COMP-330
Theory of Computation

Fall 2019 -- Prof. Claude Crépeau

Lec. 5 : NFA-DFA equivalence

Definition of NFA

- Let $N = (Q,\Sigma,\delta,q_0,F)$ be a nondeterministic finite state automaton and let $w=w_1w_2...w_n$ ($n \geq 0$) be a string where each symbol $w_i \in \Sigma$.

- N <u>**accepts**</u> w if $\exists\ m \geq n$, $\exists\ s_0,s_1,\ldots,s_m$ and $\exists\ y_1y_2...y_m = w$, with each $y_i \in \Sigma_\varepsilon$ s.t.
  1. $s_0 = q_0$
  2. $s_{i+1} \in \delta(s_i,y_{i+1})$ for $i = 0 \ldots m-1$, and
  3. $s_m \in F$

NFA-DFA equivalence

Regular Languages

THEOREM 1.39

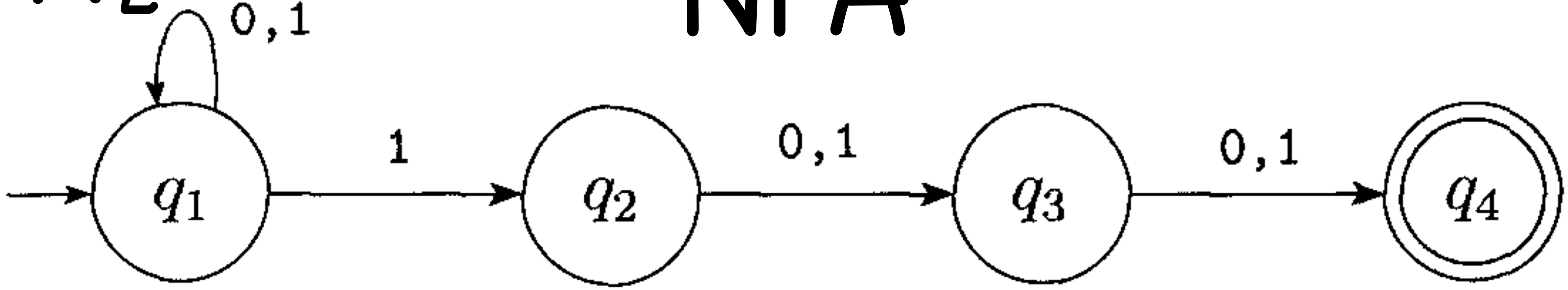
Every nondeterministic finite automaton has an equivalent deterministic finite automaton.

COROLLARY 1.40

A language is regular if and only if some nondeterministic finite automaton recognizes it.

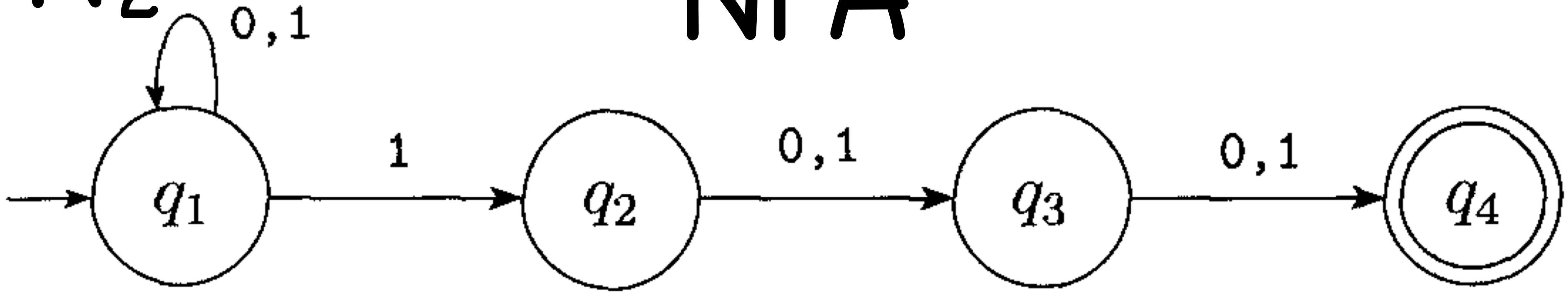
NFA

N_2



N_2

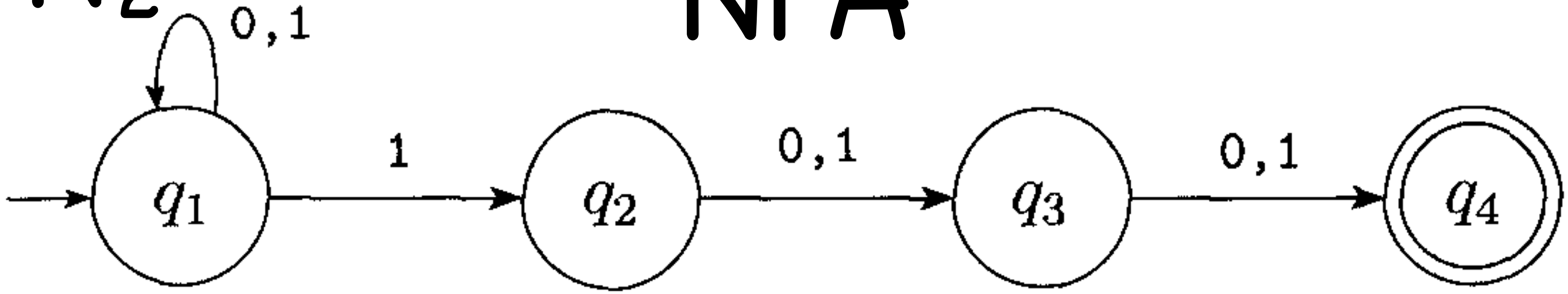
NFA



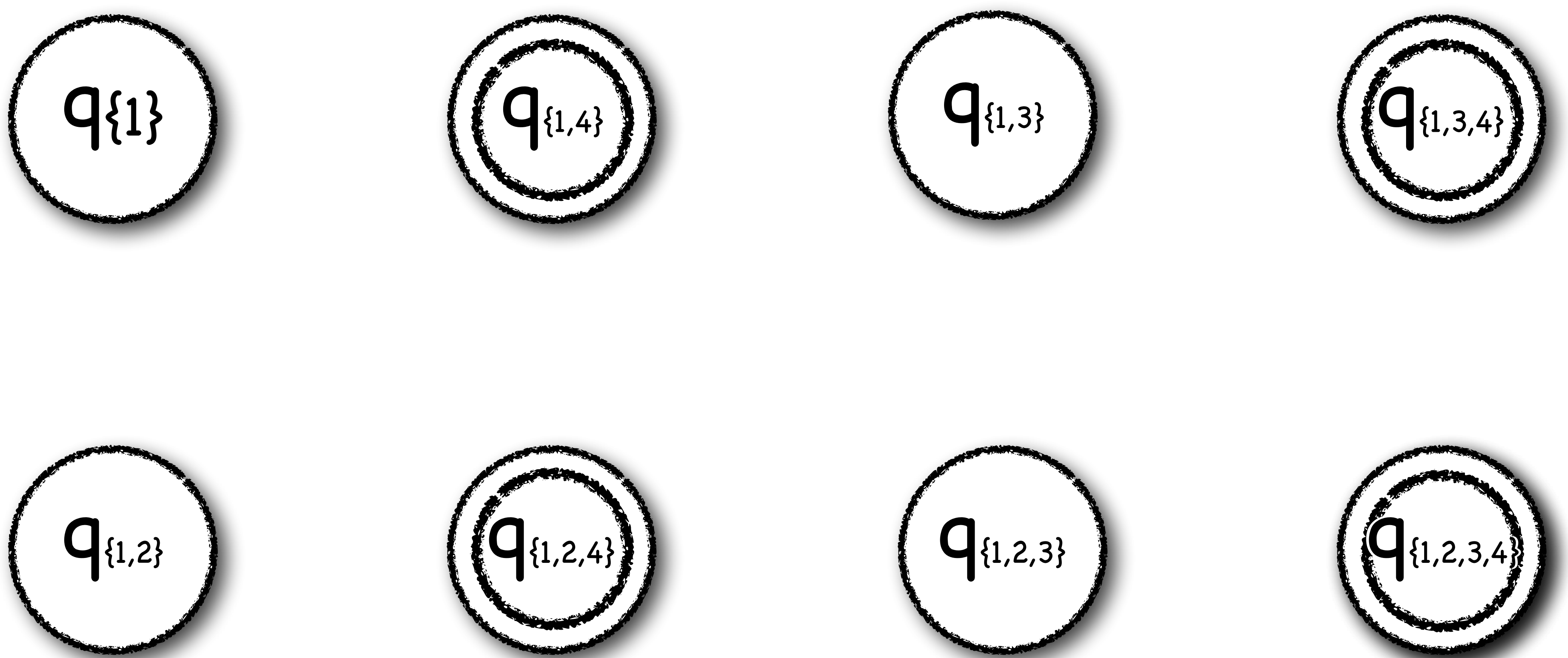
DFA

N2

NFA

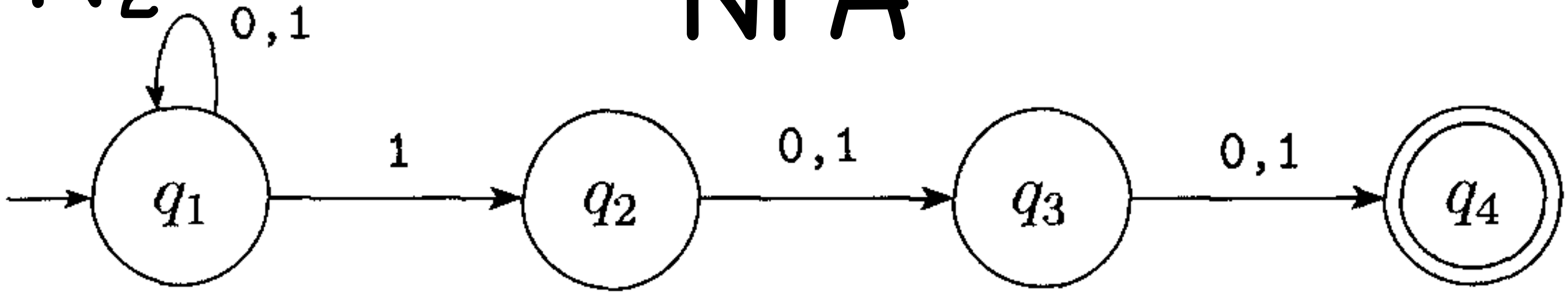


DFA



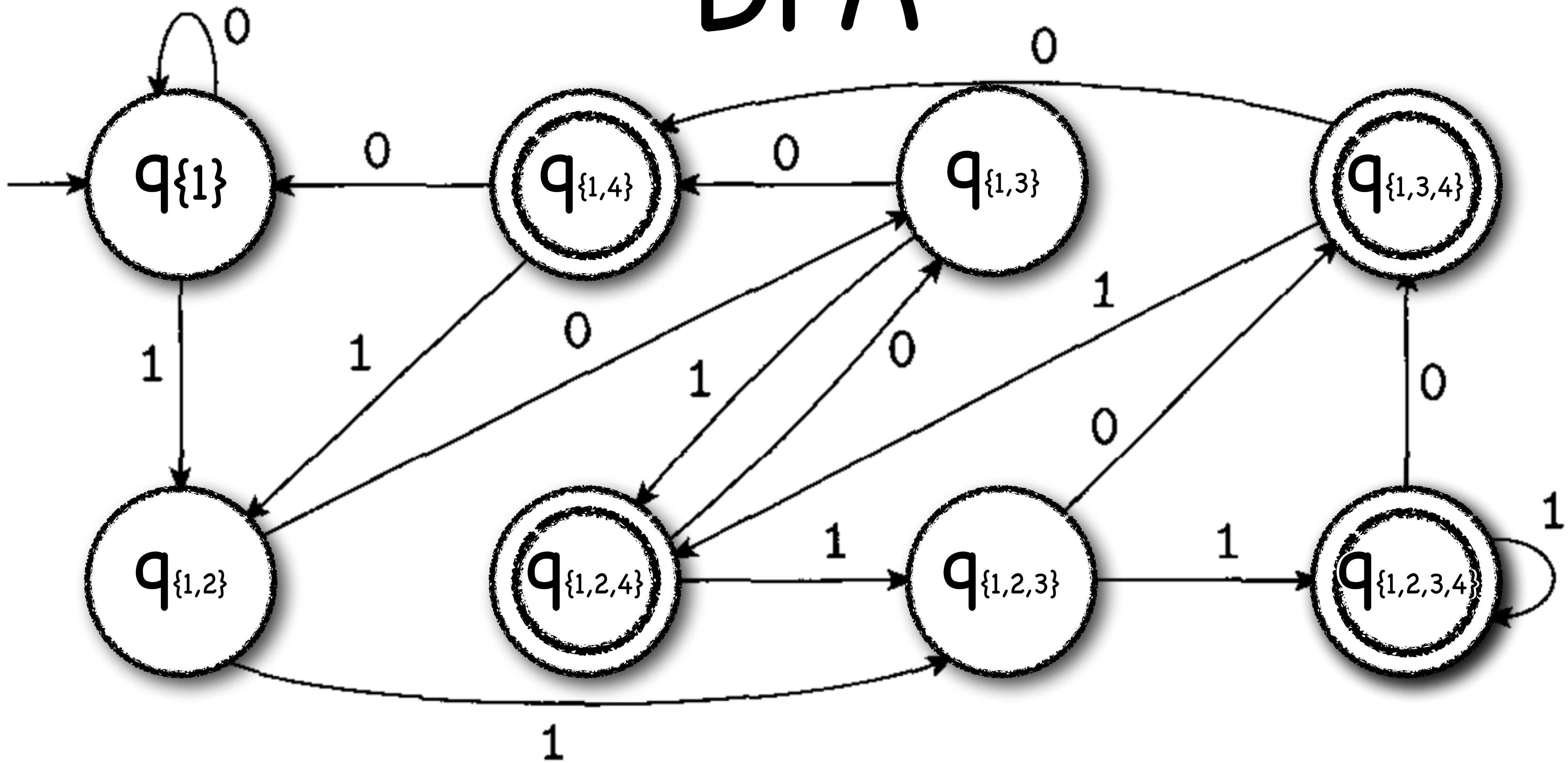
N_2

NFA



0,1
1
0,1
0,1
q_1
q_2
q_3
q_4

DFA



0
0
0
0
1
1
0
1
0
1
0
0
1
1
1
1
1
$q_{\{1\}}$
$q_{\{1,4\}}$
$q_{\{1,3\}}$
$q_{\{1,3,4\}}$
$q_{\{1,2\}}$
$q_{\{1,2,4\}}$
$q_{\{1,2,3\}}$
$q_{\{1,2,3,4\}}$

NFA-DFA equivalence

(without empty transitions)

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(without empty transitions)

- Let $N = (Q,\Sigma,\delta,q_0,F)$ be an NFA (without empty transitions) accepting language A. We show a DFA $M = (Q',\Sigma,\delta',q_0',F')$ accepting A.

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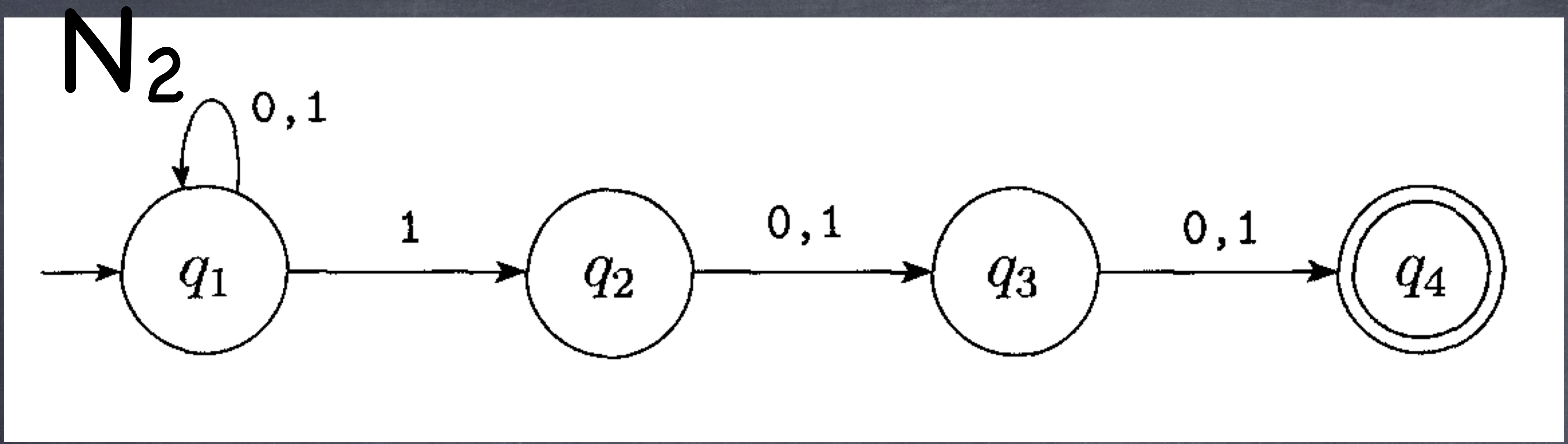
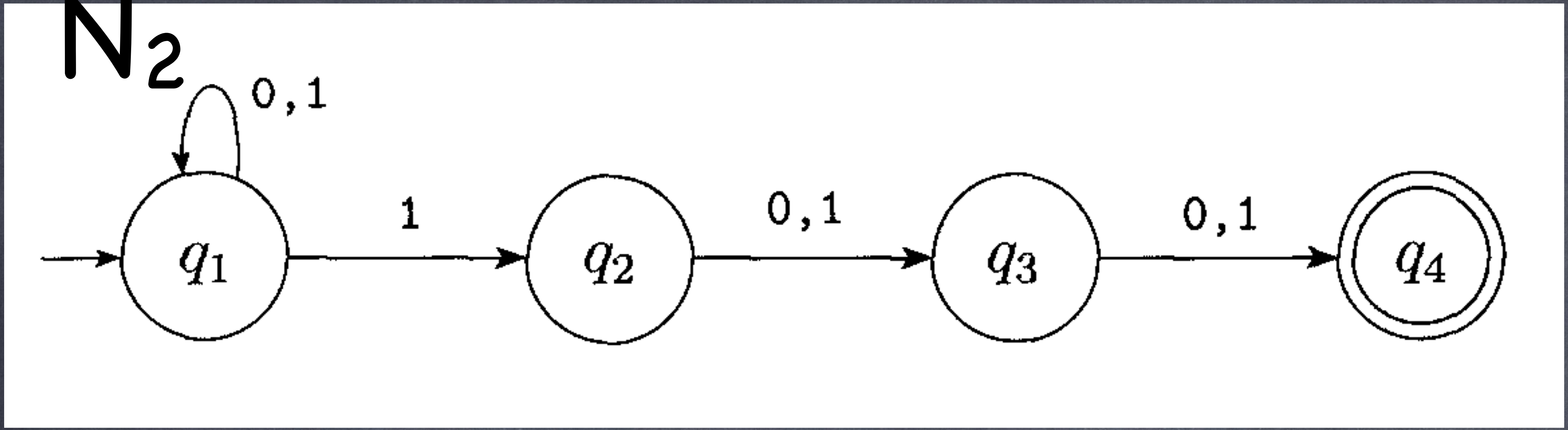


$q_{0000}=q_{\emptyset}$	$q_{0001}=q\{1\}$	$q_{0111}=q_{\{1,2,3\}}$	$q_{0110}=q_{\{2,3\}}$
$q_{0011}=q_{\{1,2\}}$	$q_{0010}=q\{2\}$	$q_{1011}=q_{\{1,2,4\}}$	$q_{1010}=q_{\{2,4\}}$
$q_{0101}=q_{\{1,3\}}$	$q_{0100}=q\{3\}$	$q_{1101}=q_{\{1,3,4\}}$	$q_{1100}=q_{\{3,4\}}$
$q_{1001}=q_{\{1,4\}}$	$q_{1000}=q\{4\}$	$q_{1110}=q_{\{2,3,4\}}$	$q_{1111}=q_{\{1,2,3,4\}}$



$q_{0000} = q_{\varnothing}$ $q_{0001} = q_{\{1\}}$ $q_{0111} = q_{\{1,2,3\}}$ $q_{0110} = q_{\{2,3\}}$

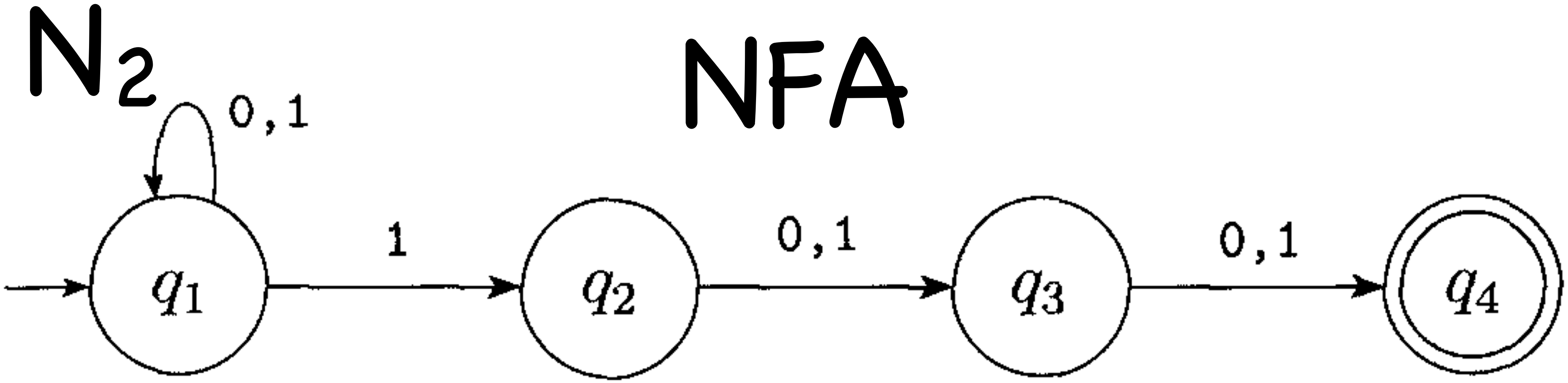
$q_{0011} = q_{\{1,2\}}$ $q_{0010} = q_{\{2\}}$ $q_{1011} = q_{\{1,2,4\}}$ $q_{1010} = q_{\{2,4\}}$

$$q_{w_4 w_3 w_2 w_1} = q_R \;:\; (w_i = 1 \Leftrightarrow i \in R)$$

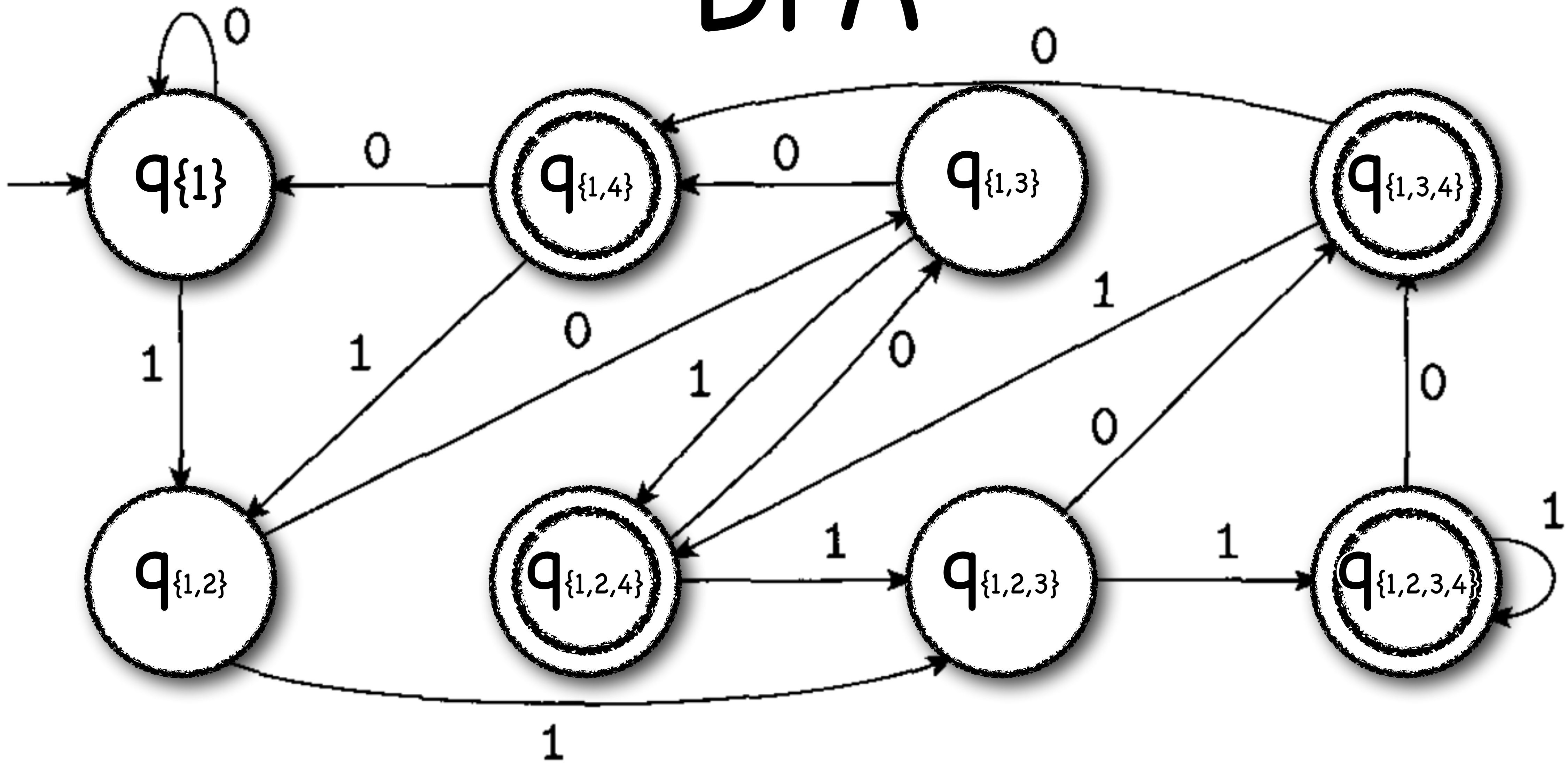
$q_{0101} = q_{\{1,3\}}$ $q_{0100} = q_{\{3\}}$ $q_{1101} = q_{\{1,3,4\}}$ $q_{1100} = q_{\{3,4\}}$

$q_{1001} = q_{\{1,4\}}$ $q_{1000} = q_{\{4\}}$ $q_{1110} = q_{\{2,3,4\}}$ $q_{1111} = q_{\{1,2,3,4\}}$

NFA

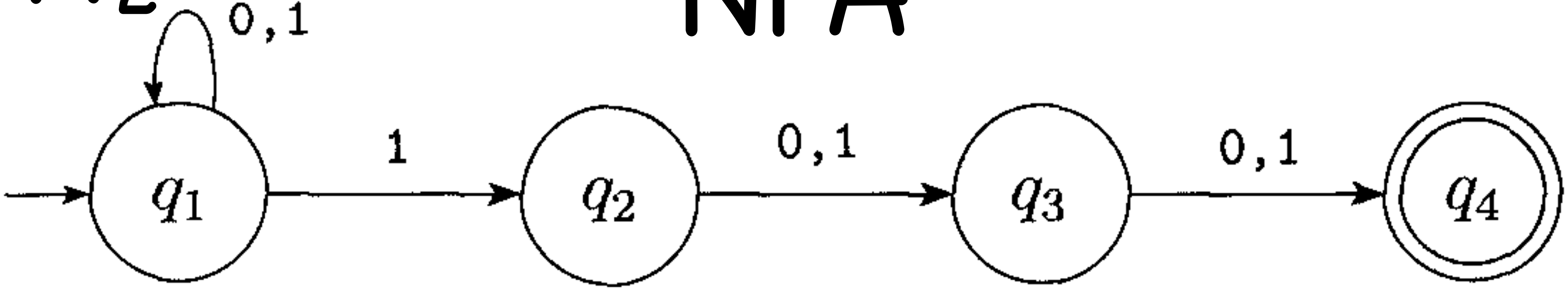


DFA

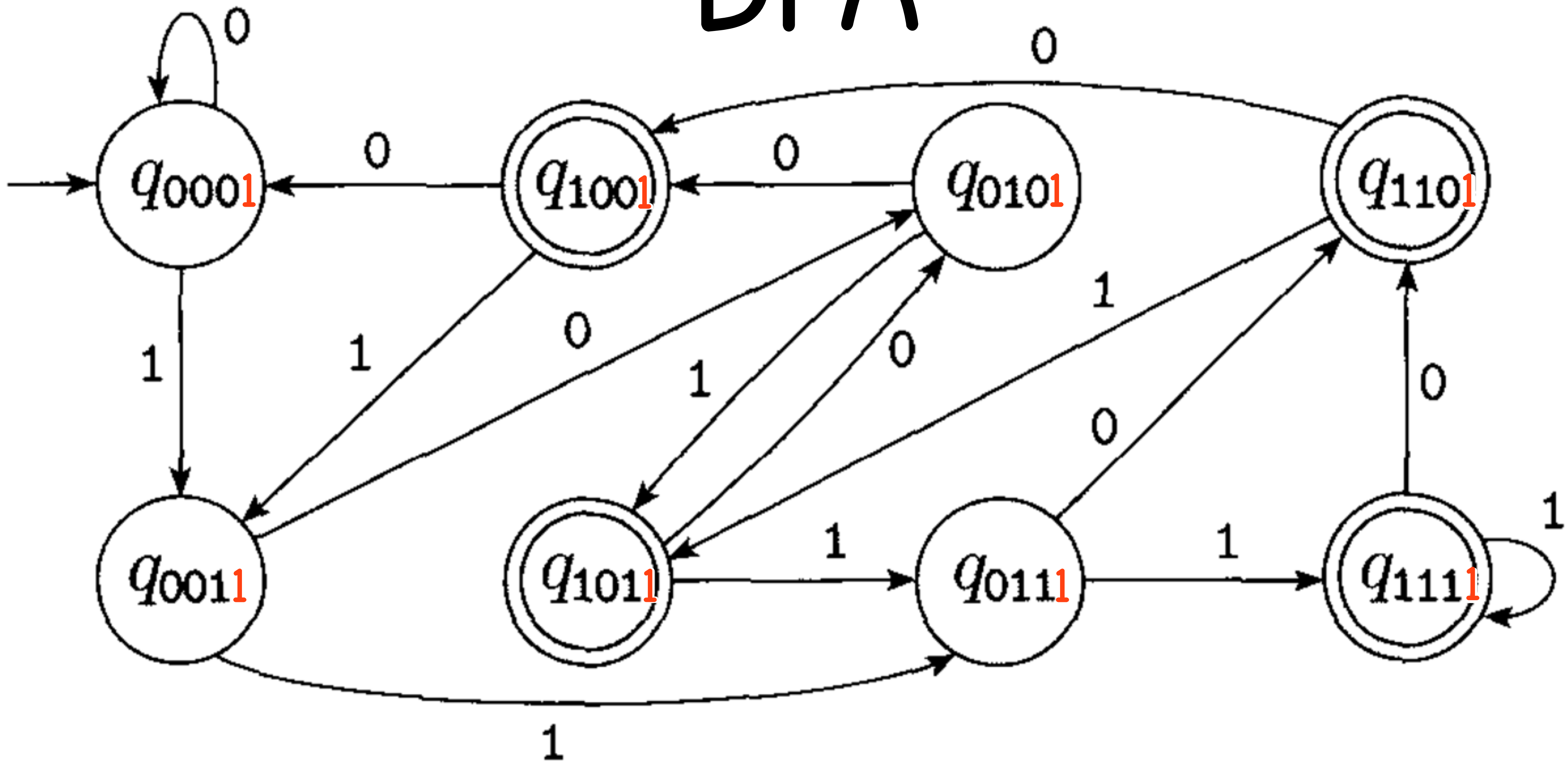


N_2

NFA

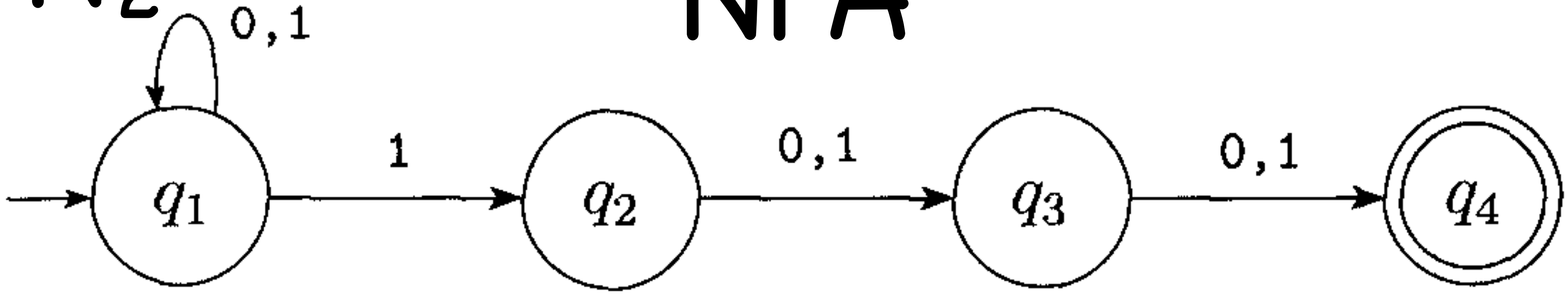

0,1
1
0,1
0,1
q_1
q_2
q_3
q_4

DFA

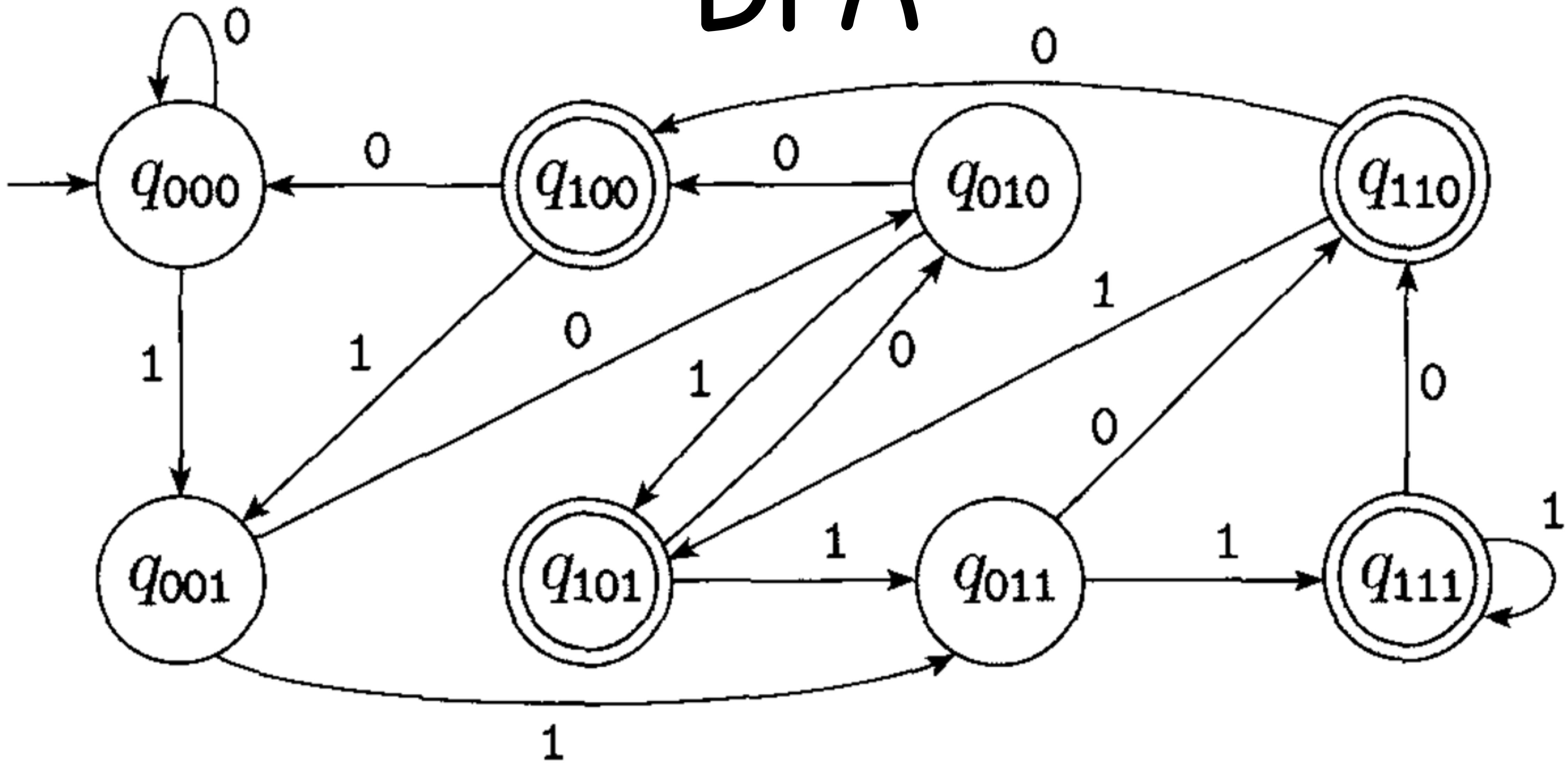

q_{0001}
q_{1001}
q_{0101}
q_{1101}
q_{0011}
q_{1011}
q_{0111}
q_{1111}
0
1

N_2

NFA


0,1
1
0,1
0,1
q_1
q_2
q_3
q_4

DFA


0
0
0
0
q_{000}
q_{100}
q_{010}
q_{110}
1
1
0
1
0
1
0
0
1
1
1
q_{001}
q_{101}
q_{011}
q_{111}
1

NFA-DFA equivalence

(with empty transitions)

$E(R) = \{q \mid q$ can be reached from R by traveling along 0 or more ε arrows$\}$.

