Algorithms

COMP 102, lecture 6

Example: from integer to binary representation

- **input** val (val is the integer whose binary
  ex:=0 representation is $B_nB_{n-1}...B_0$,
  pw:=1 where each B_i is $0 \le B_i \le 1$, $B_n=1$)
  while val≥2*pw **do**
    ex:=ex+1; pw:=pw*2
  for i:=ex **downto** 0 **do**
    if val<pw **then** B_i:=0 **else** B_i:=1; val:=val-pw
    pw:=pw/2
  n:=ex
  output $B_nB_{n-1}...B_0$

val	pw	ex	n	i	
37					
B	...				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    val≥2*pw ⊢ 37≥2*1 ⊢ 37≥2 ⊢ true
      ex:=ex+1 ⊢ ex:=1 ; pw:=pw*2 ⊢ pw:=1*2 ⊢ pw:=2
    val≥2*pw ⊢ 37≥2*2 ⊢ 37≥4 ⊢ true
      ex:=ex+1 ⊢ ex:=2 ; pw:=pw*2 ⊢ pw:=2*2 ⊢ pw:=4
    val≥2*pw ⊢ 37≥2*4 ⊢ 37≥8 ⊢ true
      ex:=ex+1 ⊢ ex:=3 ; pw:=pw*2 ⊢ pw:=4*2 ⊢ pw:=8
    val≥2*pw ⊢ 37≥2*8 ⊢ 37≥16 ⊢ true
      ex:=ex+1 ⊢ ex:=4 ; pw:=pw*2 ⊢ pw:=8*2 ⊢ pw:=16
    val≥2*pw ⊢ 37≥2*16 ⊢ 37≥32 ⊢ true
      ex:=ex+1 ⊢ ex:=5 ; pw:=pw*2 ⊢ pw:=16*2 ⊢ pw:=32
    val≥2*pw ⊢ 37≥2*32 ⊢ 37≥64 ⊢ false

val	pw	ex	n	i	
37		0			
B	...				

Example

- **input** val (= 37)
  ex:=0

val	pw	ex	n	i	
37	1	0			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1

val	pw	ex	n	i	
37	1	0			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**

val	pw	ex	n	i	
37	1	0			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    val≥2*pw

val	pw	ex	n	i	
37	1	0			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    val≥2*pw ⊢ 37≥2*1

val	pw	ex	n	i	
37	1	0			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    val≥2*pw ⊢ 37≥2*1 ⊢ 37≥2

val	pw	ex	n	i	
37	1	0			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    val≥2*pw ⊢ 37≥2*1 ⊢ 37≥2 ⊢ true

val	pw	ex	n	i	
37	1	0			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true

val	pw	ex	n	i	
37	1	0			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=ex+1

val	pw	ex	n	i	
37	1	0			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=ex+1 ⊢ ex:=1

val	pw	ex	n	i	
37	1	1			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1

val	pw	ex	n	i	
37	1	1			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=pw*2

val	pw	ex	n	i	
37	1	1			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=pw*2 ⊢ pw:=1*2

val	pw	ex	n	i	
37	1	1			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=pw*2 ⊢ pw:=1*2 ⊢ pw:=2

val	pw	ex	n	i	
37	2	1			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2

val	pw	ex	n	i	
37	2	1			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
      val≥2*pw

val	pw	ex	n	i	
37	2	1			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
  val≥2*pw ⊢ 37≥2*2

val	pw	ex	n	i	
37	2	1			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2

  val≥2*pw ⊢ 37≥2*2 ⊢ 37≥4

val	pw	ex	n	i	
37	2	1			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2

val≥2*pw ⊢ 37≥2*2 ⊢ 37≥4 ⊢ true

val	pw	ex	n	i	
37	2	1			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
  true

val	pw	ex	n	i	
37	2	1			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=ex+1

val	pw	ex	n	i	
37	2	1			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=ex+1 ⊢ ex:=2

