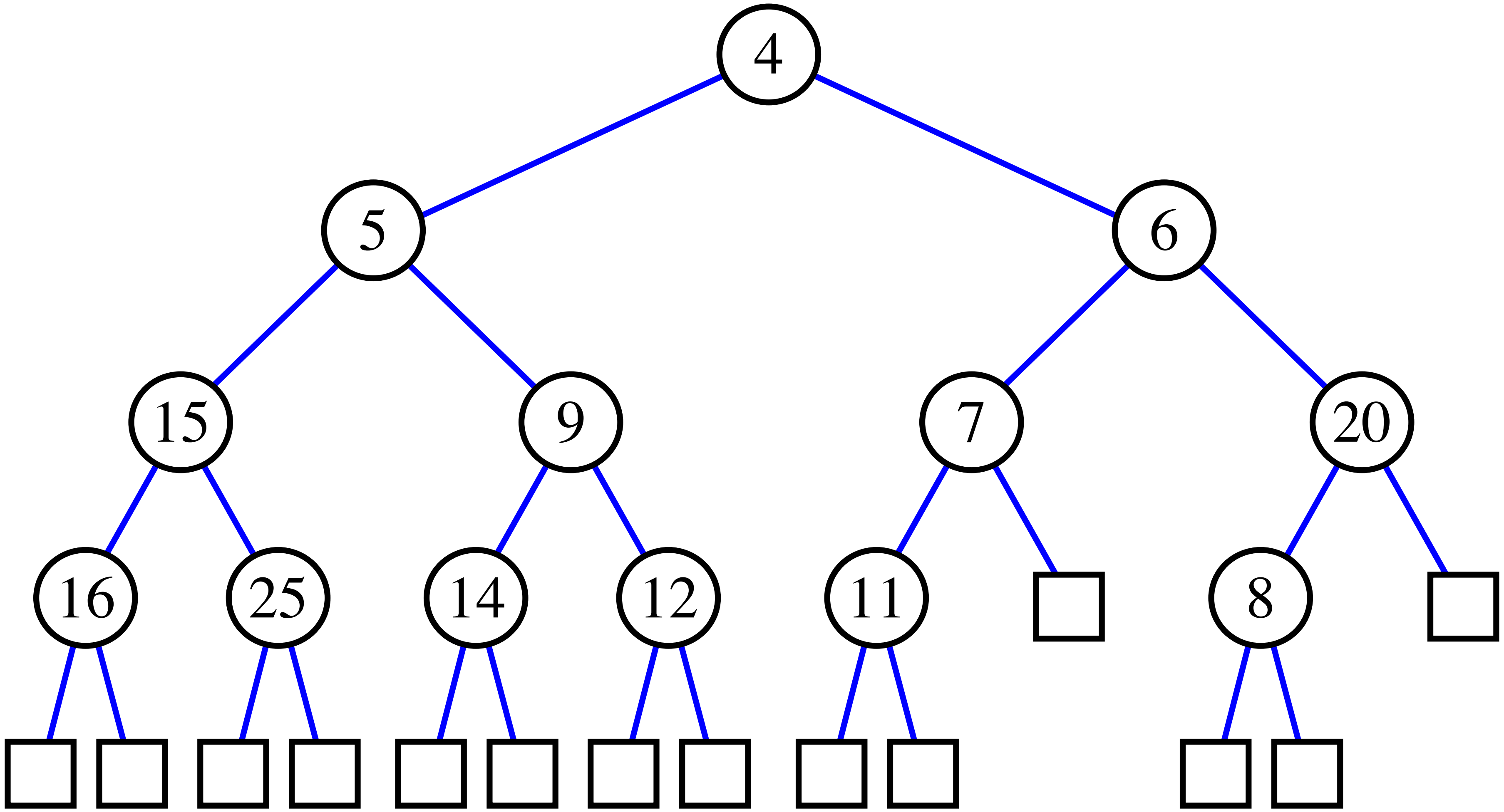
Heaps

- A ***heap*** is a binary tree *T* that stores a collection of keys (or key-element pairs) at its internal nodes and that satisfies two additional properties:
  - ***Order Property:*** key(parent) ≤ key(child)
  - ***Structural Property***: all levels are full, except the last one, which is left-filled (***complete binary tree***)



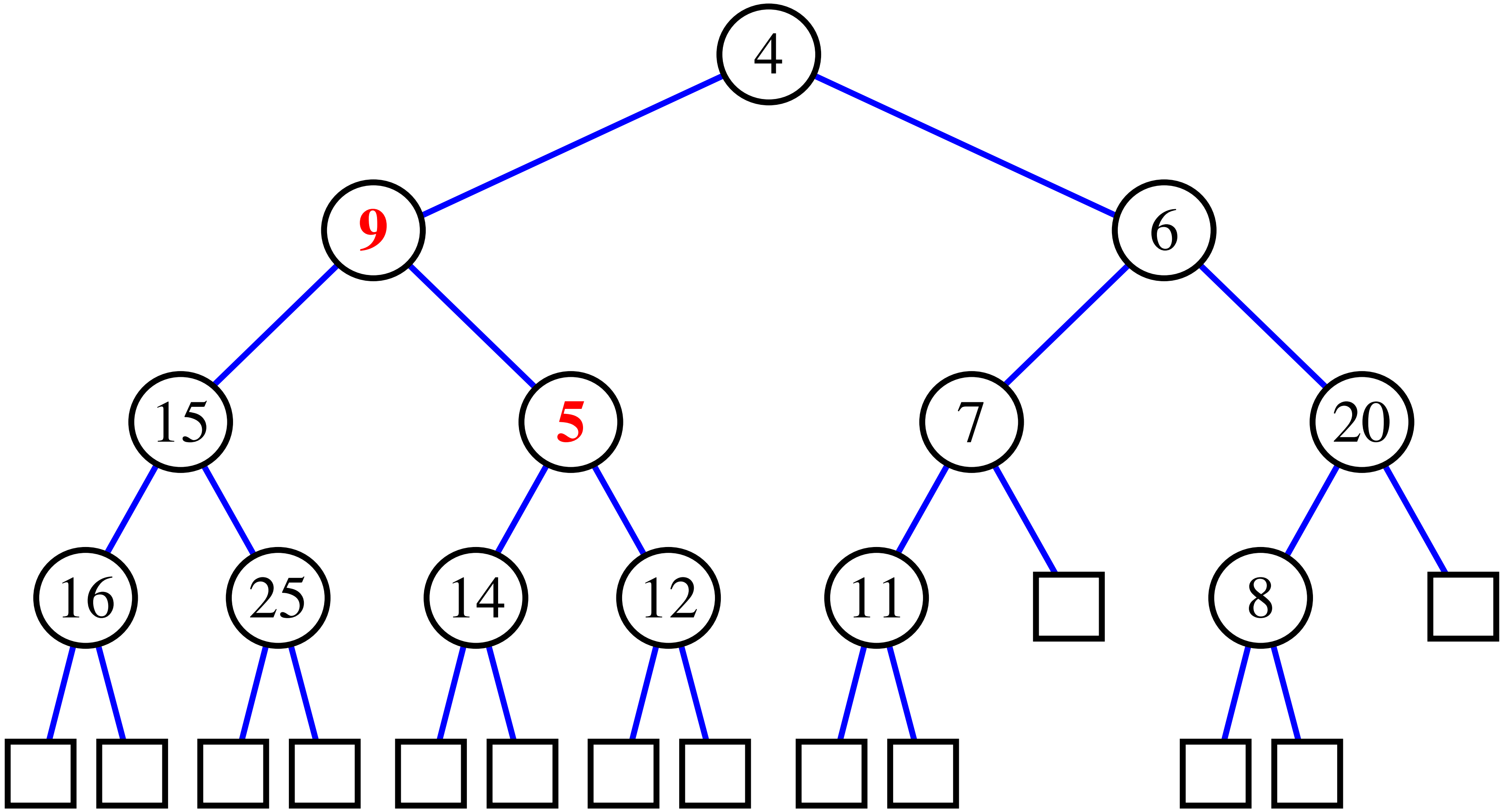
Not Heaps

- bottom level is not left-filled



Not Heaps

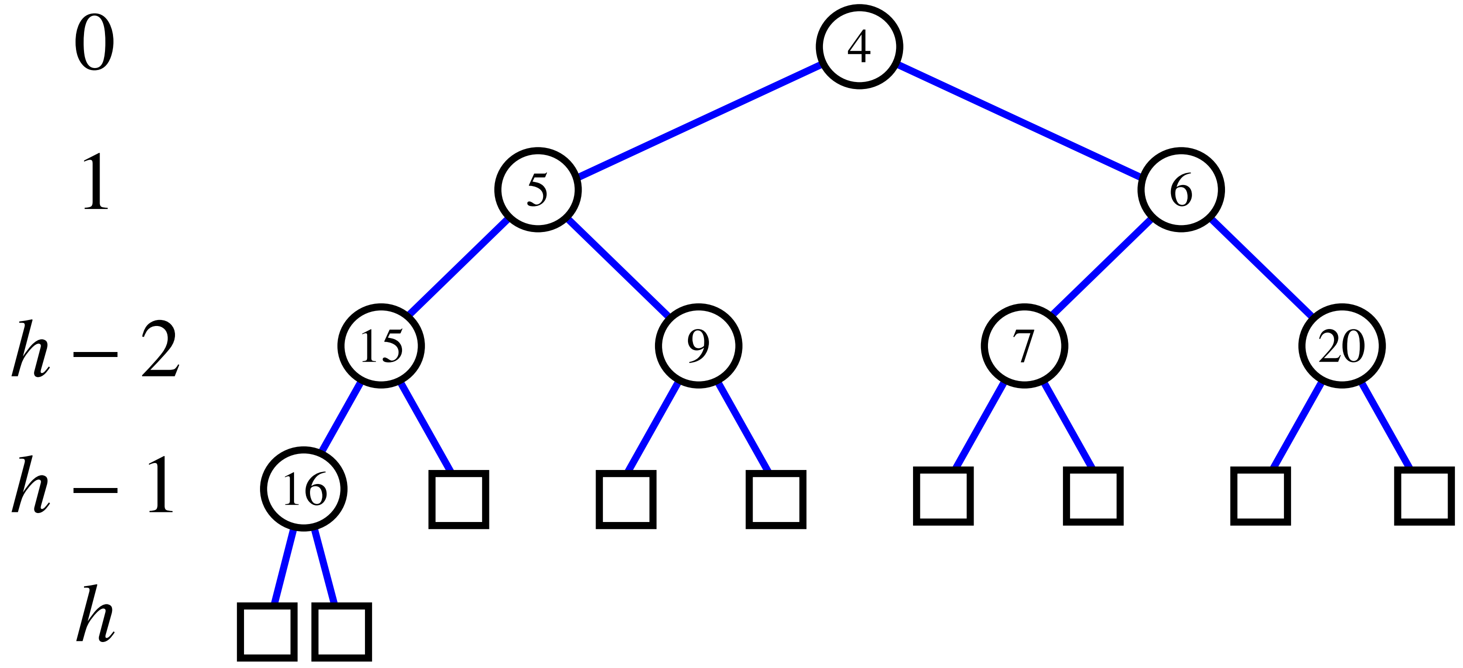
- key(parent)> key(child)



Height of a Heap

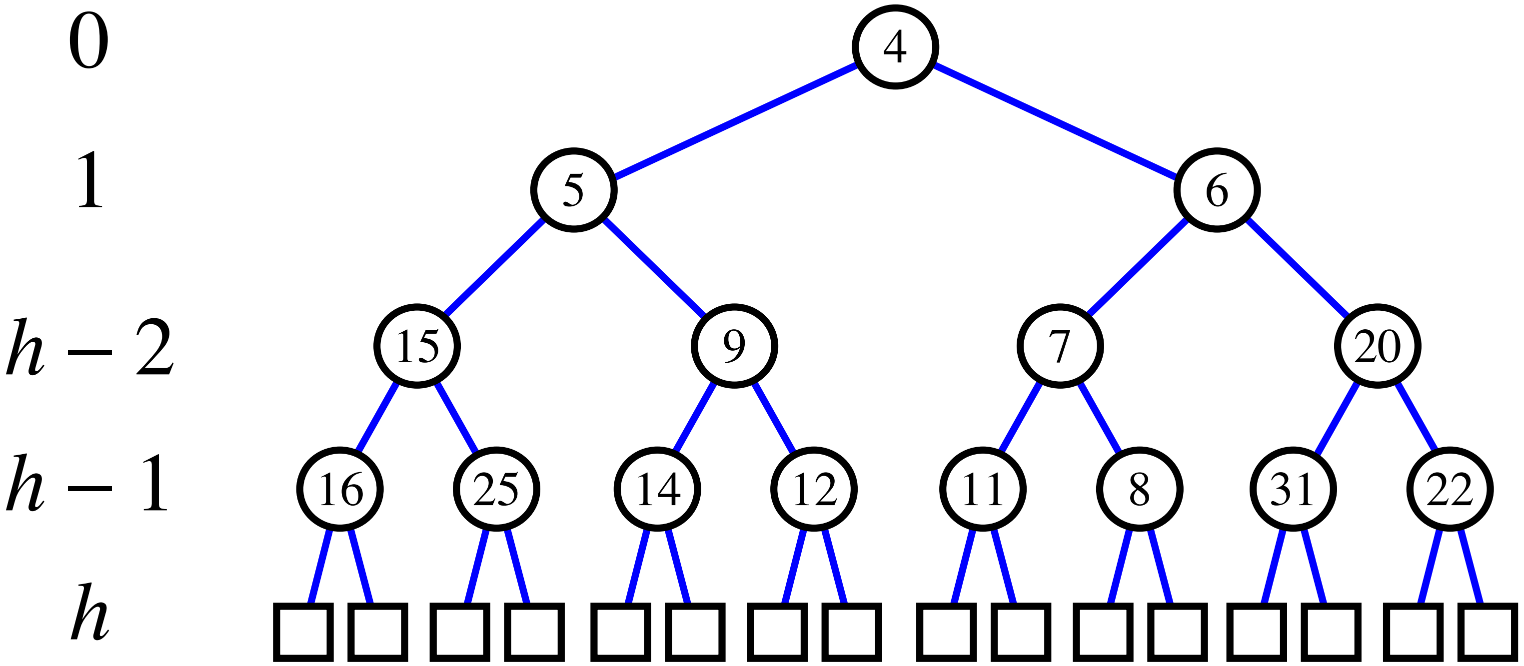
A heap T storing n keys has height $h = \lceil \log(n + 1) \rceil$, which is $O(\log n)$