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- Let $N = (Q,\Sigma,\delta,q_0,F)$ be an NFA accepting language A. We construct a DFA $M = (Q',\Sigma,\delta',q_0',F')$ accepting A as well.

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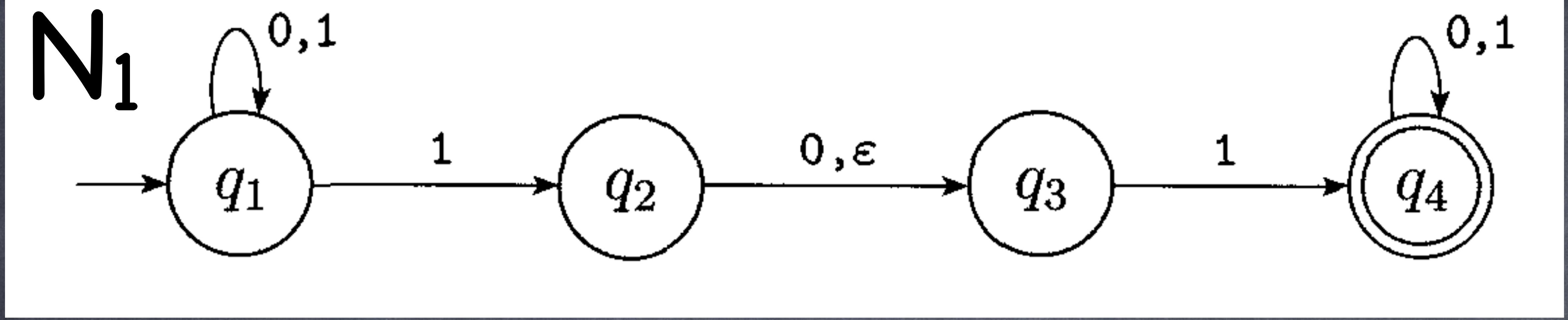
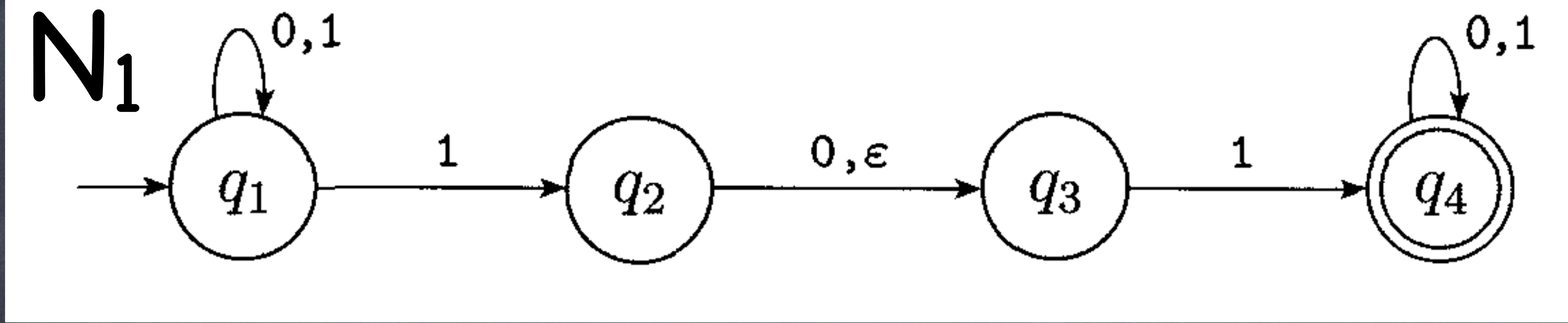
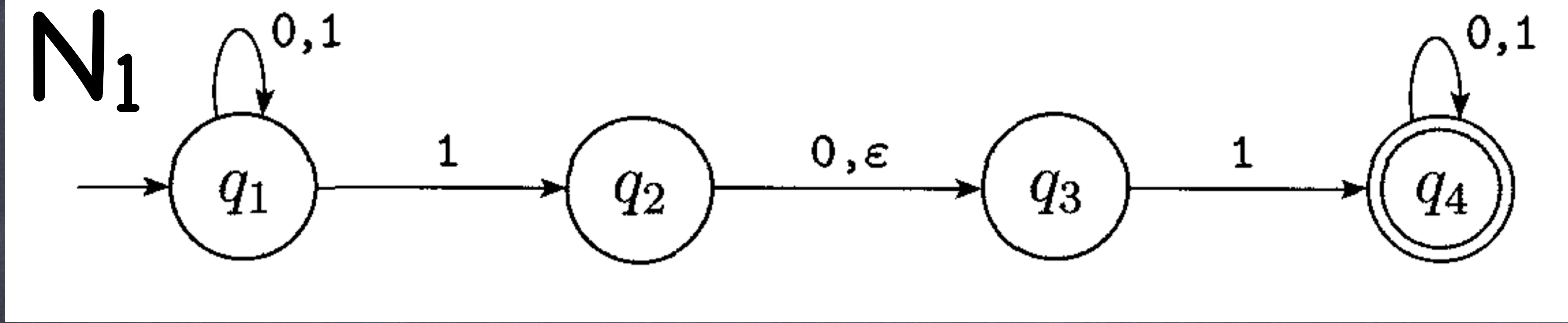