val	pw	ex	n	i	
37	2	2			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2

val	pw	ex	n	i	
37	2	2			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=pw*2

val	pw	ex	n	i	
37	2	2			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=pw*2 ⊢ pw:=2*2

val	pw	ex	n	i	
37	2	2			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=pw*2 ⊢ pw:=2*2 ⊢ pw:=4

val	pw	ex	n	i	
37	4	2			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4

val	pw	ex	n	i	
37	4	2			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
  val≥2*pw

val	pw	ex	n	i	
37	4	2			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
  true
    ex:=2 ; pw:=4
  val≥2*pw ⊢ 37≥2*4

val	pw	ex	n	i	
37	4	2			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
  true
    ex:=2 ; pw:=4
  val≥2*pw ⊢ 37≥2*4 ⊢ 37≥8

val	pw	ex	n	i	
37	4	2			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
  true
    ex:=2 ; pw:=4
  val≥2*pw ⊢ 37≥2*4 ⊢ 37≥8 ⊢ true

val	pw	ex	n	i	
37	4	2			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
  true

val	pw	ex	n	i	
37	4	2			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=ex+1

val	pw	ex	n	i	
37	4	2			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=ex+1 ⊢ ex:=3

val	pw	ex	n	i	
37	4	3			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3

val	pw	ex	n	i	
37	4	3			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=pw*2

val	pw	ex	n	i	
37	4	3			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=pw*2 ⊢ pw:=4*2

val	pw	ex	n	i	
37	4	3			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=pw*2 ⊢ pw:=4*2 ⊢ pw:=8

val	pw	ex	n	i	
37	8	3			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8

val	pw	ex	n	i	
37	8	3			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
  val≥2*pw

val	pw	ex	n	i	
37	8	3			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
  val≥2*pw ⊢ 37≥2*8

val	pw	ex	n	i	
37	8	3			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
  val≥2*pw ⊢ 37≥2*8 ⊢ 37≥16

val	pw	ex	n	i	
37	8	3			
B	…				

Example

- **input** val (= 37)

  ex:=0 ; pw:=1

  while val≥2*pw **do**

    true

      ex:=1 ; pw:=2

    true

      ex:=2 ; pw:=4

    true

      ex:=3 ; pw:=8

  val≥2*pw ⊢ 37≥2*8 ⊢ 37≥16 ⊢ true

val	pw	ex	n	i	
37	8	3			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true

val	pw	ex	n	i	
37	8	3			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=ex+1

val	pw	ex	n	i	
37	8	3			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=ex+1 ⊢ ex:=4

val	pw	ex	n	i	
37	8	4			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4

val	pw	ex	n	i	
37	8	4			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=pw*2

val	pw	ex	n	i	
37	8	4			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=pw*2 ⊢ pw:=8*2

val	pw	ex	n	i	
37	8	4			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=pw*2 ⊢ pw:=8*2 ⊢ pw:=16

val	pw	ex	n	i	
37	16	4			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16

val	pw	ex	n	i	
37	16	4			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16
  val≥2*pw

val	pw	ex	n	i	
37	16	4			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16
  val≥2*pw ⊢ 37≥2*16

val	pw	ex	n	i	
37	16	4			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16
  val≥2*pw ⊢ 37≥2*16 ⊢ 37≥32

val	pw	ex	n	i	
37	16	4			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16

  val≥2*pw ⊢ 37≥2*16 ⊢ 37≥32 ⊢ true

val	pw	ex	n	i	
37	16	4			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16
    true

val	pw	ex	n	i	
37	16	4			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16
    true
      ex:=ex+1

val	pw	ex	n	i	
37	16	4			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16
    true
      ex:=ex+1 ⊢ ex:=5

val	pw	ex	n	i	
37	16	5			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16
    true
      ex:=5

val	pw	ex	n	i	
37	16	5			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16
    true
      ex:=5 ; pw:=pw*2

val	pw	ex	n	i	
37	16	5			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16
    true
      ex:=5 ; pw:=pw*2 ⊢ pw:=16*2