- $n \geq 1 + 2 + 4 + \ldots + 2^{h-2} + 1 = 2^{h-1} - 1 + 1 = 2^{h-1}$



Height of a Heap

- $n \leq 1 + 2 + 4 + \ldots + 2^{h-1} = 2^h - 1$

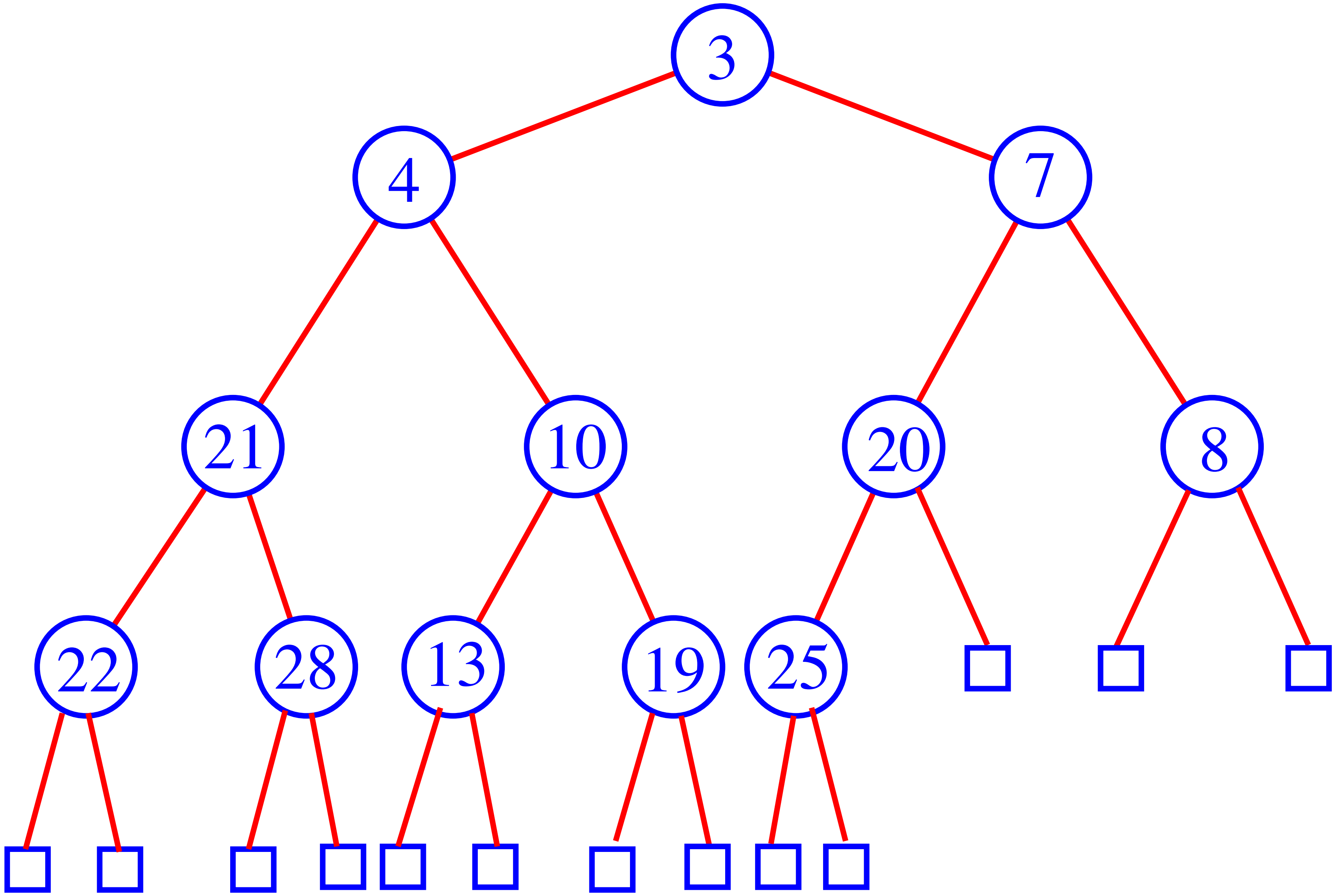


- Therefore $2^{h-1} \leq n \leq 2^h - 1$
- Taking logs, we get $\log(n + 1) \leq h \leq \log n + 1$
- Which implies $\boldsymbol{h = \lceil \log(n+1) \rceil}$

Heap Insertion

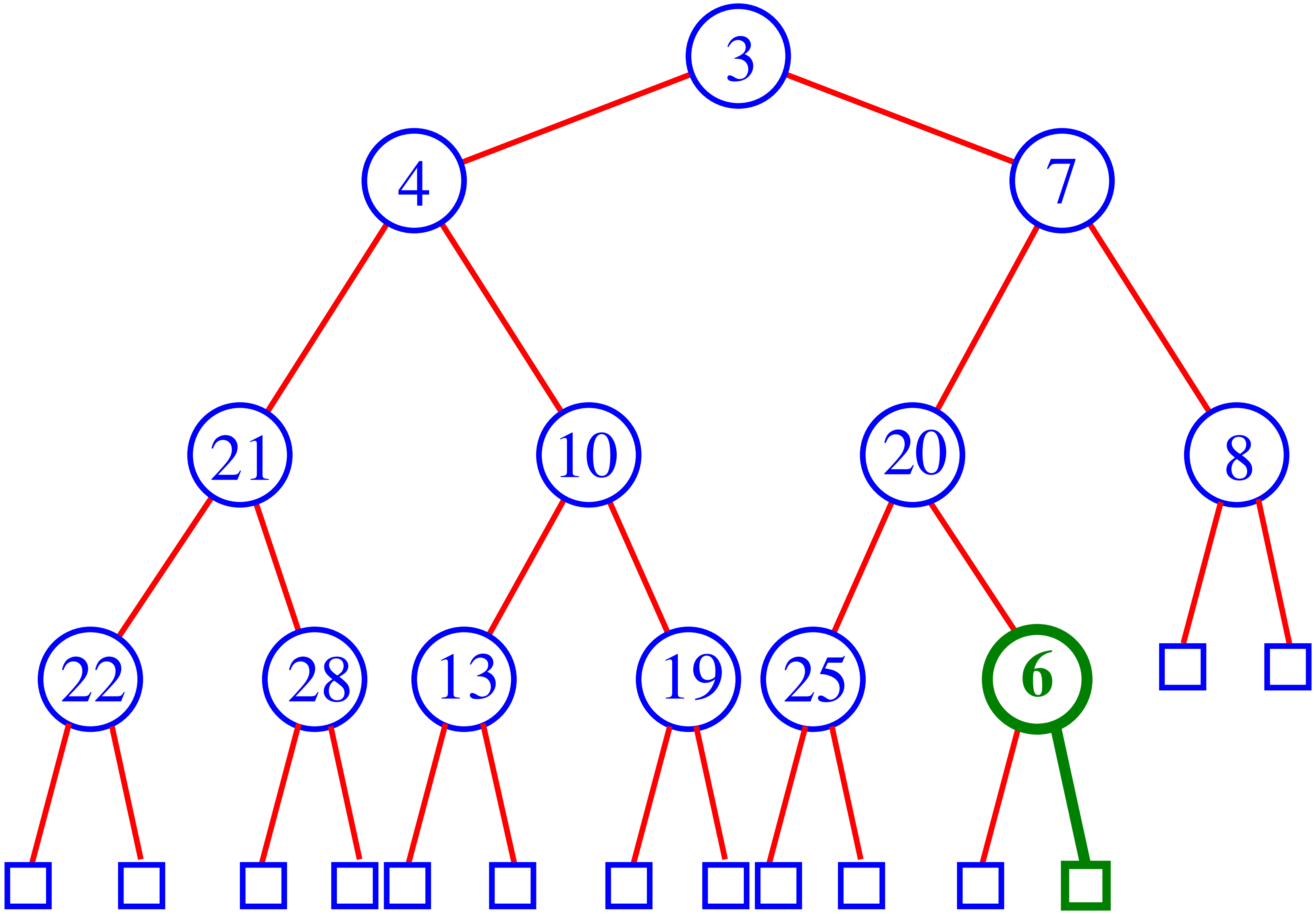
So here we go ...

The key to insert is **6**



Heap Insertion

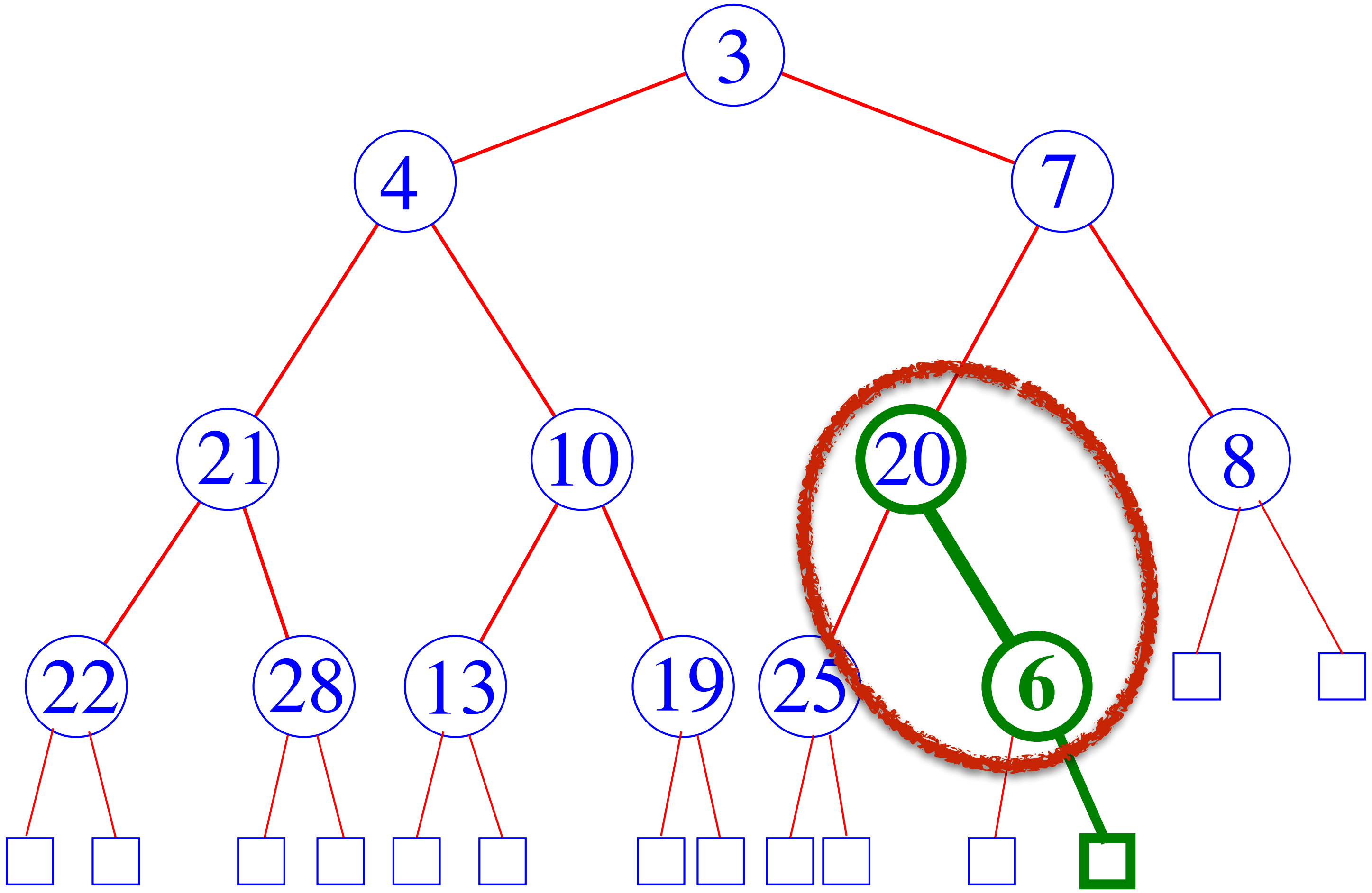
Add the key in the ***next available position*** in the heap.



Now begin ***Upheap***.