N1
0,1
1
0,ε
1
0,1
q_1
q_2
q_3
q_4

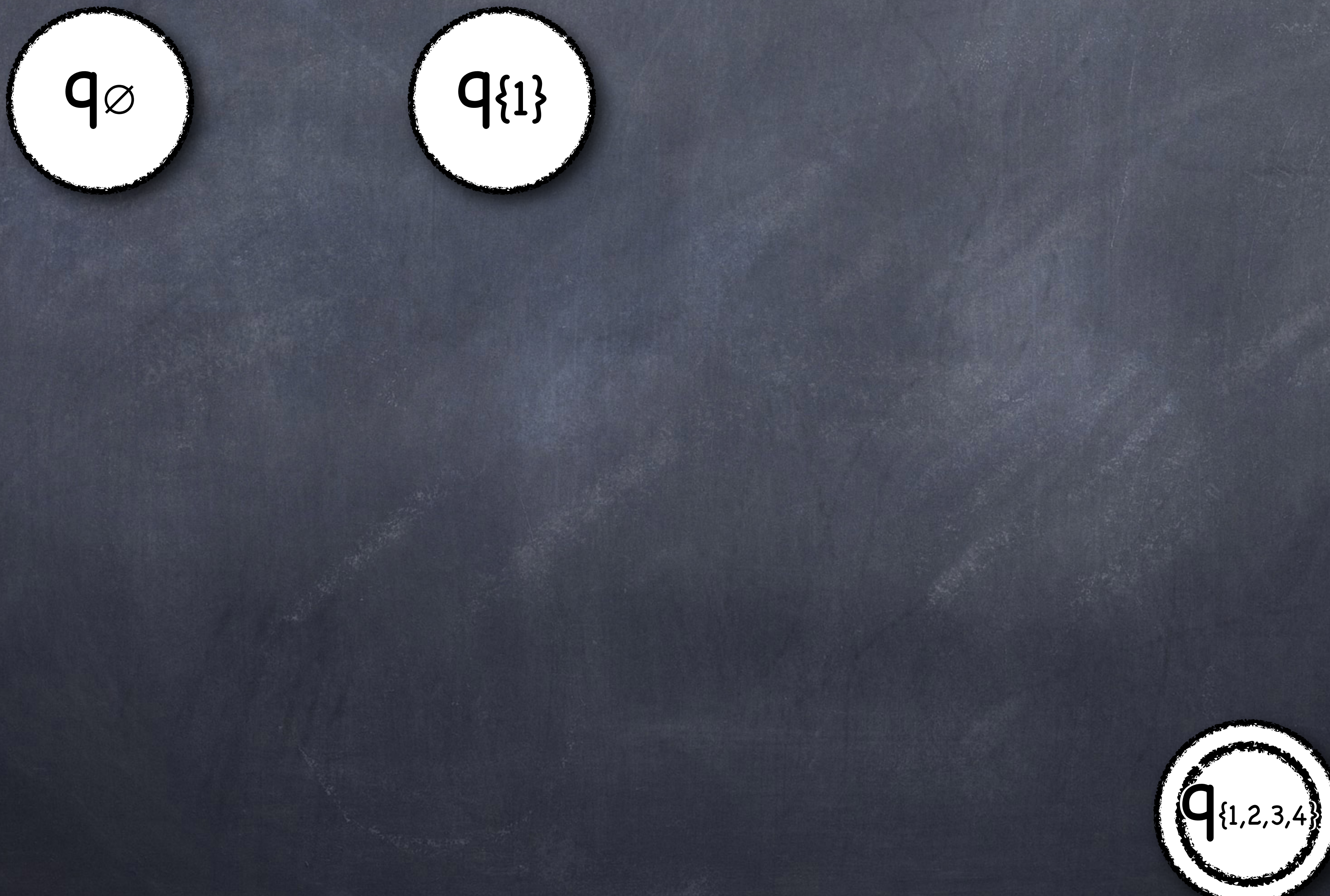

q{1,2,3,4}

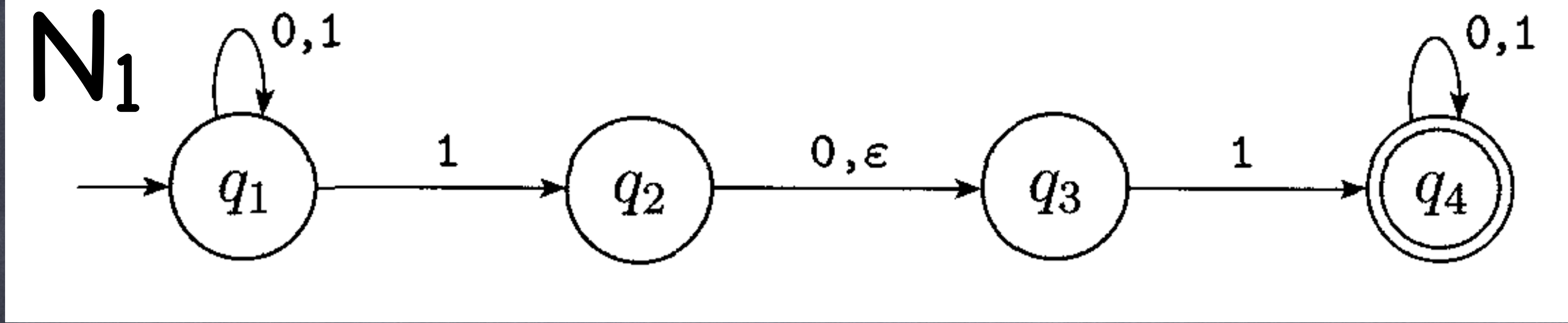

N1
0,1
1
0,ε
1
0,1
q1
q2
q3
q4

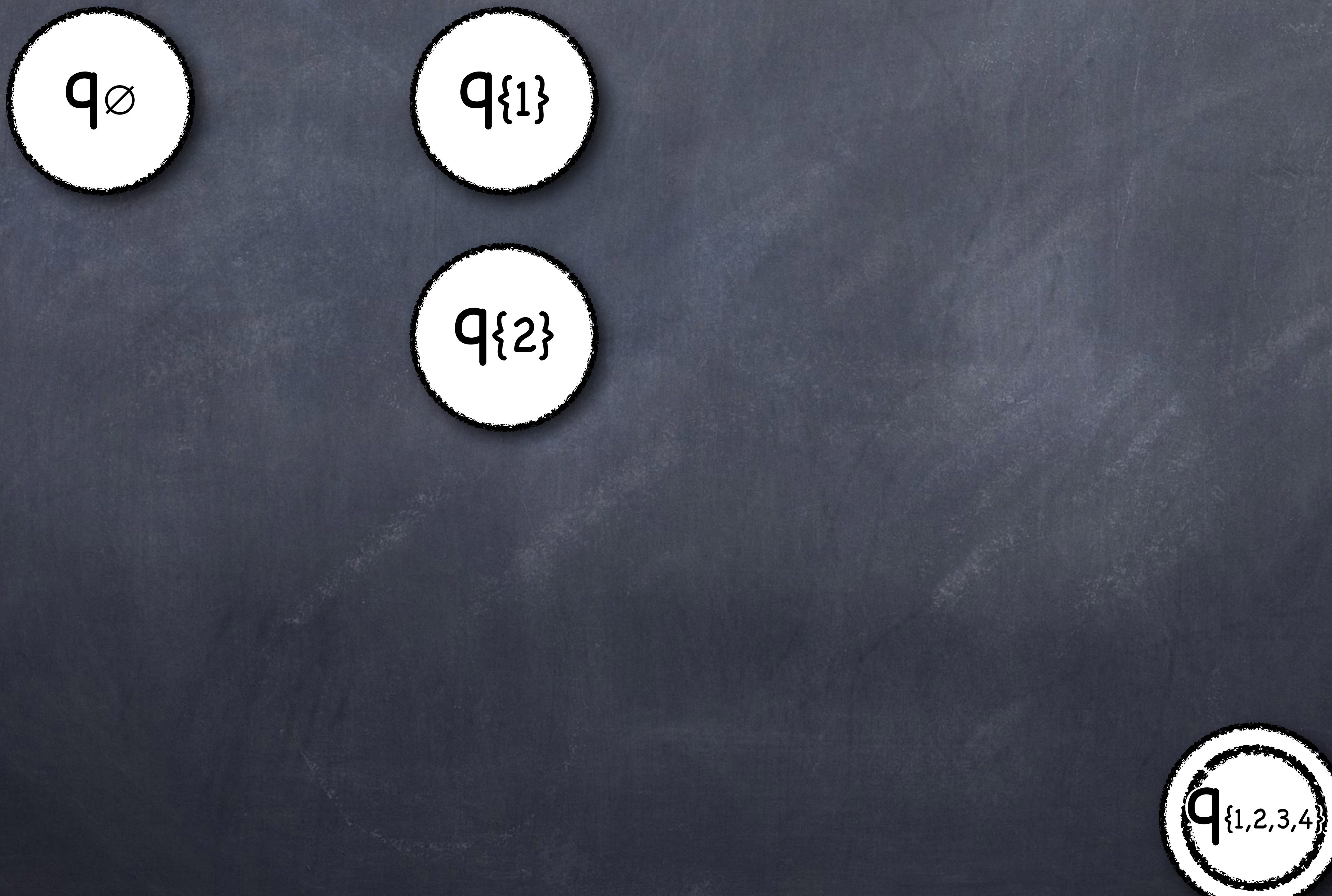

q∅

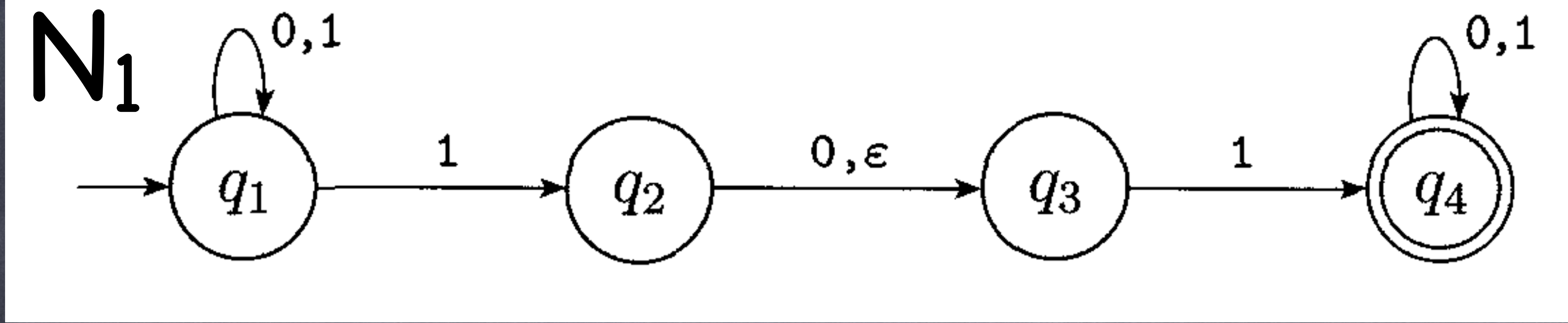

q{1,2,3,4}

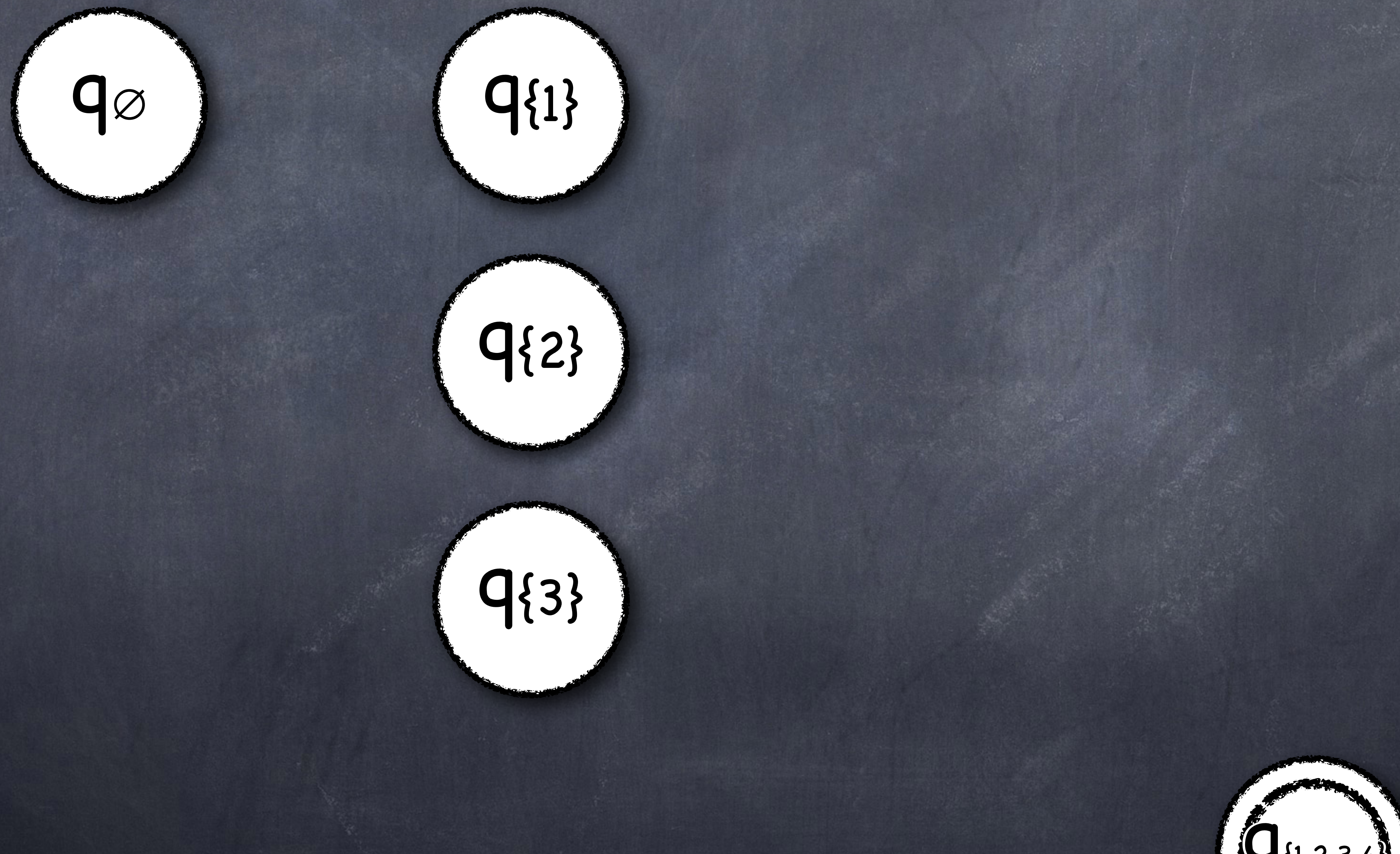

N1
0,1
1
0,ε
1
0,1
q1
q2
q3
q4

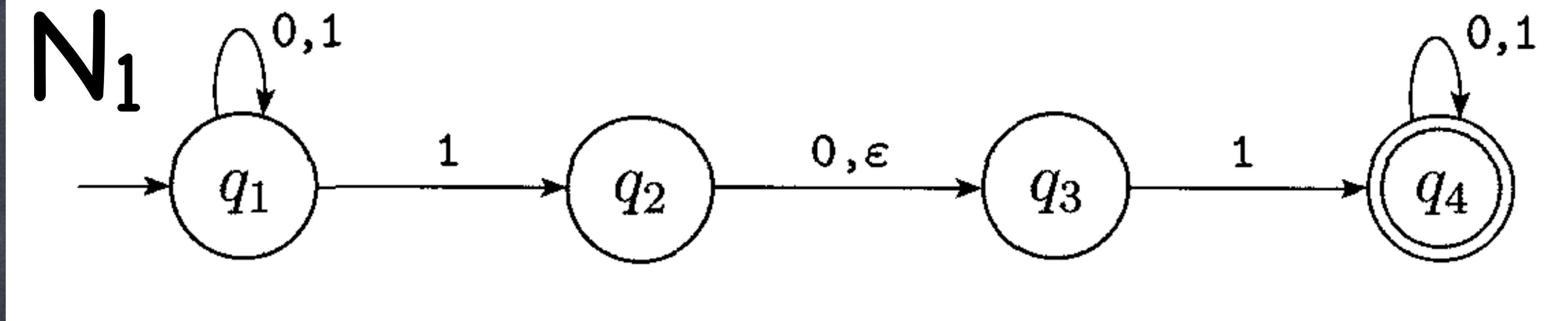

q∅
q{1}
q{1,2,3,4}

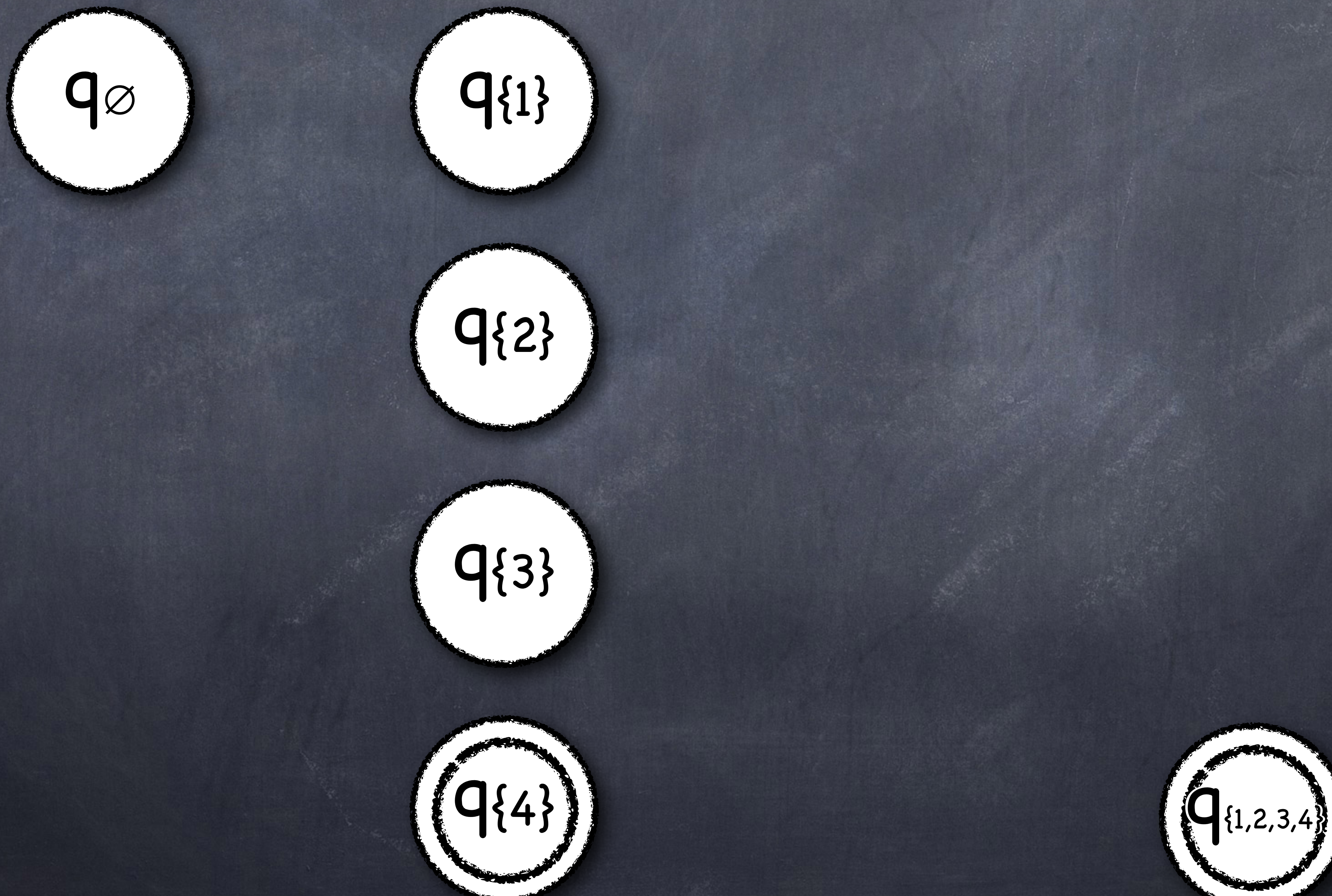

N1
0,1
1
0,ε
1
0,1
q1
q2
q3
q4

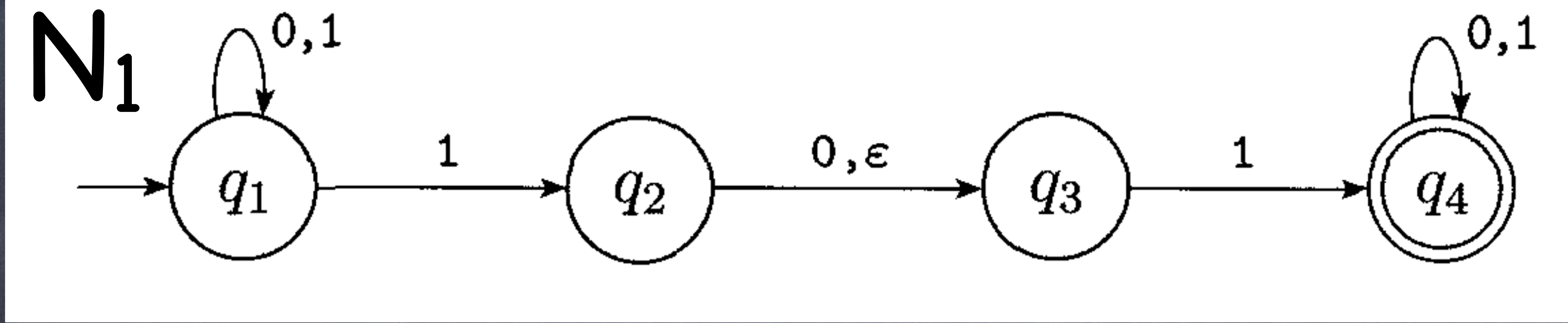

q∅
q{1}
q{2}
q{1,2,3,4}

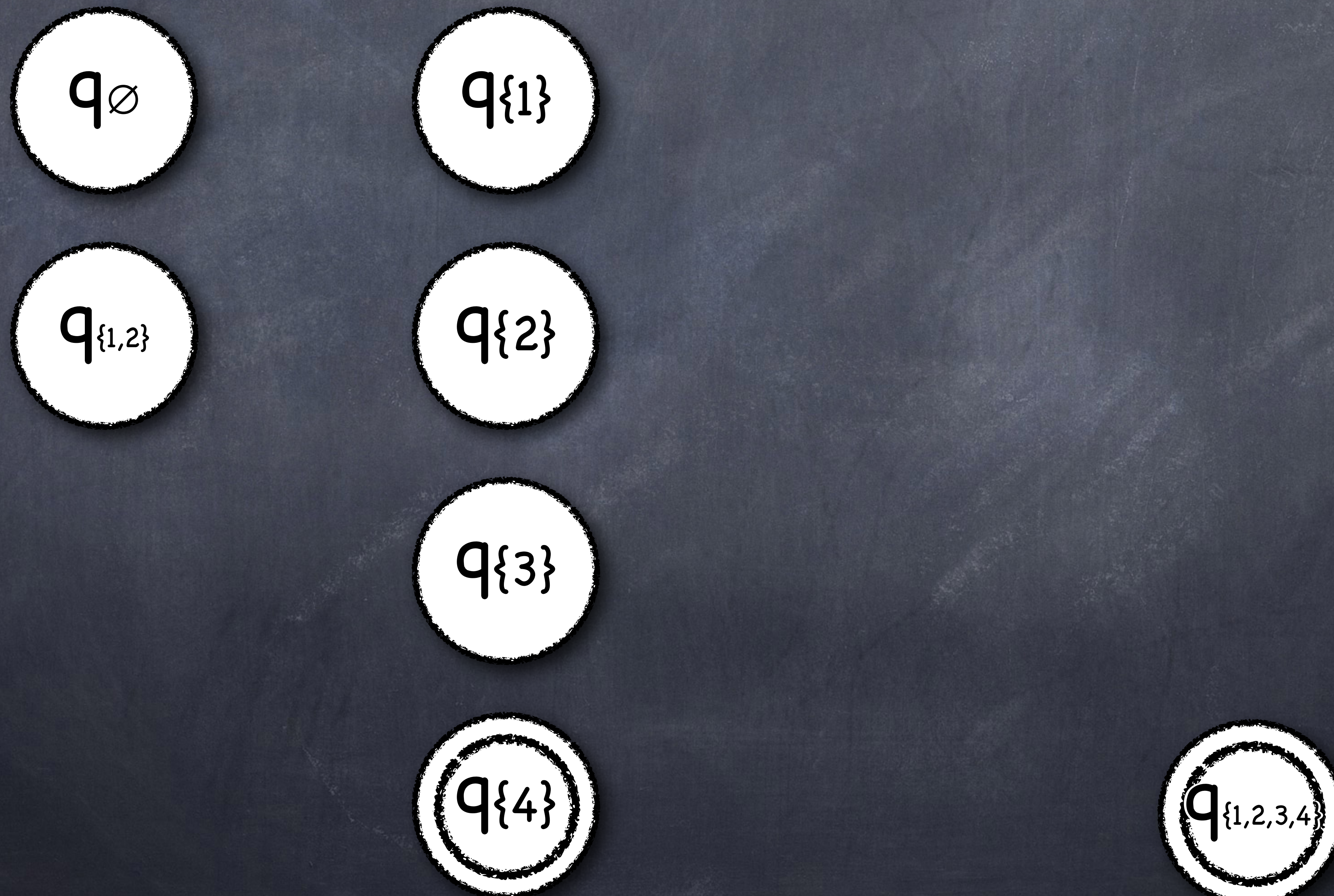

N1
0,1
1
0,ε
1
0,1
q1
q2
q3
q4

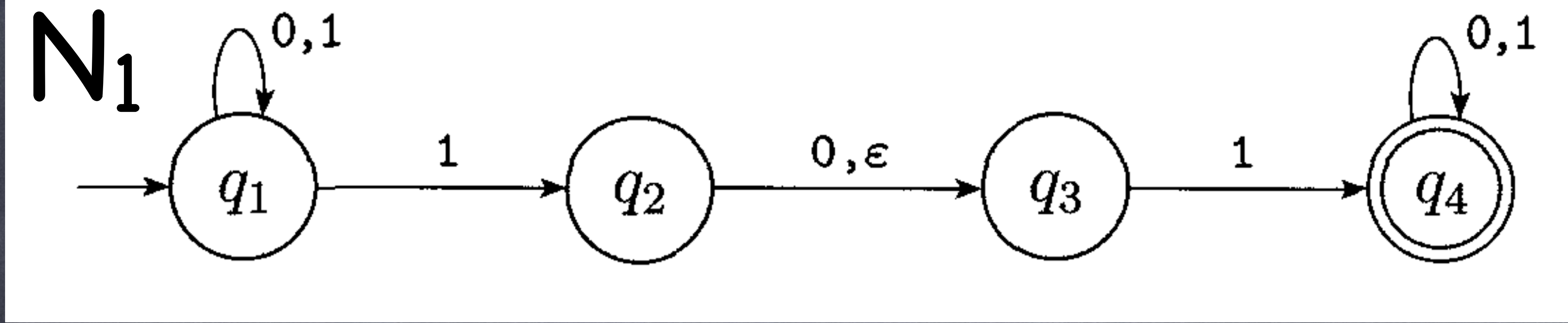

q∅
q{1}
q{2}
q{3}
q{1,2,3,4}

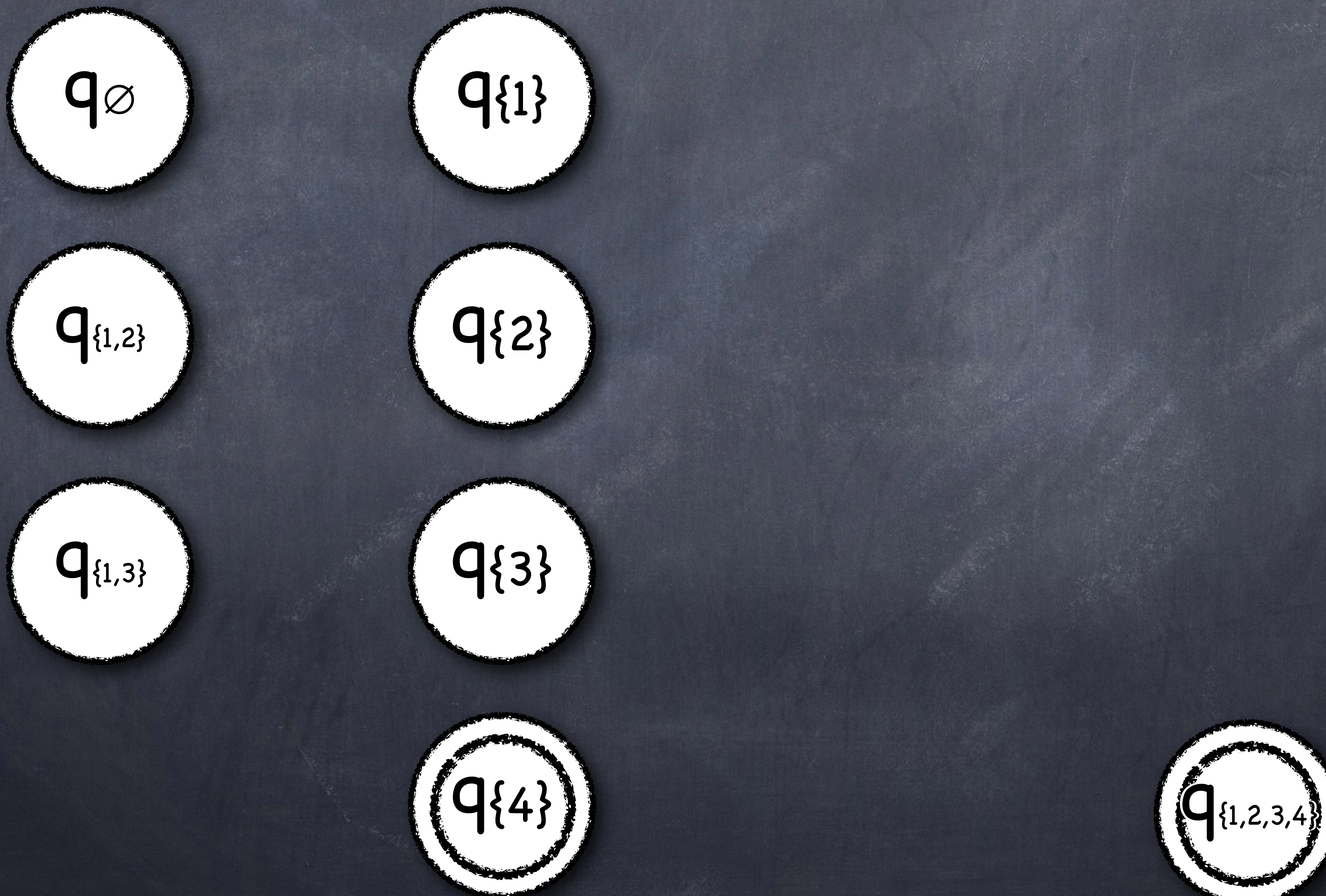

N1
0,1
1
0,ε
1
0,1
q1
q2
q3
q4

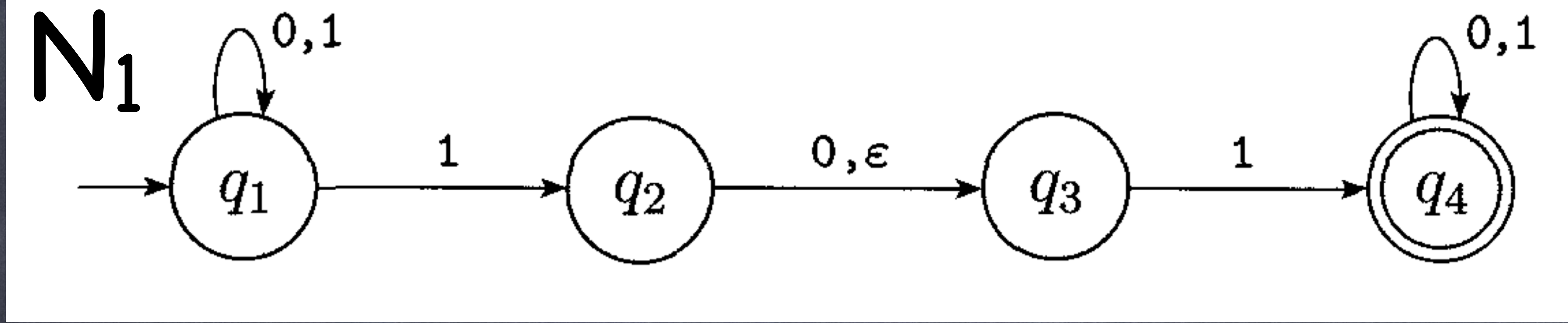

q∅
q{1}
q{2}
q{3}
q{4}
q{1,2,3,4}

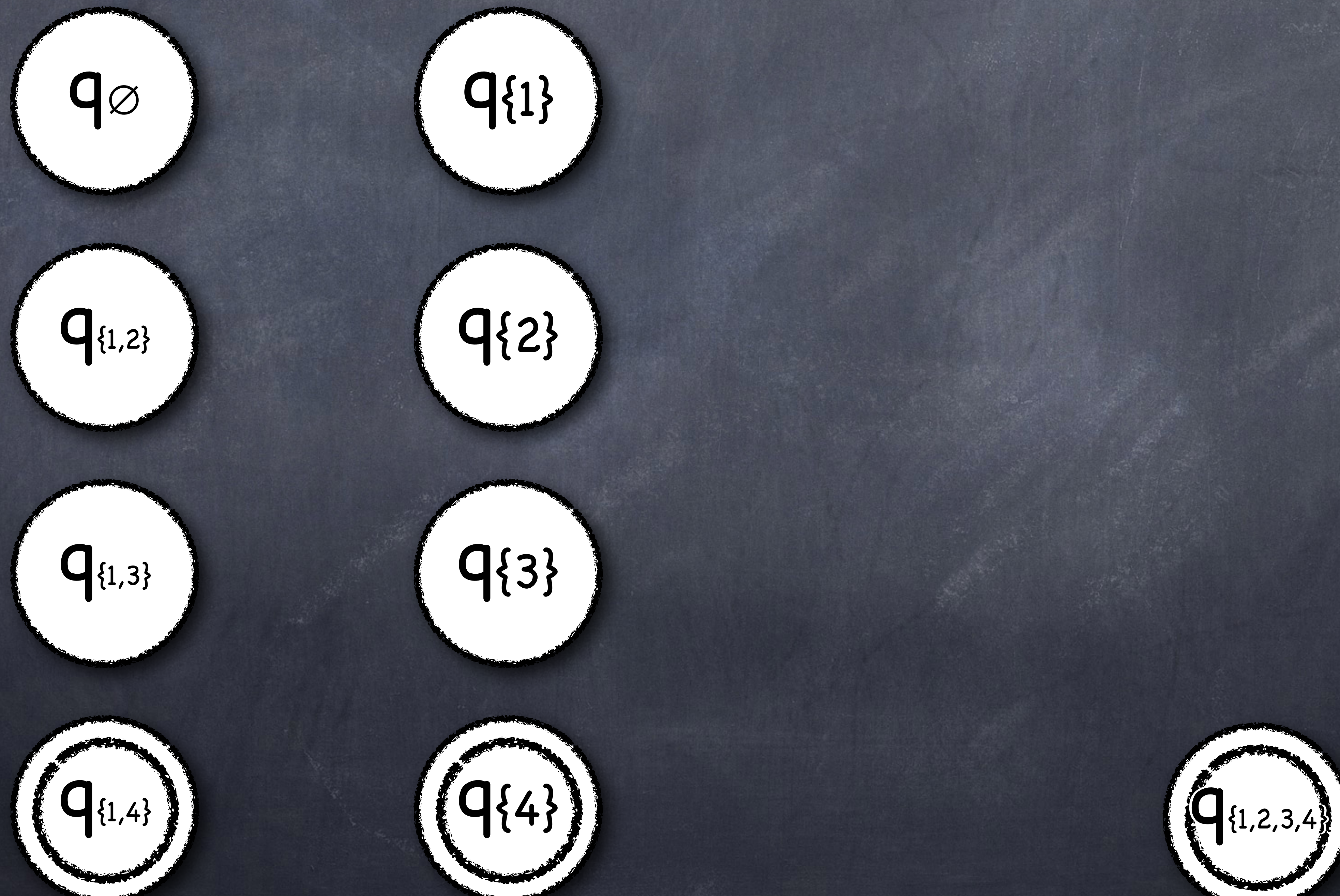

N1
0,1
q1
1
q2
0,ε
q3
1
q4
0,1

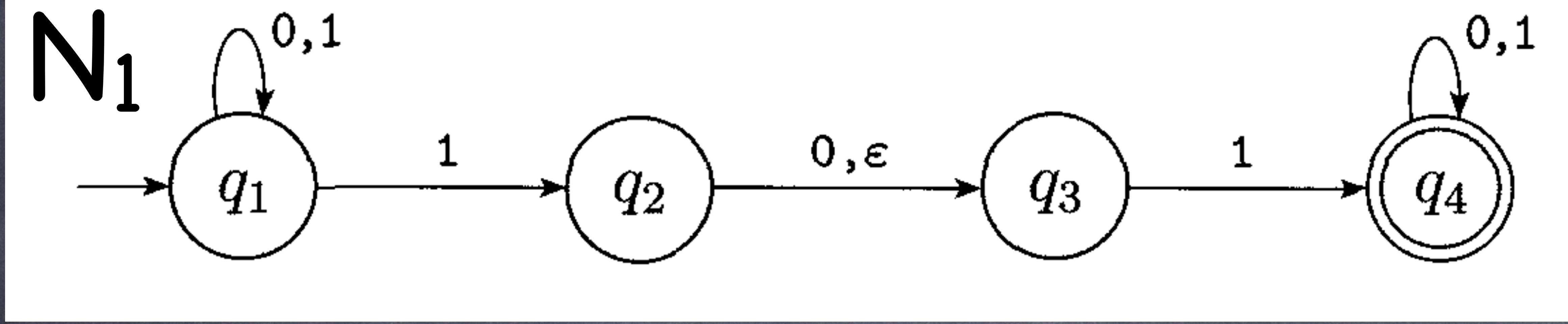

q∅
q{1}
q{1,2}
q{2}
q{3}
q{4}
q{1,2,3,4}

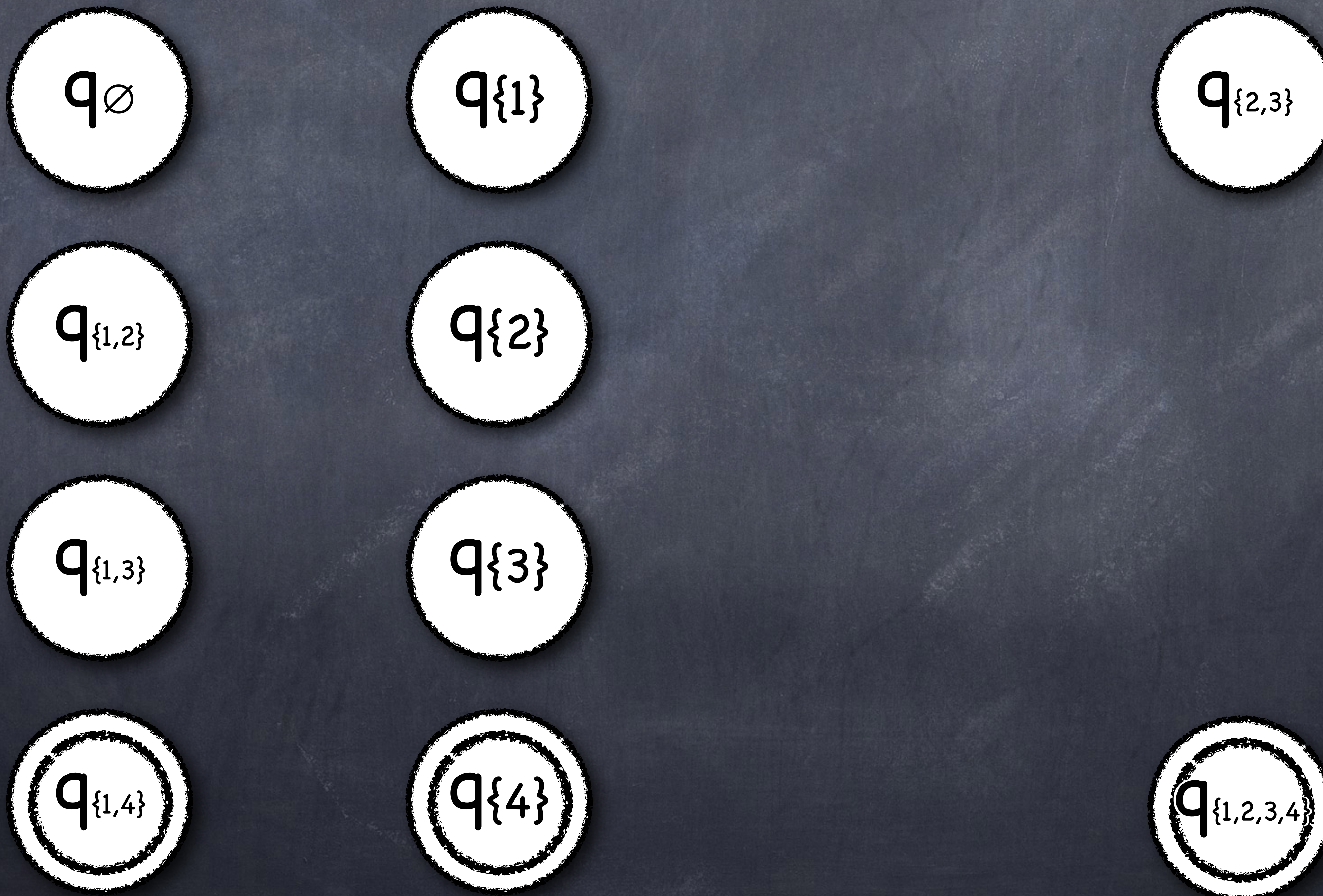

N1
0,1
1
0,ε
1
0,1
q1
q2
q3
q4

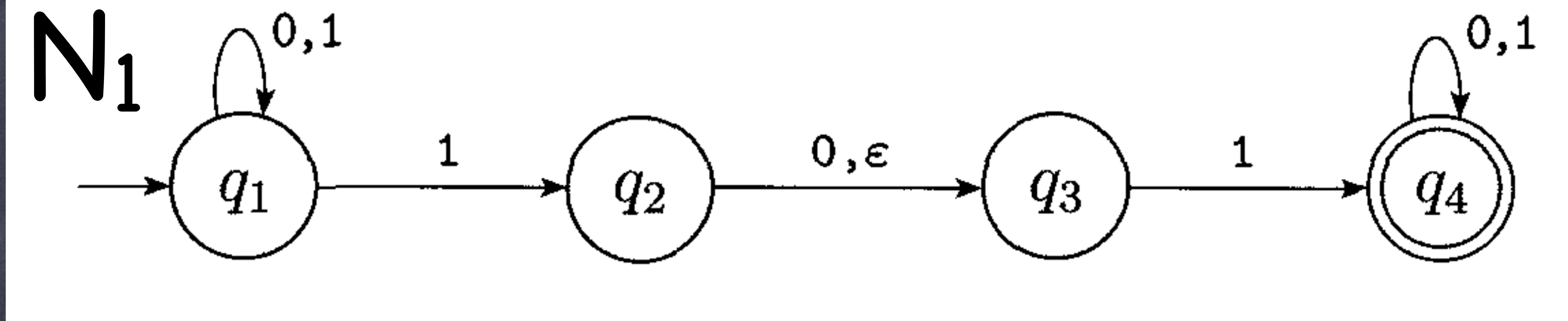

q∅
q{1}
q{1,2}
q{2}
q{1,3}
q{3}
q{4}
q{1,2,3,4}


N_1
0,1
1
0,ε
1
0,1
q_1
q_2
q_3
q_4

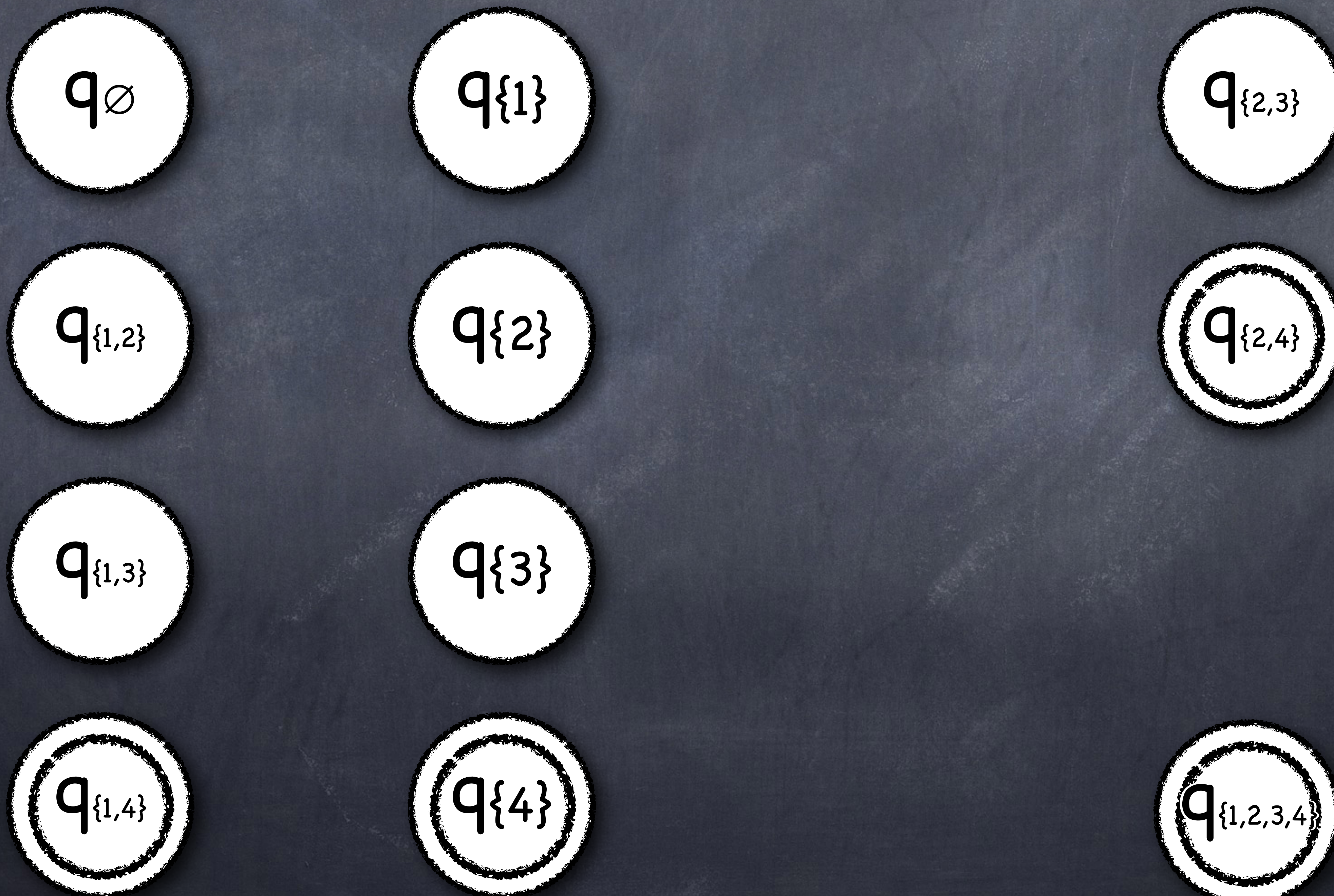

$q_\varnothing$
$q_{\{1\}}$
$q_{\{1,2\}}$
$q_{\{2\}}$
$q_{\{1,3\}}$
$q_{\{3\}}$
$q_{\{1,4\}}$
$q_{\{4\}}$
$q_{\{1,2,3,4\}}$


N1
0,1
1
0,ε
1
0,1
q1
q2
q3
q4

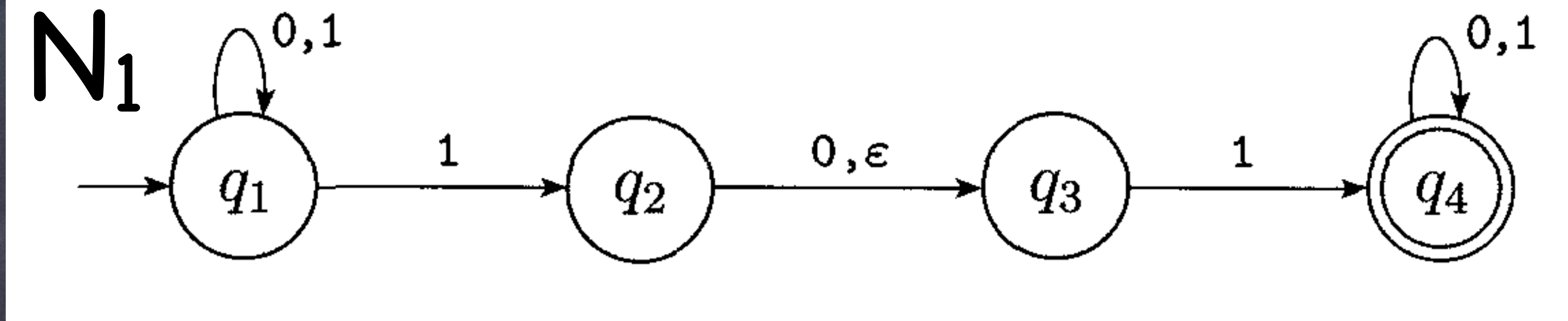

q∅
q{1}
q{2,3}
q{1,2}
q{2}
q{1,3}
q{3}
q{1,4}
q{4}
q{1,2,3,4}

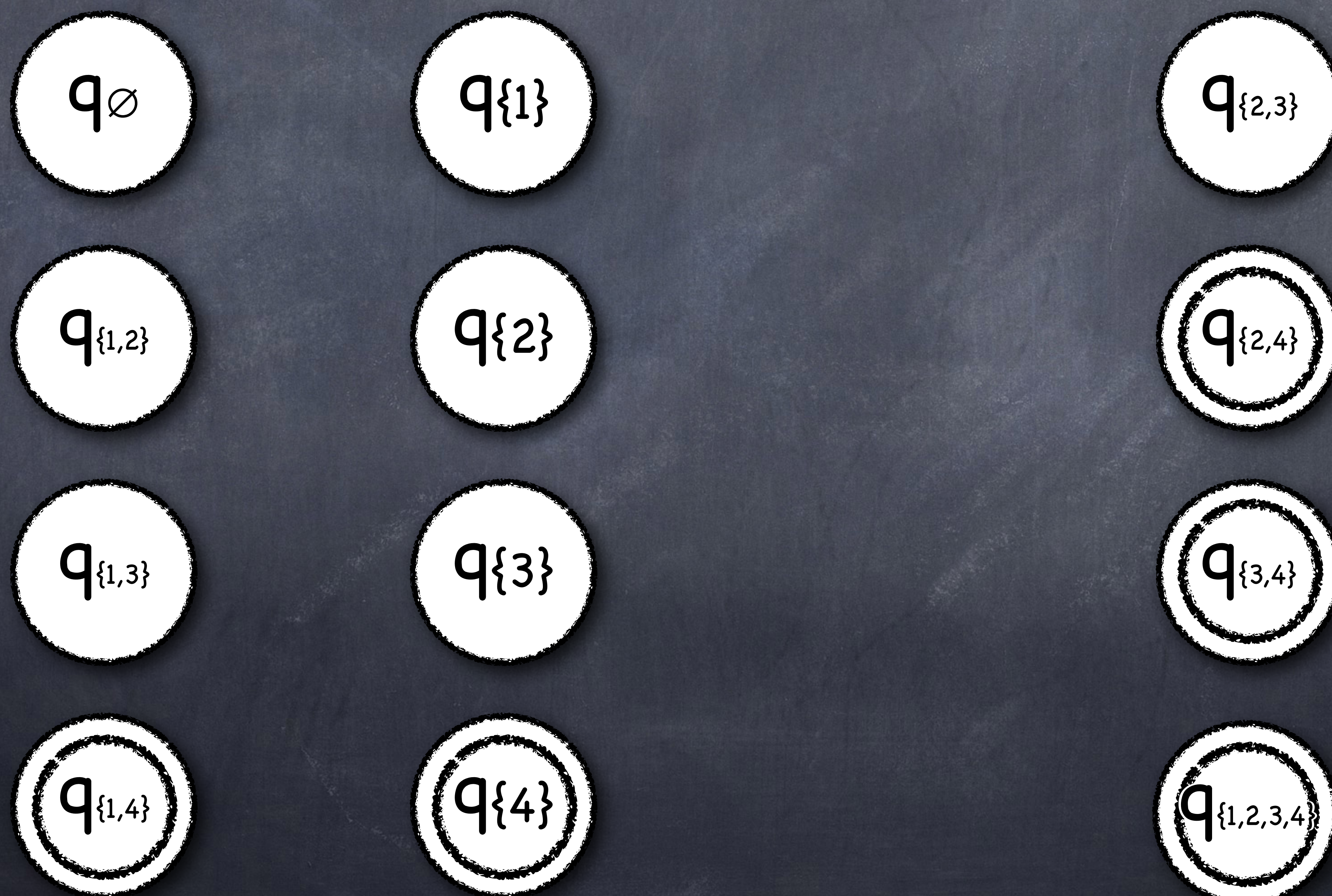


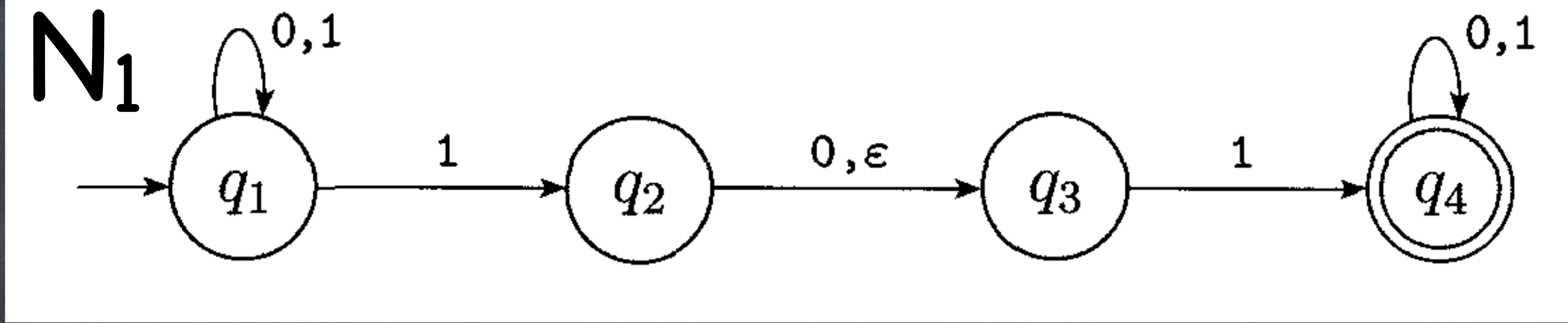
N1
0,1
1
0,ε
1
0,1
q1
q2
q3
q4

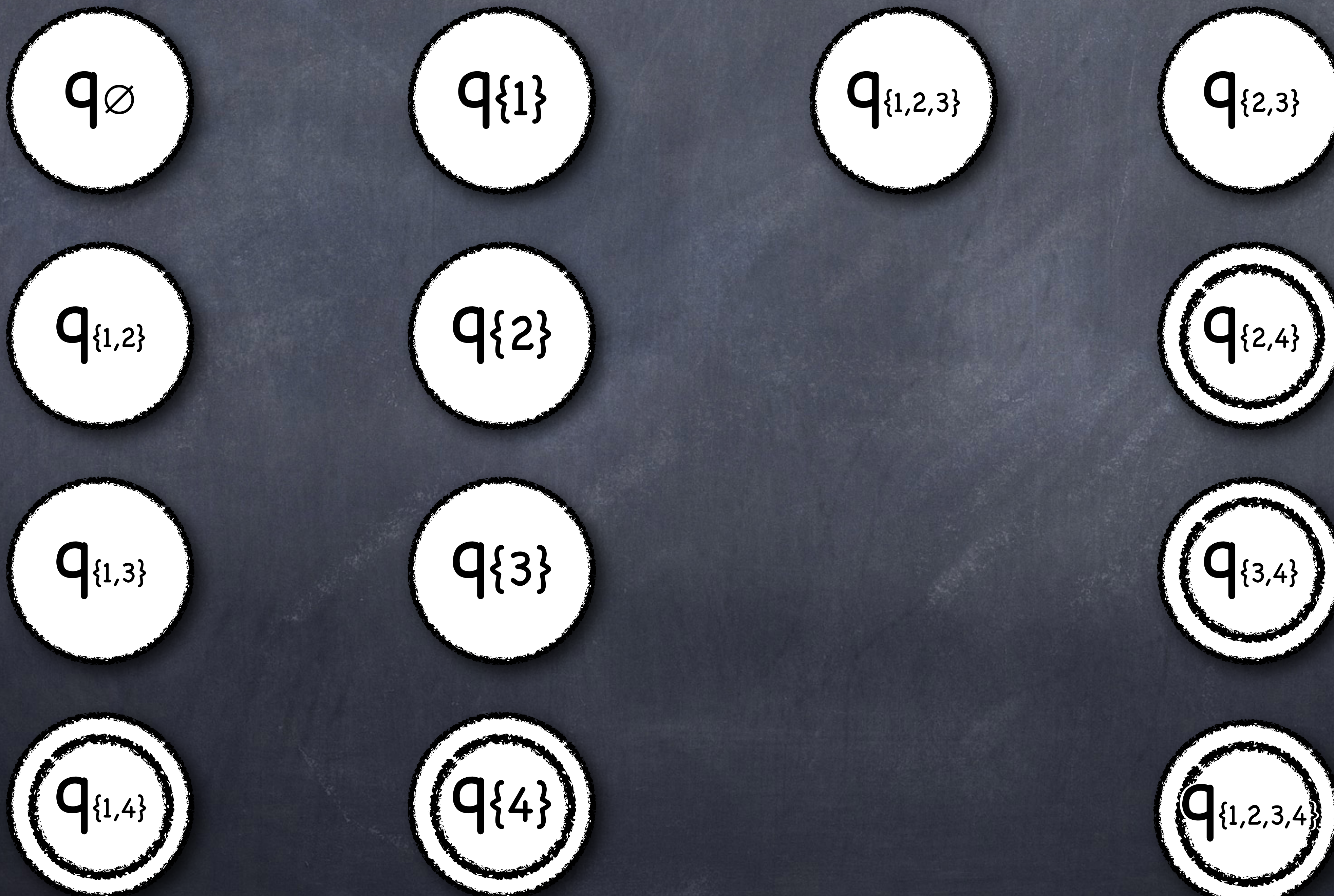


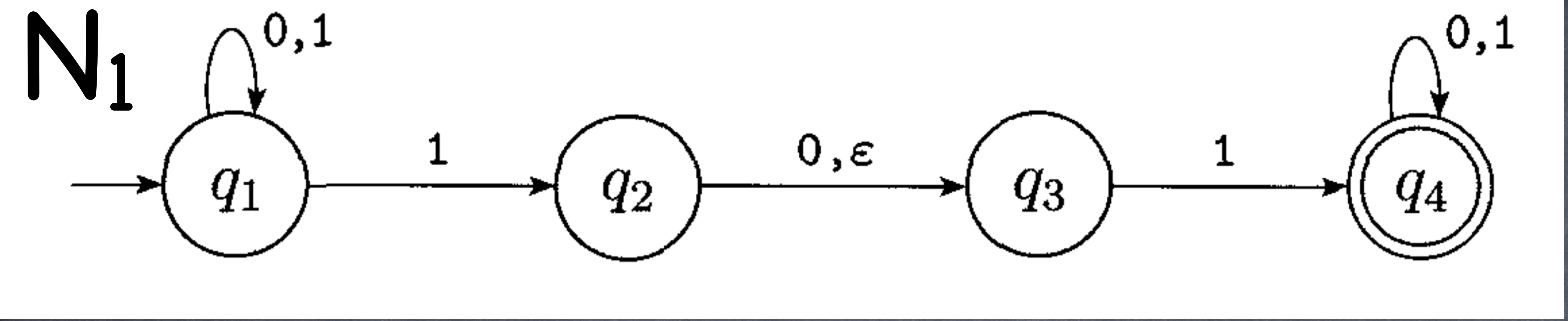
q∅
q{1}
q{2,3}
q{1,2}
q{2}
q{2,4}
q{1,3}
q{3}
q{1,4}
q{4}
q{1,2,3,4}