val	pw	ex	n	i	
37	16	5			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16
    true
      ex:=5 ; pw:=pw*2 ⊢ pw:=16*2 ⊢ pw:=32

val	pw	ex	n	i	
37	32	5			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16
    true
      ex:=5 ; pw:=32

val	pw	ex	n	i	
37	32	5			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16
    true
      ex:=5 ; pw:=32
  val≥2*pw

val	pw	ex	n	i	
37	32	5			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16
    true
      ex:=5 ; pw:=32
  val≥2*pw ⊢ 37≥2*32

val	pw	ex	n	i	
37	32	5			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16
    true
      ex:=5 ; pw:=32
  val≥2*pw ⊢ 37≥2*32 ⊢ 37≥64

val	pw	ex	n	i	
37	32	5			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16
    true
      ex:=5 ; pw:=32
  val≥2*pw ⊢ 37≥2*32 ⊢ 37≥64 ⊢ false

val	pw	ex	n	i	
37	32	5			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  while val≥2*pw **do**
    true
      ex:=1 ; pw:=2
    true
      ex:=2 ; pw:=4
    true
      ex:=3 ; pw:=8
    true
      ex:=4 ; pw:=16
    true
      ex:=5 ; pw:=32
    false

val	pw	ex	n	i	
37	32	5			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  ex:=5 ; pw:=32 (**while**)

val	pw	ex	n	i	
37	32	5			
B	…				

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  ex:=5 ; pw:=32 (**while**)
  for i:=ex **downto** 0 **do**

val	pw	ex	n	i	
37	32	5			
B	…				

Example

- **for** i:=5 **downto** 0 **do**

val	pw	ex	n	i	
37	32	5		5	
B	…				

Example

- **for** i:=5 **downto** 0 **do**

  i=5: **if** val<pw **then** B_5:=0 **else** B_5:=1 ; val:=val-pw

val	pw	ex	n	i	
37	32	5		5	
B	…				

Example

- **for** i:=5 **downto** 0 **do**

  i=5: **if** val<pw **then** B_5:=0 **else** B_5:=1 ; val:=val-pw

  ⊢ **if** 37<32 **then** B_5:=0 **else** B_5:=1 ; val:=val-pw

val	pw	ex	n	i	
37	32	5		5	
B	…				

Example

- **for** i:=5 **downto** 0 **do**

  i=5: **if** val<pw **then** B_5:=0 **else** B_5:=1 ; val:=val-pw

  ⊢ **if** 37<32 **then** B_5:=0 **else** B_5:=1; val:=val-pw ⊢ B_5:=1; val:=val-pw

val	pw	ex	n	i	
37	32	5		5	
B	…				B
					1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=val-pw

val	pw	ex	n	i	
37	32	5		5	
B	…				B
					1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=val-pw ⊢ val:=37-32

val	pw	ex	n	i	
37	32	5		5	
B	…				B
					1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=val-pw ⊢ val:=37-32 ⊢ val:=5

val	pw	ex	n	i	
5	32	5		5	
B	…				B
					1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

val	pw	ex	n	i	
5	32	5		5	
B	…				B
					1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=pw/2

val	pw	ex	n	i	
5	32	5		5	
B	…				B
					1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=pw/2 ⊢ pw:=16

val	pw	ex	n	i	
5	16	5		5	
B	…				B
					1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

val	pw	ex	n	i	
5	16	5		4	
B	…				B
					1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: **if** val<pw **then** B_4:=0 **else** B_4:=1 ; val:=val-pw

val	pw	ex	n	i	
5	16	5		4	
B	…				B
					1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: **if** val<pw **then** B_4:=0 **else** B_4:=1 ; val:=val-pw

  ⊢ **if** 5<16 **then** B_4:=0 **else** B_4:=1 ; val:=val-pw

val	pw	ex	n	i	
5	16	5		4	
B	…				B
					1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: **if** val<pw **then** B_4:=0 **else** B_4:=1 ; val:=val-pw