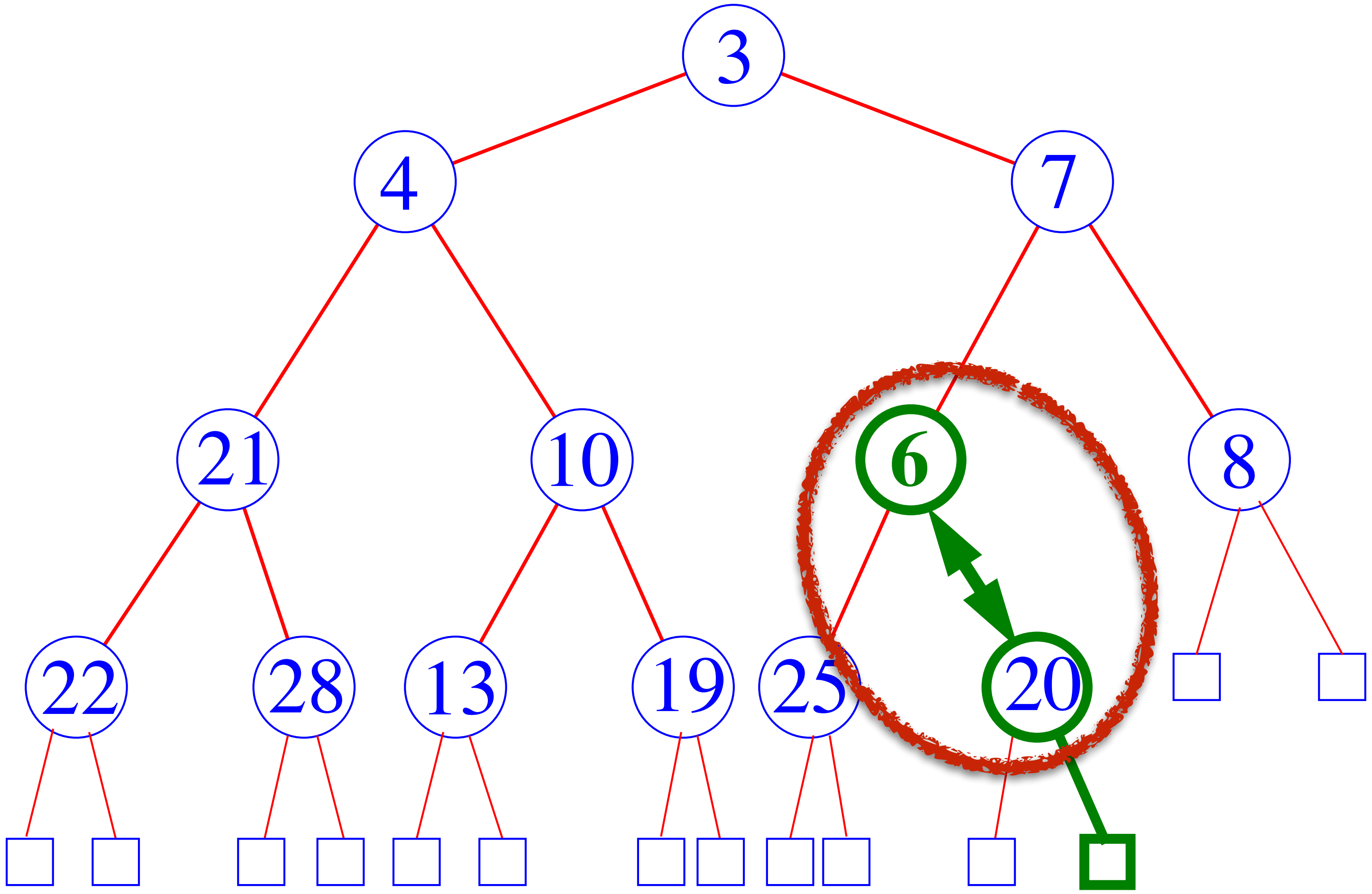
Upheap

- ***Swap parent-child keys out of order***

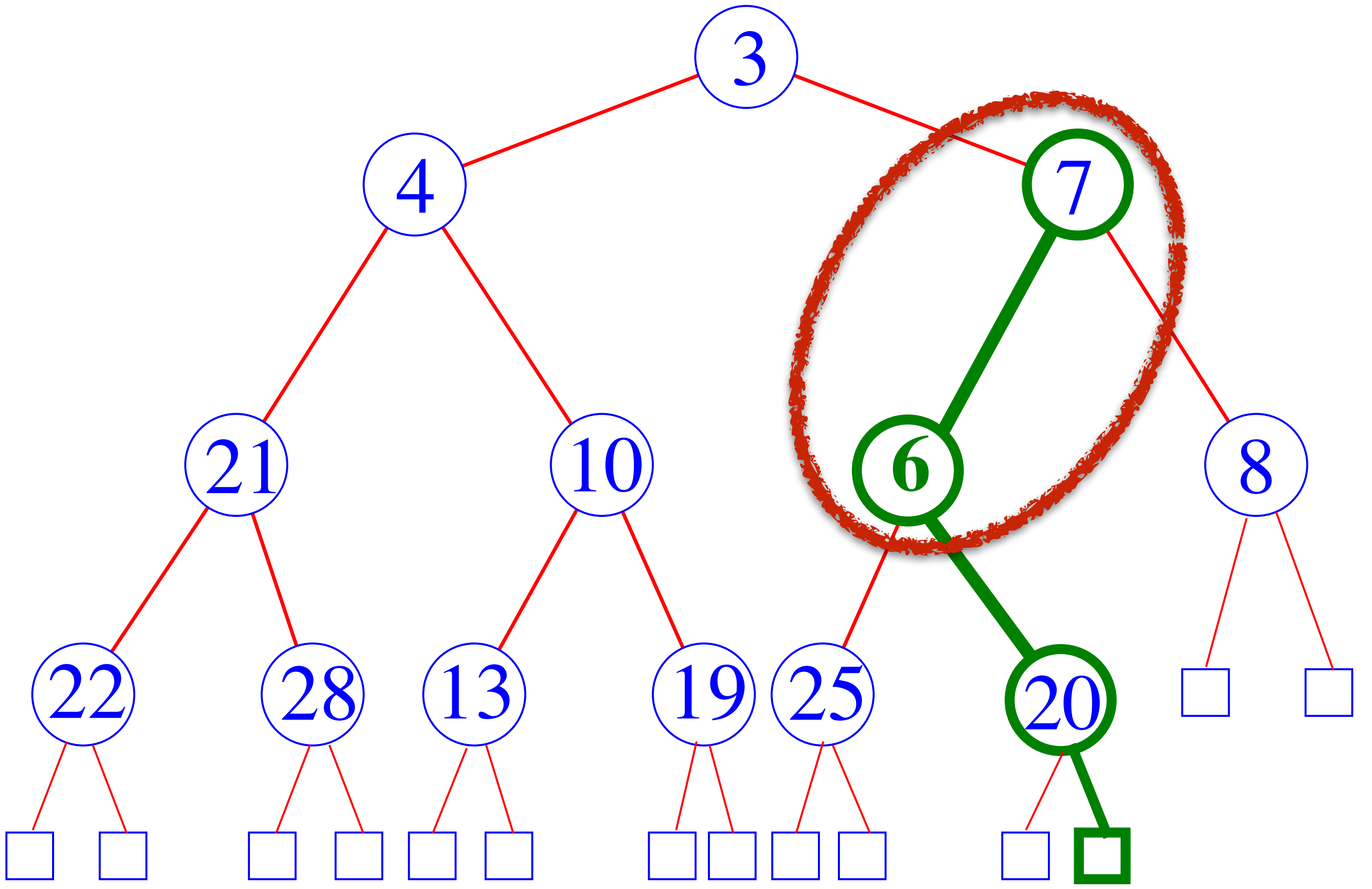


Upheap

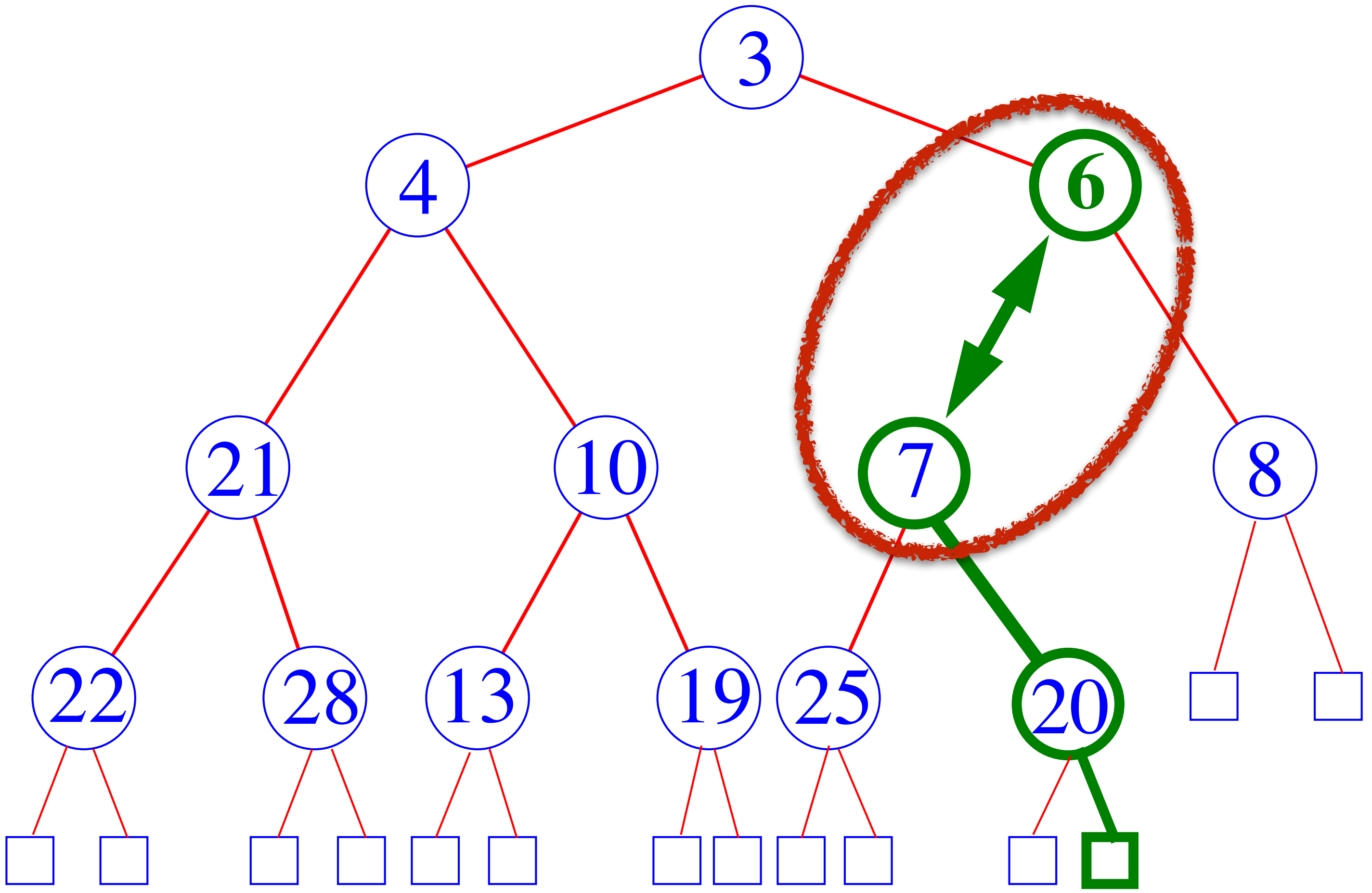
- ***Swap parent-child keys out of order***



