Q : How does one turn $(X.Y)_B$ into $(U.V\underline{W})_R$?

A :

OVERVIEW :

1) Turn $(X)_B$ into $(U)_R$:

2) Turn $(Y)_B$ into $(P)_R$.

3) Build a number $(Z)_B$ as **1** followed by length(Y) many zeros.
Turn $(Z)_B$ into $(Q)_R$.

4) Do Long-Division of P by Q (in base R).
V and W come out of that.

DETAILS :

1) **Turn $(X)_B$ into $(U)_R$** :
   - 1.1) Compute $(R)_B$ using fixed size arithmetic. (B,R≤60)
   - 1.2) Do Long-Division of X by R (in base B).
   - 1.3) This will produce one digit of U (the remainder) and a new X (the ratio).
   - 1.4) Repeat 1.2 and 1.3 (Long-Division) until X=0.

2) **Turn $(Y)_B$ into $(P)_R$.** Do this as in 1).

3) **Build a number $(Z)_B$ as 1 followed by length(Y) many zeros.**
Turn $(Z)_B$ into $(Q)_R$. Do this as in 1).
(The number of zeros is the size of Y.)
That's because $(0.Y)_B$ really means $(Y)_B$ / $(10^{\text{length}(Y)})_B$.

4) **Do Long-Division of P by Q (in base R).**
Keep track of all the remainders during division and stop when you find a remainder Z that you have seen before.
The digits of the ratio produced before the first occurrence of Z are V and those produced from the first occurrence of Z on are W.