Algorithms

COMP 102, lectures 5

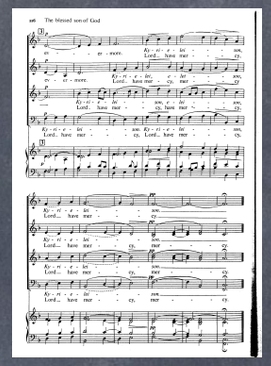
Algorithms

- Informal definition

An algorithm is the specification of a sequence of instructions to be carried out by a processor.

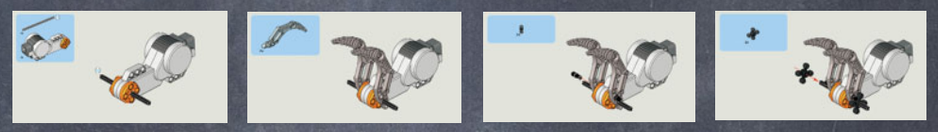
Examples

- musical score



- recipe

- assembly instructions

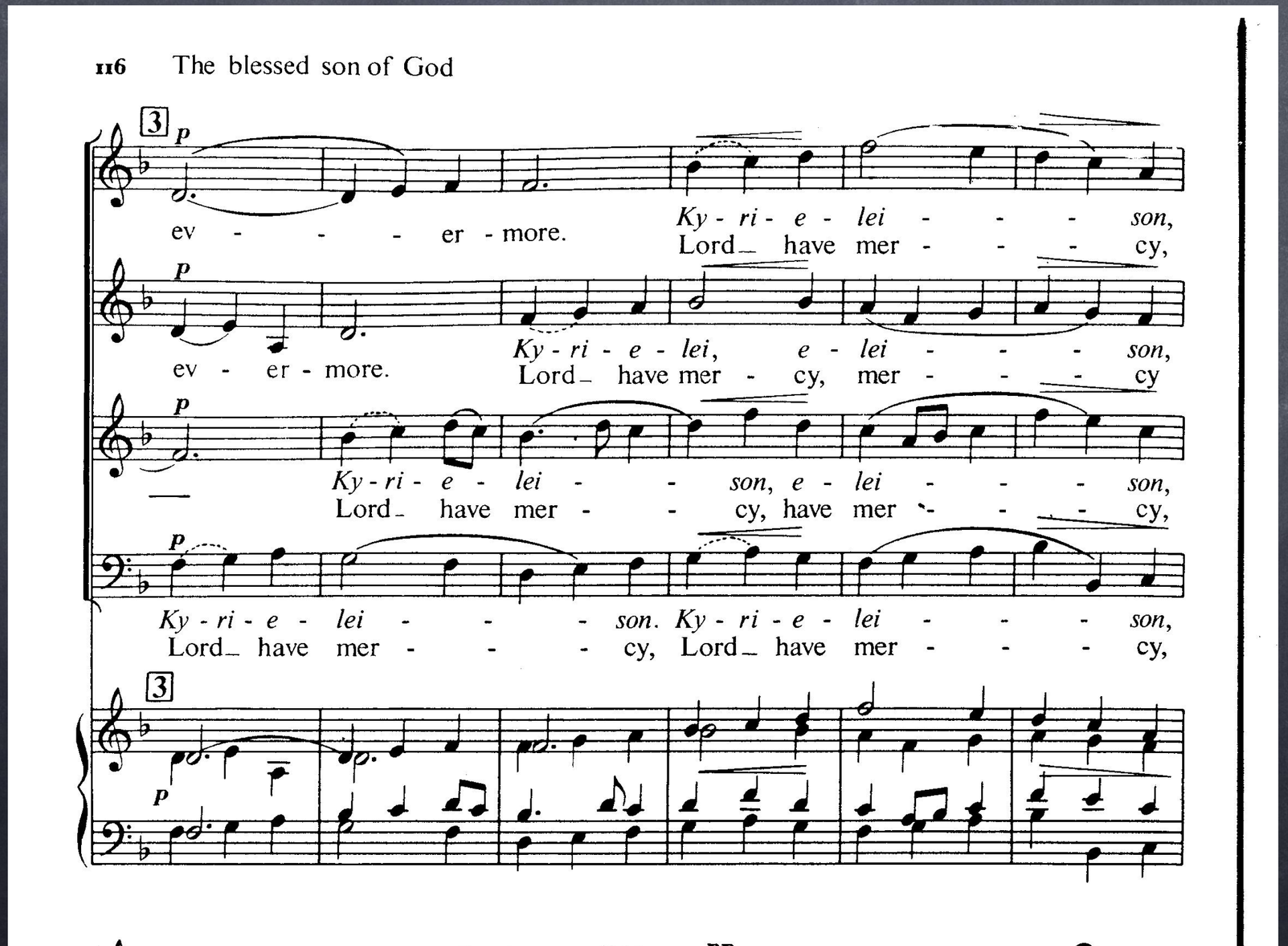


- Origami

- instructions for dialling telephone

- computer program

Music SCORE