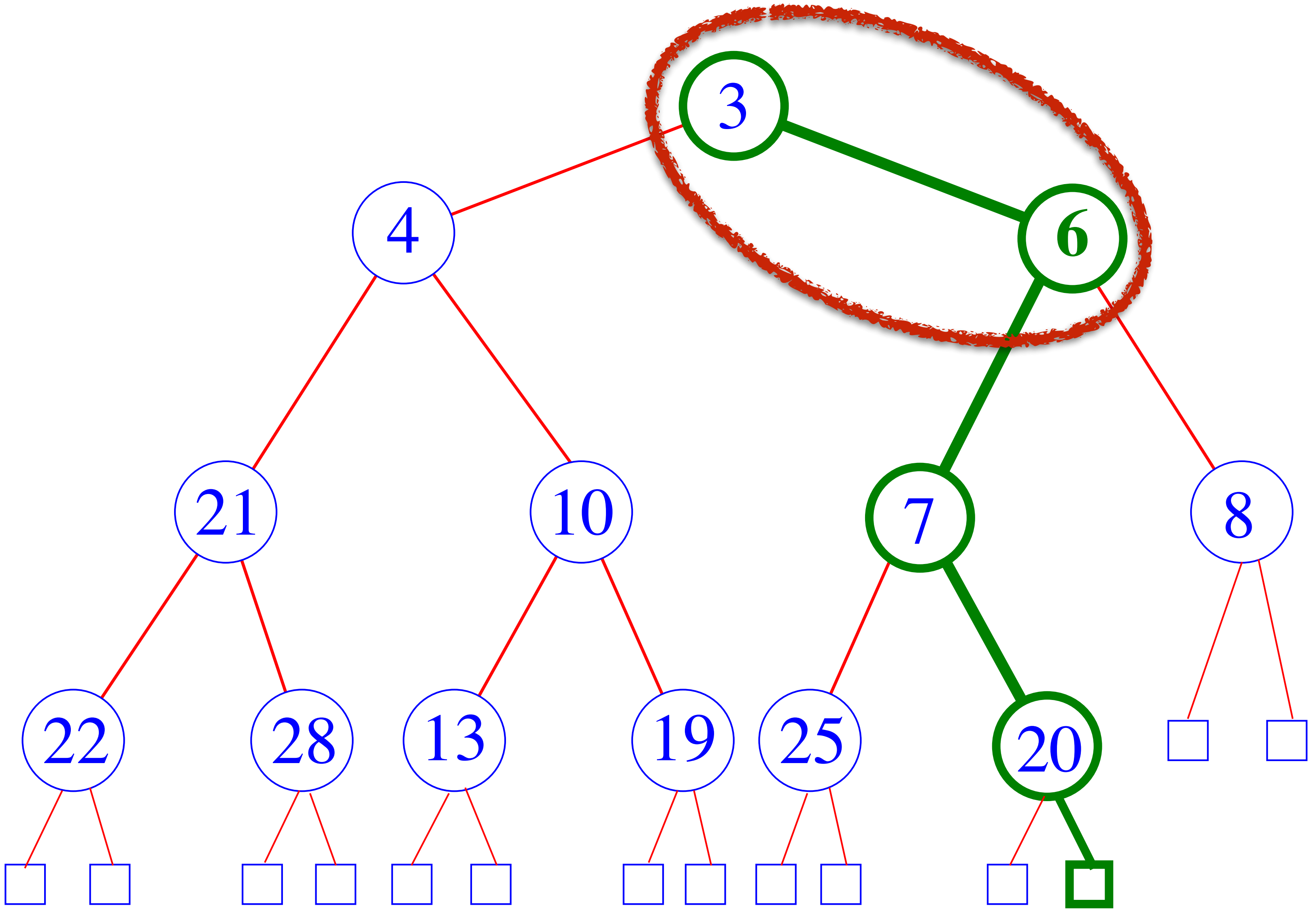
Upheap Continues



Upheap Continues



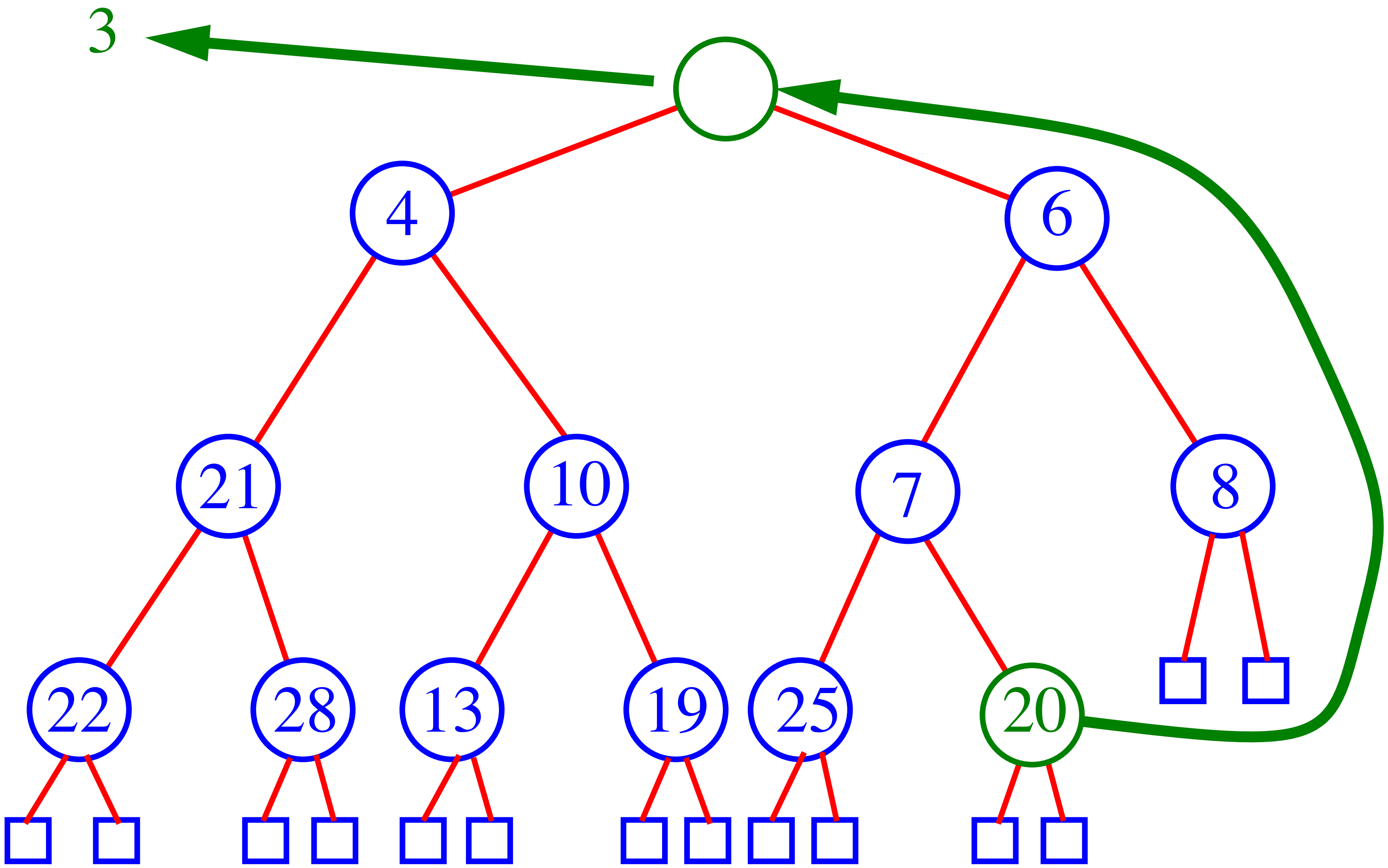
End of Upheap

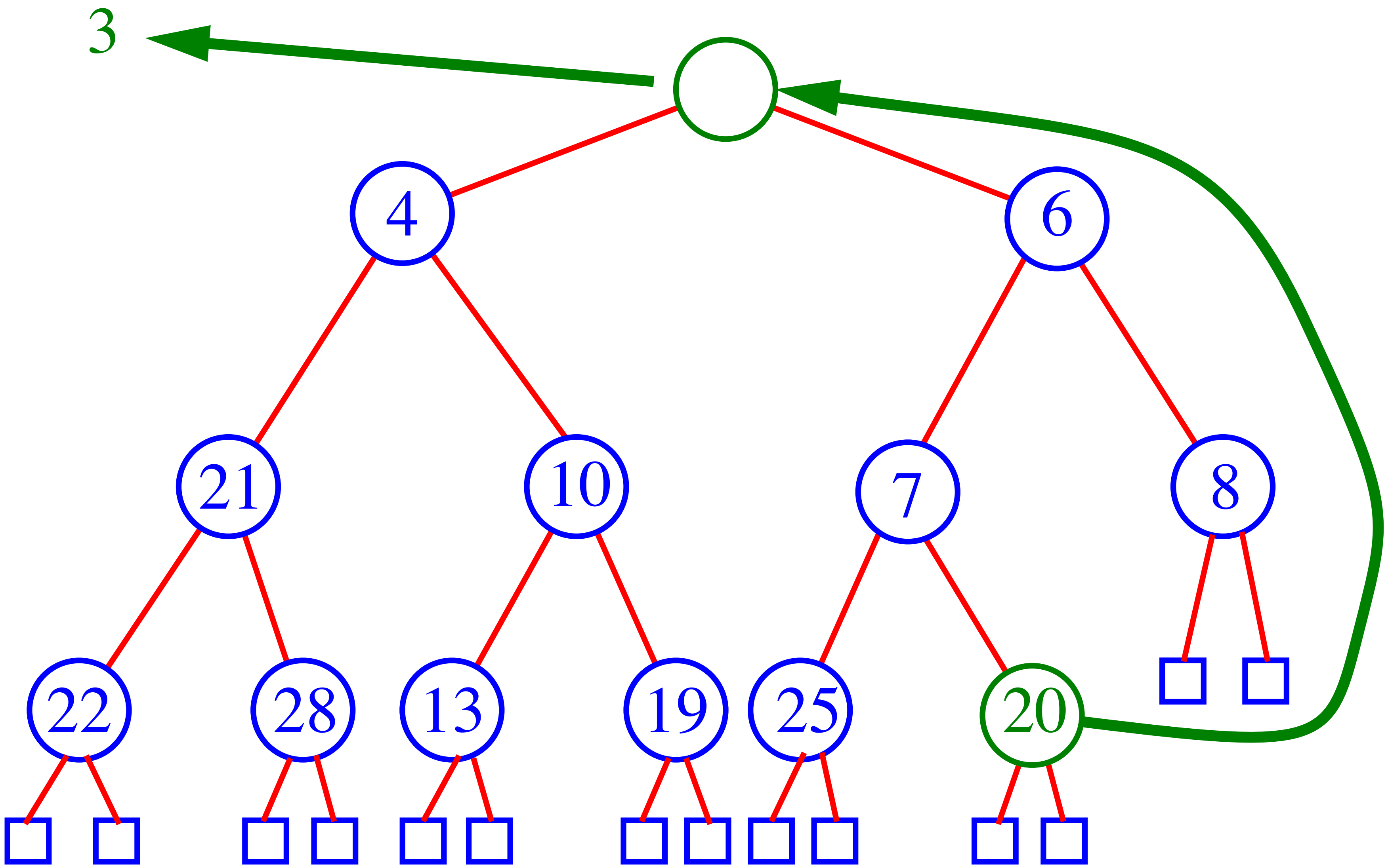


- ***Upheap*** terminates when new key is greater than the key of its parent **or** the top of the heap is reached
- (total #swaps) $\leq$ $(h - 1)$, which is $O(\log n)$

Removal From a Heap

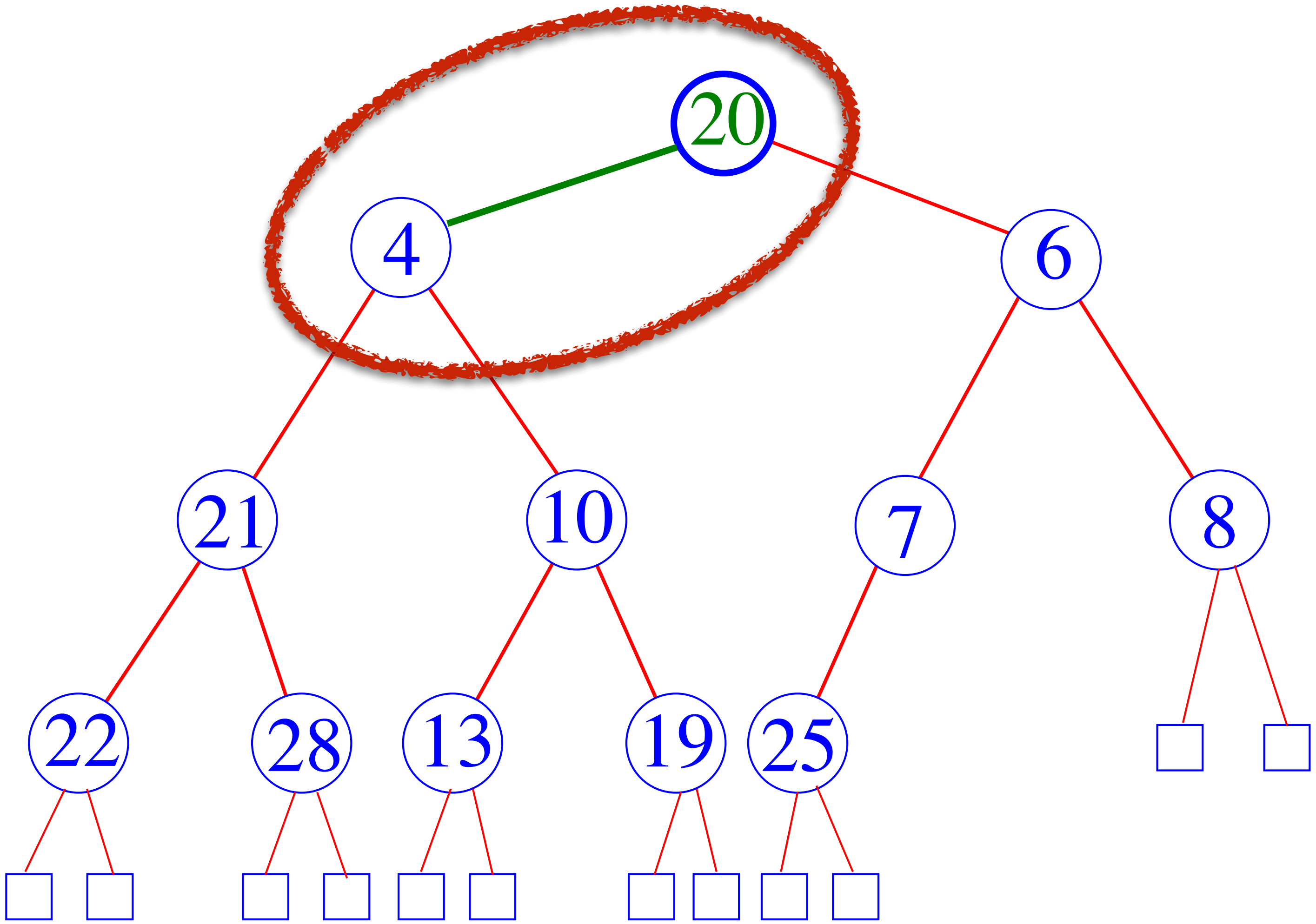
RemoveMin()



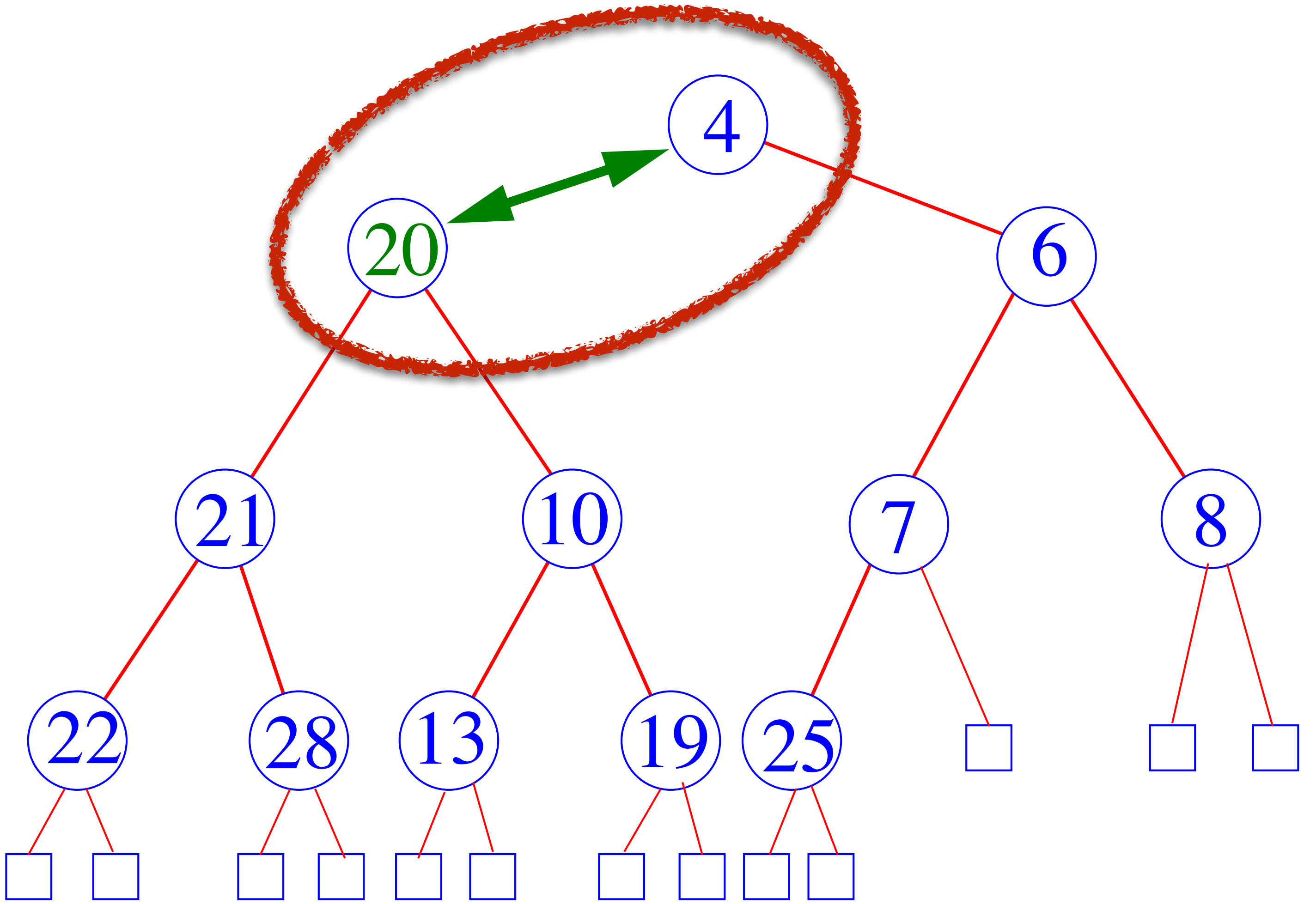


- The removal of the top key leaves a hole
- We need to fix the heap
- First, replace the hole with the last key in the heap
- Then, begin ***Downheap***