  ⊢ **if** 5<16 **then** B_4:=0 **else** B_4:=1 ; val:=val-pw ⊢ B_4:=0

val	pw	ex	n	i	
5	16	5		4	
B	…			B	B
				0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=pw/2

val	pw	ex	n	i	
5	16	5		4	
B	…			B	B
				0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=pw/2 ⊢ pw:=8

val	pw	ex	n	i	
5	8	5		4	
B	…			B	B
				0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

val	pw	ex	n	i	
5	8	5		3	
B	…			B	B
				0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: **if** val<pw **then** B_3:=0 **else** B_3:=1 ; val:=val-pw

val	pw	ex	n	i	
5	8	5		3	
B	…			B	B
				0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: **if** val<pw **then** B_3:=0 **else** B_3:=1 ; val:=val-pw

  ⊢ **if** 5<8 **then** B_3:=0 **else** B_3:=1 ; val:=val-pw

val	pw	ex	n	i	
5	8	5		3	
B	…			B	B
				0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: **if** val<pw **then** B_3:=0 **else** B_3:=1 ; val:=val-pw

  ⊢ **if** 5<8 **then** B_3:=0 **else** B_3:=1 ; val:=val-pw ⊢ B_3:=0

val	pw	ex	n	i	
5	8	5		3	
B	…		B	B	B
			0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

val	pw	ex	n	i	
5	8	5		3	
B	…		B	B	B
			0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=pw/2

val	pw	ex	n	i	
5	8	5		3	
B	…		B	B	B
			0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=pw/2 ⊢ pw:=4

val	pw	ex	n	i	
5	4	5		3	
B	…		B	B	B
			0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

val	pw	ex	n	i	
5	4	5		2	
B	…		B	B	B
			0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: **if** val<pw **then** B_2:=0 **else** B_2:=1 ; val:=val-pw

val	pw	ex	n	i	
5	4	5		2	
B	…		B	B	B
			0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: **if** val<pw **then** B_2:=0 **else** B_2:=1 ; val:=val-pw

  ⊢ **if** 5<4 **then** B_2:=0 **else** B_2:=1 ; val:=val-pw

val	pw	ex	n	i	
5	4	5		2	
B	…		B	B	B
			0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: **if** val<pw **then** B_2:=0 **else** B_2:=1 ; val:=val-pw

  ⊢ **if** 5<4 **then** B_2:=0 **else** B_2:=1 ; val:=val-pw ⊢ B_2:=1 ; val:=val-pw

val	pw	ex	n	i	
5	4	5		2	
B	…	B	B	B	B
		1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=val-pw

val	pw	ex	n	i	
5	4	5		2	
B	…	B	B	B	B
		1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=val-pw ⊢ val:=5-4

val	pw	ex	n	i	
5	4	5		2	
B	…	B	B	B	B
		1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=val-pw ⊢ val:=5-4 ⊢ val:=1

val	pw	ex	n	i	
1	4	5		2	
B	…	B	B	B	B
		1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

val	pw	ex	n	i	
1	4	5		2	
B	…	B	B	B	B
		1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=pw/2

val	pw	ex	n	i	
1	4	5		2	
B	…	B	B	B	B
		1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=pw/2 ⊢ pw:=2

val	pw	ex	n	i	
1	2	5		2	
B	…	B	B	B	B
		1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=2

val	pw	ex	n	i	
1	2	5		1	
B	…	B	B	B	B
		1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=2

  i=1: **if** val<pw **then** B_1:=0 **else** B_1:=1 ; val:=val-pw

val	pw	ex	n	i	
1	2	5		1	
B	…	B	B	B	B
		1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=2

  i=1: **if** val<pw **then** B_1:=0 **else** B_1:=1 ; val:=val-pw

  ⊢ **if** 1<2 **then** B_1:=0 **else** B_1:=1 ; val:=val-pw

val	pw	ex	n	i	
1	2	5		1	
B	…	B	B	B	B
		1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=2