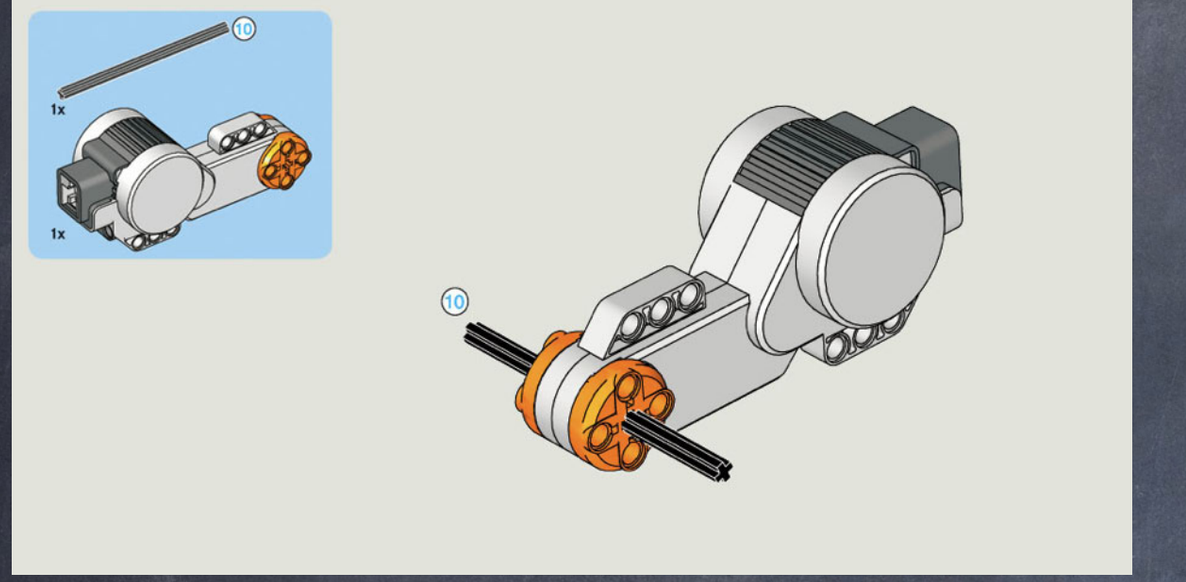

116 The blessed son of God
3
p
ev - - - - er - more.
Ky - ri - e - lei - - - - son,
Lord_ have mer - - - - cy,
p
ev - er - more.
Ky - ri - e - lei, e - lei - - - - son,
Lord_ have mer - cy, mer - - - - cy
p
Ky - ri - e - lei - - - - son, e - lei - - - - son,
Lord_ have mer - - cy, have mer - - - - cy,
p
Ky - ri - e - lei - - - - son. Ky - ri - e - lei - - - - son,
Lord_ have mer - - - - cy, Lord_ have mer - - - - cy,
3
p
pp