Downheap

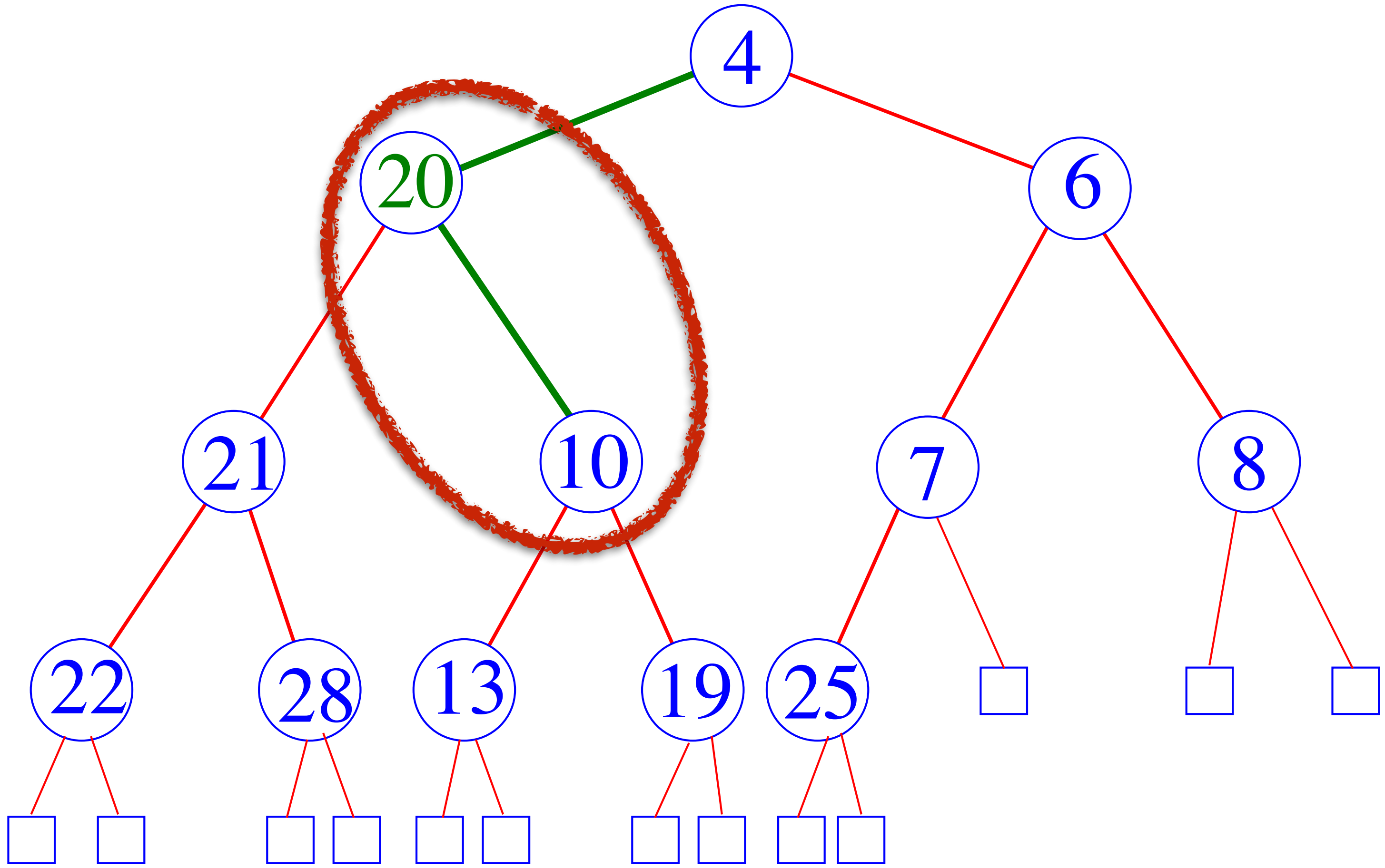


Downheap

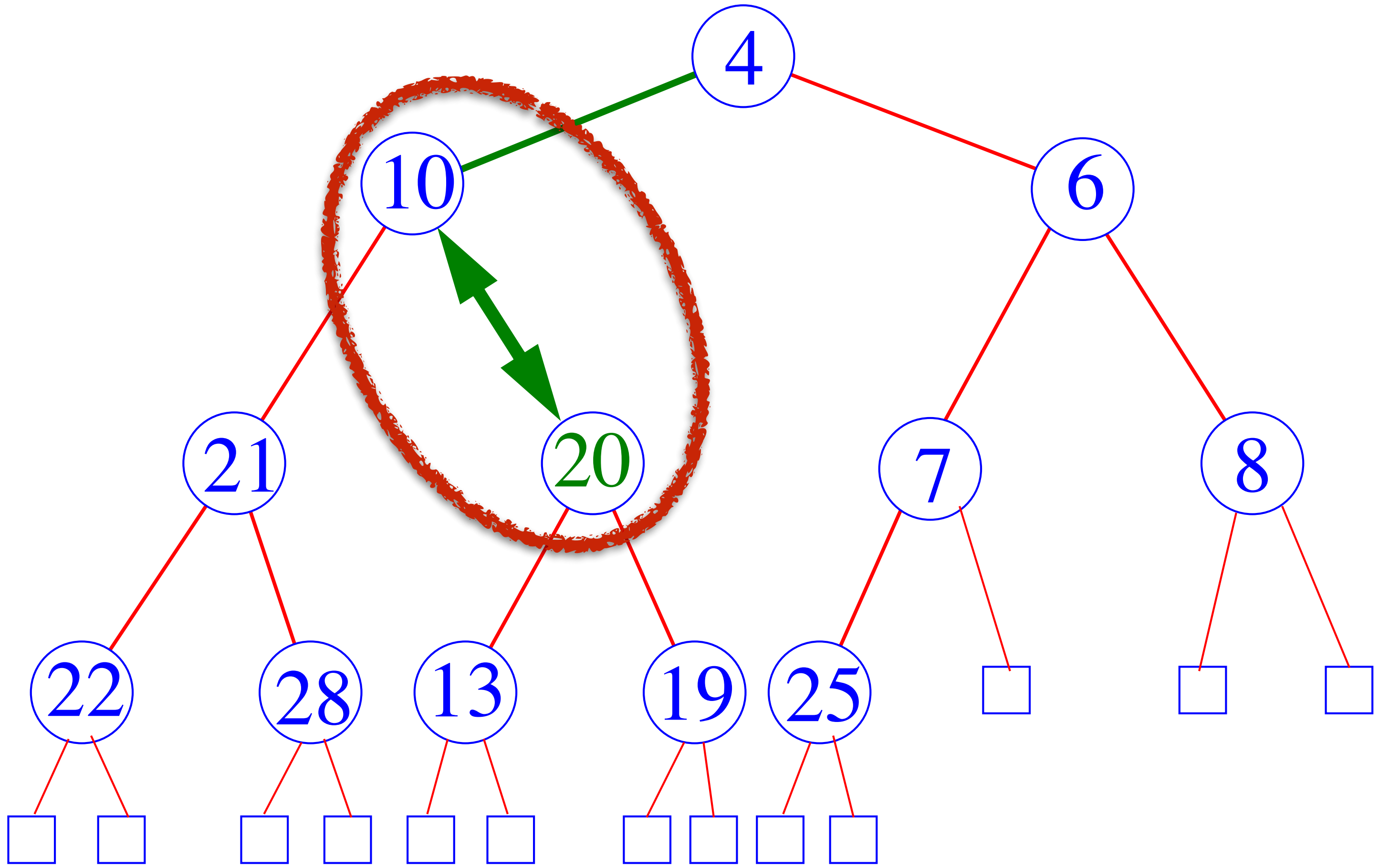


Downheap compares the parent with the smallest child. If the child is smaller, it switches the two.