  i=1: **if** val<pw **then** B_1:=0 **else** B_1:=1 ; val:=val-pw

  ⊢ **if** 1<2 **then** B_1:=0 **else** B_1:=1 ; val:=val-pw ⊢ B_1:=0

val	pw	ex	n	i	
1	2	5		1	
B	B	B	B	B	B
	0	1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=2

  i=1: B_1:=0

val	pw	ex	n	i	
1	2	5		1	
B	B	B	B	B	B
	0	1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=2

  i=1: B_1:=0

  pw:=pw/2

val	pw	ex	n	i	
1	2	5		1	
B	B	B	B	B	B
	0	1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=2

  i=1: B_1:=0

  pw:=pw/2 ⊢ pw:=1

val	pw	ex	n	i	
1	1	5		1	
B	B	B	B	B	B
	0	1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=2

  i=1: B_1:=0

  pw:=1

val	pw	ex	n	i	
1	1	5		0	
B	B	B	B	B	B
	0	1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=2

  i=1: B_1:=0

  pw:=1

  i=0: **if** val<pw **then** B_0:=0 **else** B_0:=1 ; val:=val-pw

val	pw	ex	n	i	
1	1	5		0	
B	B	B	B	B	B
	0	1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=2

  i=1: B_1:=0

  pw:=1

  i=0: **if** val<pw **then** B_0:=0 **else** B_0:=1 ; val:=val-pw

  ⊢ **if** 1<1 **then** B_0:=0 **else** B_0:=1 ; val:=val-pw ⊢ B_0:=1 ; val:=val-pw

val	pw	ex	n	i	
1	1	5		0	
B	B	B	B	B	B
1	0	1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=2

  i=1: B_1:=0

  pw:=1

  i=0: B_0:=1 ; val:=val-pw

val	pw	ex	n	i	
1	1	5		0	
B	B	B	B	B	B
1	0	1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=2

  i=1: B_1:=0

  pw:=1

  i=0: B_0:=1 ; val:=val-pw ⊢ val:=1-1

val	pw	ex	n	i	
1	1	5		0	
B	B	B	B	B	B
1	0	1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=2

  i=1: B_1:=0

  pw:=1

  i=0: B_0:=1 ; val:=val-pw ⊢ val:=1-1 ⊢ val:=0

val	pw	ex	n	i	
0	1	5		0	
B	B	B	B	B	B
1	0	1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=2

  i=1: B_1:=0

  pw:=1

  i=0: B_0:=1 ; val:=0

val	pw	ex	n	i	
0	1	5		0	
B	B	B	B	B	B
1	0	1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=2

  i=1: B_1:=0

  pw:=1

  i=0: B_0:=1 ; val:=0

  pw:=pw/2

val	pw	ex	n	i	
0	1	5		0	
B	B	B	B	B	B
1	0	1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=2

  i=1: B_1:=0

  pw:=1

  i=0: B_0:=1 ; val:=0

  pw:=pw/2 ⊢ pw:=0

val	pw	ex	n	i	
0	0	5		0	
B	B	B	B	B	B
1	0	1	0	0	1

Example

- **for** i:=5 **downto** 0 **do**

  i=5: B_5:=1 ; val:=5

  pw:=16

  i=4: B_4:=0

  pw:=8

  i=3: B_3:=0

  pw:=4

  i=2: B_2:=1 ; val:=1

  pw:=2

  i=1: B_1:=0

  pw:=1

  i=0: B_0:=1 ; val:=0

  pw:=0

val	pw	ex	n	i	
0	0	5		0	
B	B	B	B	B	B
1	0	1	0	0	1

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  ex:=5 ; pw:=32 (**while**)
  B_5:=1 ; B_4:=0 ; B_3:=0 ; B_2:=1 ; B_1:=0 ; B_0:=1 ; val:=0 ; pw:=0 (**for**)

val	pw	ex	n	i	
0	0	5		0	
B	B	B	B	B	B
1	0	1	0	0	1

Example

- **input** val (= 37)

  ex:=0 ; pw:=1

  ex:=5 ; pw:=32 (**while**)