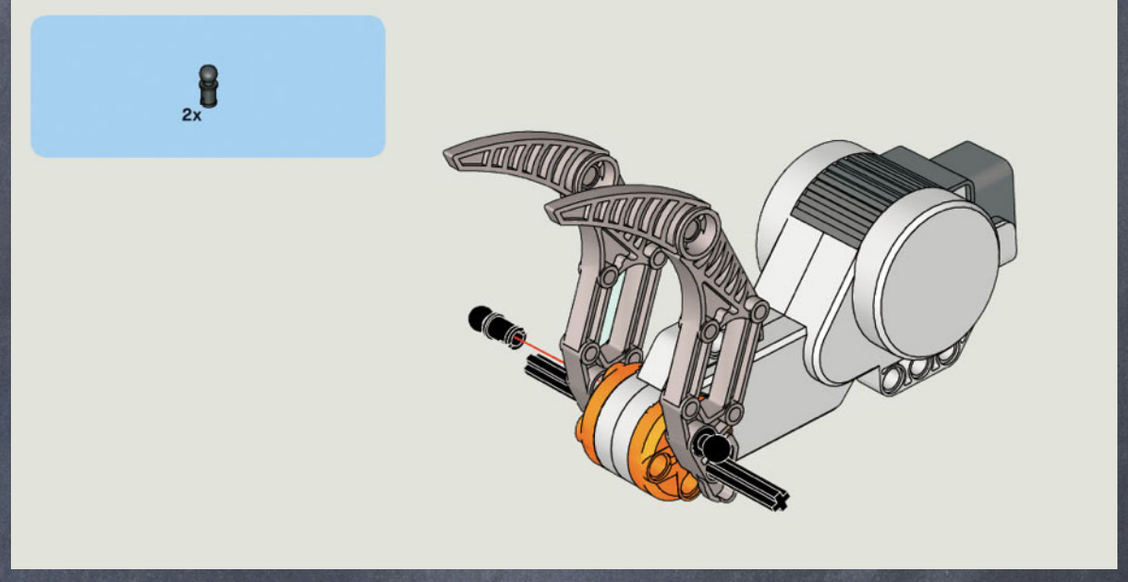
Recipe:
"Gâteau aux Carottes à Johanne"

- Instructions

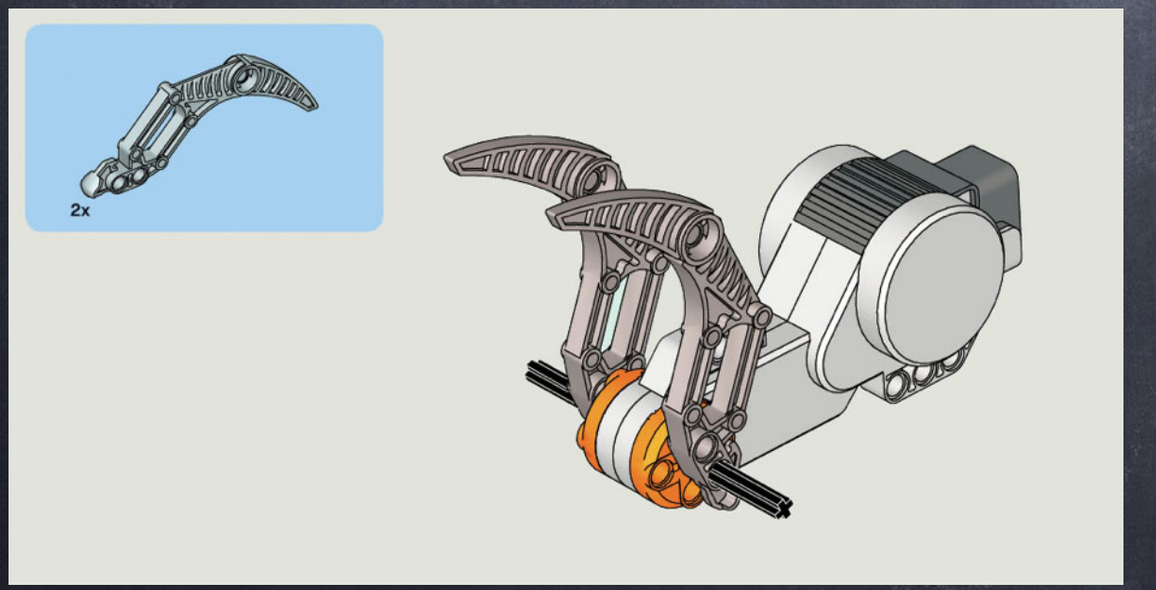
Chauffer le four à 450 F
Beurrer 2 plats de 9 pouces
Dans un grand bol battre le sucre, huile, oeufs et vanille.
Dans un autre bol mélanger farine, soda à pâte, épices et sel
Réunir le tout, ajouter les ananas, carottes et raisins
Placer dans les moules, cuire pendant 40 minutes.
Préparer le glaçage
Laisser refroidir gâteau avant de glacer.