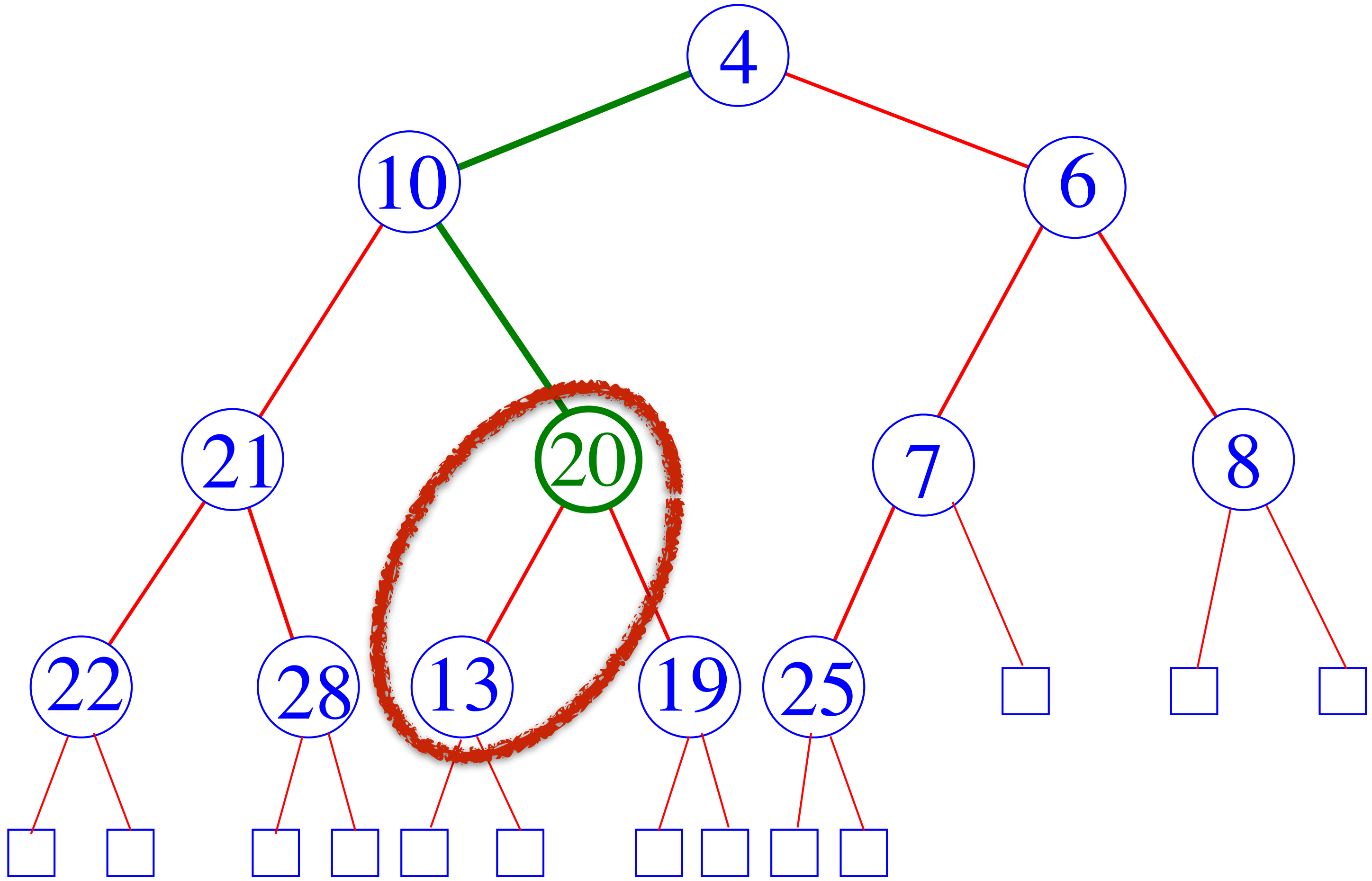
Downheap Continues



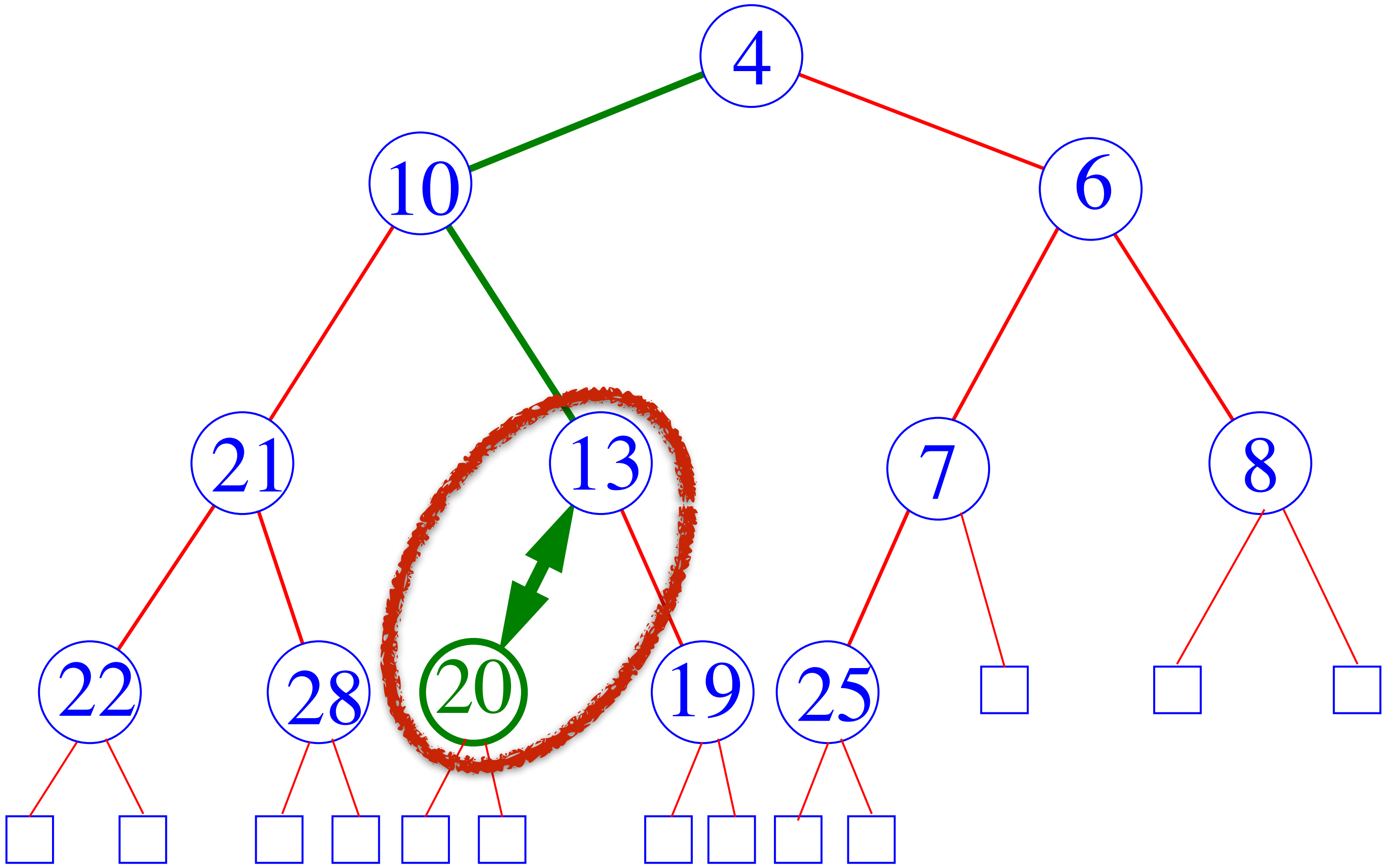
Downheap Continues



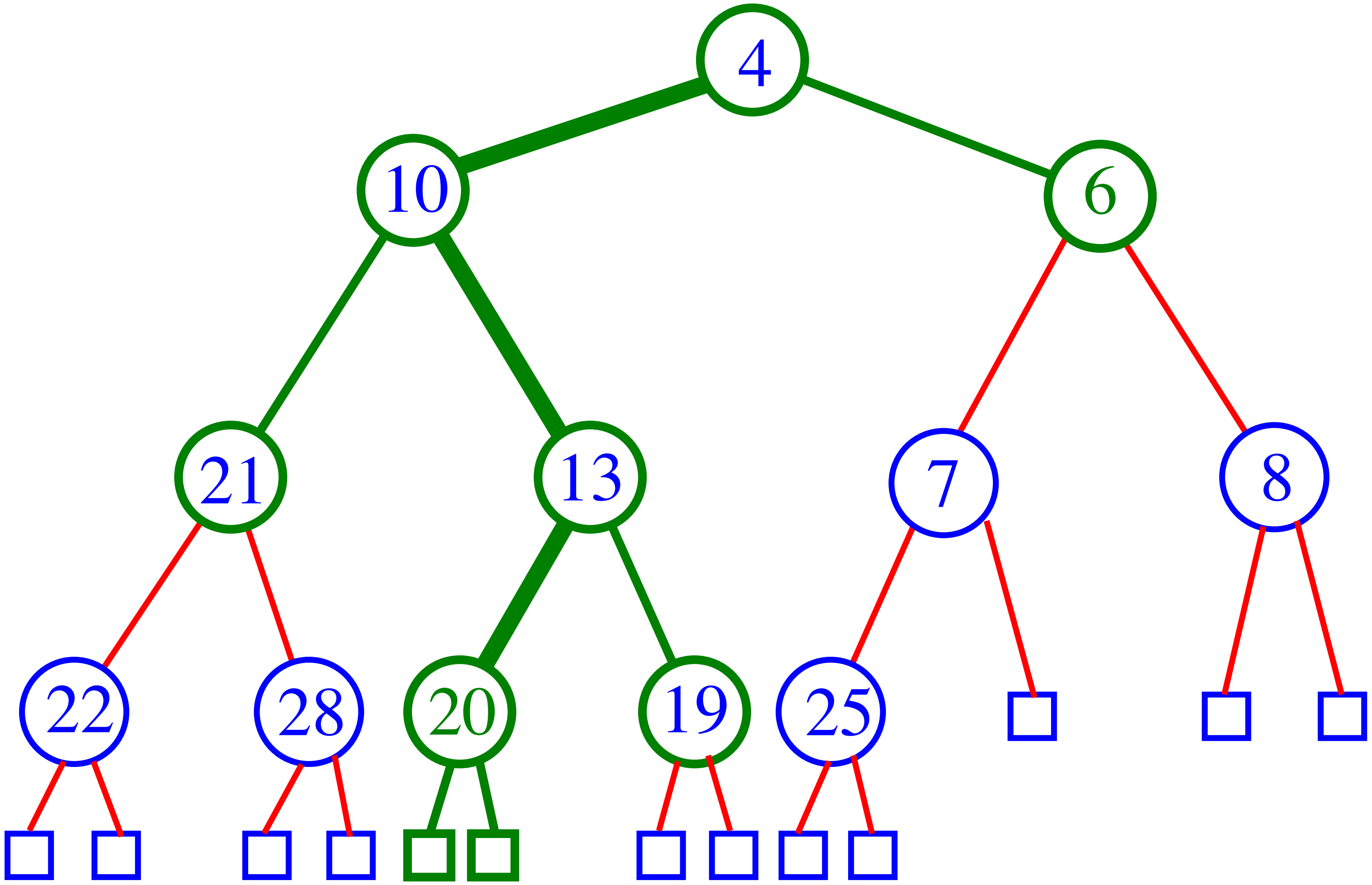
Downheap Continues



Downheap Continues



End of Downheap

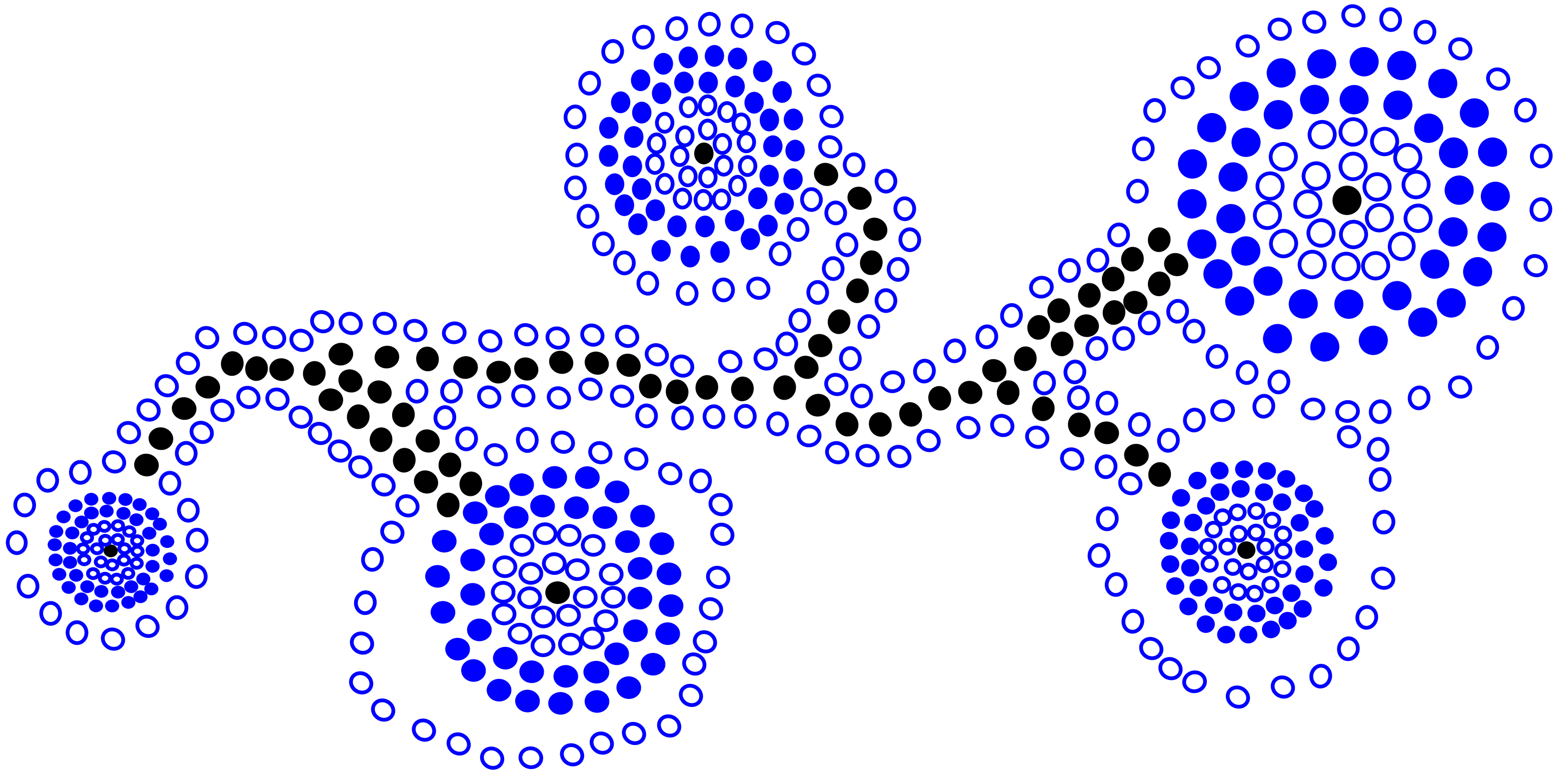


- ***Downheap*** terminates when the key is greater than the keys of both its children **or** the bottom of the heap is reached.

- (total #swaps) $\leq$ ($h - 1$), which is O(log n)

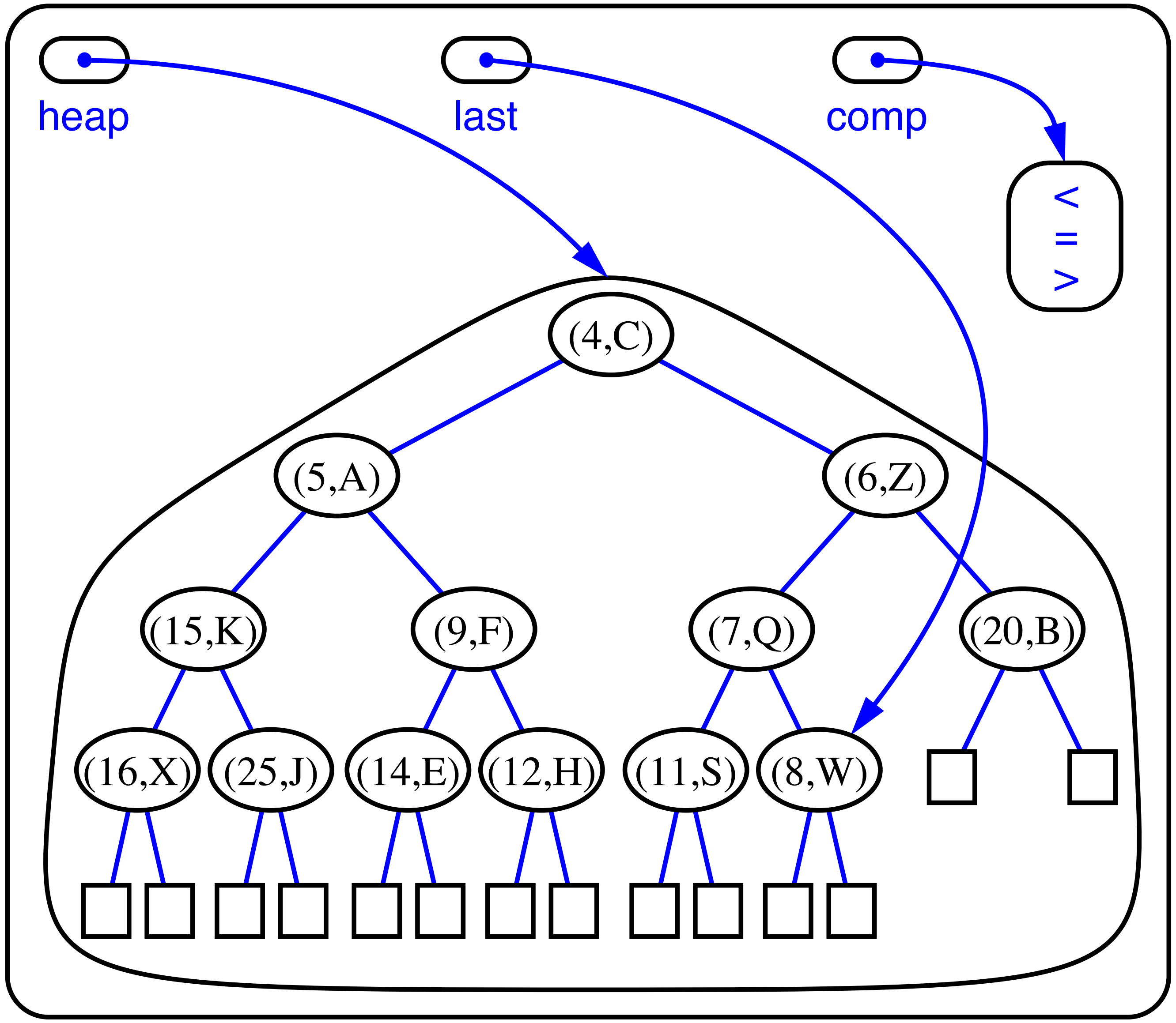
HEAPS II

- Implementation
- HeapSort
- Bottom-Up Heap Construction
- Locators



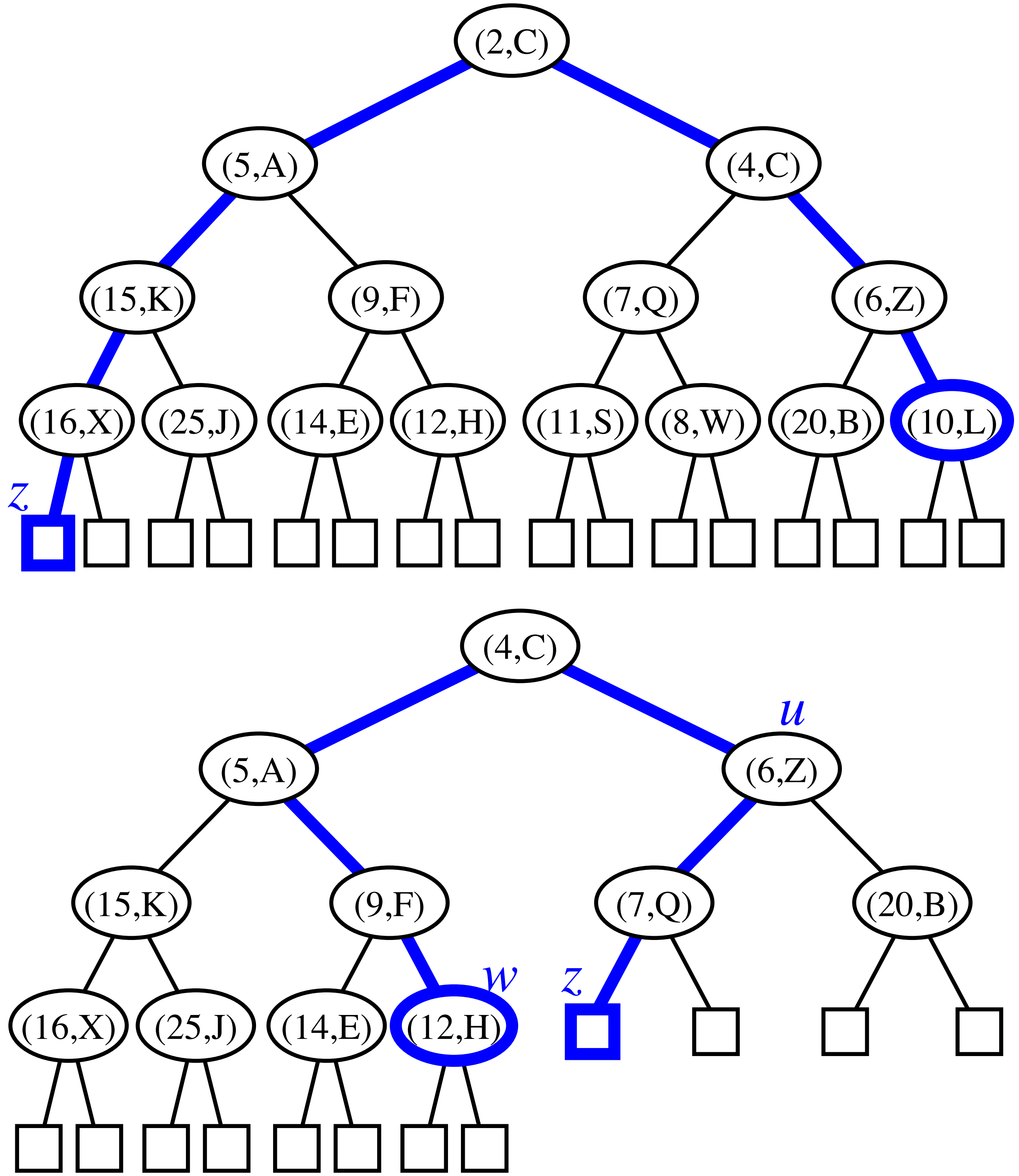
Implementation of a Heap

```
public class HeapPriorityQueue implements PriorityQueue
{
   BinaryTree T;
   Position last;
   Comparator comparator;
   ...
}
```