  B_5:=1 ; B_4:=0 ; B_3:=0 ; B_2:=1 ; B_1:=0 ; B_0:=1 ; val:=0 ; pw:=0 **(for)**

  n:=ex

val	pw	ex	n	i	
0	0	5		0	
B	B	B	B	B	B
1	0	1	0	0	1

Example

- **input** val (= 37)

  ex:=0 ; pw:=1

  ex:=5 ; pw:=32 (**while**)

  B_5:=1 ; B_4:=0 ; B_3:=0 ; B_2:=1 ; B_1:=0 ; B_0:=1 ; val:=0 ; pw:=0 **(for)**

  n:=ex $\vdash$ n:=5

val	pw	ex	n	i	
0	0	5	5	0	
B	B	B	B	B	B
1	0	1	0	0	1

Example

- **input** val (= 37)
  ex:=0 ; pw:=1
  ex:=5 ; pw:=32 (**while**)
  B_5:=1 ; B_4:=0 ; B_3:=0 ; B_2:=1 ; B_1:=0 ; B_0:=1 ; val:=0 ; pw:=0 **(for)**
  n:=5

val	pw	ex	n	i	
0	0	5	5	0	
B	B	B	B	B	B
1	0	1	0	0	1

Example

- **input** val (= 37)

  ex:=0 ; pw:=1

  ex:=5 ; pw:=32 (**while**)

  B_5:=1 ; B_4:=0 ; B_3:=0 ; B_2:=1 ; B_1:=0 ; B_0:=1 ; val:=0 ; pw:=0 **(for)**

  n:=5

  output B_n B_{n-1} .. B_1 B_0

val	pw	ex	n	i	
0	0	5	5	0	
B	B	B	B	B	B
1	0	1	0	0	1

Example

- **input** val (= 37)

  ex:=0 ; pw:=1

  ex:=5 ; pw:=32 (**while**)

  B_5:=1 ; B_4:=0 ; B_3:=0 ; B_2:=1 ; B_1:=0 ; B_0:=1 ; val:=0 ; pw:=0 **(for)**

  n:=5

  output B_5 B_4 B_3 B_2 B_1 B_0 (= 1 0 0 1 0 1)

Basic Pseudo-code Instructions

- :=
- for
- while
- if
- input / output

- assignment
- definite iteration
- indefinite iteration
- conditionals
- data transfer

Assignment

- **Syntax:**

  <variable> := <expression>

- **Semantics**:

  the variable whose name is <variable> stores the value of the <expression> on the RHS.

Conditionals

- **Syntax**:

  if <boolean expression> **then** <instructions$_0$>
  else <instructions$_1$>

- **Semantics**:

  if the <boolean expression> evaluates to true then execute the sequence of instructions in <instructions$_0$> and those in <instructions$_1$> when it evaluates to false.

Definite Iteration

- **Syntax**:

  for <variable> := <expression$_0$> **to** <expression$_1$>
  do <instructions>

- **Semantics**:

  The instructions in <instructions> are executed several times, once for each value of the variable named <variable>, starting at the value obtained by evaluating <expression$_0$>, increasing one by one (<expression$_0$+1>), and ending with <expression$_1$>.

Definite Iteration

- **Syntax**:

  for <variable> := <expression$_0$> **downto** <expression$_1$>
  do <instructions>

- **Semantics**:

  The instructions in <instructions> are executed several times, once for each value of the variable named <variable>, starting at the value obtained by evaluating <expression$_0$>, decreasing one by one (<expression$_0$-1>), and ending with <expression$_1$>.

Indefinite Iteration

- **Syntax**:

  while <boolean expression> **do** <instructions>

- **Semantics**:

  The instructions in <instructions> are executed several times, as long as the value obtained by evaluating <boolean expression> is true. It terminates when this dynamic <boolean expression> evaluates to false.