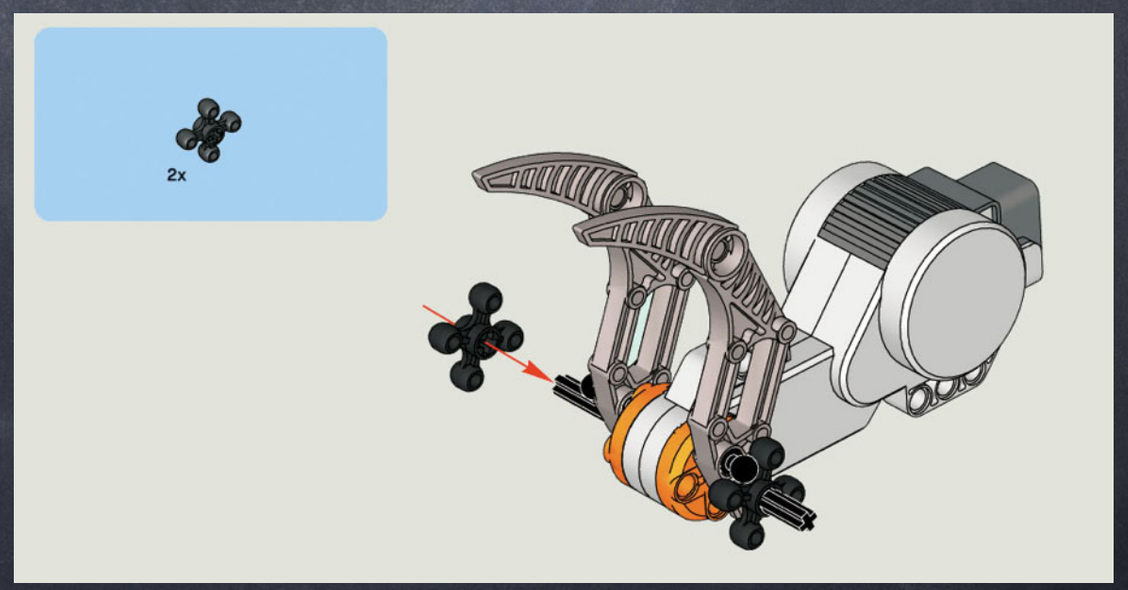
Assembly Instruction





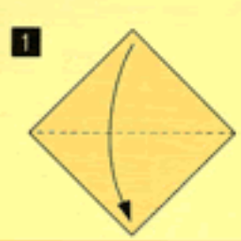
- LEGO (RoboArm (Machine)) instructions

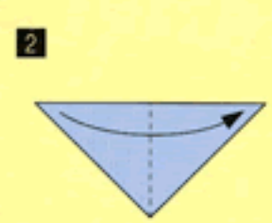


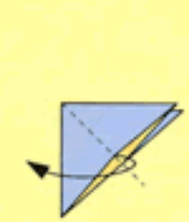


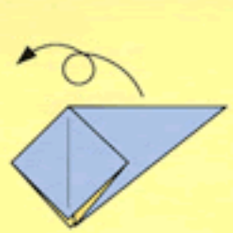
Origami

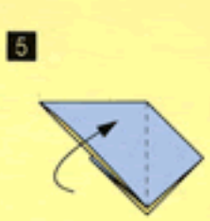
Comment faire une grue en papier

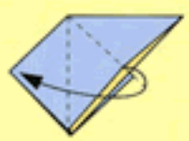












10 Dépliez les rabats.

11 *Voyez au bas de la page.