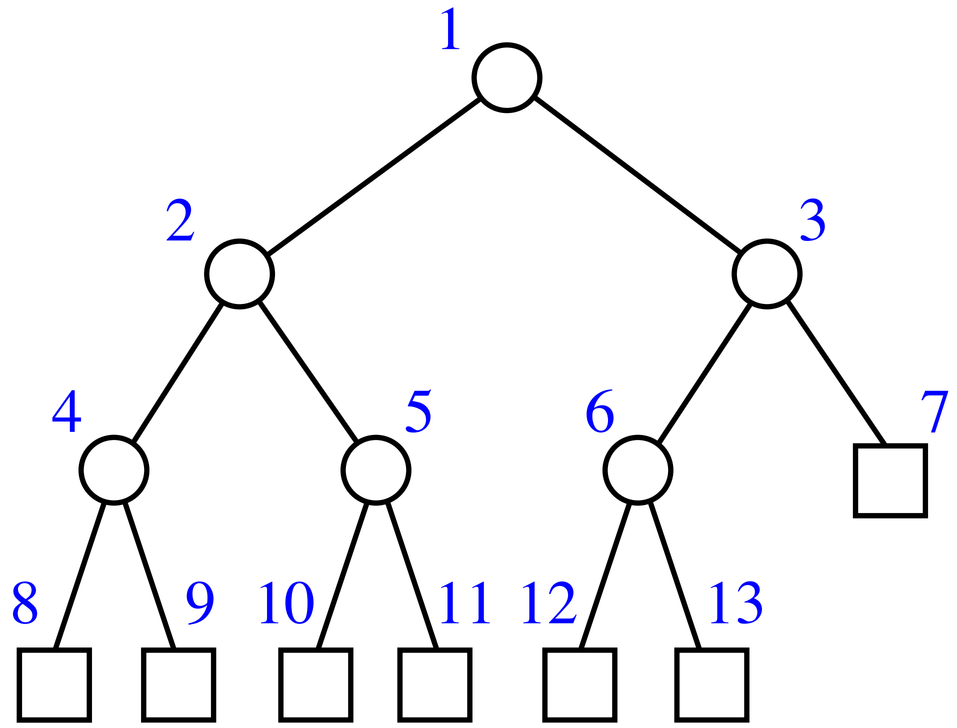
Implementation of a Heap(cont.)

- Two ways to find the insertion position z in a heap:



Vector Based Implementation

- Updates in the underlying tree occur only at the "last element"
- A heap can be represented by a vector, where the node at rank *i* has
  - left child at rank 2*i* and
  - right child at rank 2*i* + 1



- The leaves do no need to be explicitly stored
- Insertion and removals into/from the heap correspond to `insertLast` and `removeLast` on the vector, respectively

Heap Sort

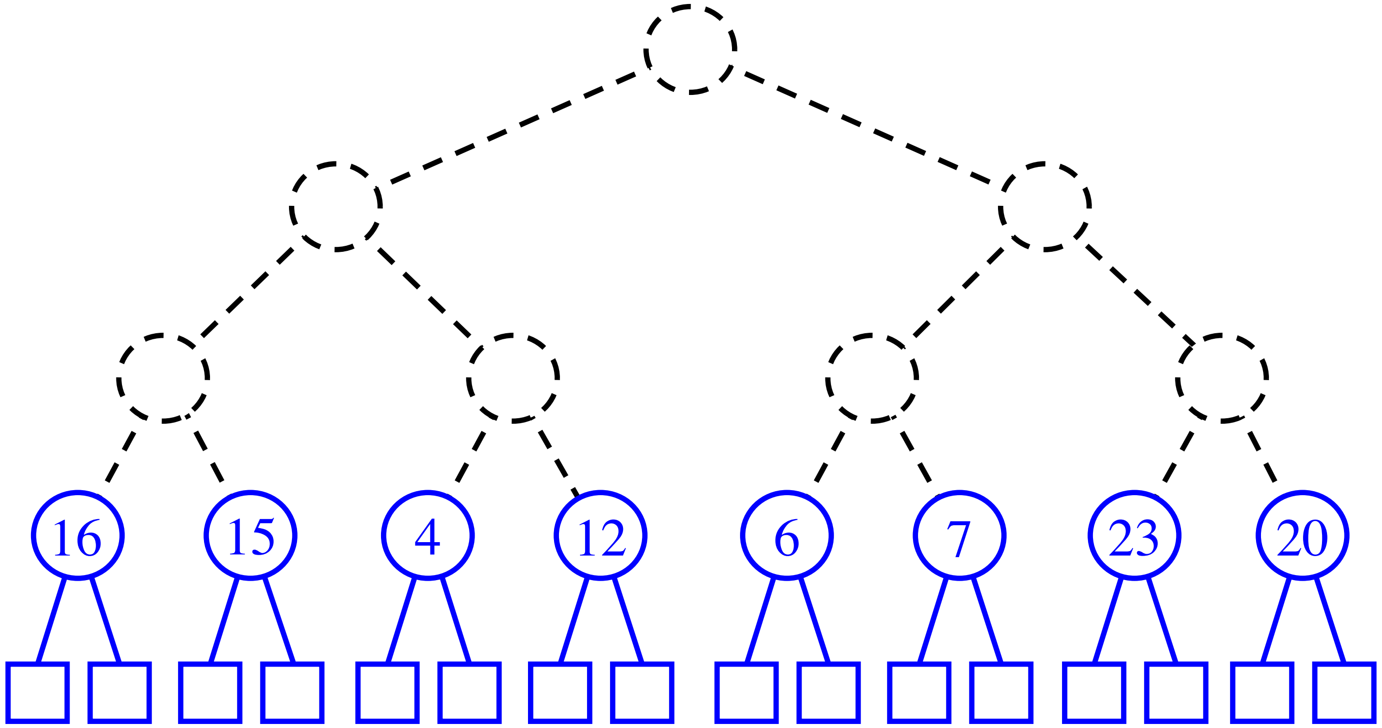
- All heap methods run in logarithmic time or better
- If we implement PriorityQueueSort using a heap for our priority queue, `insertItem` and `removeMin` each take O(log k), k being the number of elements in the heap at a given time.
- We always have at most n elements in the heap, so the worst case time complexity of these methods is O(log n).
- Thus each phase takes O(n log n) time, so the algorithm runs in O(n log n) time also.
- This sort is known as ***heap-sort***.
- The O(n log n) run time of heap-sort is much better than the O(n^2) run time of selection and insertion sort.

In-Place Heap-Sort

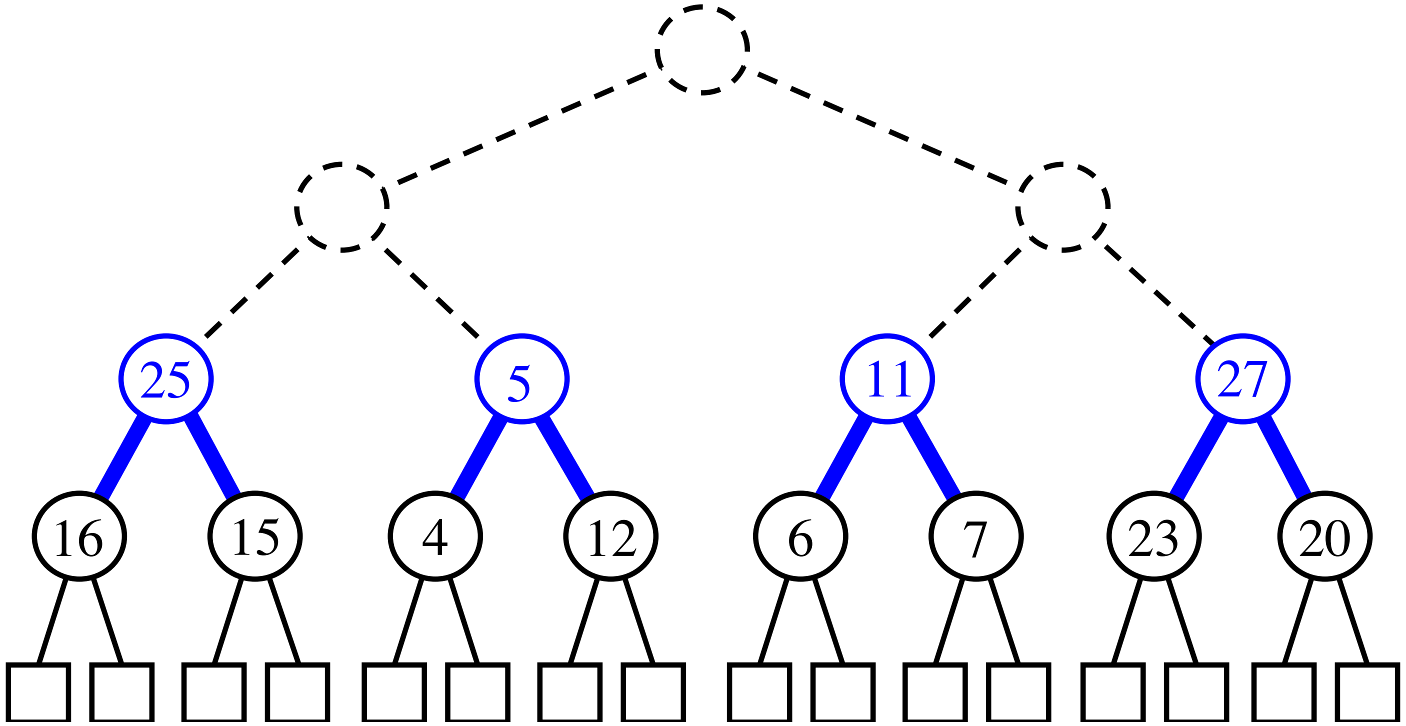
- Do not use an external heap
- Embed the heap into the sequence, using the vector representation