Instructions

Instructions

- **Syntax**:

  <instructions> : <instruction>

  : <instructions>
    <instruction>

  : <instructions> ; <instruction>

Instructions

- **Syntax**:

  <instructions> : <instruction>
  : <instructions>
  <instruction>
  : <instructions> ; <instruction>

- **Semantics**:

  Sequence of instructions.

Instruction

- **Syntax:**

  <instruction>

  : <variable> := <expression>

  : **if** <boolean expression> **then** <instructions$_0$>

  else <instructions$_1$>

  : **for** <variable> := <expression$_0$> **(down)to**

  <expression$_1$> **do** <instructions>

  : **while** <boolean expression> **do** <instructions>

  : **input** <list-of-variables>

  : **output** <list-of-expressions>

Data Transfers

- Syntax

  input <list-of-variables>

- Semantics

  The variables listed in <list-of-variables> are assigned values provided externally to the algorithm. These values should be of the correct type for the algorithm to work right.

Data Transfers

- Syntax

  output <list-of-expressions>

- Semantics

  The expressions listed in <list-of-expressions> are provided externally from the algorithm. These values may be of any type.

Lists

- Syntax

  <list-of-things> : <>
  : <thing> <list-of-things>
  : <thing>,<list-of-things>
  <thing> : <variable>
  : <expression>

- Semantics

  Lists of variables or expressions.

To Do

- The following have not yet been fully defined

<variable>
<expression>
<boolean expression>

Addition base ten

$x_n x_{n-1} \ldots x_0 + y_n y_{n-1} \ldots y_0 = z_{n+1} z_n z_{n-1} \ldots z_0$

- **input** $x_n x_{n-1} \ldots x_0, y_n y_{n-1} \ldots y_0$

  carry:=0

  for i:=0 **to** n **do**

    Digit:=x_i+y_i+carry

    if Digit≥10 **then** z_i:=Digit-10; carry:=1

    else z_i:=Digit; carry:=0

  z_{n+1}:=carry

  output $z_{n+1} z_n z_{n-1} \ldots z_0$

Addition base B

$x_n x_{n-1} \ldots x_0 + y_n y_{n-1} \ldots y_0 = z_{n+1} z_n \ldots z_0$

- **input** $B, x_n x_{n-1} \ldots x_0, y_n y_{n-1} \ldots y_0$

  carry:=0

  for i:=0 **to** n **do**

    Bigit:=x_i+y_i+carry

    if Bigit≥B **then** z_i:=Bigit-B; carry:=1

    else z_i:=Bigit; carry:=0

  z_{n+1}:=carry

  output $z_{n+1} z_n z_{n-1} \ldots z_0$

Abstracting Complex Operations

Finding Minimum and Sorting

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \le x_j$, $1 \le j \le n$

- **input** $x_1x_2...x_n$
  mini:=1; min:=x_1
  for i:=2 **to** n **do**
    if x_i<min **then** min:=x_i; mini:=i
  output mini

Find Minimum

given $x_1 x_2 \ldots x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)

  mini:=1; min:=x_1 $\vdash$ min:=8

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8
  for i:=2 **to** n **do**

Find Minimum

given $x_1 x_2 \ldots x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8
  for i:=2 **to** 5 **do**
  if x_i<min **then** min:=x_i; mini:=i

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8

  for i:=2 **to** 5 **do**

  i=2: **if** x_i<min **then** min:=x_i; mini:=i

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8

  for i:=2 **to** 5 **do**

  i=2: **if** x_i<min **then** min:=x_i; mini:=i ⊢

  if x_2<min **then** min:=x_2; mini:=2

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8

  for i:=2 **to** 5 **do**

  i=2: **if** x_i<min **then** min:=x_i; mini:=i ⊢

  if x_2<min **then** min:=x_2; mini:=2 ⊢

  if 5<8 **then** min:=5; mini:=2

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8

  for i:=2 **to** 5 **do**

  i=2: **if** x_i<min **then** min:=x_i; mini:=i ⊢

  if x_2<min **then** min:=x_2; mini:=2 ⊢

  if 5<8 **then** min:=5; mini:=2 ⊢

  min:=5; mini:=2

Find Minimum

given $x_1 x_2 \ldots x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8
  for i:=2 **to** 5 **do**
  i=2: min:=5; mini:=2