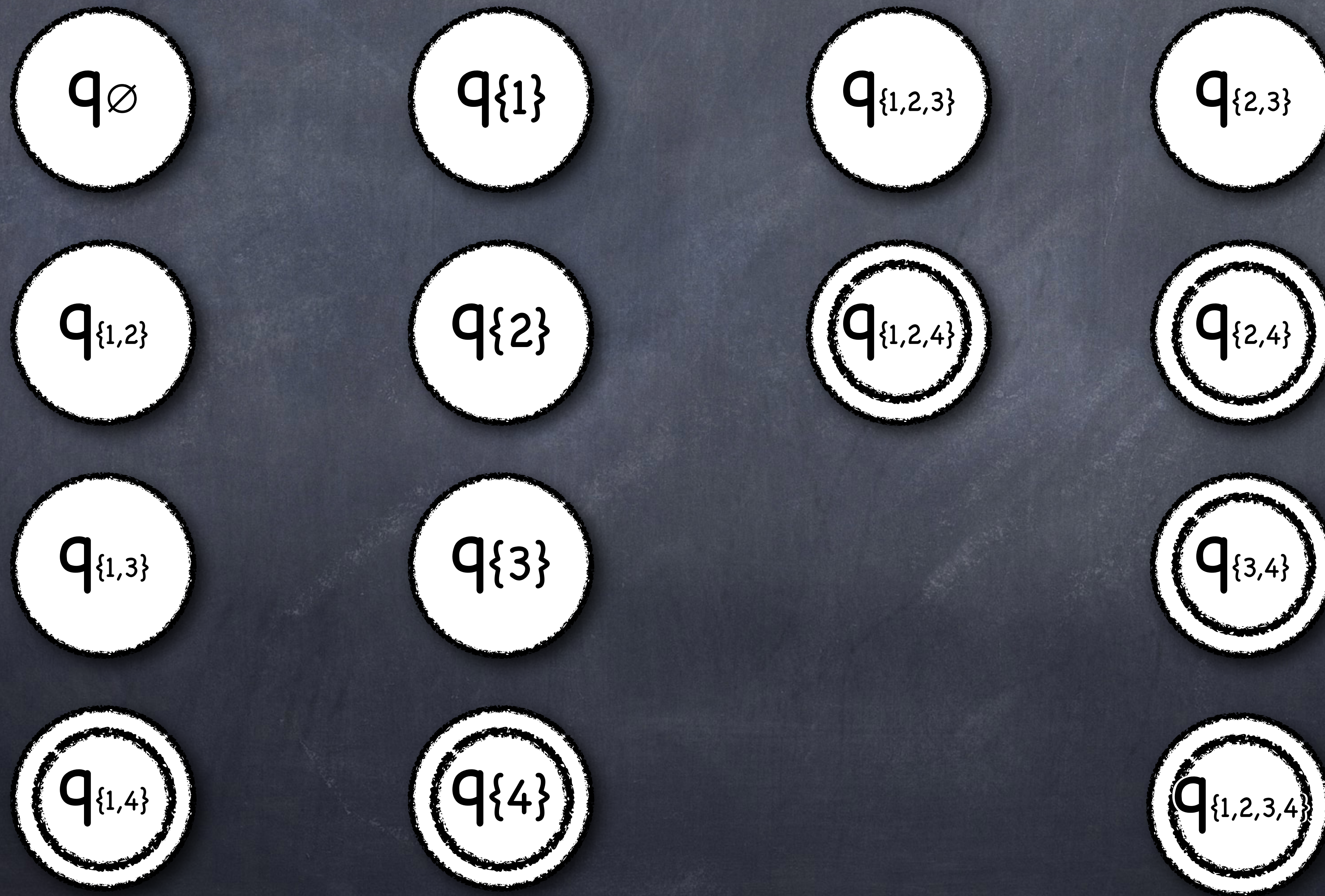

N1
0,1
1
0,ε
1
0,1
q1
q2
q3
q4


q∅
q{1}
q{2,3}
q{1,2}
q{2}
q{2,4}
q{1,3}
q{3}
q{3,4}
q{1,4}
q{4}
q{1,2,3,4}


N_1
0,1
1
0,ε
1
0,1
q_1
q_2
q_3
q_4


$q_\varnothing$
$q_{\{1\}}$
$q_{\{1,2,3\}}$
$q_{\{2,3\}}$
$q_{\{1,2\}}$
$q_{\{2\}}$
$q_{\{2,4\}}$
$q_{\{1,3\}}$
$q_{\{3\}}$
$q_{\{3,4\}}$
$q_{\{1,4\}}$
$q_{\{4\}}$
$q_{\{1,2,3,4\}}$


N1
0,1
1
0,ε
1
0,1
q1
q2
q3
q4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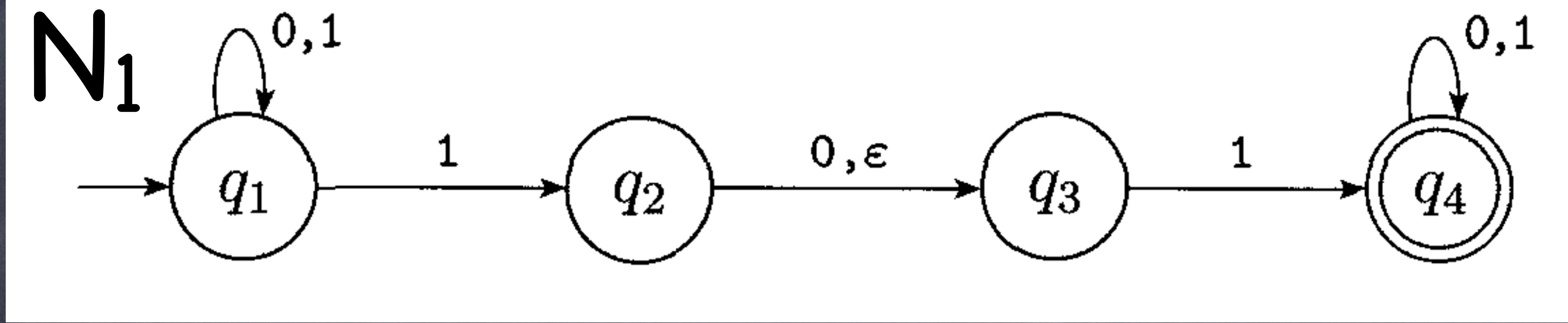
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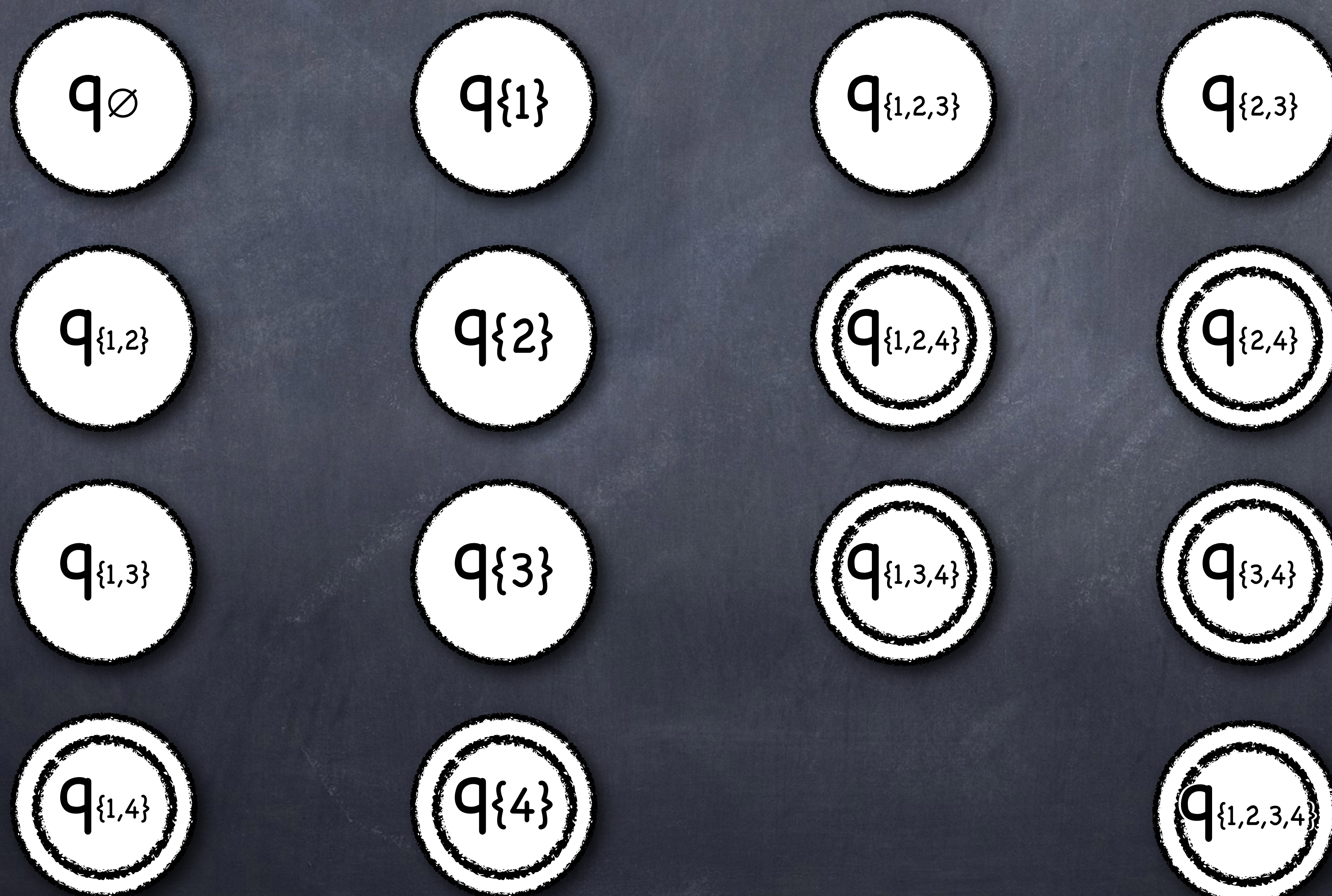
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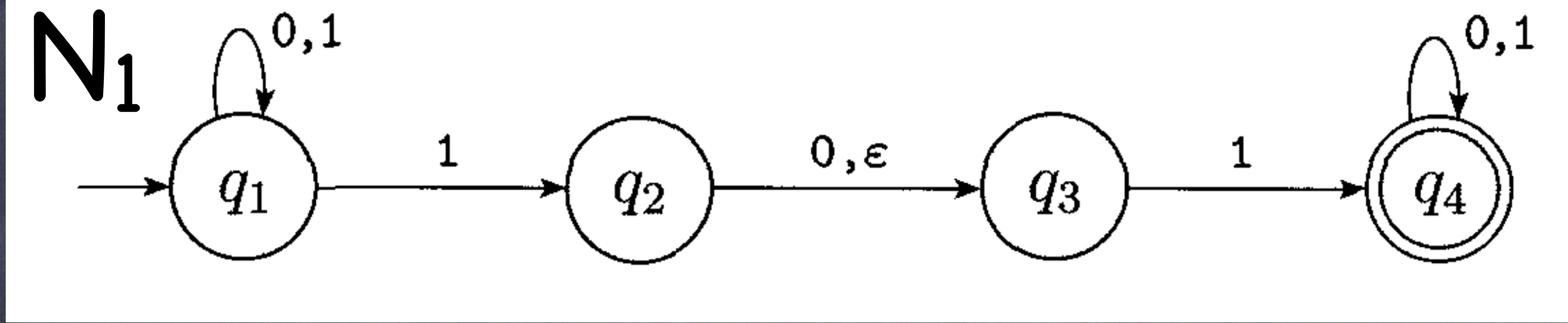

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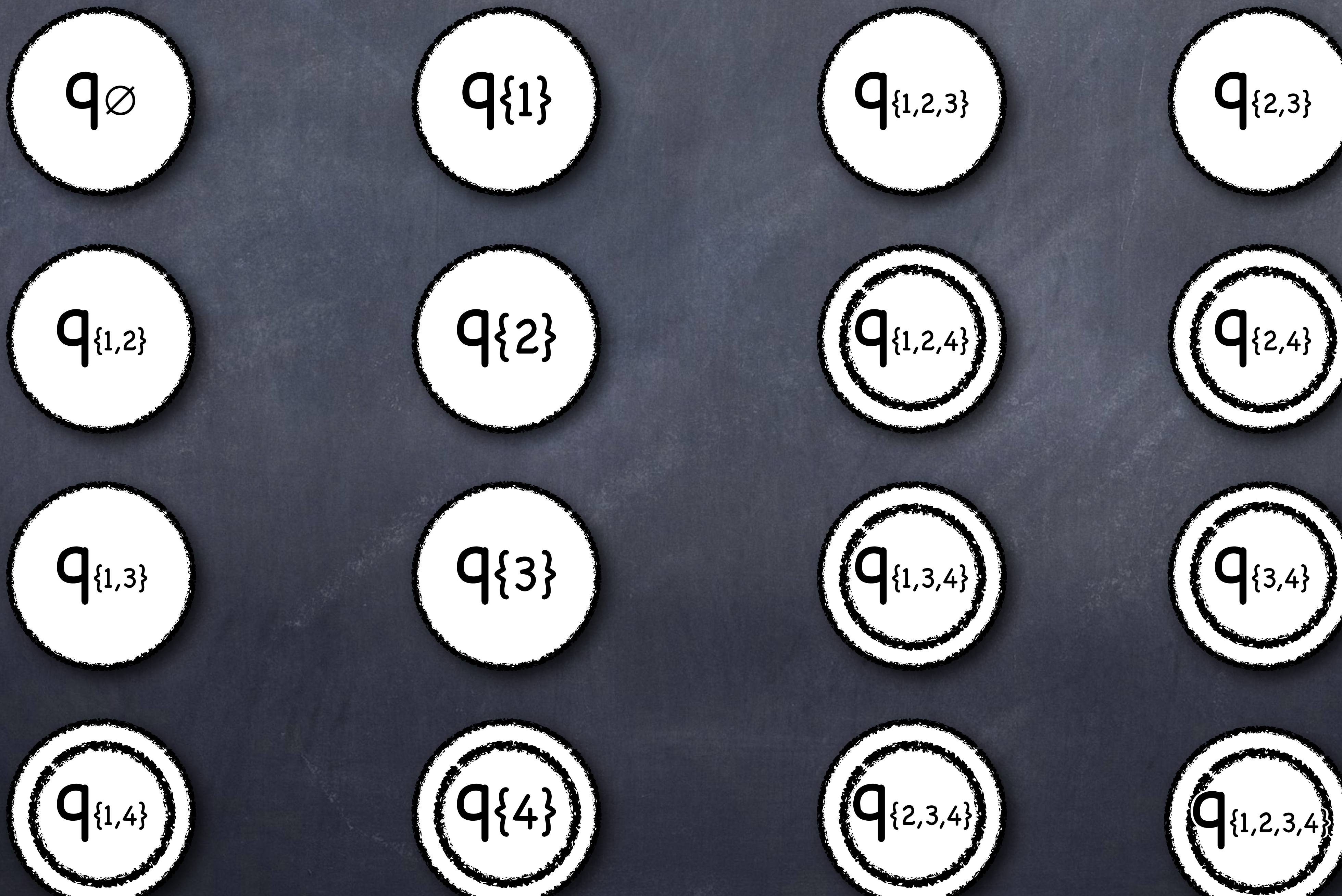

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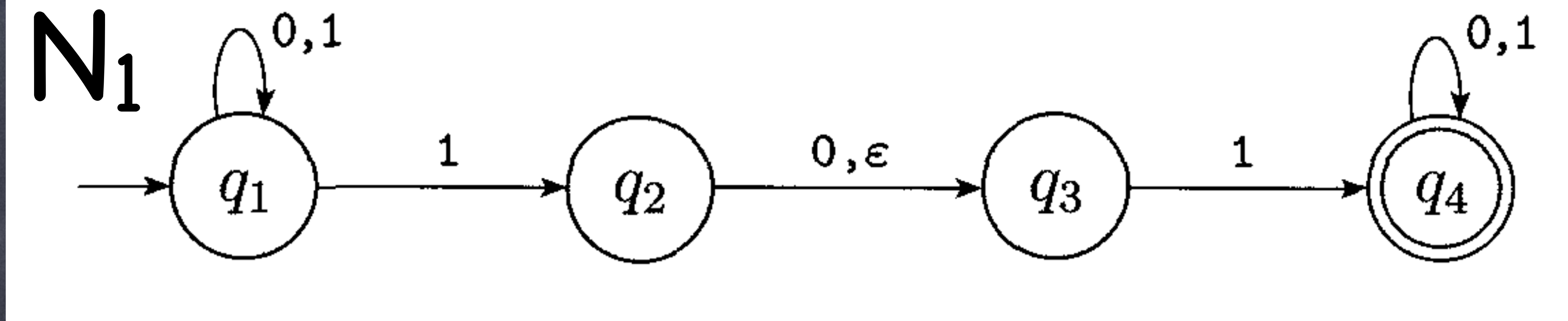
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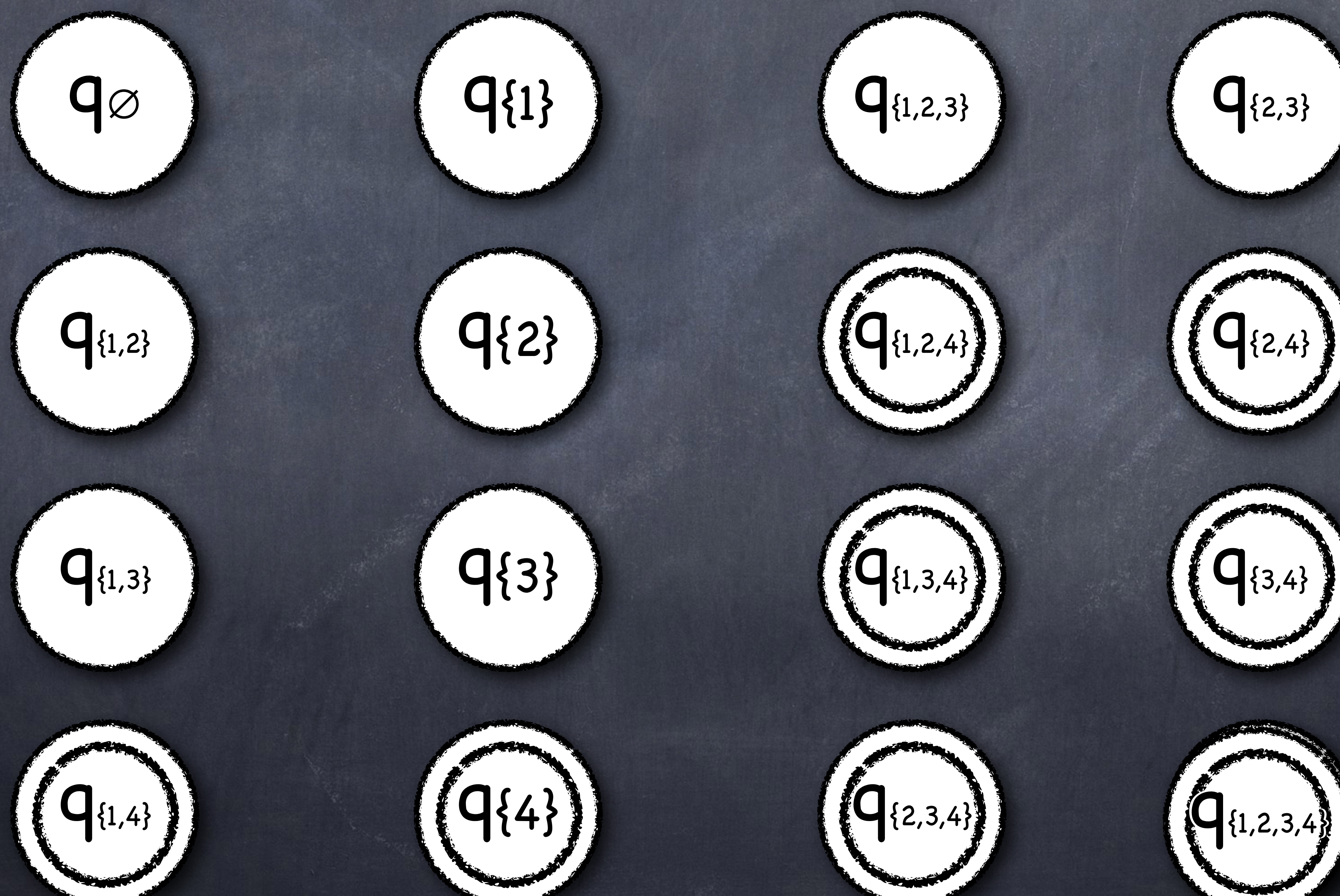
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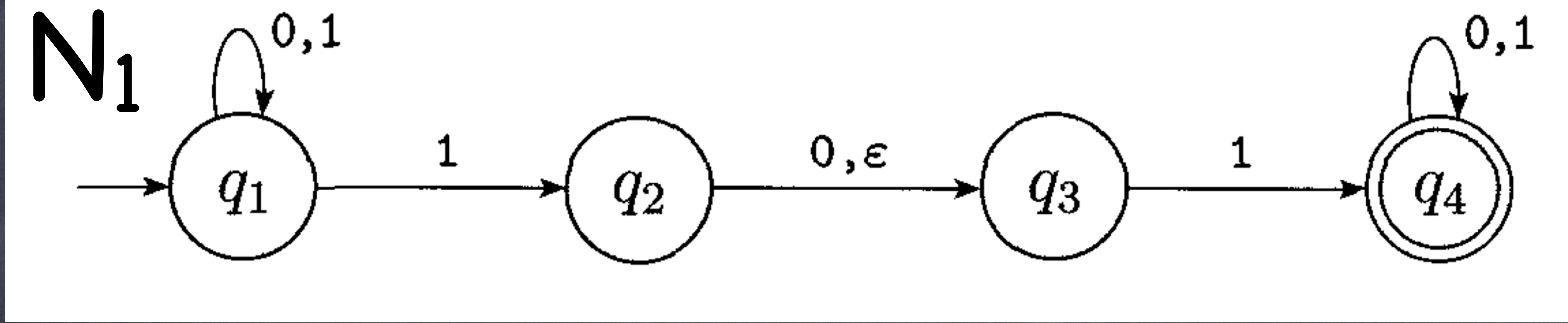
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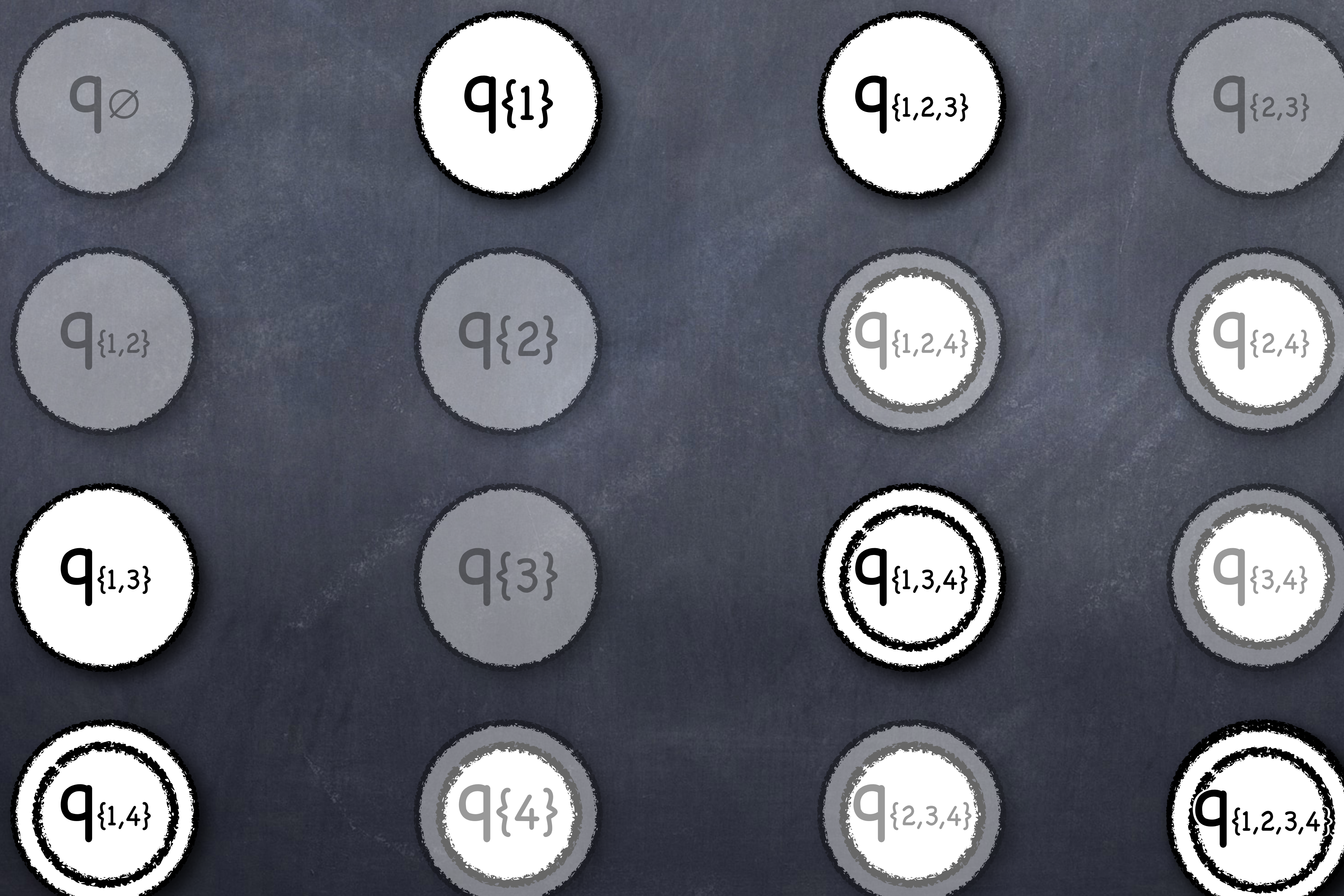
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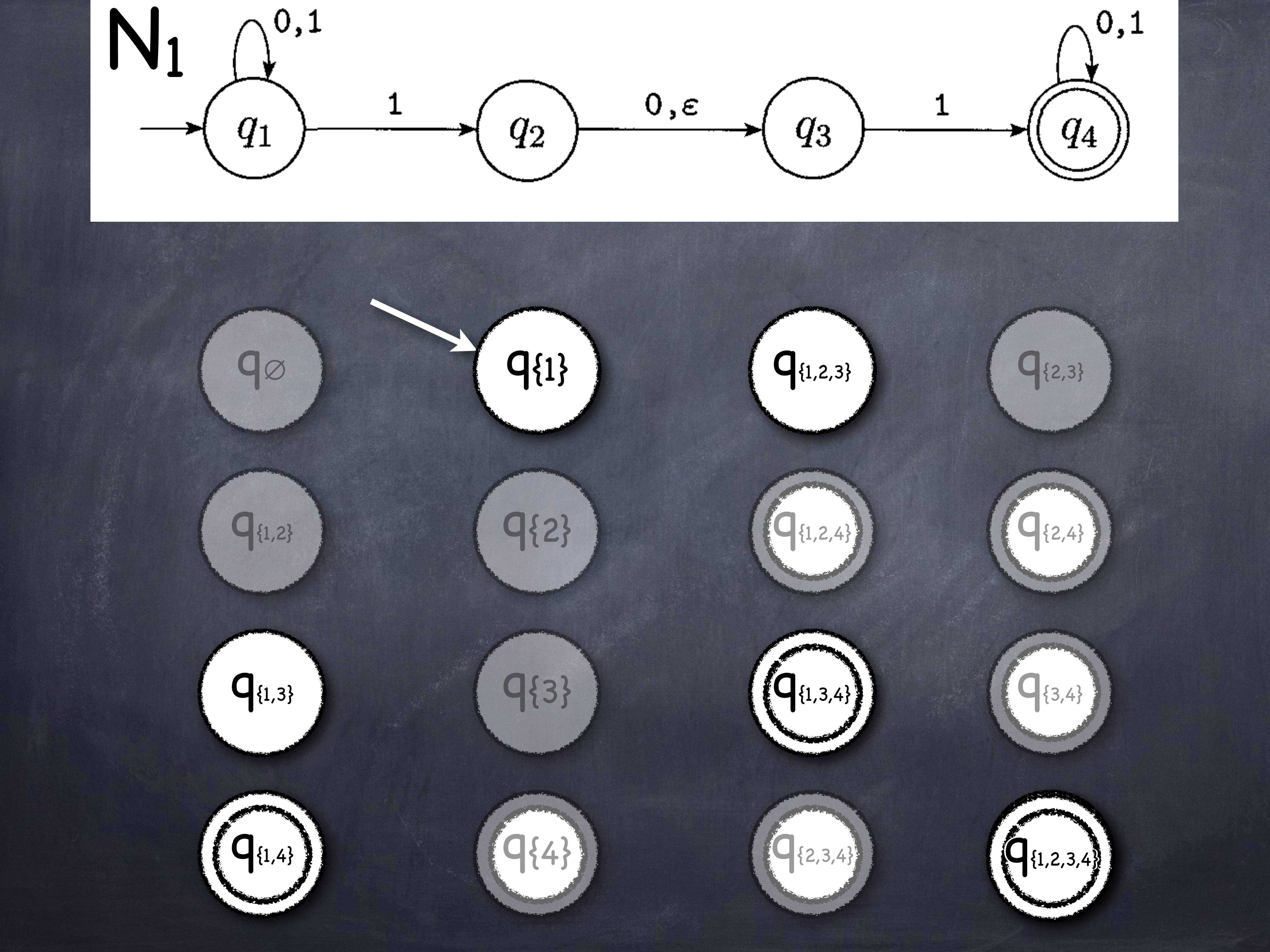

