COMP-547 2017 Homework set #2

Due Wednesday February 15, 2017 23:59

Exercises from Katz and Lindell's book(1.1, 1.5, 1.6, 1.7, 2.8, 2.9, 2.13)

Historic Cryptography

[10%] 1.1 Decrypt the ciphertext provided at the end of the section on mono-alphabetic substitution ciphers.

```
JGRMQOYGHMVBJWRWQFPWHGFFDQGFPFZRKBEEBJIZQQOCIBZKLFAFGQVFZFWWE
OGWOPFGFHWOLPHLRLOLFDMFGQWBLWBWQOLKFWBYLBLYLFSFLJGRMQBOLWJVFP
FWQVHQWFFPQOQVFPQOCFPOGFWFJIGFQVHLHLROQVFGWJVFPFOLFHGQVQVFILE
OGQILHQFQGIQVVOSFAFGBWQVHQWIJVWJVFPFWHGFIWIHZZRQGBABHZQOCGFHX
```

[15%] 1.5 Show that the shift, substitution, and Vigenère ciphers are all trivial to break using a chosen-plaintext attack. How much chosen plaintext is needed to recover the key for each of the ciphers?

[5%] 1.6 Assume an attacker knows that a user's password is either `abcd` or `bedg`. Say the user encrypts his password using the shift cipher, and the attacker sees the resulting ciphertext. Show how the attacker can determine the user's password, or explain why this is not possible.

[15%] 1.7 Repeat the previous exercise for the Vigenère cipher using period 2, using period 3, and using period 4.

Perfect Secrecy

[15%] Let (Gen_1,Enc_1,Dec_1), (Gen_2,Enc_2,Dec_2), and (Gen_3,Enc_3,Dec_3) be three encryption schemes over the same message space M. Consider the composed encryption scheme (Gen_c,Enc_c,Dec_c) over message space M defined as

$Gen_c = (Gen_1,Gen_2,Gen_3)$

$Enc_c(m)$ = **pick** at random $u,v \in M$;
return $(Enc_1(u), Enc_2(v), Enc_3(u \oplus v \oplus m))$

$Dec_c(u,v,w)$ = **return** $(Dec_1(u) \oplus Dec_2(v) \oplus Dec_3(w))$

Prove that if any of the three encryption schemes (Gen_s,Enc_s,Dec_s), $1 \le s \le 3$, is *perfectly secret* then so is (Gen_c,Enc_c,Dec_c).

[10%] 2.8 Let Π denote the Vigenère cipher where the message space consists of all 3-character strings (over the English alphabet), and the key is generated by first choosing the period t uniformly from $\{1, 2, 3\}$ and then letting the key be a uniform string of length t.

(a) Define $\mathcal{A}$ as follows: $\mathcal{A}$ outputs $m_0 = \texttt{aab}$ and $m_1 = \texttt{abb}$. When given a ciphertext c, it outputs 0 if the first character of c is the same as the second character of c, and outputs 1 otherwise. Compute $\Pr[\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A},\Pi} = 1]$.

(b) Construct and analyze an adversary $\mathcal{A}'$ for which $\Pr[\mathsf{PrivK}^{\mathsf{eav}}_{\mathcal{A}',\Pi} = 1]$ is greater than your answer from part (a).

[15%] 2.9 In this exercise, we look at different conditions under which the shift, mono-alphabetic substitution, and Vigenère ciphers are perfectly secret:

(a) Prove that if only a single character is encrypted, then the shift cipher is perfectly secret.

(b) What is the largest message space $\mathcal{M}$ for which the mono-alphabetic substitution cipher provides perfect secrecy?

(c) Prove that the Vigenère cipher using (fixed) period t is perfectly secret when used to encrypt messages of length t.

Reconcile this with the attacks shown in the previous chapter.

2.13 In this problem we consider definitions of perfect secrecy for the encryption of *two* messages (using the same key). Here we consider distributions over *pairs* of messages from the message space $\mathcal{M}$; we let M_1, M_2 be random variables denoting the first and second message, respectively. (We stress that these random variables are not assumed to be independent.) We generate a (single) key k, sample a pair of messages (m_1, m_2) according to the given distribution, and then compute ciphertexts $c_1 \leftarrow \mathsf{Enc}_k(m_1)$ and $c_2 \leftarrow \mathsf{Enc}_k(m_2)$; this induces a distribution over pairs of ciphertexts and we let C_1, C_2 be the corresponding random variables.

[5%]

(a) Say encryption scheme $(\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ is *perfectly secret for two messages* if for all distributions over $\mathcal{M} \times \mathcal{M}$, all $m_1, m_2 \in \mathcal{M}$, and all ciphertexts $c_1, c_2 \in \mathcal{C}$ with $\Pr[C_1 = c_1 \wedge C_2 = c_2] > 0$:

$$\Pr[M_1 = m_1 \wedge M_2 = m_2 \mid C_1 = c_1 \wedge C_2 = c_2] = \Pr[M_1 = m_1 \wedge M_2 = m_2].$$

Prove that *no* encryption scheme can satisfy this definition.

Hint: Take $c_1 = c_2$.

[10%]

(b) Say encryption scheme $(\mathsf{Gen}, \mathsf{Enc}, \mathsf{Dec})$ is *perfectly secret for two distinct messages* if for all distributions over $\mathcal{M} \times \mathcal{M}$ where the first and second messages are guaranteed to be different (i.e., distributions over pairs of *distinct* messages), all $m_1, m_2 \in \mathcal{M}$, and all $c_1, c_2 \in \mathcal{C}$ with $\Pr[C_1 = c_1 \wedge C_2 = c_2] > 0$:

$$\begin{aligned}\Pr[M_1 = m_1 \wedge M_2 = m_2 \mid C_1 = c_1 \wedge C_2 = c_2] \\ = \Pr[M_1 = m_1 \wedge M_2 = m_2].\end{aligned}$$

Show an encryption scheme that provably satisfies this definition.

Hint: The encryption scheme you propose need not be efficient, although an efficient solution is possible.