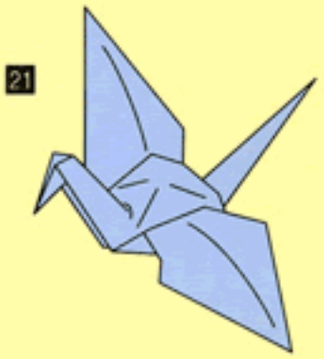
13 Répétez les étapes 9 à 10.

14 Comme à l'étape 11.

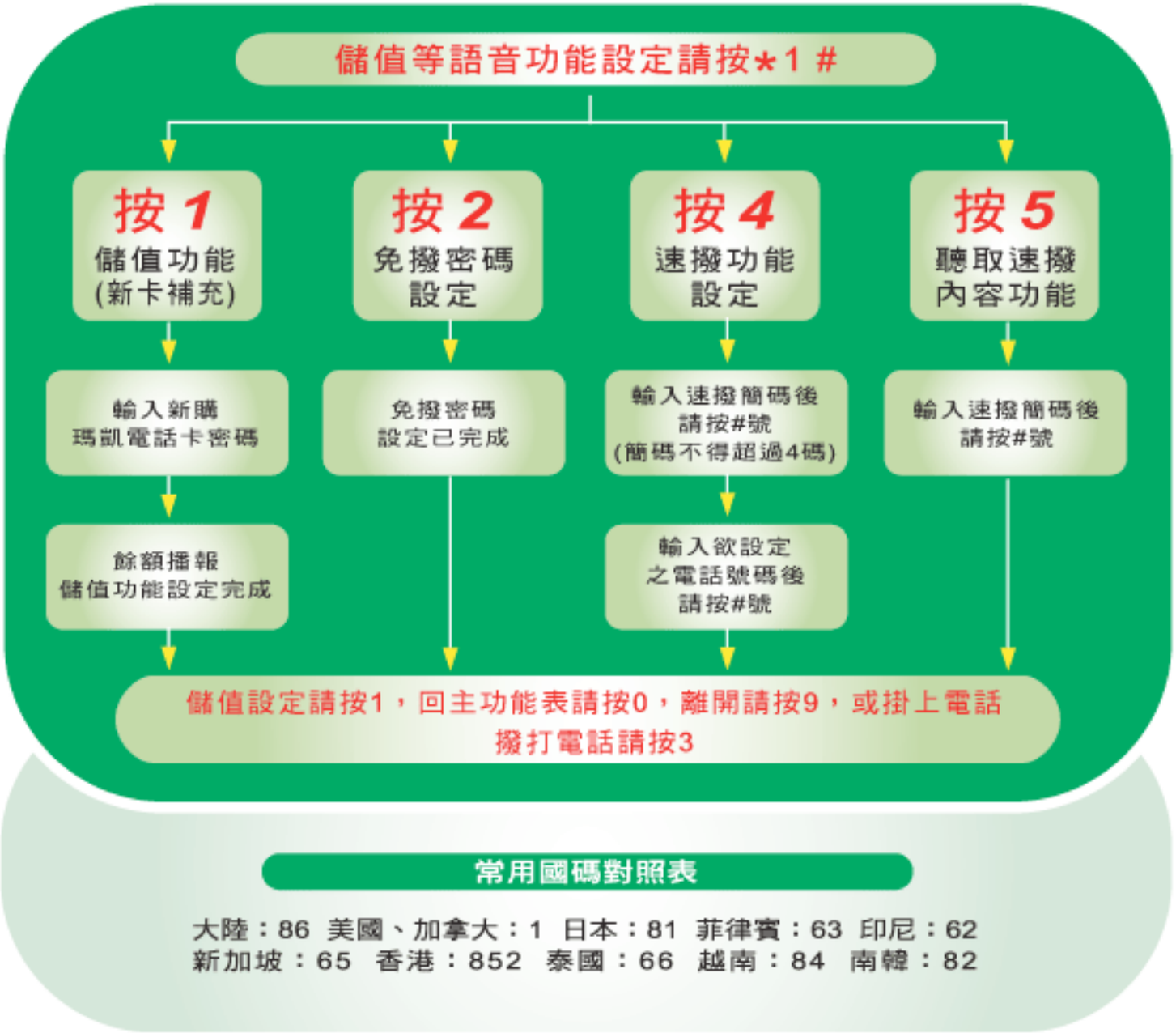
* Instructions pour 11

Tirer le coin inférieur de la feuille du dessus vers le haut, en pliant sur la ligne pointillée afin de former une pointe au sommet, et en pliant les côtés vers l'intérieur.

21



Dialling Instructions

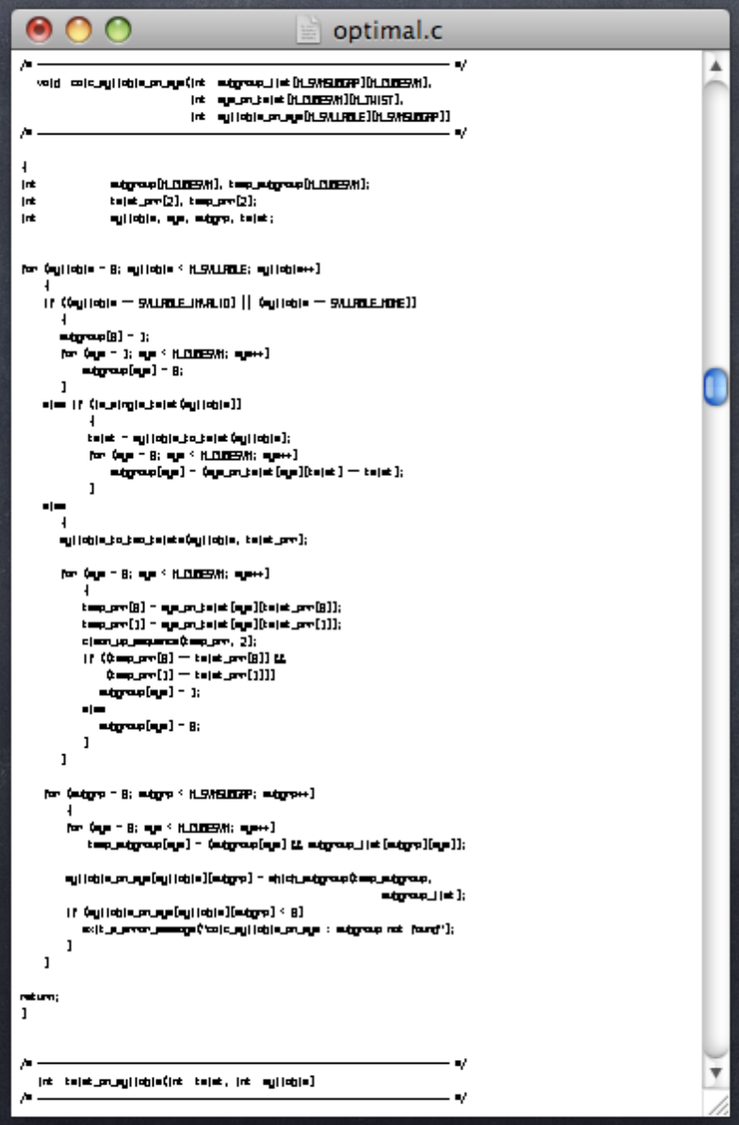


常用國碼對照表

大陸：86　美國、加拿大：1　日本：81　菲律賓：63　印尼：62
新加坡：65　香港：852　泰國：66　越南：84　南韓：82

Computer Program

- C program



Computer Science

- **Computer Science** is the study of algorithms for computing machines.
- <u>(Formal) Definition of an Algorithm</u>

  A **well-ordered** collection of **unambiguous effectively computable** operations that when executed produces a **result** and halts in a **finite** amount of time.