Find Minimum

given $x_1 x_2 \ldots x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8
  for i:=2 **to** 5 **do**
  i=2: min:=5; mini:=2
  i=3: **if** x_i<min **then** min:=x_i; mini:=i

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8

  for i:=2 **to** 5 **do**

  i=2: min:=5; mini:=2

  i=3: **if** x_i<min **then** min:=x_i; mini:=i ⊢

  if x_3<min **then** min:=x_3; mini:=3

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8

  for i:=2 **to** 5 **do**

  i=2: min:=5; mini:=2

  i=3: **if** x_i<min **then** min:=x_i; mini:=i ⊢

  if x_3<min **then** min:=x_3; mini:=3 ⊢

  if 1<5 **then** min:=1; mini:=3

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8

  for i:=2 **to** 5 **do**

  i=2: min:=5; mini:=2

  i=3: **if** x_i<min **then** min:=x_i; mini:=i ⊢

  if x_3<min **then** min:=x_3; mini:=3 ⊢

  if 1<5 **then** min:=1; mini:=3 ⊢

  min:=1; mini:=3

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8

  for i:=2 **to** 5 **do**

  i=2: min:=5; mini:=2

  i=3: min:=1; mini:=3

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8
  for i:=2 **to** 5 **do**
  i=2: min:=5; mini:=2
  i=3: min:=1; mini:=3
  i=4: **if** x_i<min **then** min:=x_i; mini:=i

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8

  for i:=2 **to** 5 **do**

  i=2: min:=5; mini:=2

  i=3: min:=1; mini:=3

  i=4: **if** x_i<min **then** min:=x_i; mini:=i ⊢

  if x_4<min **then** min:=x_4; mini:=4

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8

  for i:=2 **to** 5 **do**

  i=2: min:=5; mini:=2

  i=3: min:=1; mini:=3

  i=4: **if** x_i<min **then** min:=x_i; mini:=i ⊢

  if x_4<min **then** min:=x_4; mini:=4 ⊢

  if 11<1 **then** min:=11; mini:=4 ⊢

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$, $1 \leq j \leq n$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8

  for i:=2 **to** 5 **do**

  i=2: min:=5; mini:=2

  i=3: min:=1; mini:=3

  i=4:

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8

  for i:=2 **to** 5 **do**

  i=2: min:=5; mini:=2

  i=3: min:=1; mini:=3

  i=4:

  i=5: **if** x_i<min **then** min:=x_i; mini:=i

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8

  for i:=2 **to** 5 **do**

  i=2: min:=5; mini:=2

  i=3: min:=1; mini:=3

  i=4:

  i=5: **if** x_i<min **then** min:=x_i; mini:=i ⊢

  if x_5<min **then** min:=x_5; mini:=5

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8

  for i:=2 **to** 5 **do**

  i=2: min:=5; mini:=2

  i=3: min:=1; mini:=3

  i=4:

  i=5: **if** x_i<min **then** min:=x_i; mini:=i ⊢

  if x_5<min **then** min:=x_5; mini:=5 ⊢

  if 8<1 **then** min:=8; mini:=5 ⊢

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8

  for i:=2 **to** 5 **do**

  i=2: min:=5; mini:=2

  i=3: min:=1; mini:=3

  i=4:

  i=5:

Find Minimum

given $x_1x_2...x_n$ find i such that $x_i \leq x_j$

- **input** x_1 (=8) x_2 (=5) x_3 (=1) x_4 (=11) x_5 (=8)
  mini:=1; min:=8

  for i:=2 **to** 5 **do**

  i=2: min:=5; mini:=2

  i=3: min:=1; mini:=3

  i=4:

  i=5:

  output mini (=3)