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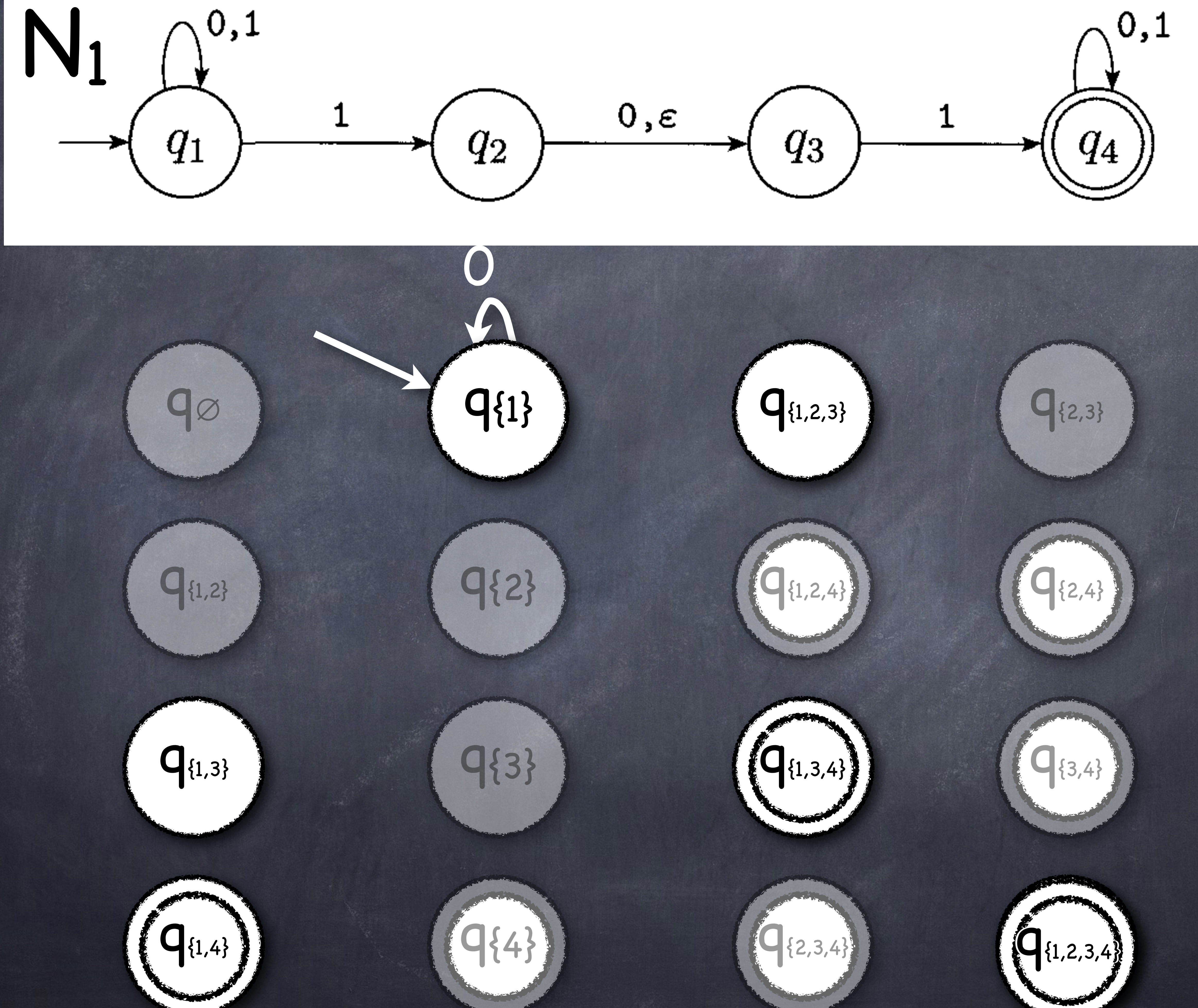
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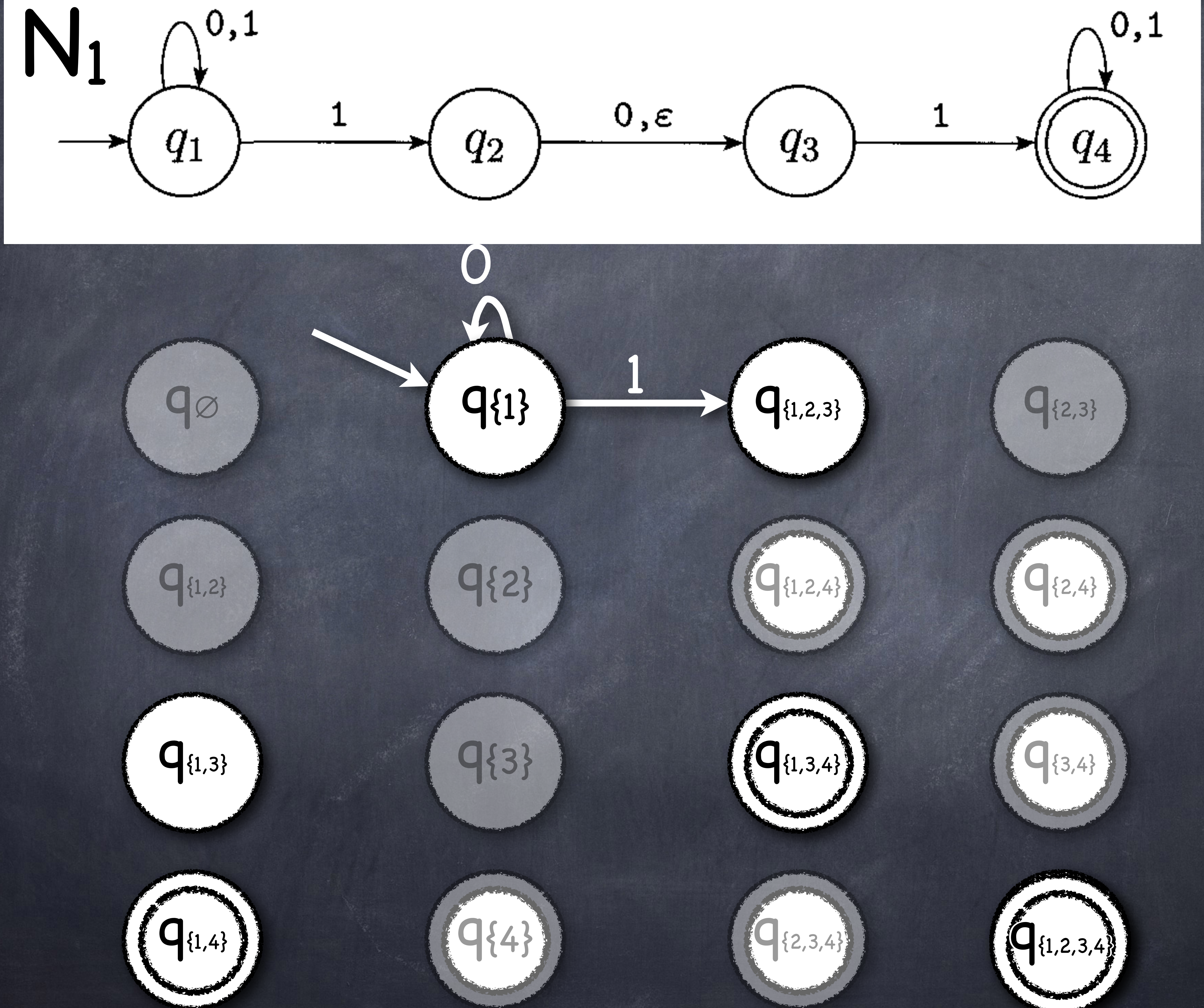
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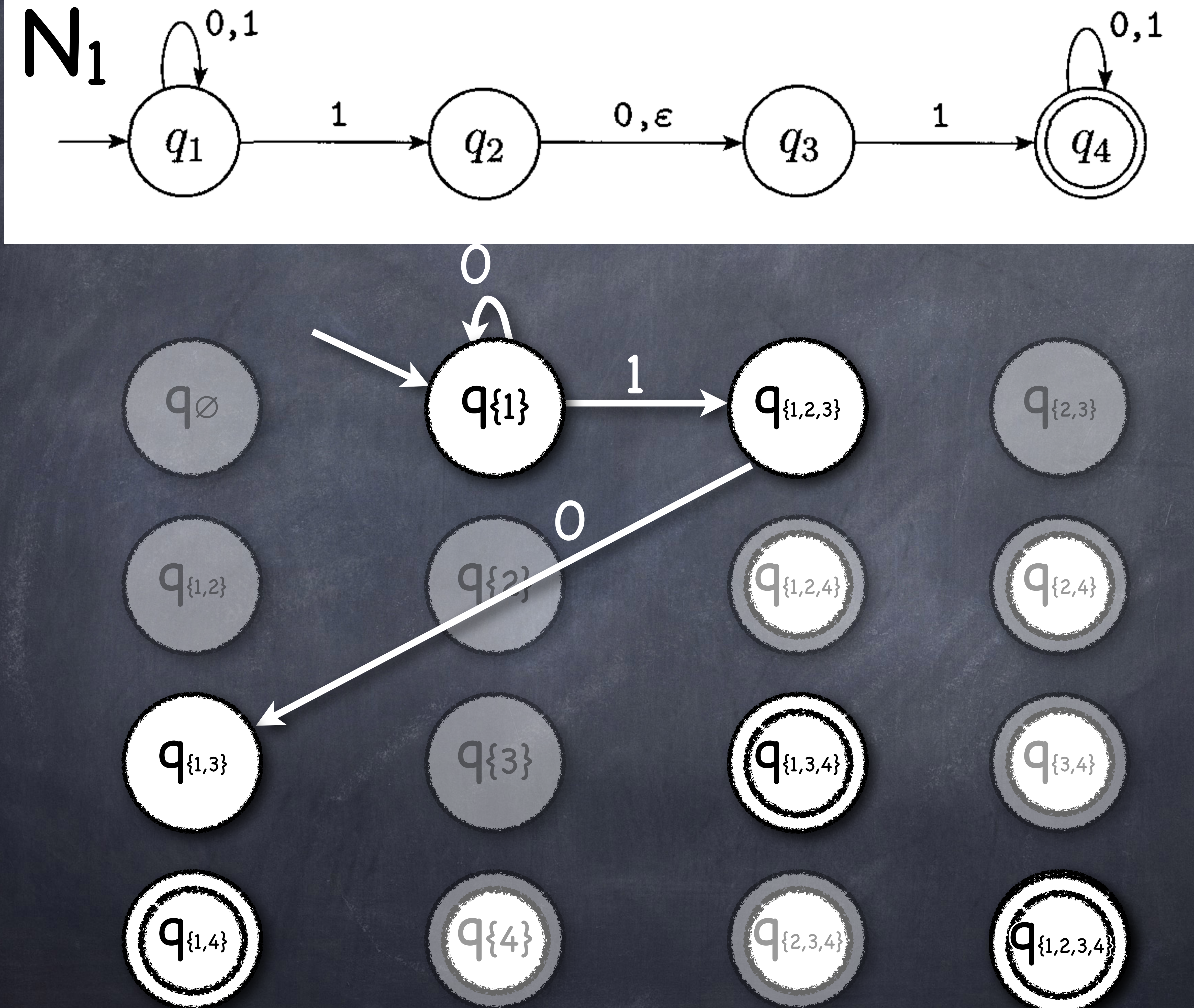
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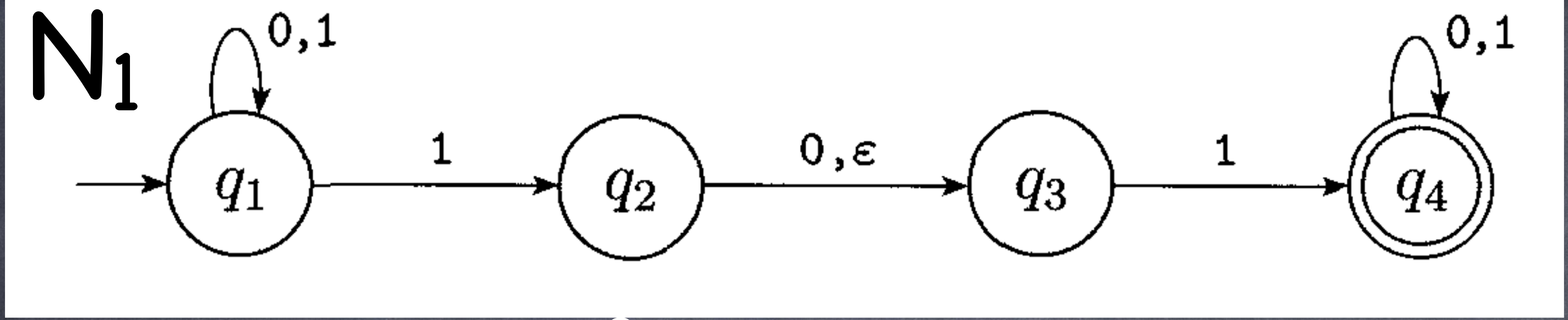
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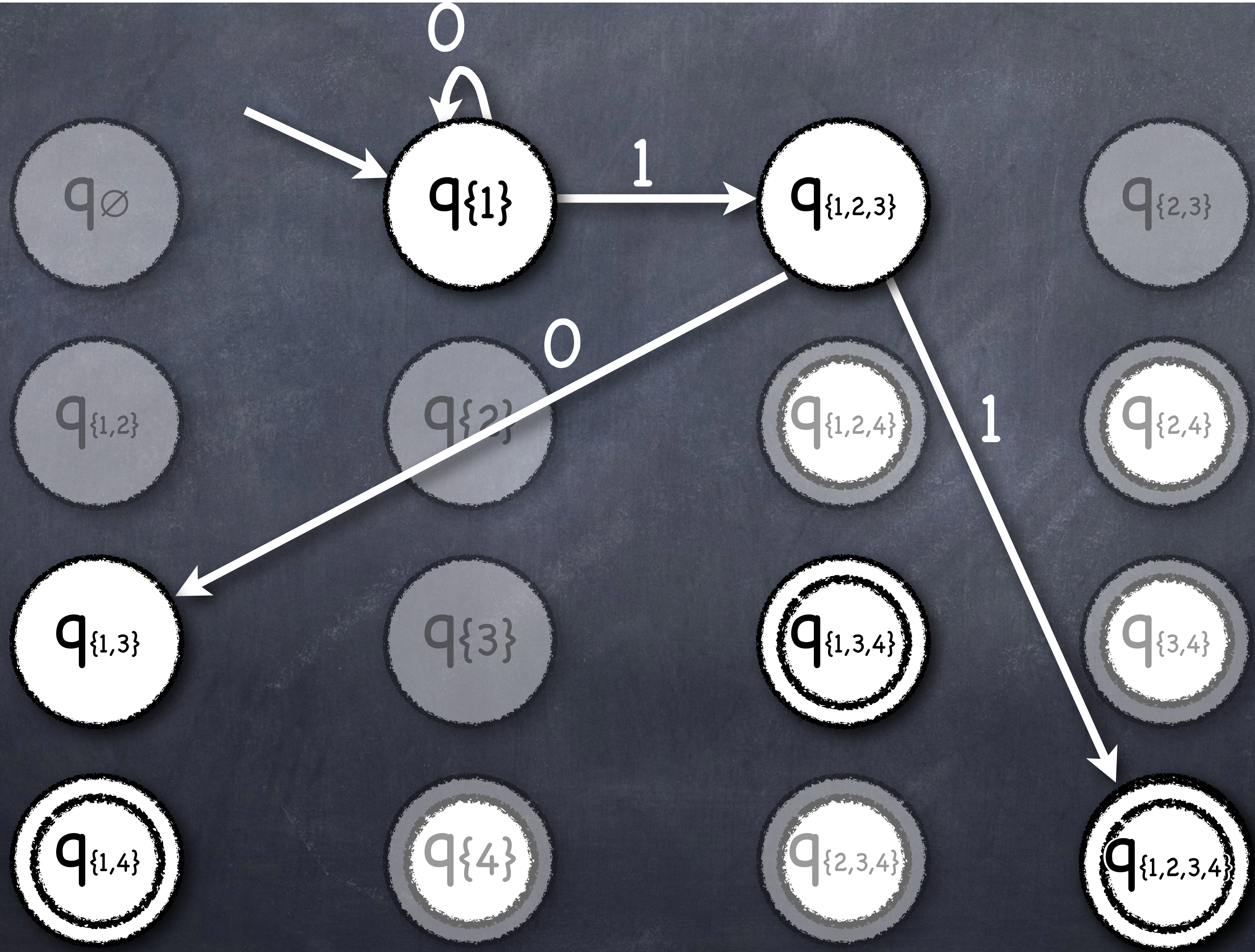
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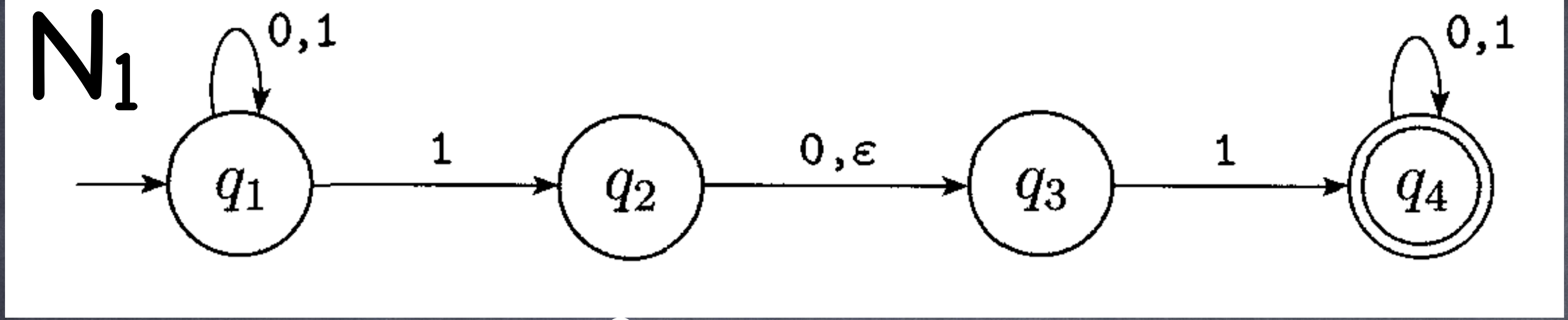
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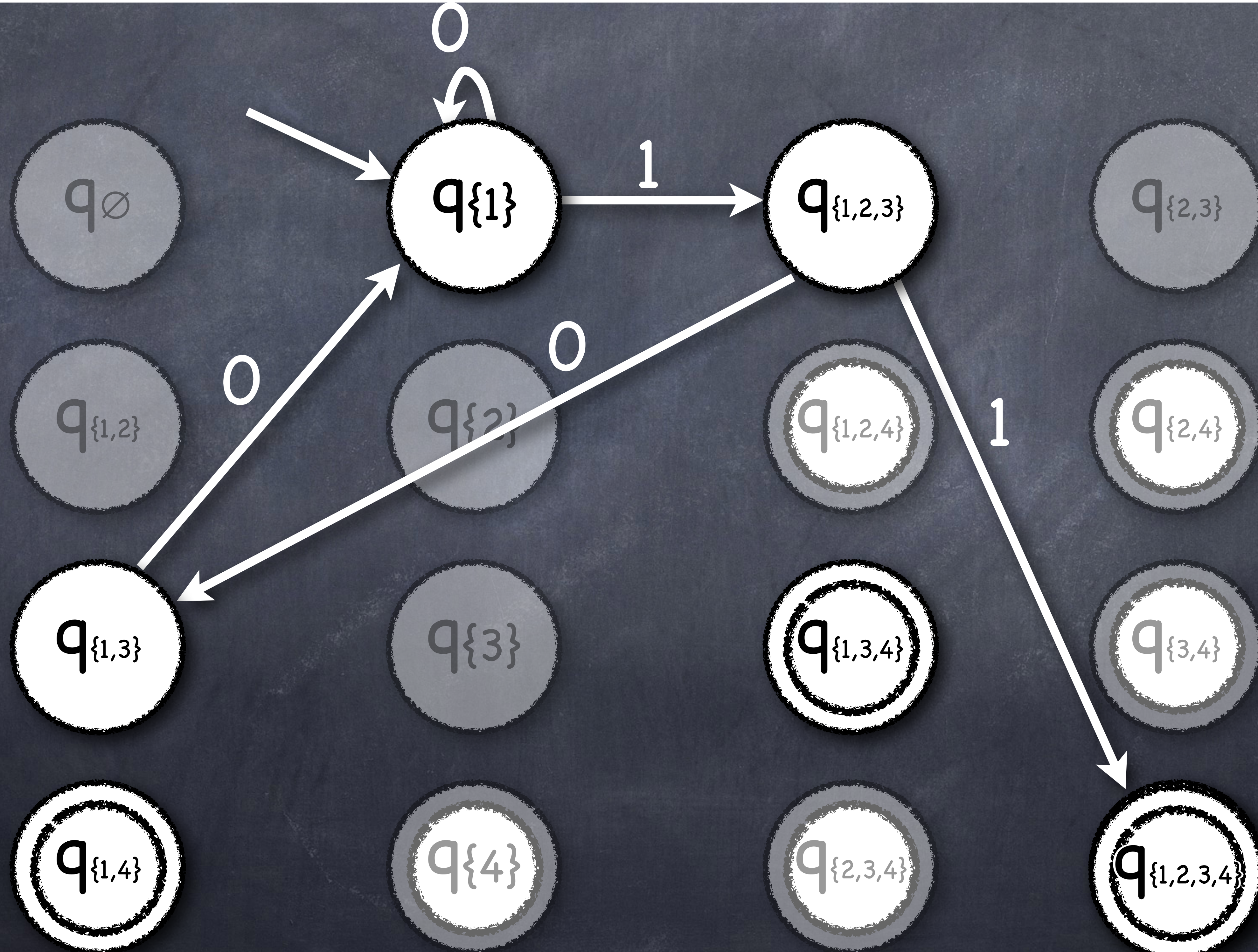
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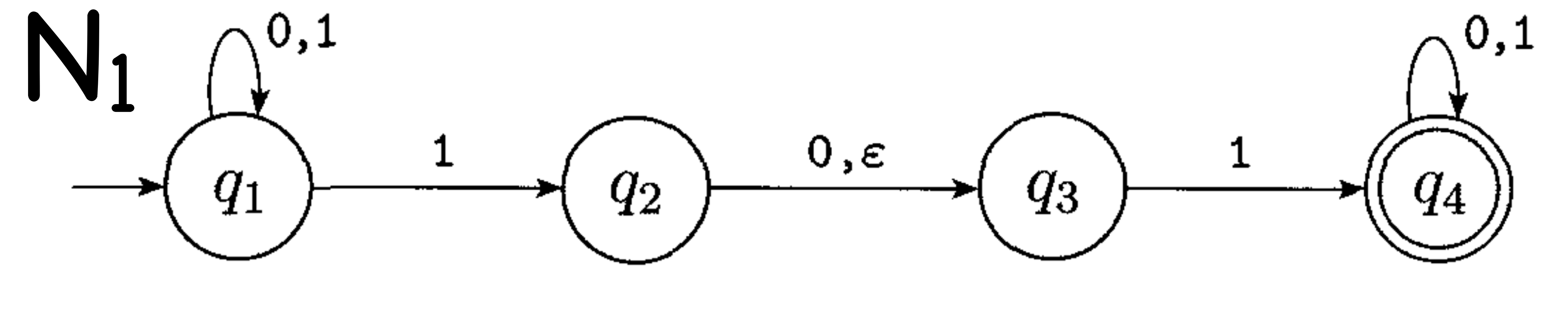
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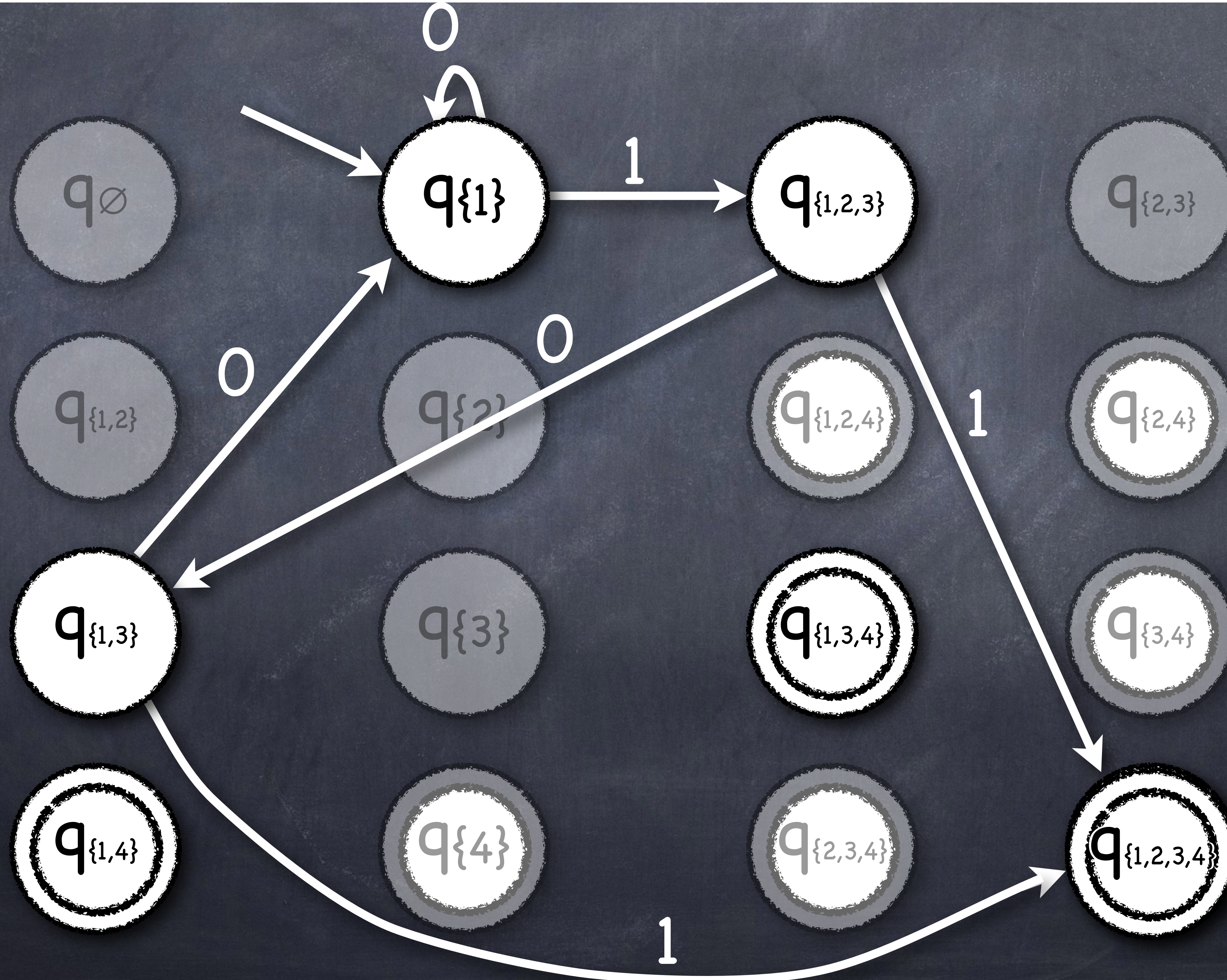
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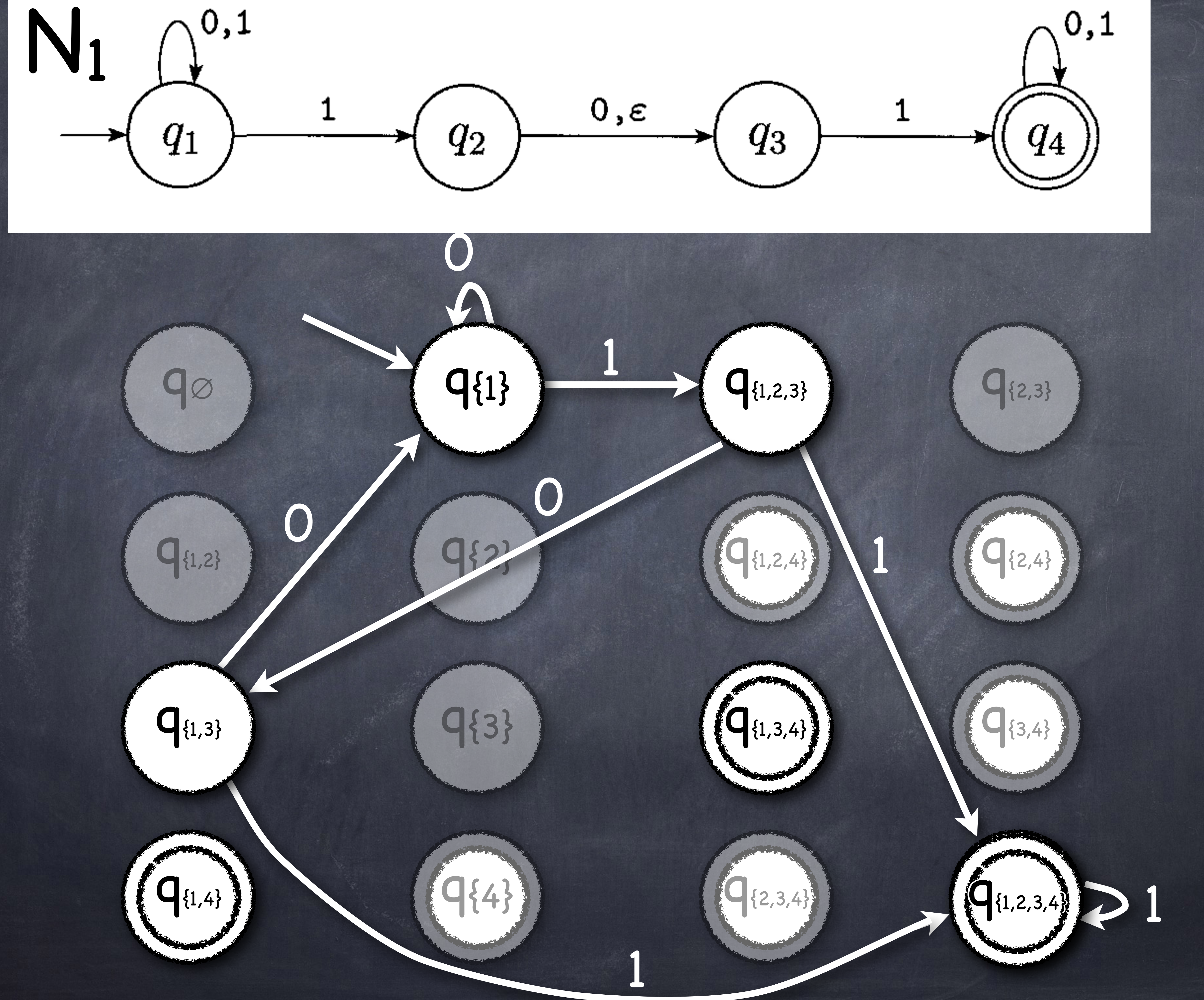

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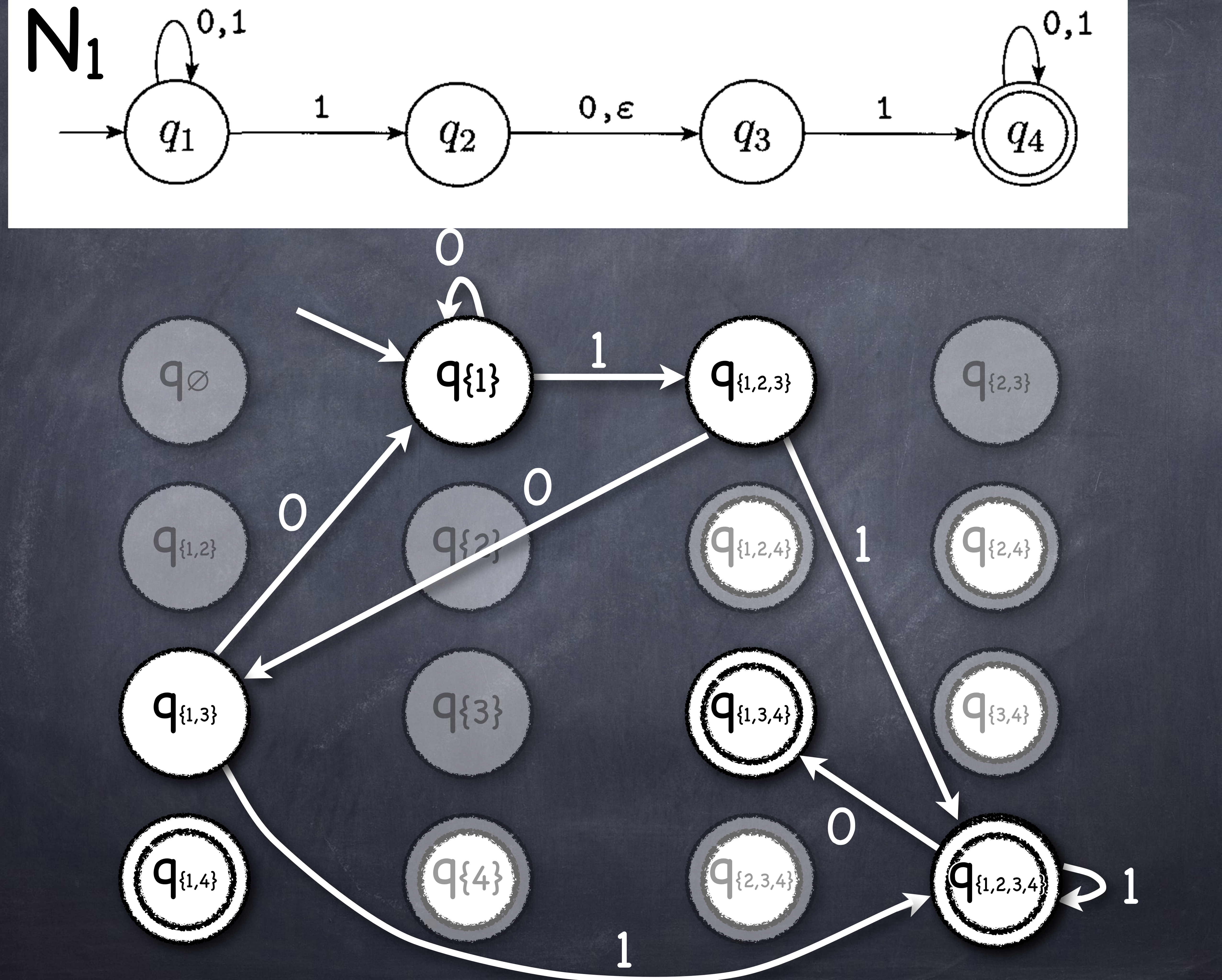

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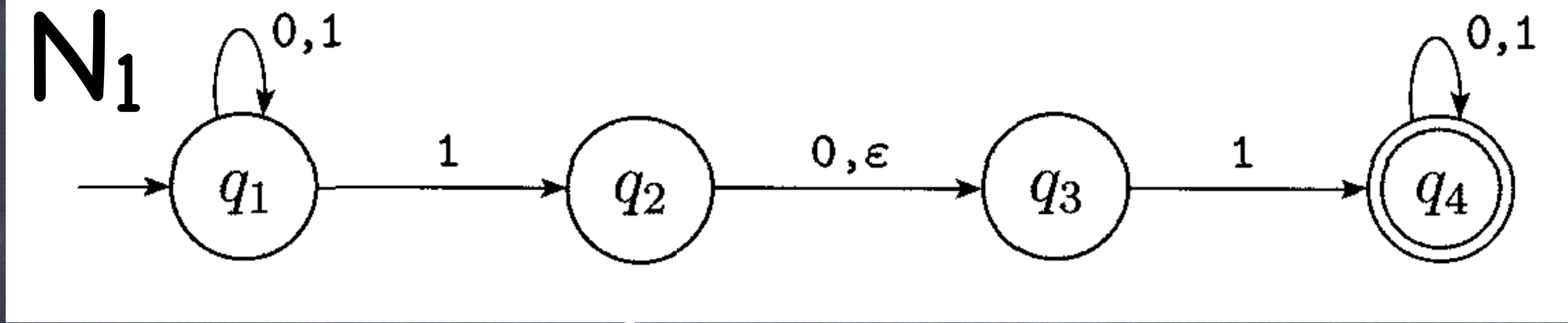