What distinguishes computer algorithms ?

- Instructions are executed very fast
- Little or no human interaction is possible
- Algorithm must be fully specified before execution
- Algorithm must be unambiguously specified

Example: from decimal representation to integer

- **input** $D_n D_{n-1} \ldots D_0$ (where each D_i is $0 \le D_i \le 9$)

```
val:=0
pw:=1
for i:=0 to n do
  val:=val+pw*Di
  pw:=pw*10
output val
```

(val is the integer whose decimal representation is $D_n D_{n-1} \ldots D_0$)

D	D	D	D	val	pw	i
5	5	9	1			

Example

- **input** 5 5 9 1 (= $D_3\ D_2\ D_1\ D_0$)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=val+pw*D_i ⊢ val:=0+1*D_0 ⊢ val:=0+1*1 ⊢ val:=1
  pw:=pw*10 ⊢ pw:=10
  i=1: val:=val+pw*D_i ⊢ val:=1+10*D_1 ⊢ val:=1+10*9 ⊢ val:=91
  pw:=pw*10 ⊢ pw:=100
  i=2: val:=val+pw*D_i ⊢ val:=91+100*D_2 ⊢ val:=91+100*5 ⊢ val:=591
  pw:=pw*10 ⊢ pw:=1000
  i=3: val:=val+pw*D_i ⊢ val:=591+1000*D_3 ⊢ val:=591+1000*5 ⊢ val:=5591
  pw:=pw*10 ⊢ pw:=10000
  output val (= 5591)

D	D	D	D	val	pw	i
5	5	9	1			

Example

- **input** 5 5 9 1 (= $D_3\ D_2\ D_1\ D_0$)

D	D	D	D	val	pw	i
5	5	9	1	0		

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0

D	D	D	D	val	pw	i
5	5	9	1	0	1	

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1

D	D	D	D	val	pw	i
5	5	9	1	0	1	0

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**

D	D	D	D	val	pw	i
5	5	9	1	0	1	0

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=val+pw*D_i

D	D	D	D	val	pw	i
5	5	9	1	0	1	0

Example

- **input** 5 5 9 1 (= $D_3\ D_2\ D_1\ D_0$)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=val+pw*D_i ⊢ val:=0+1*D_0

D	D	D	D	val	pw	i
5	5	9	1	0	1	0

Example

- **input** 5 5 9 1 (= $D_3\ D_2\ D_1\ D_0$)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=val+pw*D_i ⊢ val:=0+1*D_0 ⊢ val:=0+1*1

D	D	D	D	val	pw	i
5	5	9	1	0	1	0

Example

- **input** 5 5 9 1 (= $D_3\ D_2\ D_1\ D_0$)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=val+pw*D_i ⊢ val:=0+1*D_0 ⊢ val:=0+1*1 ⊢ val:=1

D	D	D	D	val	pw	i
5	5	9	1	1	1	0

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1

D	D	D	D	val	pw	i
5	5	9	1	1	1	0

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=pw*10

D	D	D	D	val	pw	i
5	5	9	1	1	1	0

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=pw*10 ⊢ pw:=10

D	D	D	D	val	pw	i
5	5	9	1	1	10	0

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10

D	D	D	D	val	pw	i
5	5	9	1	1	10	1

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=val+pw*D_i

D	D	D	D	val	pw	i
5	5	9	1	1	10	1

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=val+pw*D_i ⊢ val:=1+10*D_1

D	D	D	D	val	pw	i
5	5	9	1	1	10	1

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=val+pw*D_i ⊢ val:=1+10*D_1 ⊢ val:=1+10*9

D	D	D	D	val	pw	i
5	5	9	1	1	10	1

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**

i=0: val:=1
  pw:=10
i=1: val:=val+pw*D_i ⊢ val:=1+10*D_1 ⊢ val:=1+10*9 ⊢ val:=91