Bottom-Up Heap Construction

- build $(n + 1)/2$ trivial one-element heaps

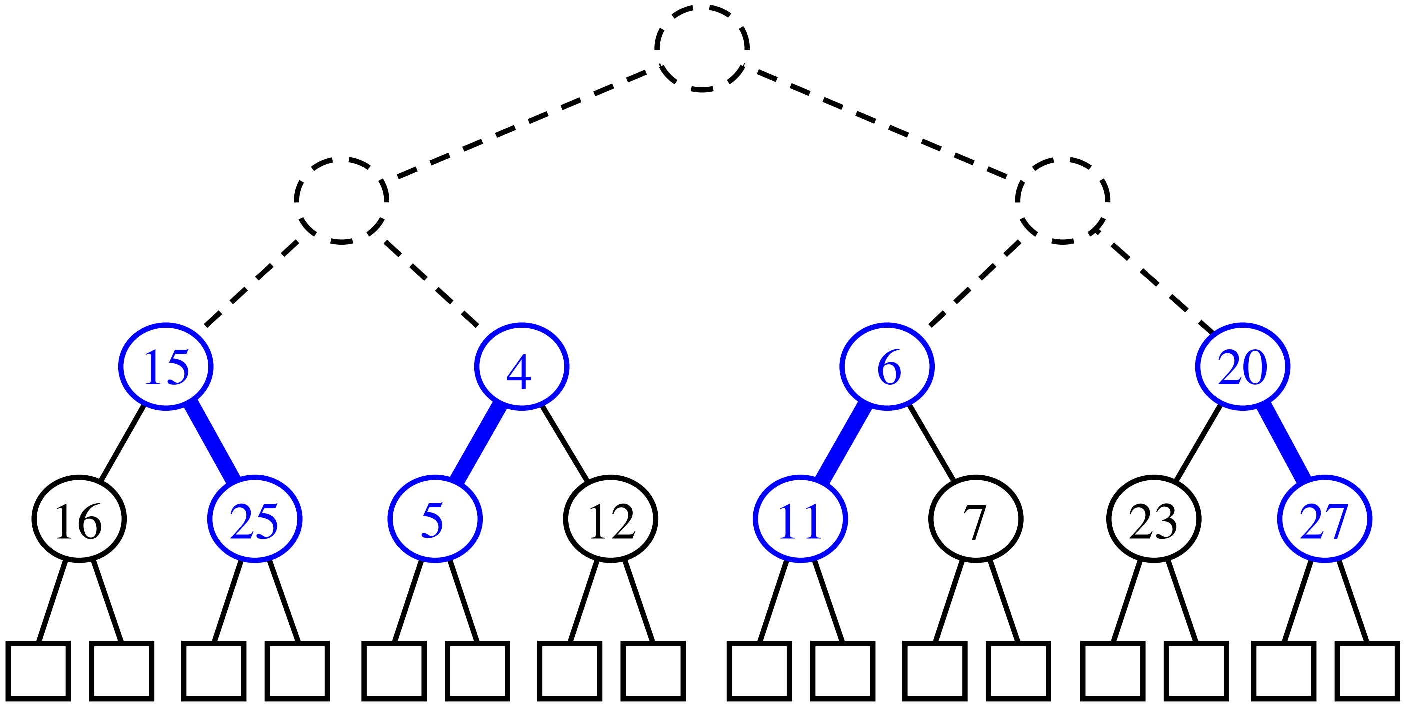


- now build three-element heaps on top of them

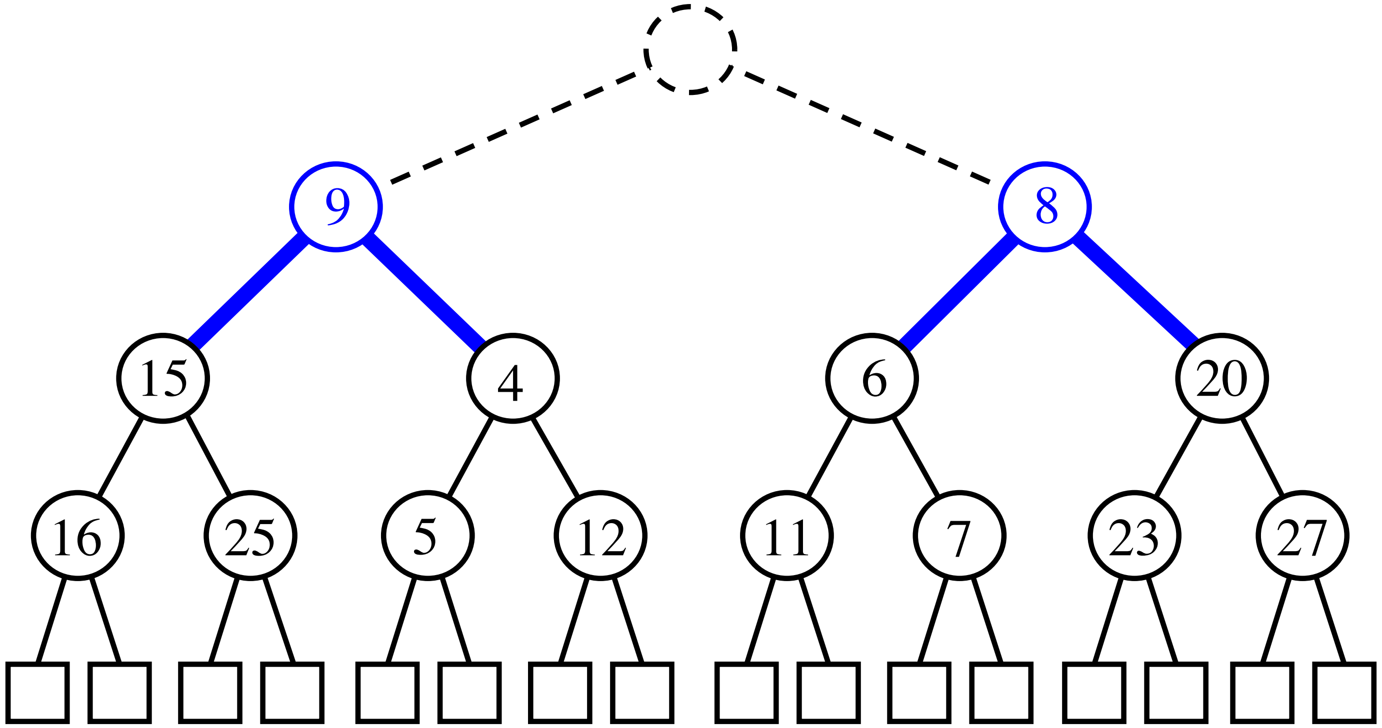


Bottom-Up Heap Construction

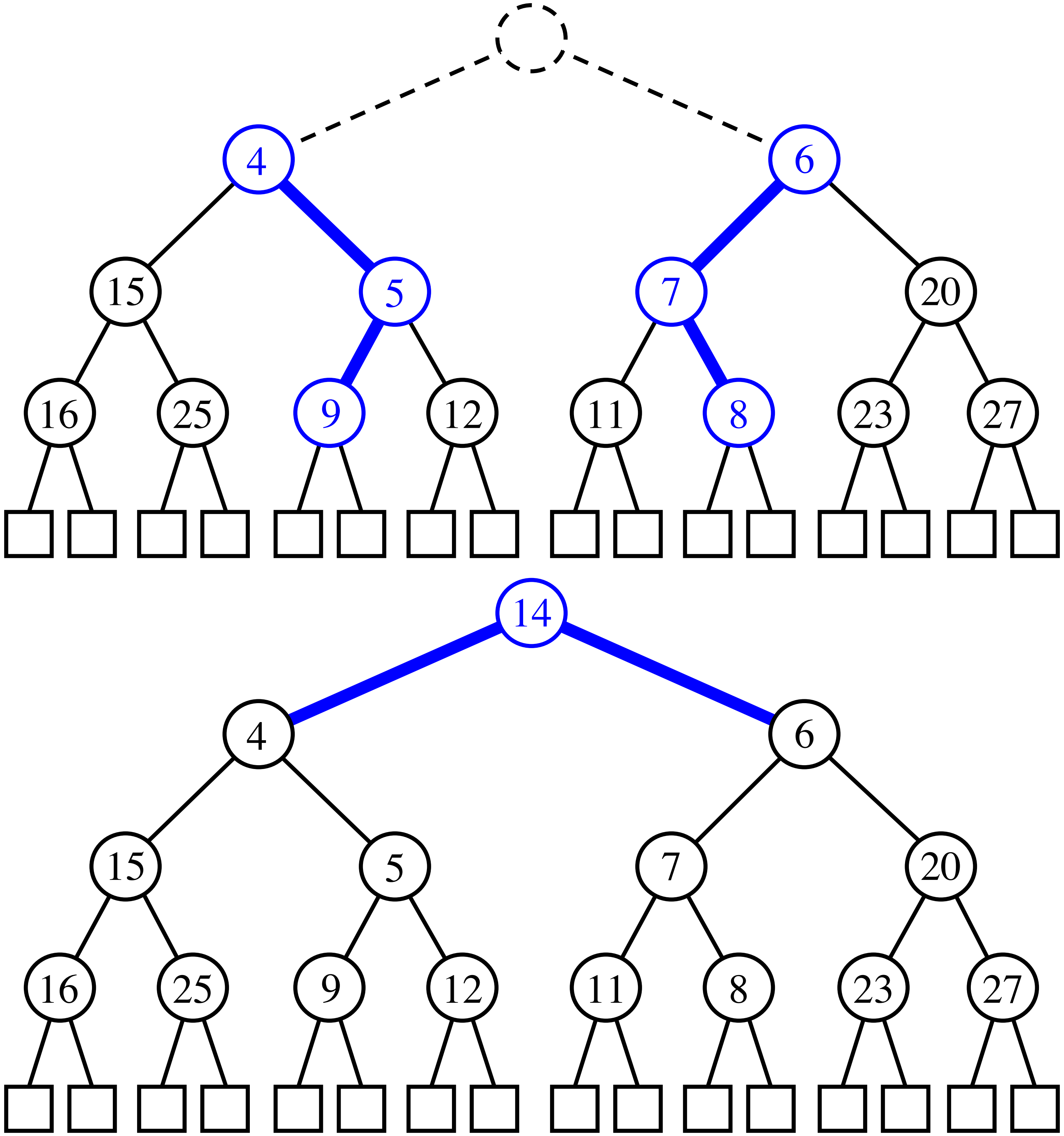
- ***downheap*** to preserve the order property



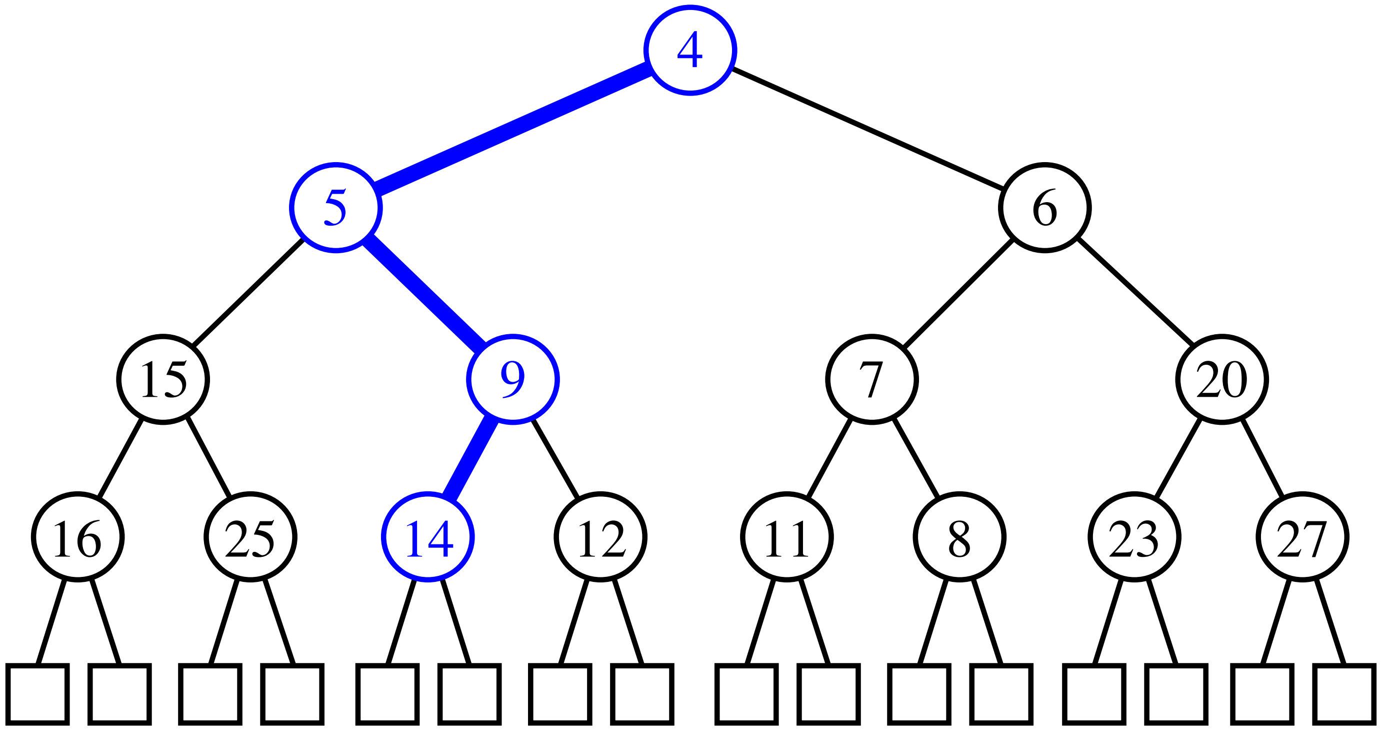
- now form seven-element heaps



Bottom-Up Heap Construction (cont.)



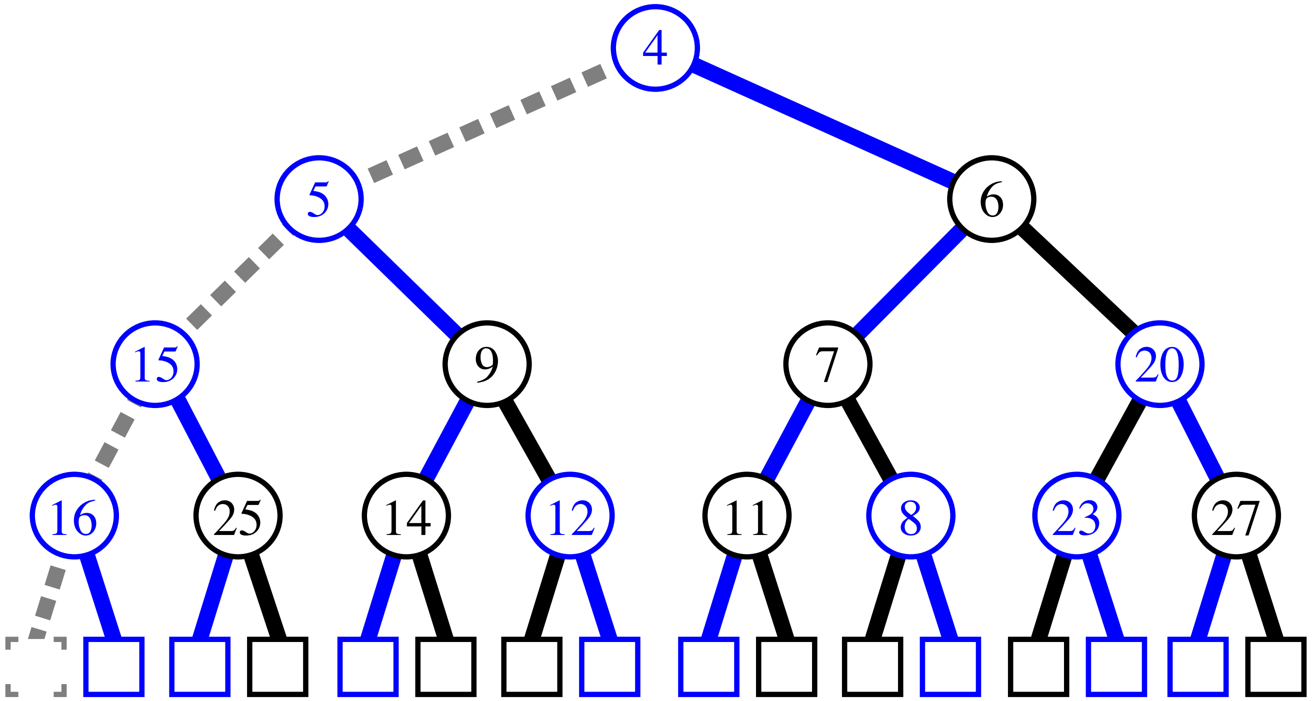
Bottom-Up Heap Construction (cont.)



The End

Analysis of Bottom-Up Heap Construction

- Proposition: Bottom-up heap construction with n keys takes $O(n)$ time.
  - Insert $(n + 1)/2$ nodes
  - Insert $(n + 1)/4$ nodes and downheap them
  - Insert $(n + 1)/8$ nodes and downheap them
  - ...
  - visual analysis:



- n inserts, $n/2$ upheaps with total $O(n)$ running time

Locators

- Locators can be used to keep track of elements as they are moved around inside a container.
- A ***locator*** sticks with a specific element, even if that element changes positions in the container.
- The locator ADT supports the following fundamental methods:
  - element(): return the element of the item associated with the locator.
  - key(): return the key of the item assocated with the locator.
- Using locators, we define additional methods for the priority queue ADT
  - insert(k,e): insert (k,e) into P and return its locator
  - min(): return the locator of an element witih smallest key
  - remove(l): remove the element with locator l
- In the stock trading application, we return a locator when an order is placed. The locator allows to specify unambiguously an order when a cancellation is requested

Positions and Locators

- At this point, you may be wondering what the difference is between locators and positions, and why we need to distinguish between them.
- It’s true that they have very similar methods
- The difference is in their primary usage
- Positions abstract the specific implementation of accessors to elements (indices vs. nodes).
- Positions are defined relatively to each other (e.g., previous-next, parent-child)
- Locators keep track of where elements are stored. In the implementation of an ADT withy locators, a locator typically holds the current position of the element.
- Locators associate elements with their keys

Locators and Positions at Work

- For example, consider the CS16 Valet Parking Service (started by the TA staff because they had too much free time on their hands).
- When they began their business, Andy and Devin decided to create a data structure to keep track of where exactly the cars were.
- Andy suggested having a ***position*** represent what ***parking space*** the car was in.
- However, Devin knew that the TAs were driving the customers' cars around campus and would not always park them back into the same spot.
- So they decided to install a ***locator*** (a ***wireless tracking device***) in each car. Each locator had a unique code, which was written on the claim check.
- When a customer demanded her car, the HTAs activated the locator. The horn of the car would honk and the lights would flash.
- If the car was parked, Andy and Devin would know where to retrieve it in the lot.
- Otherwise, the TA driving the car knew it was time to bring it back.

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