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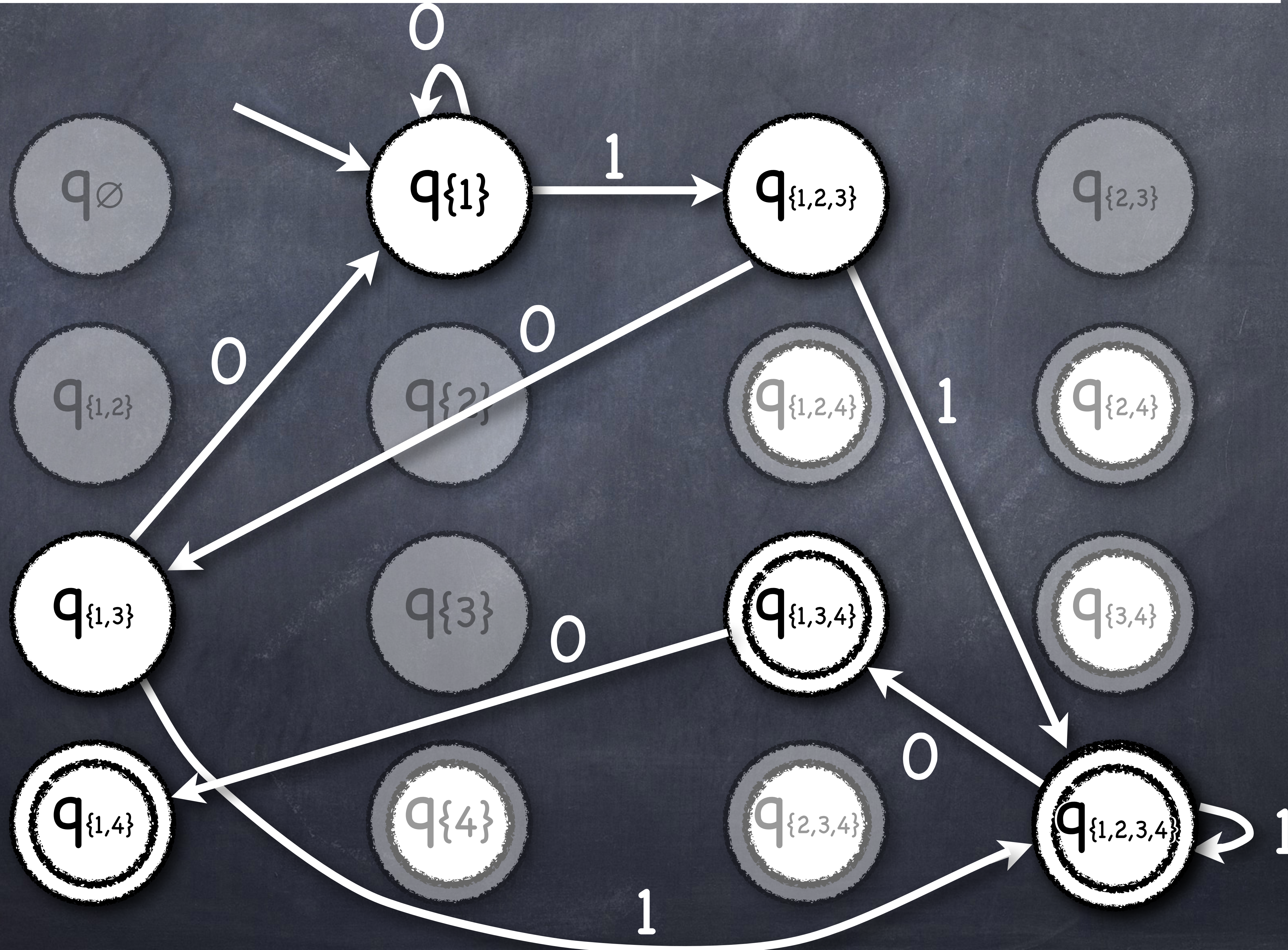

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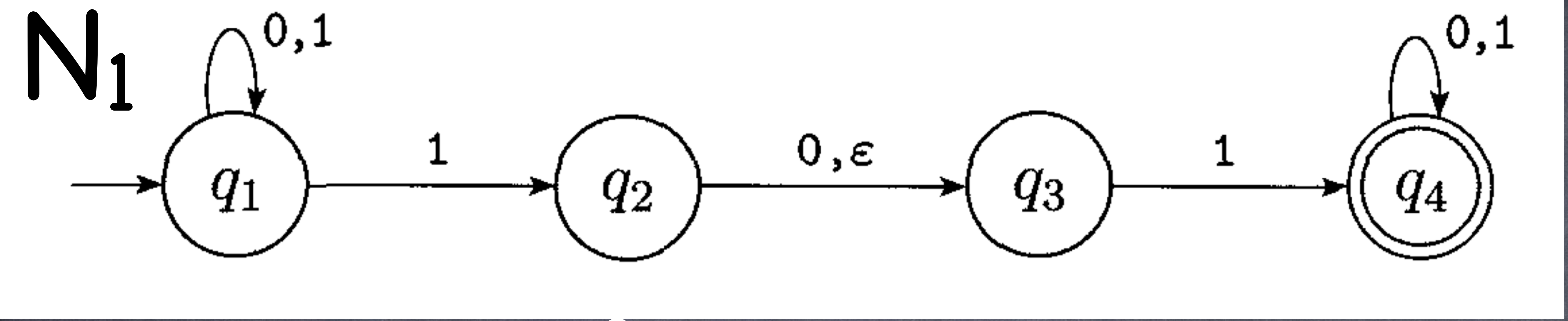
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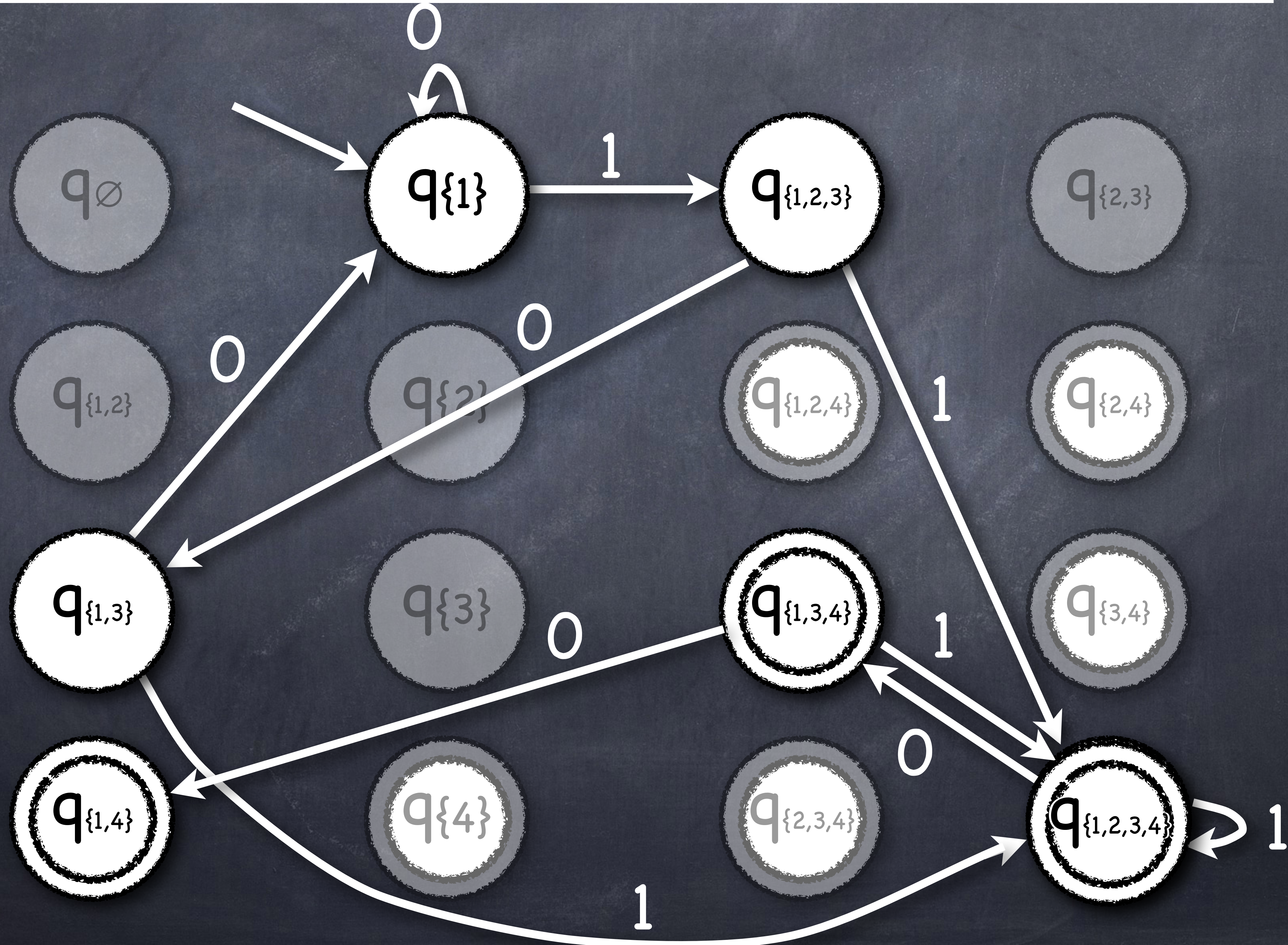
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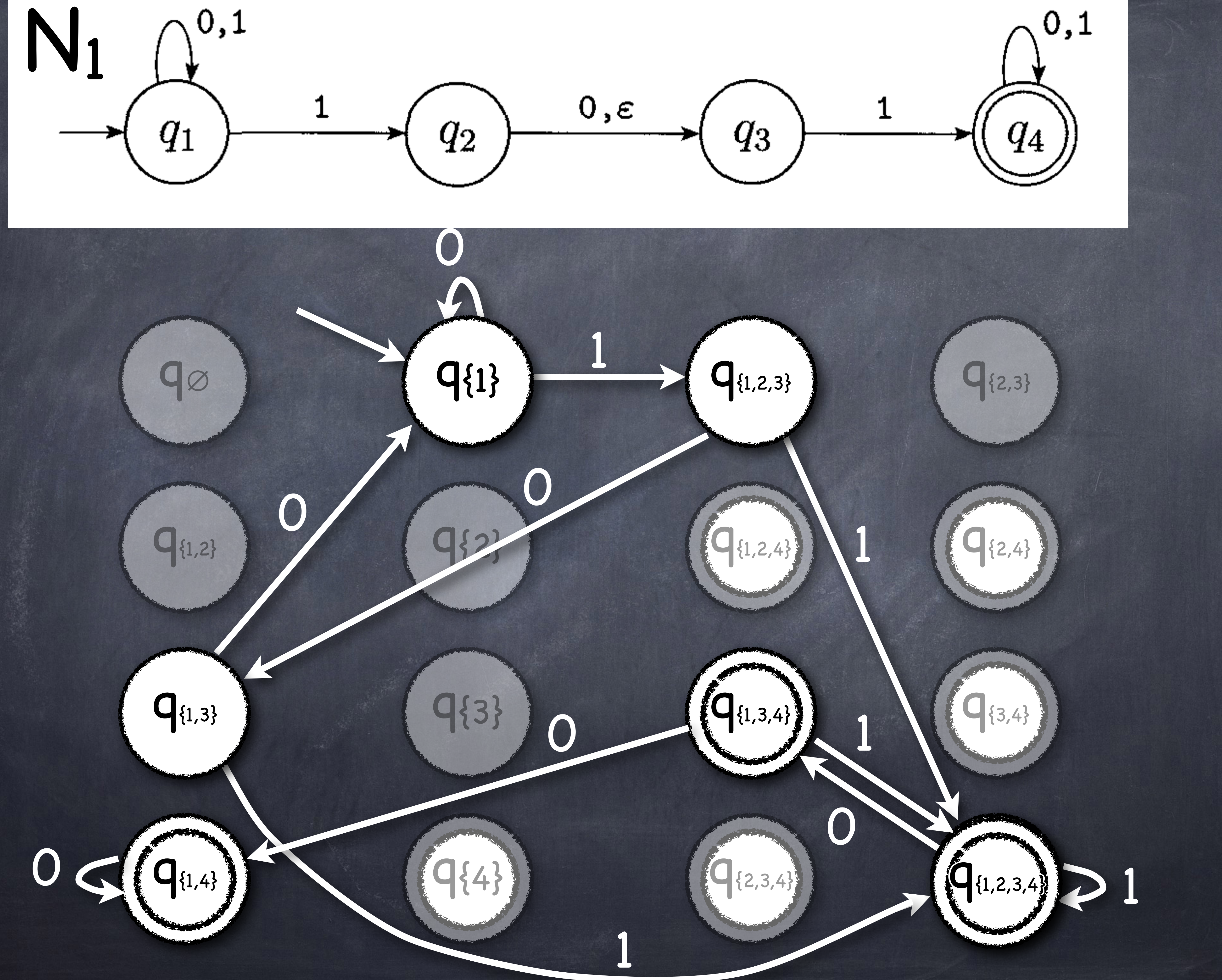
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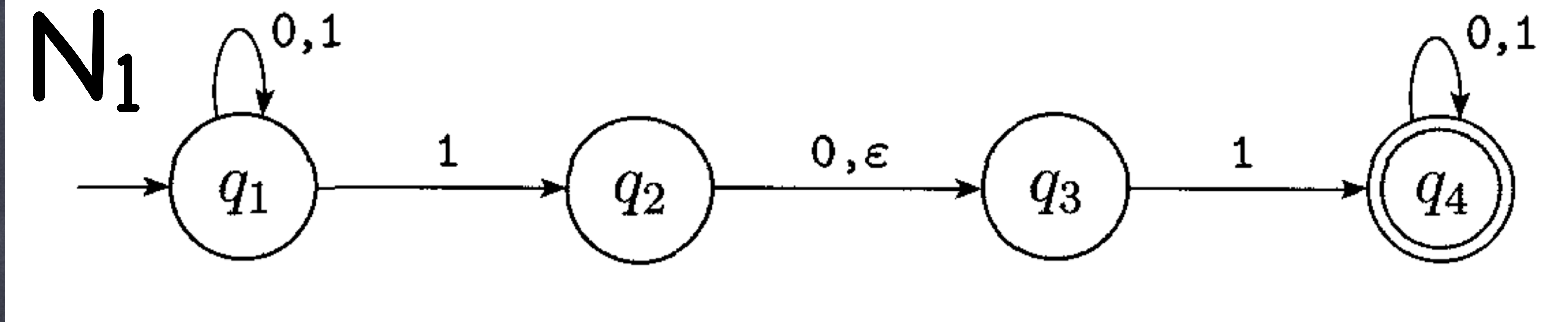
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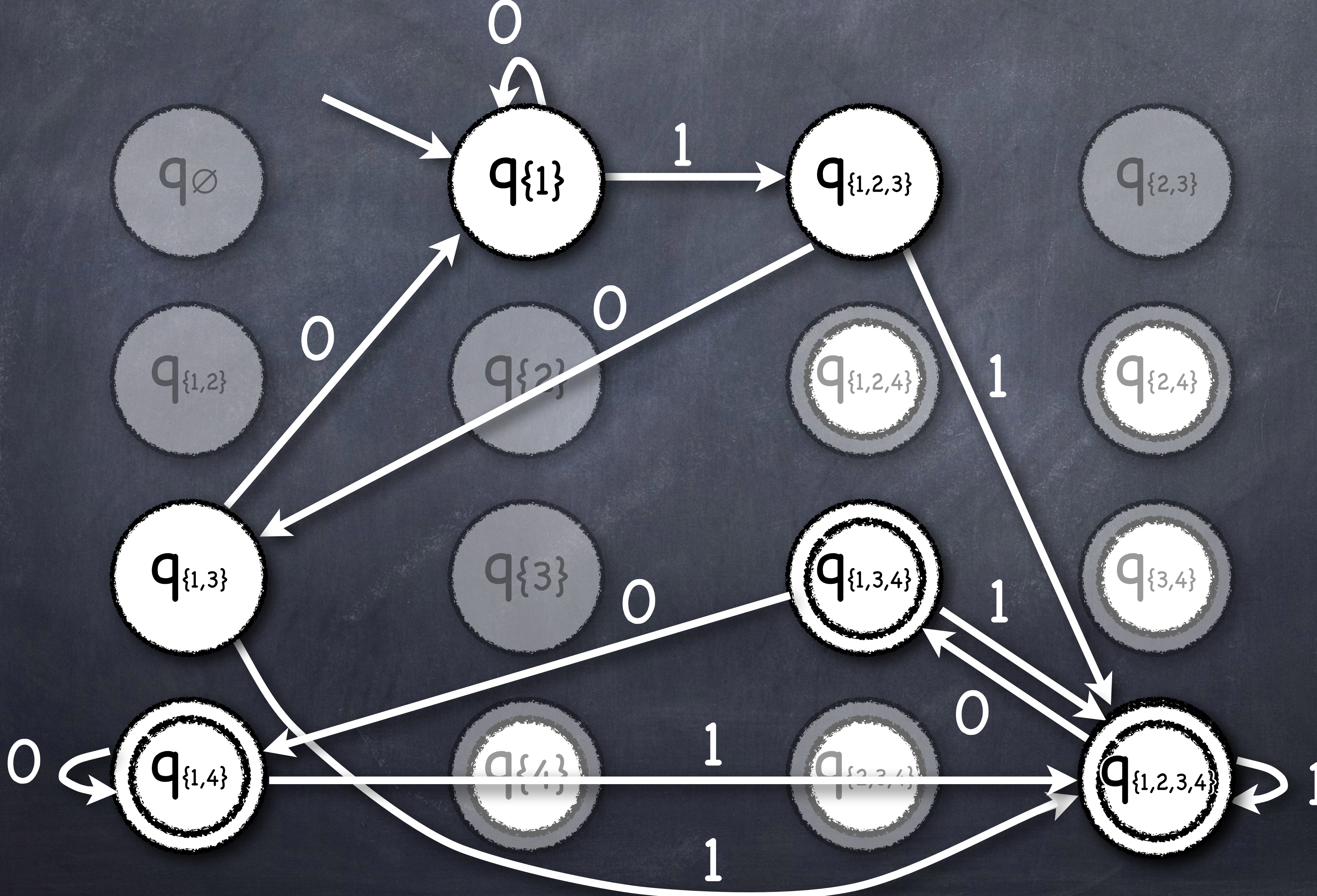
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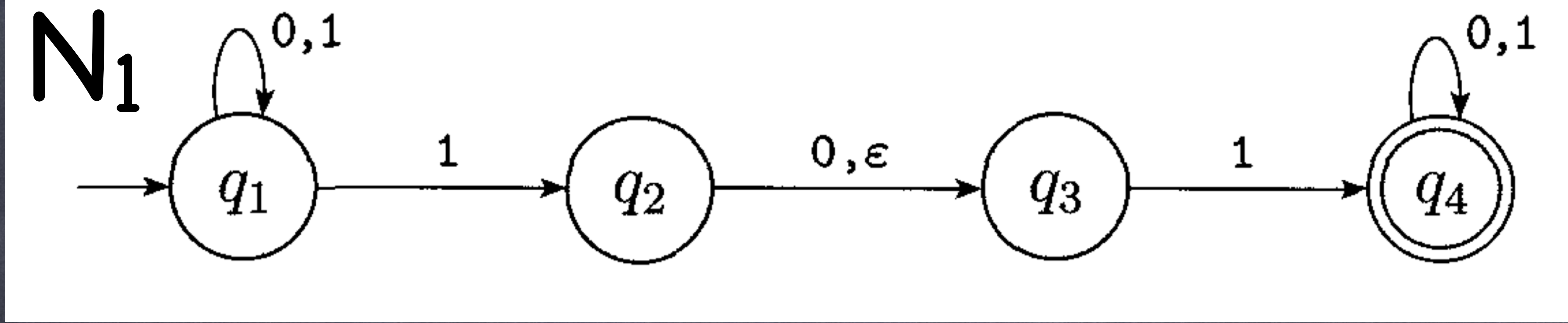
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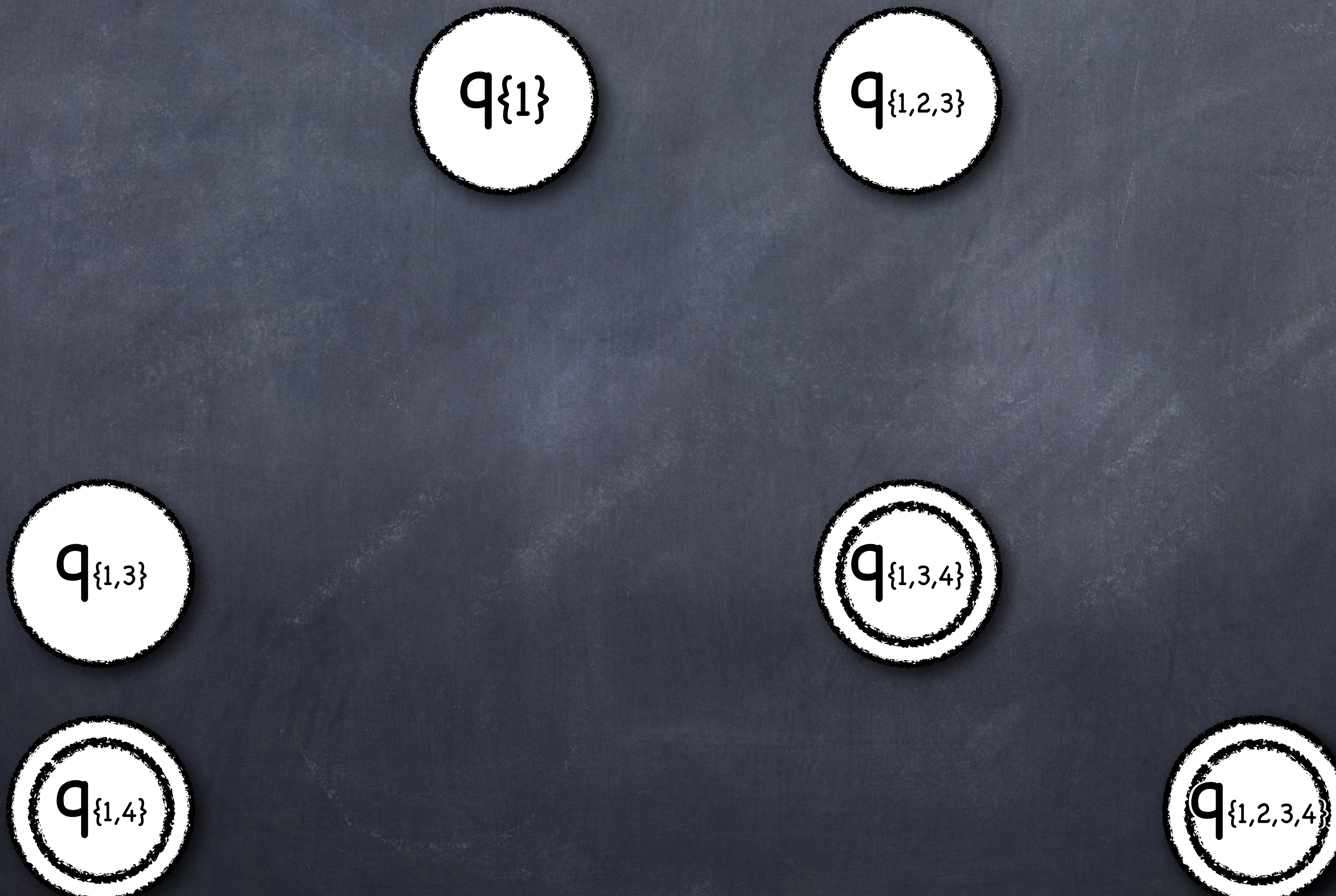
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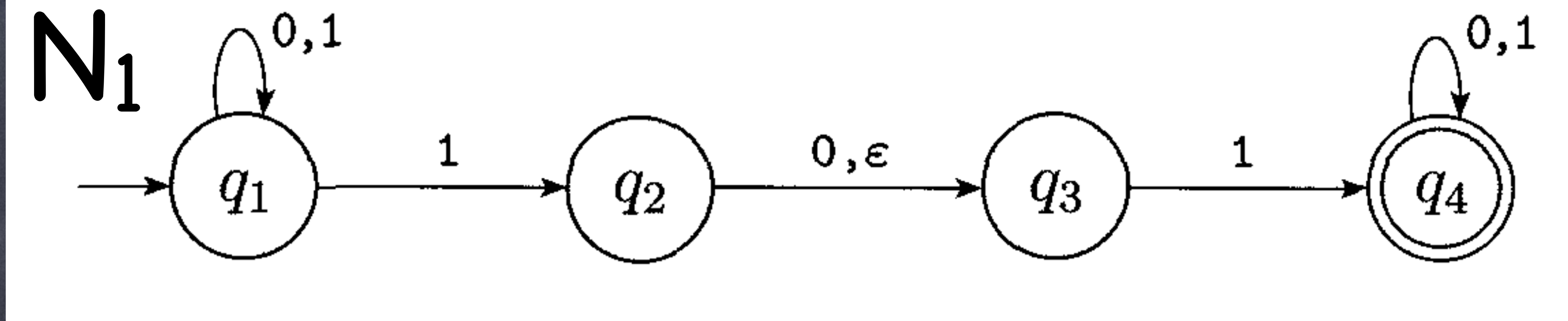

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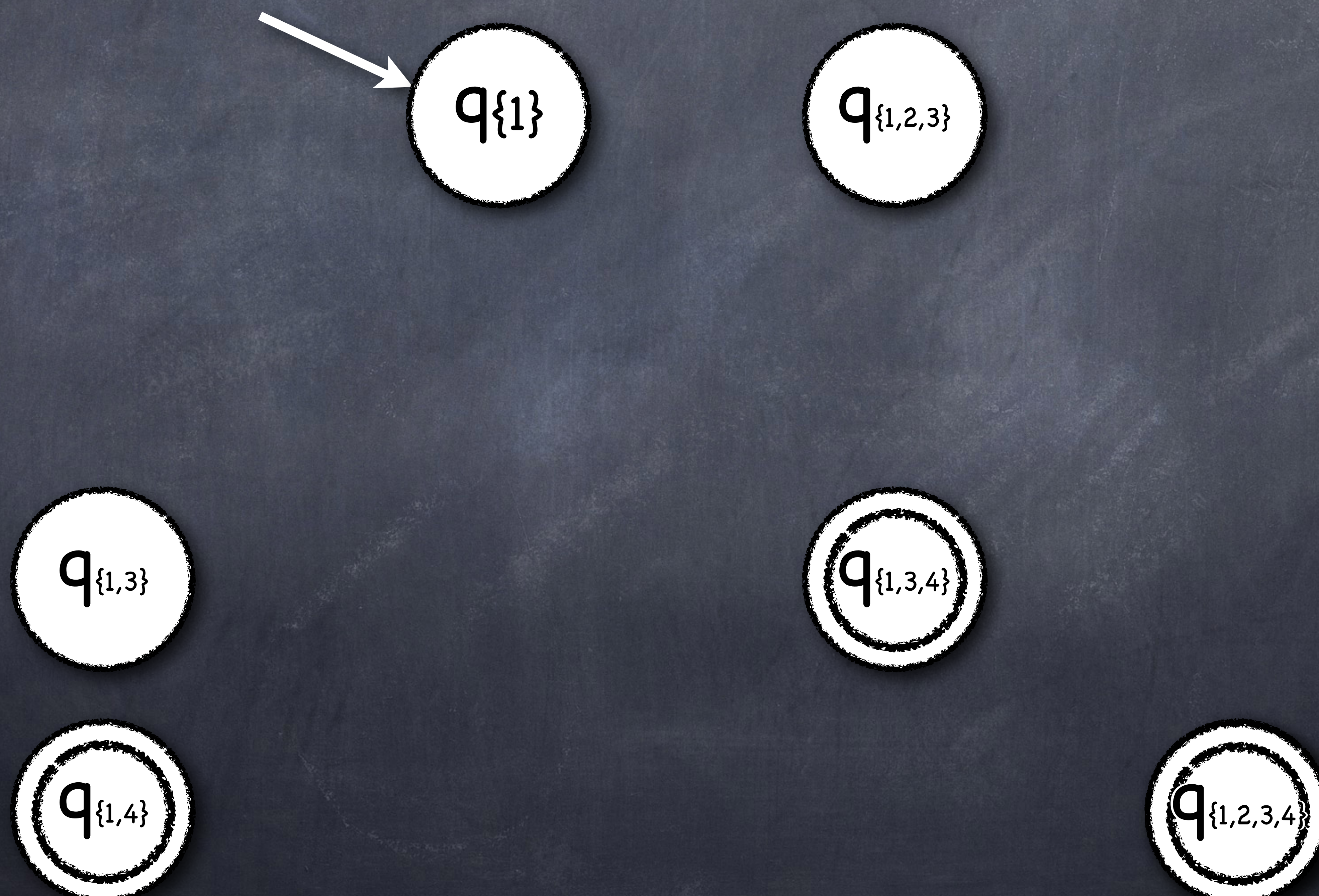