D	D	D	D	val	pw	i
5	5	9	1	91	10	1

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91

D	D	D	D	val	pw	i
5	5	9	1	91	10	1

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=pw*10

D	D	D	D	val	pw	i
5	5	9	1	91	10	1

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=pw*10 ⊢ pw:=100

D	D	D	D	val	pw	i
5	5	9	1	91	100	1

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100

D	D	D	D	val	pw	i
5	5	9	1	91	100	2

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100
  i=2: val:=val+pw*D_i

D	D	D	D	val	pw	i
5	5	9	1	91	100	2

Example

- **input** 5 5 9 1 (= $D_3\ D_2\ D_1\ D_0$)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100
  i=2: val:=val+pw*D_i ⊢ val:=91+100*D_2

D	D	D	D	val	pw	i
5	5	9	1	91	100	2

Example

- **input** 5 5 9 1 (= $D_3\ D_2\ D_1\ D_0$)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100
  i=2: val:=val+pw*D_i ⊢ val:=91+100*D_2 ⊢ val:=91+100*5

D	D	D	D	val	pw	i
5	5	9	1	91	100	2

Example

- **input** 5 5 9 1 (= $D_3\ D_2\ D_1\ D_0$)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100
  i=2: val:=val+pw*D_i $\vdash$ val:=91+100*D_2 $\vdash$ val:=91+100*5 $\vdash$ val:=591

D	D	D	D	val	pw	i
5	5	9	1	591	100	2

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100
  i=2: val:=591

D	D	D	D	val	pw	i
5	5	9	1	591	100	2

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100
  i=2: val:=591
  pw:=pw*10

D	D	D	D	val	pw	i
5	5	9	1	591	100	2

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100
  i=2: val:=591
  pw:=pw*10 ⊢ pw:=1000

D	D	D	D	val	pw	i
5	5	9	1	591	1000	2

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100
  i=2: val:=591
  pw:=1000

D	D	D	D	val	pw	i
5	5	9	1	591	1000	3

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100
  i=2: val:=591
  pw:=1000
  i=3: val:=val+pw*D_i

D	D	D	D	val	pw	i
5	5	9	1	591	1000	3

Example

- **input** 5 5 9 1 (= $D_3\ D_2\ D_1\ D_0$)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100
  i=2: val:=591
  pw:=1000
  i=3: val:=val+pw*D_i ⊢ val:=591+1000*D_3

D	D	D	D	val	pw	i
5	5	9	1	591	1000	3

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100
  i=2: val:=591
  pw:=1000
  i=3: val:=val+pw*D_i ⊢ val:=591+1000*D_3 ⊢ val:=591+1000*5

D	D	D	D	val	pw	i
5	5	9	1	591	1000	3

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100
  i=2: val:=591
  pw:=1000
  i=3: val:=val+pw*D_i ⊢ val:=591+1000*D_3 ⊢ val:=591+1000*5 ⊢ val:=5591

D	D	D	D	val	pw	i
5	5	9	1	5591	1000	3

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100
  i=2: val:=591
  pw:=1000
  i=3: val:=5591

D	D	D	D	val	pw	i
5	5	9	1	5591	1000	3

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100
  i=2: val:=591
  pw:=1000
  i=3: val:=5591
  pw:=pw*10

D	D	D	D	val	pw	i
5	5	9	1	5591	1000	3

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100
  i=2: val:=591
  pw:=1000
  i=3: val:=5591
  pw:=pw*10 ⊢ pw:=10000

D	D	D	D	val	pw	i
5	5	9	1	5591	10000	3

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100
  i=2: val:=591
  pw:=1000
  i=3: val:=5591
  pw:=10000

D	D	D	D	val	pw	i
5	5	9	1	5591	10000	3

Example

- **input** 5 5 9 1 (= D_3 D_2 D_1 D_0)
  val:=0
  pw:=1
  for i:=0 **to** 3 **do**
  i=0: val:=1
  pw:=10
  i=1: val:=91
  pw:=100
  i=2: val:=591
  pw:=1000
  i=3: val:=5591
  pw:=10000
  output val (= 5591)