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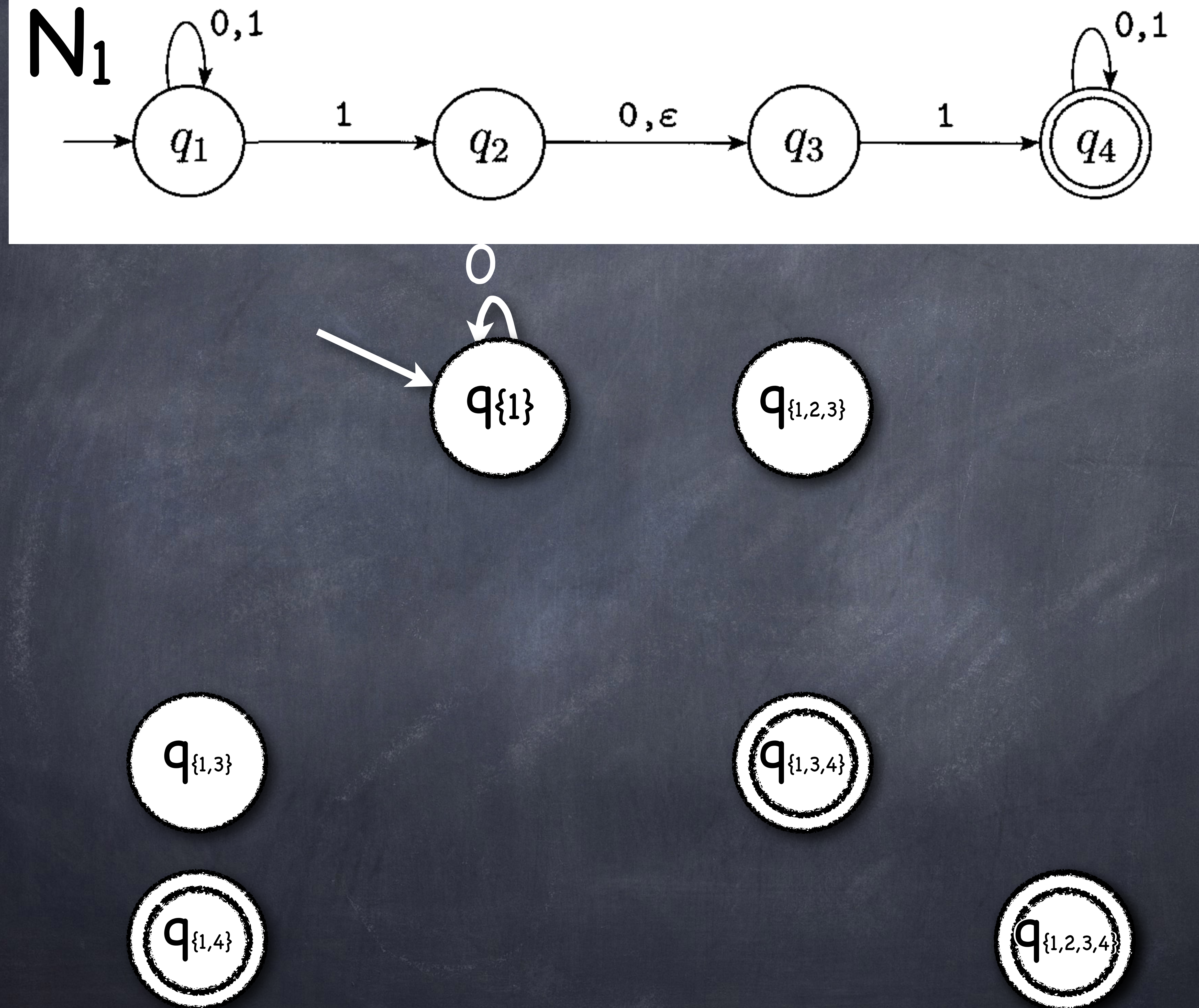
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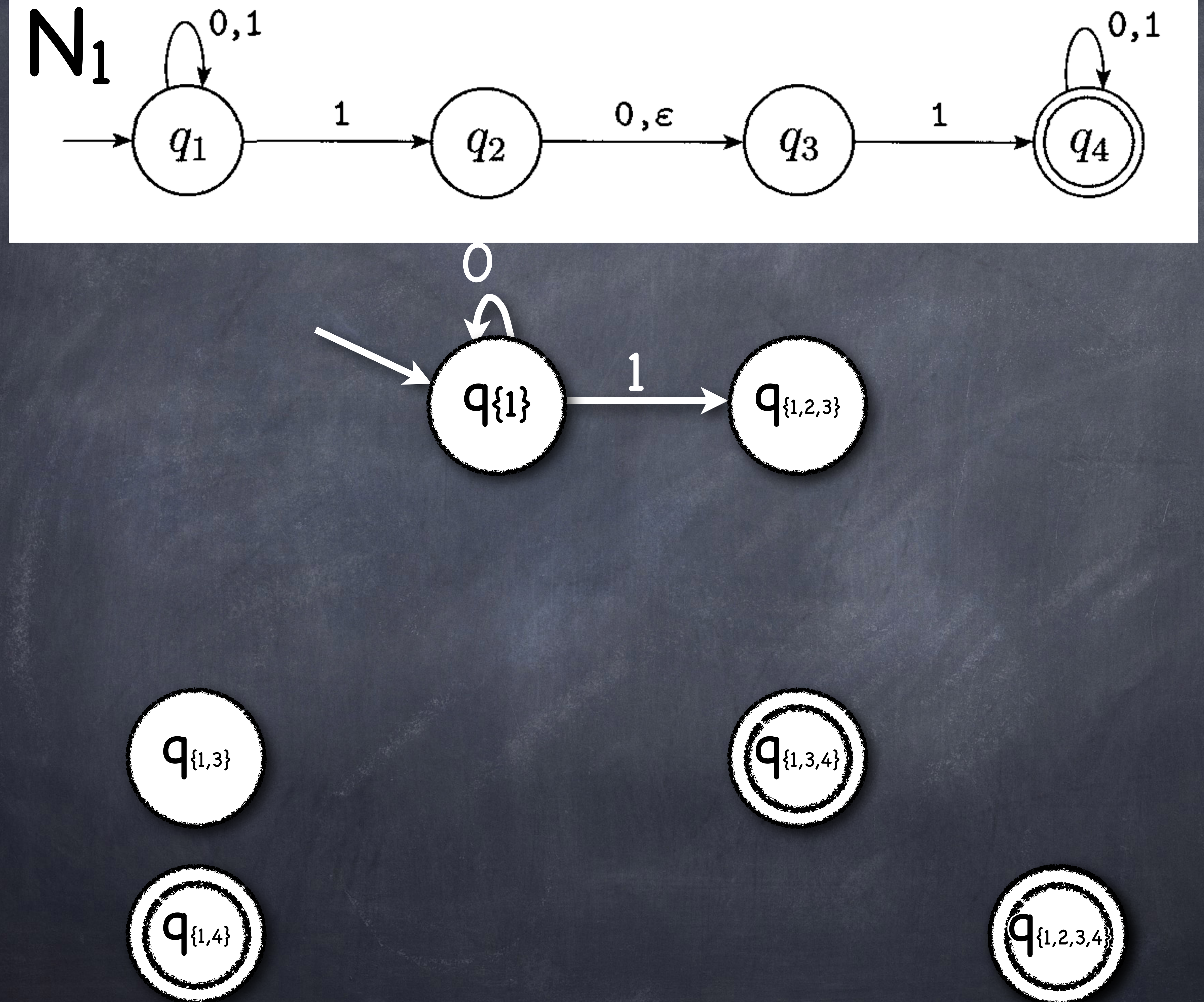
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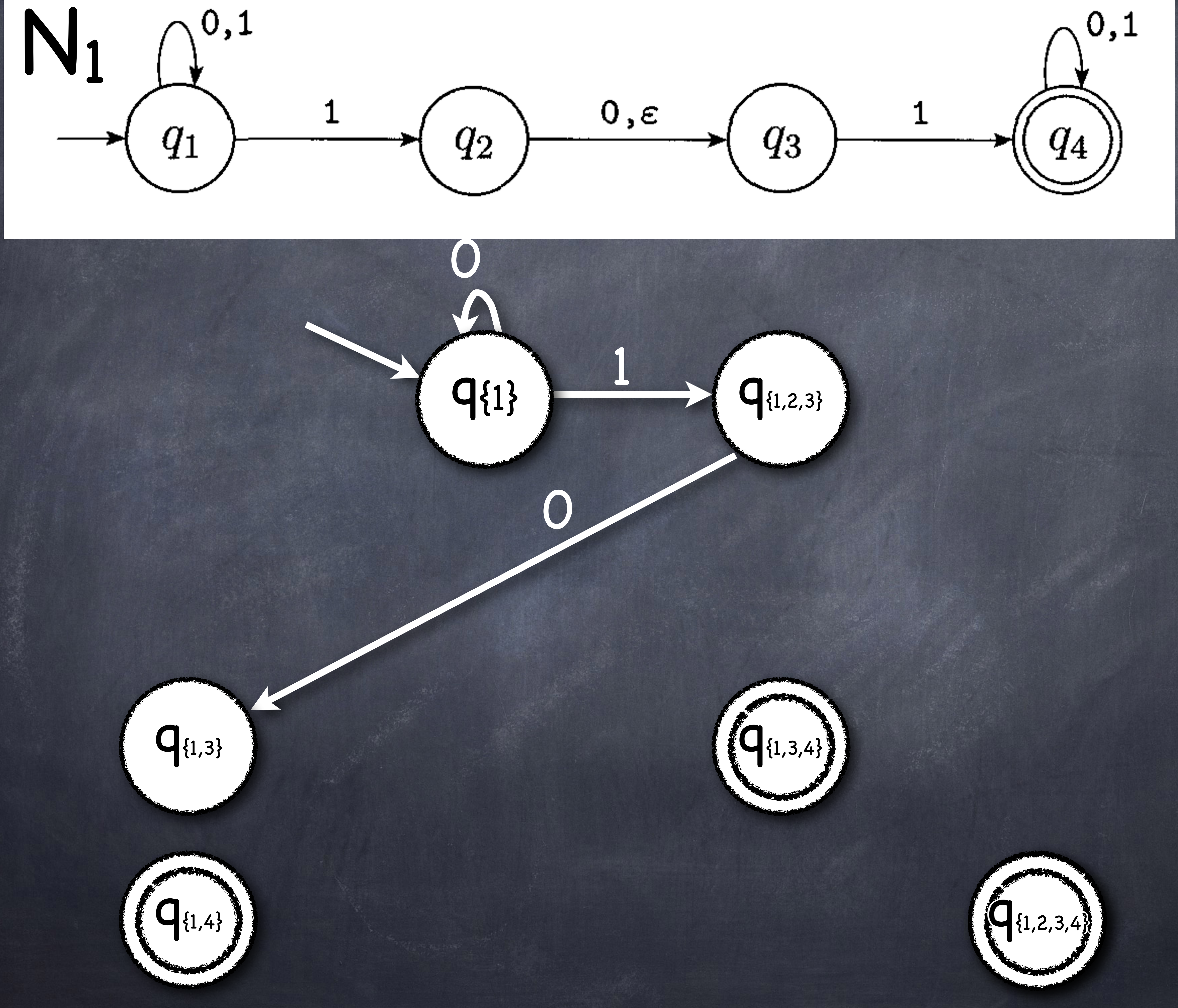

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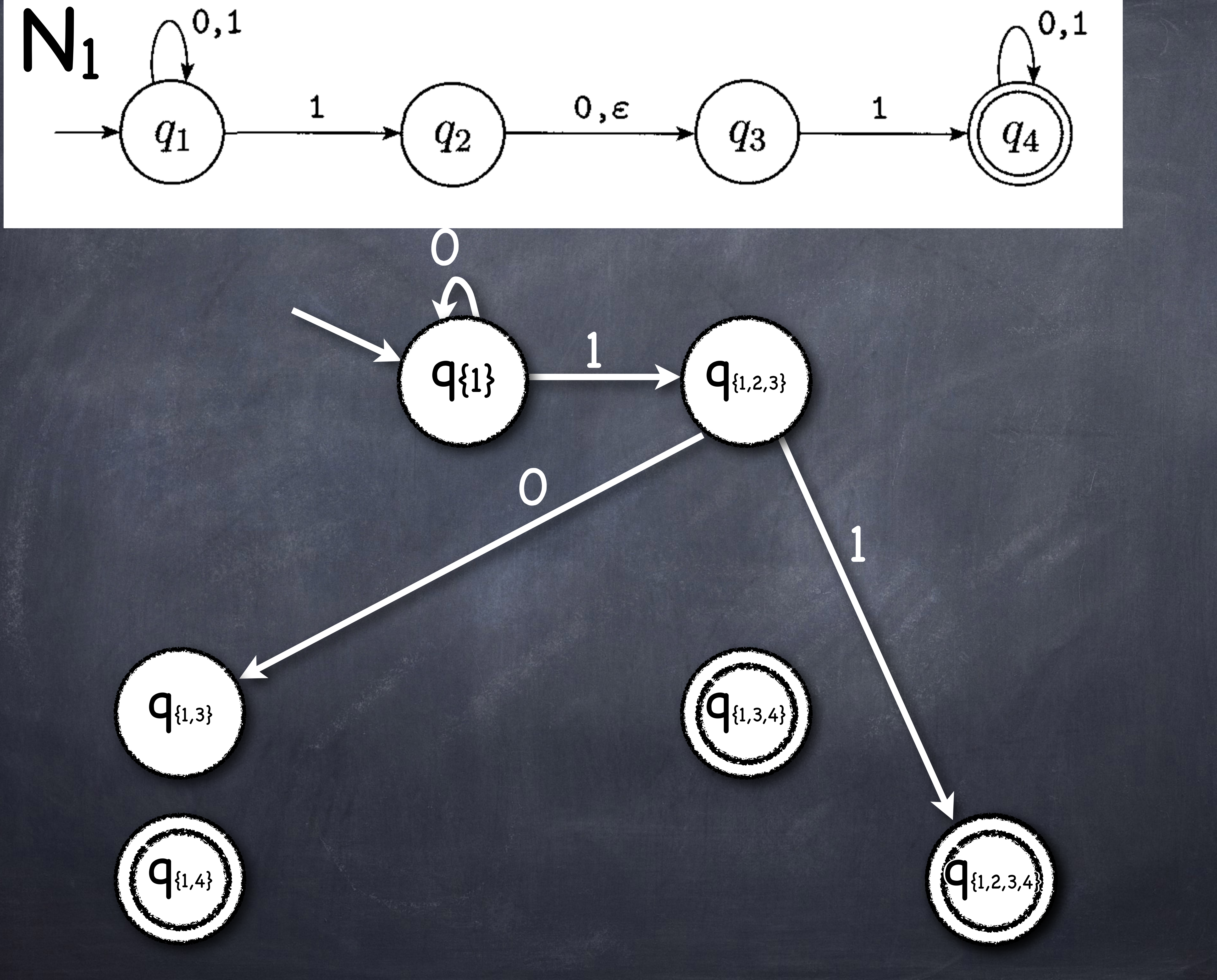
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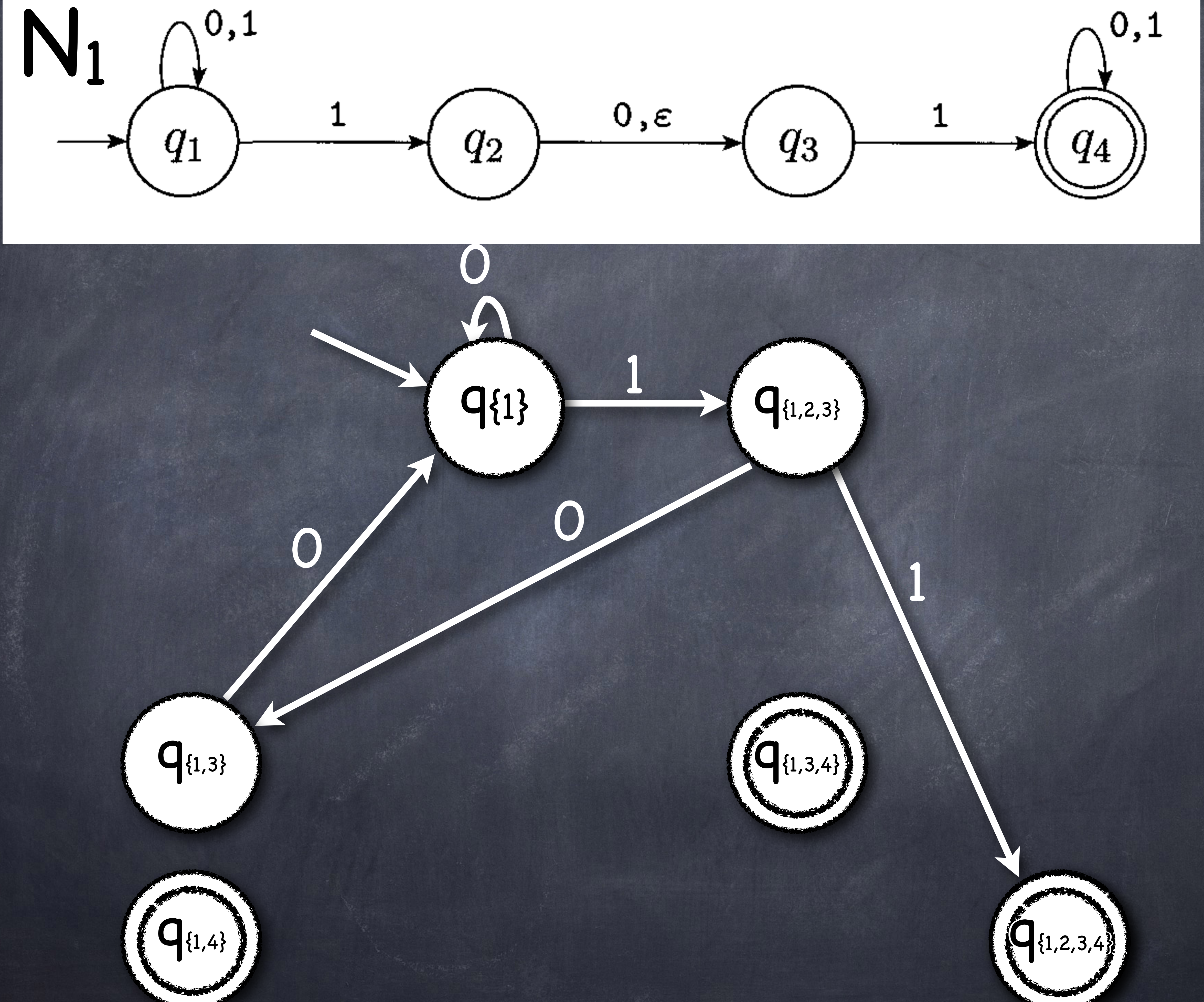
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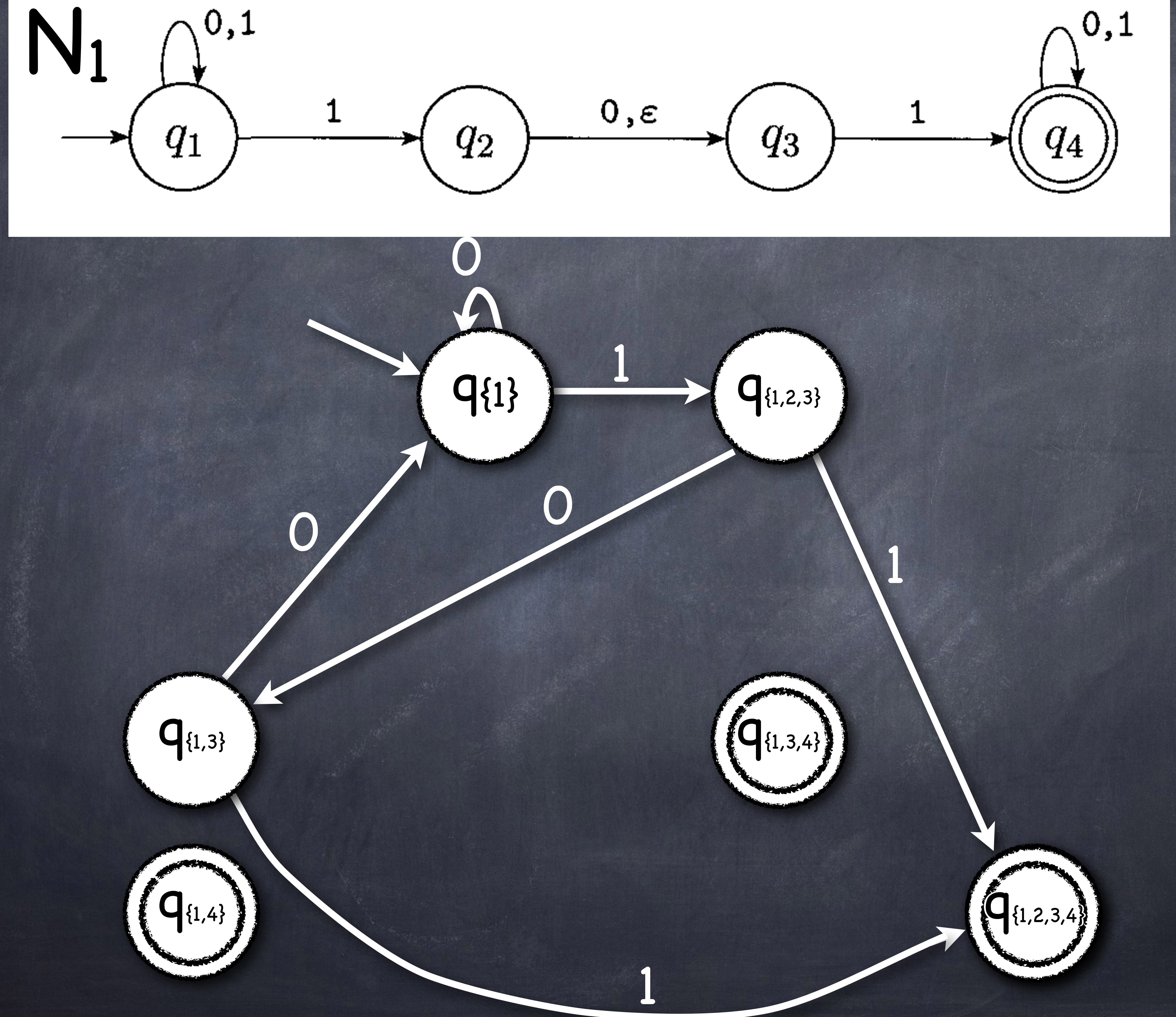
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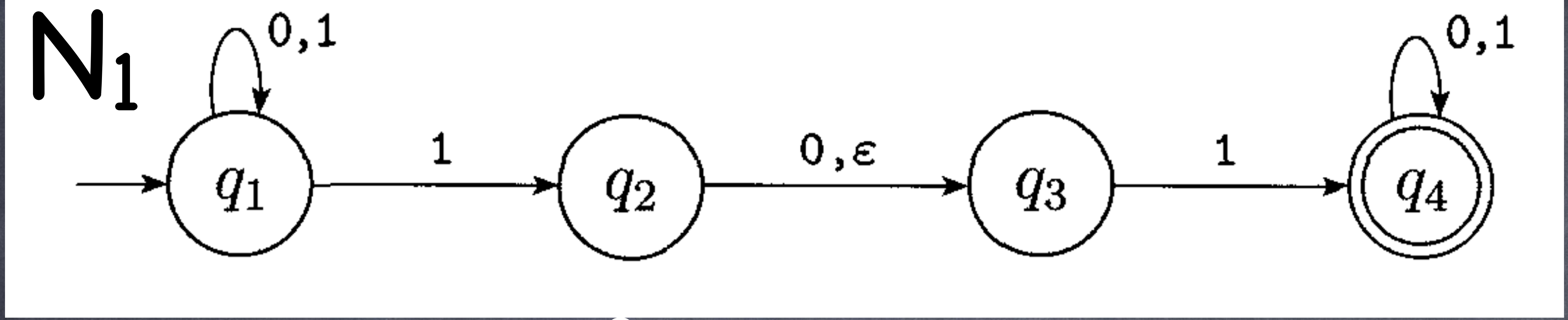
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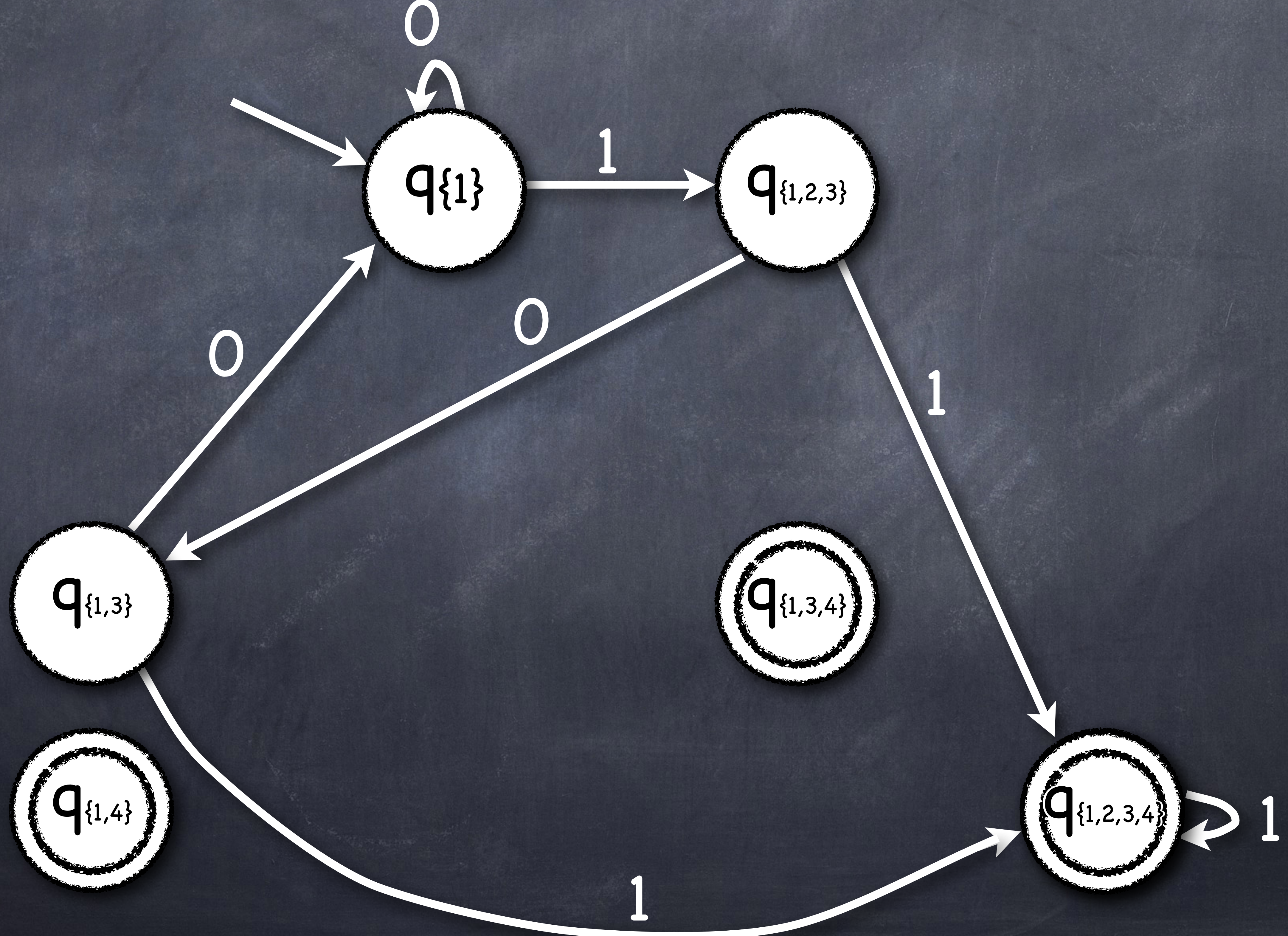
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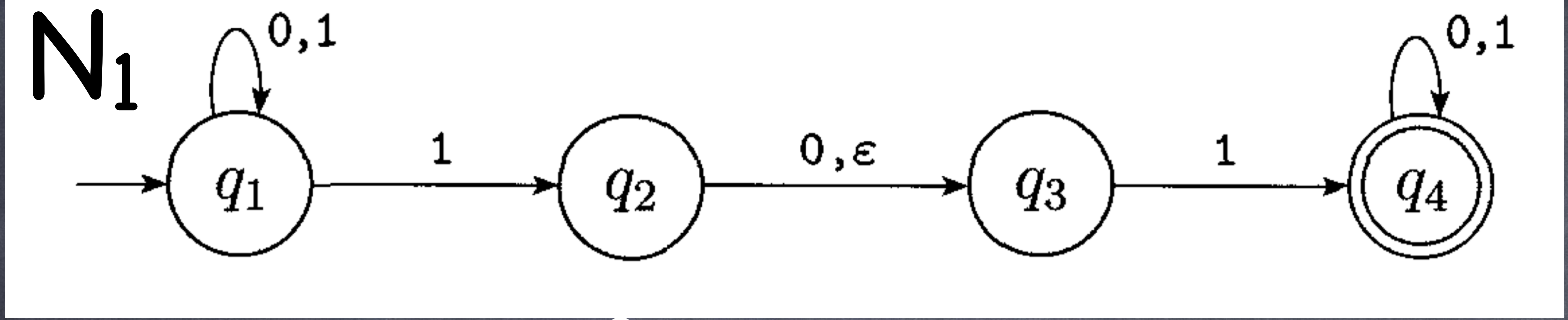
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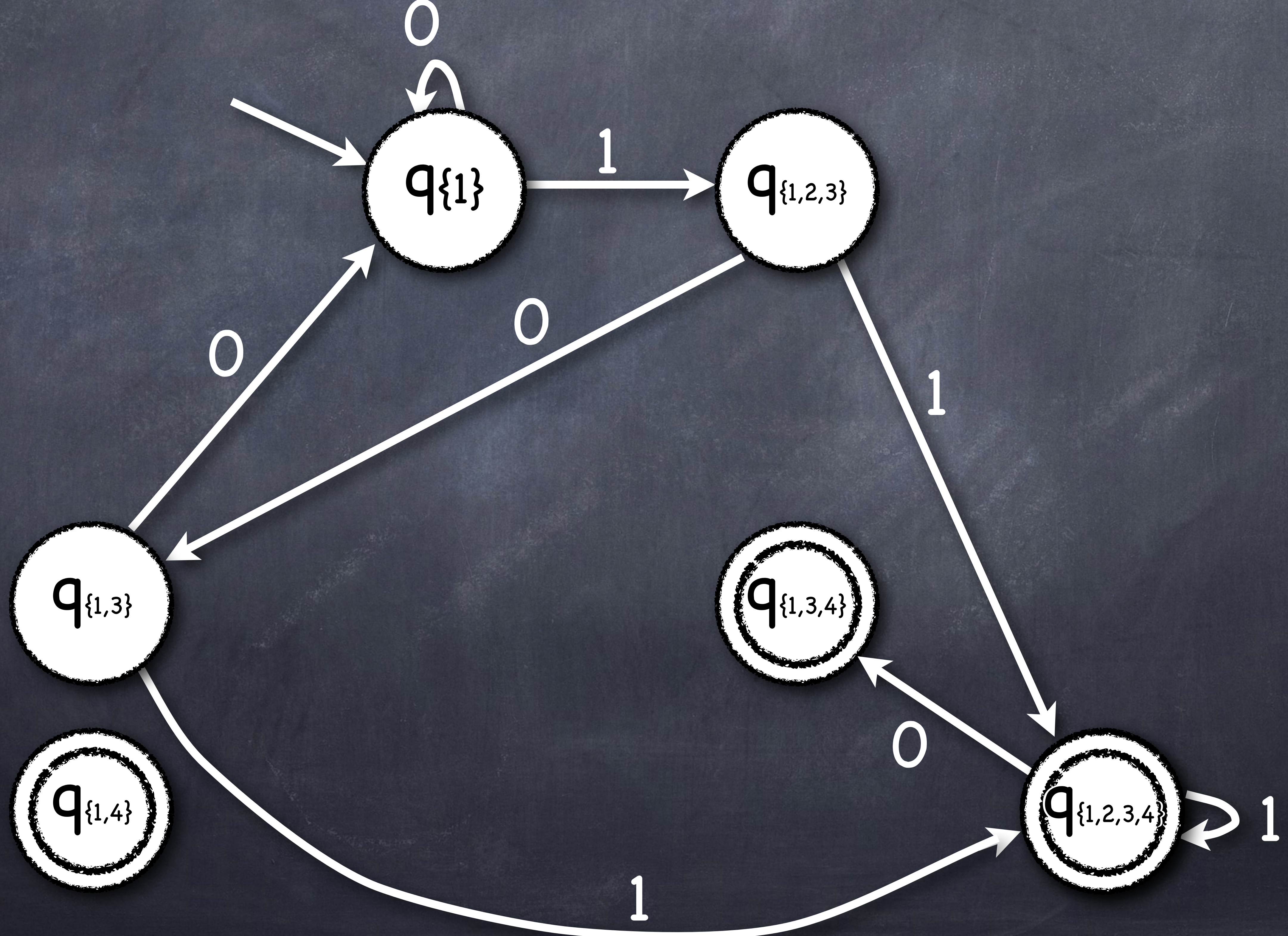
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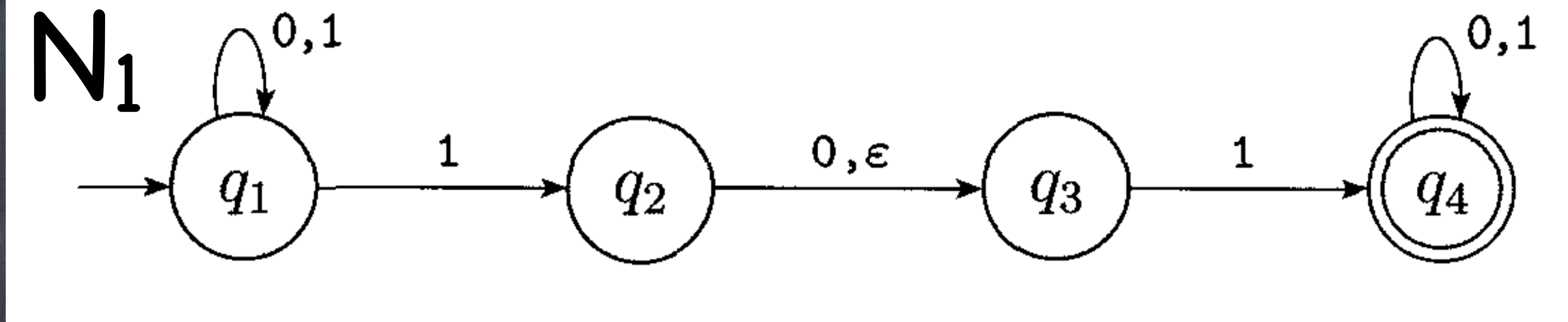
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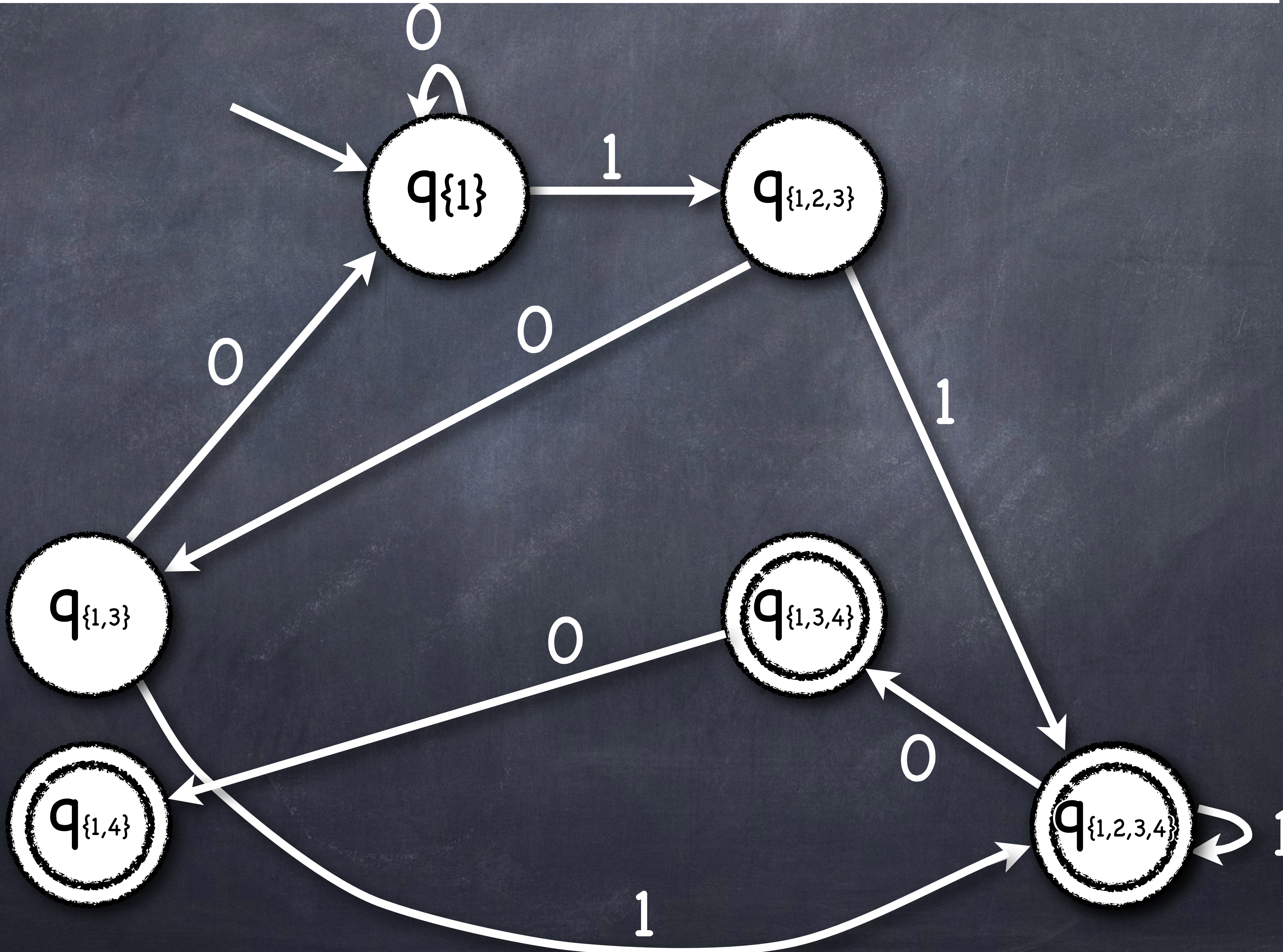
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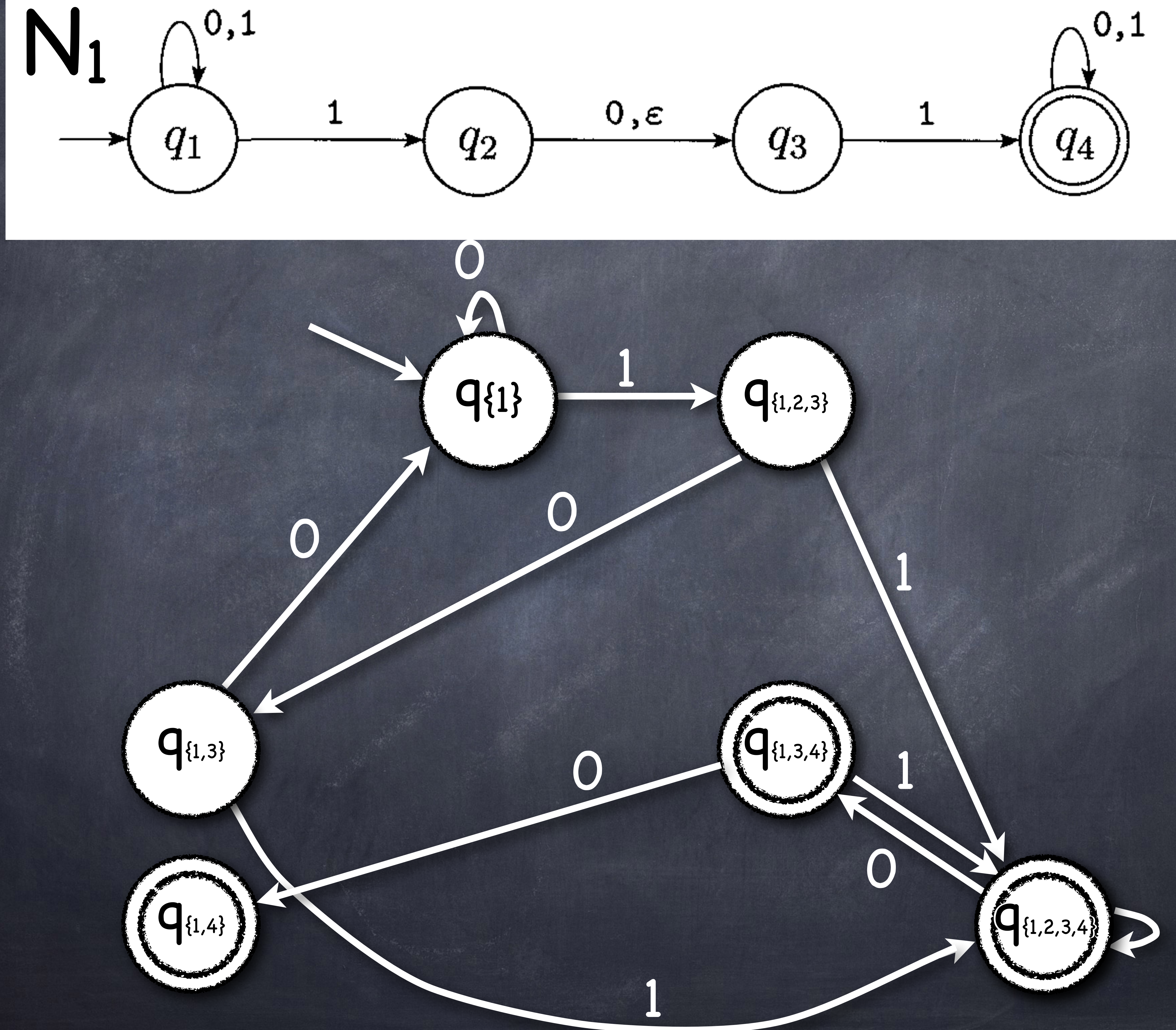
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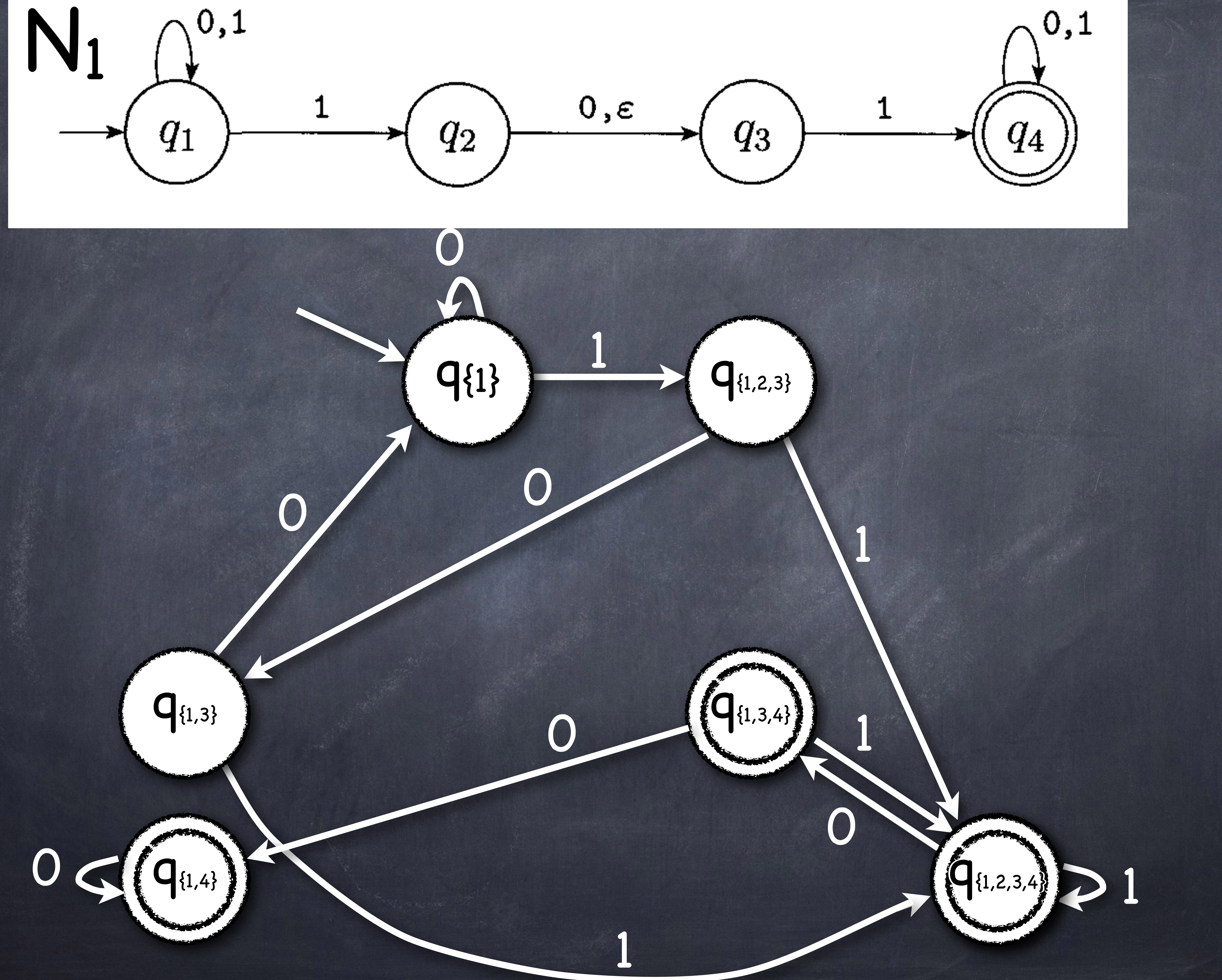
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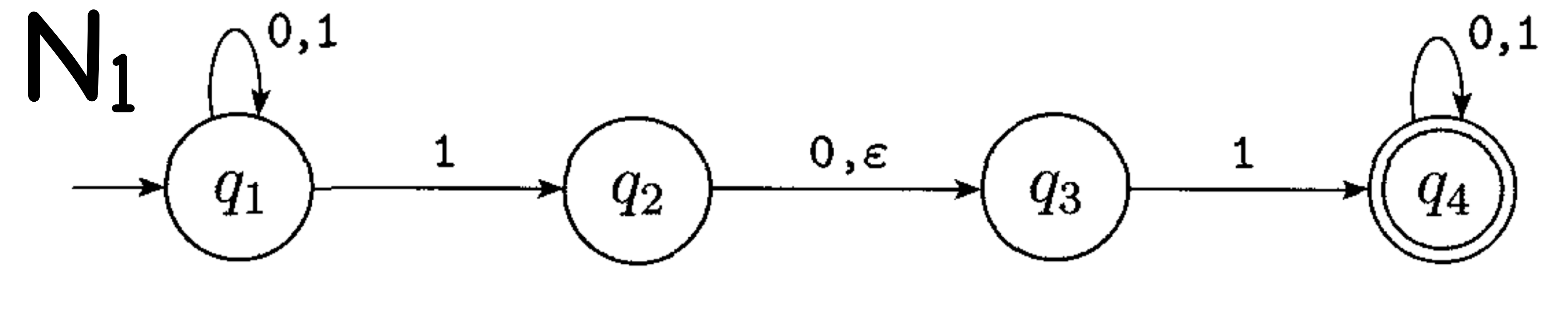
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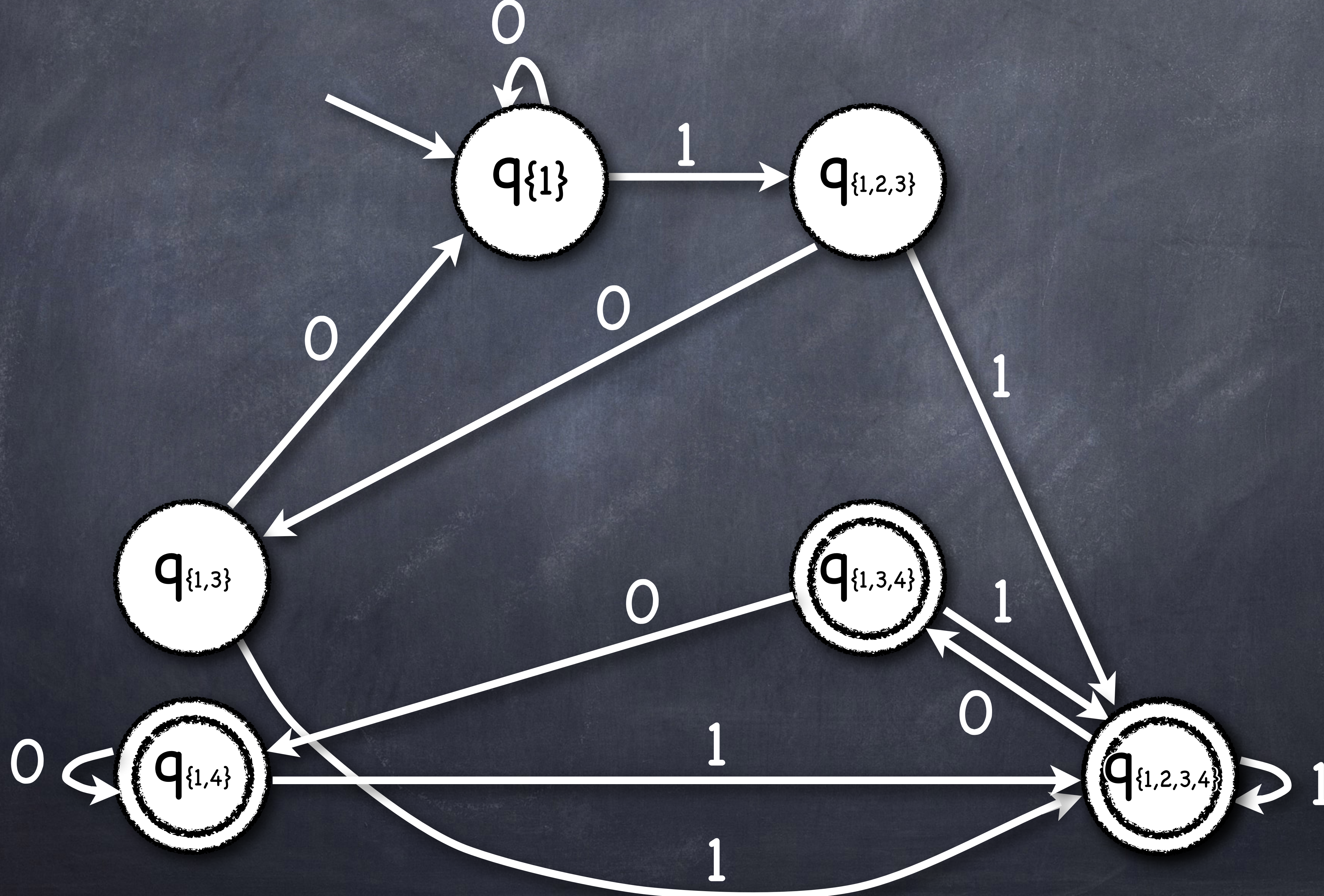
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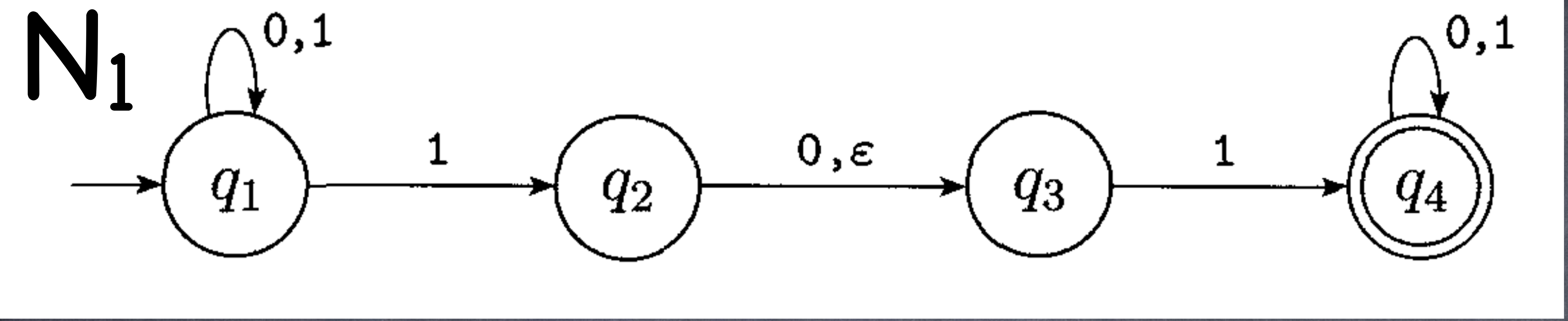

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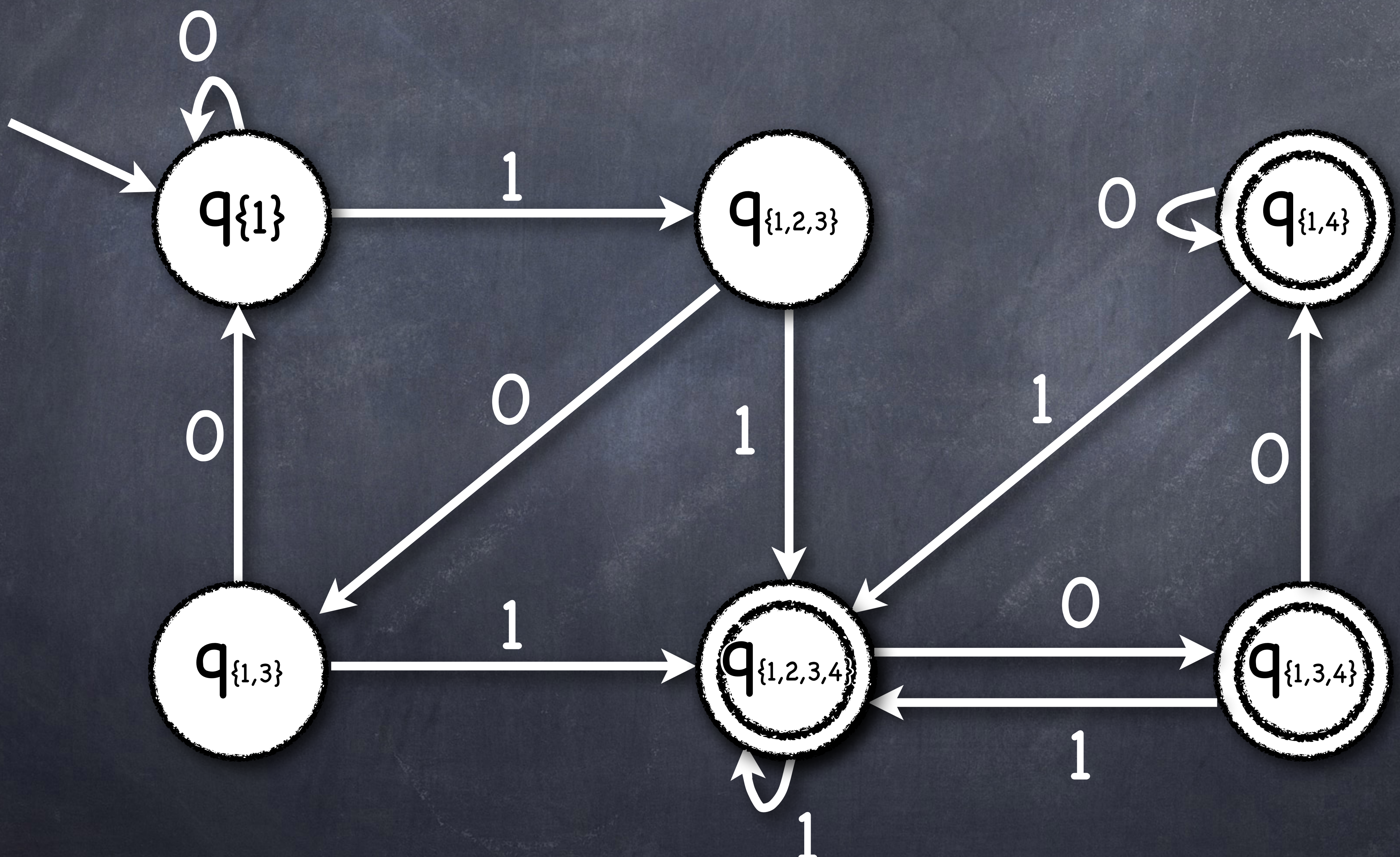

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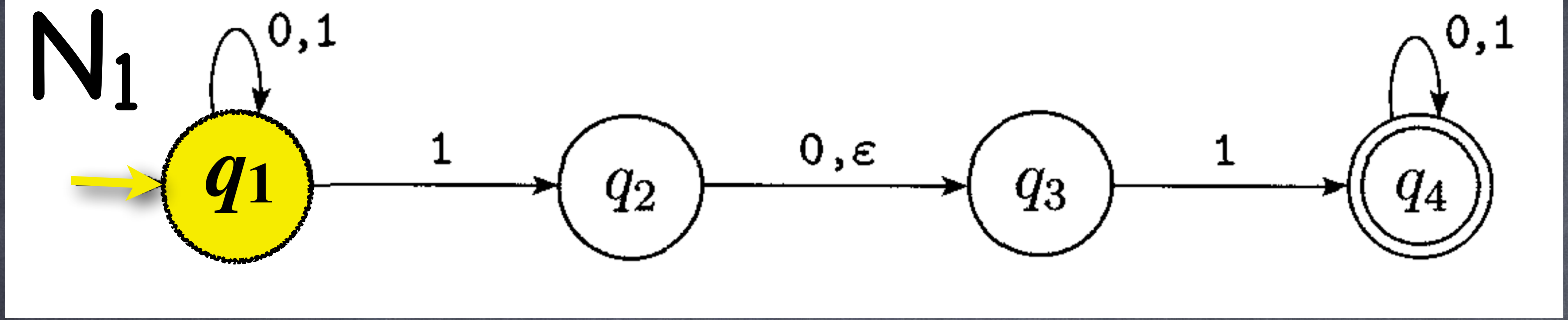
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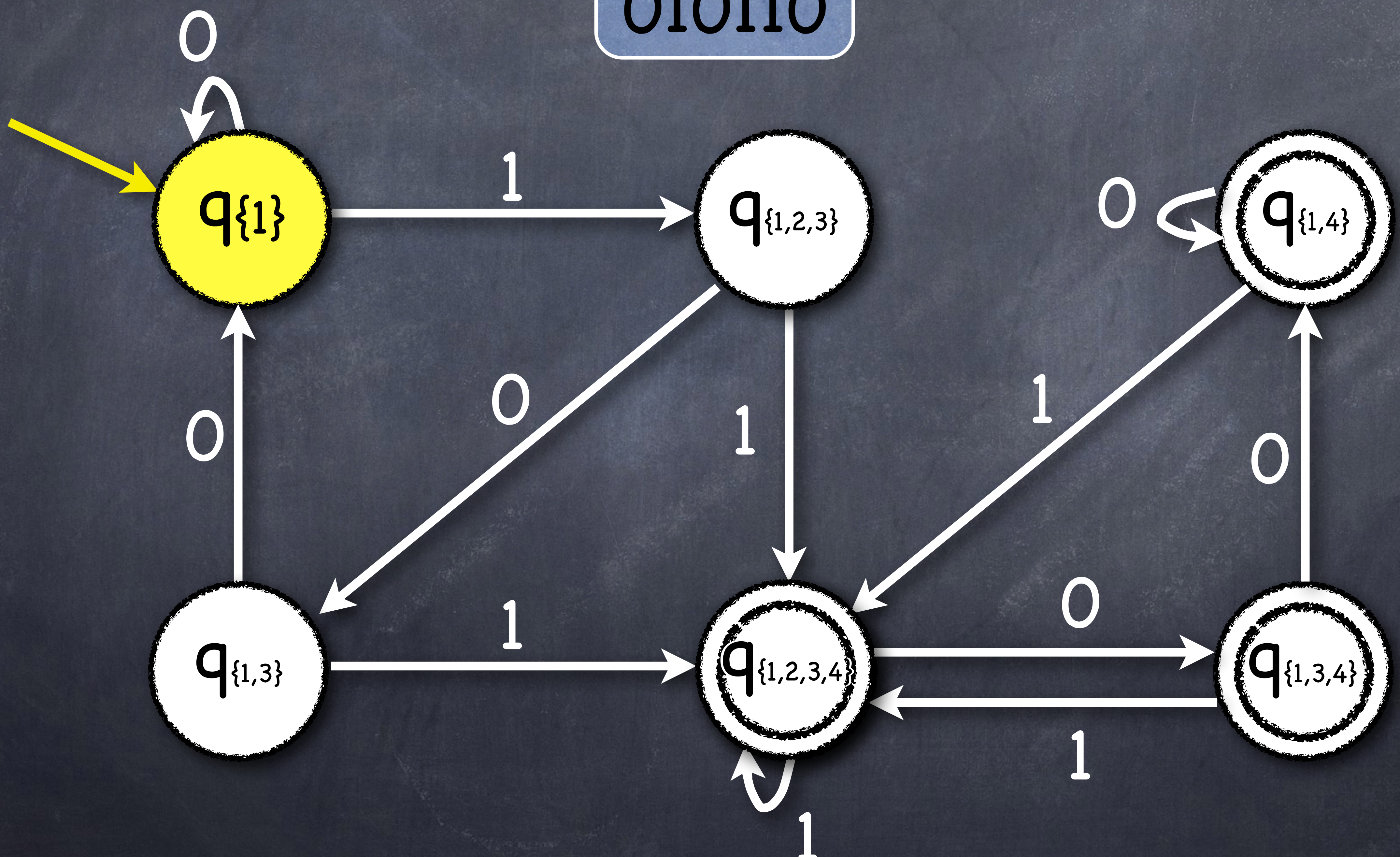
010110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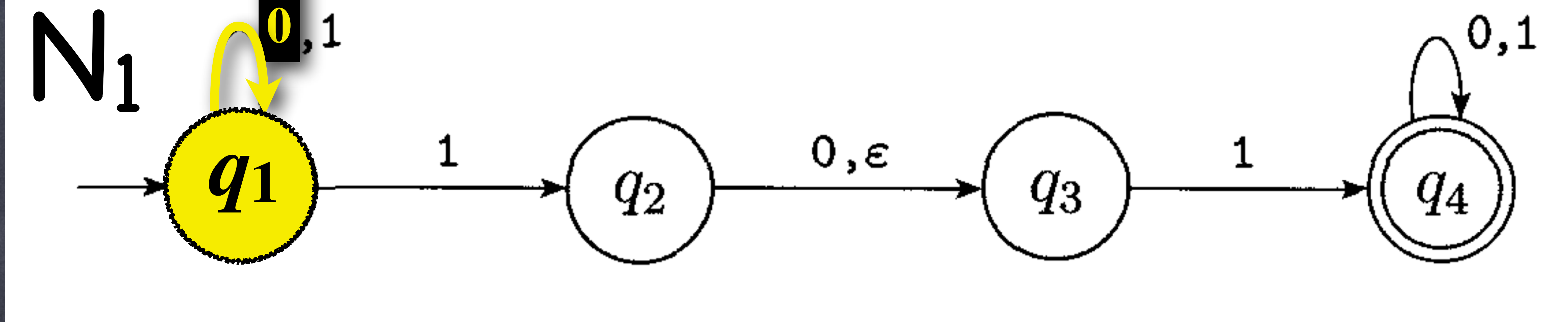
N1
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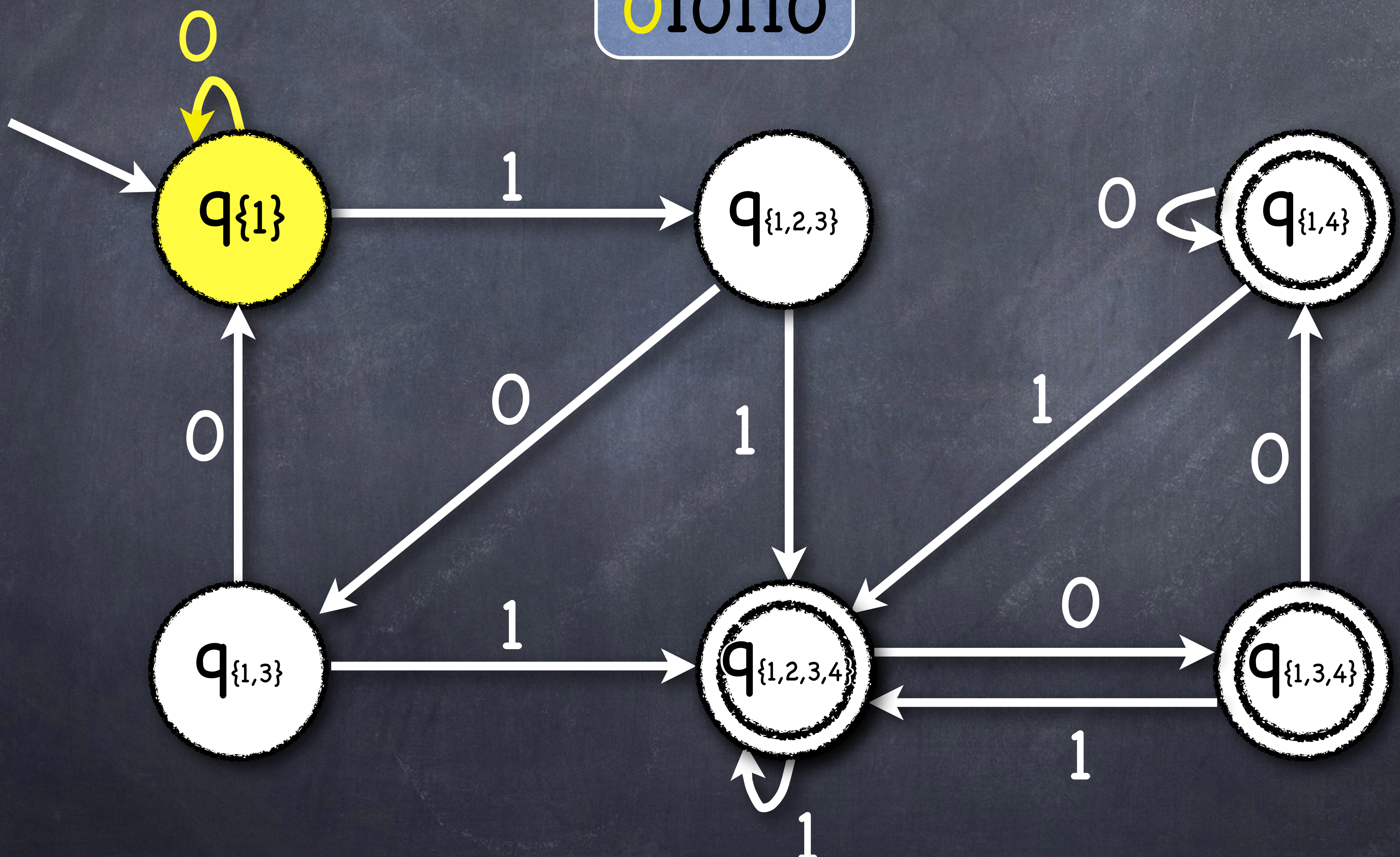
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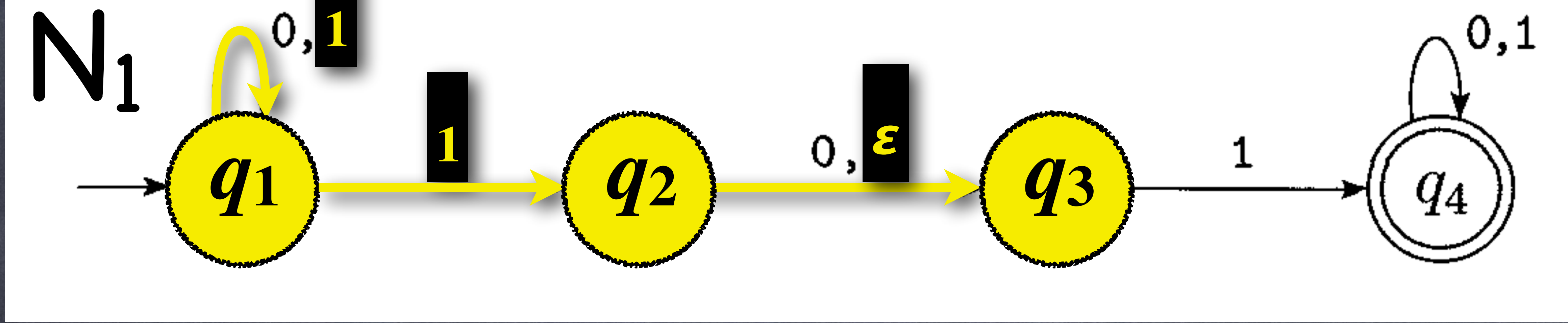

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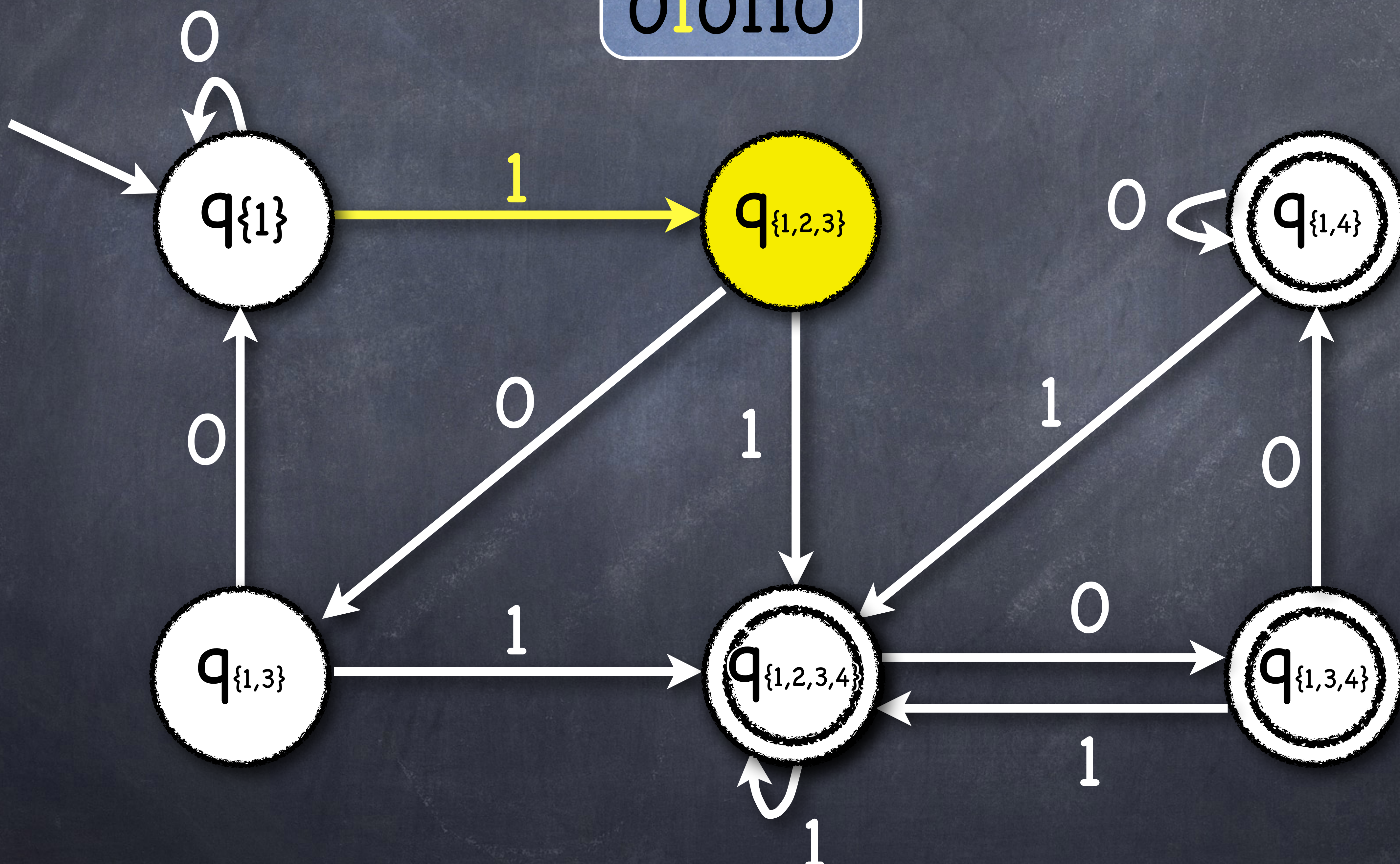

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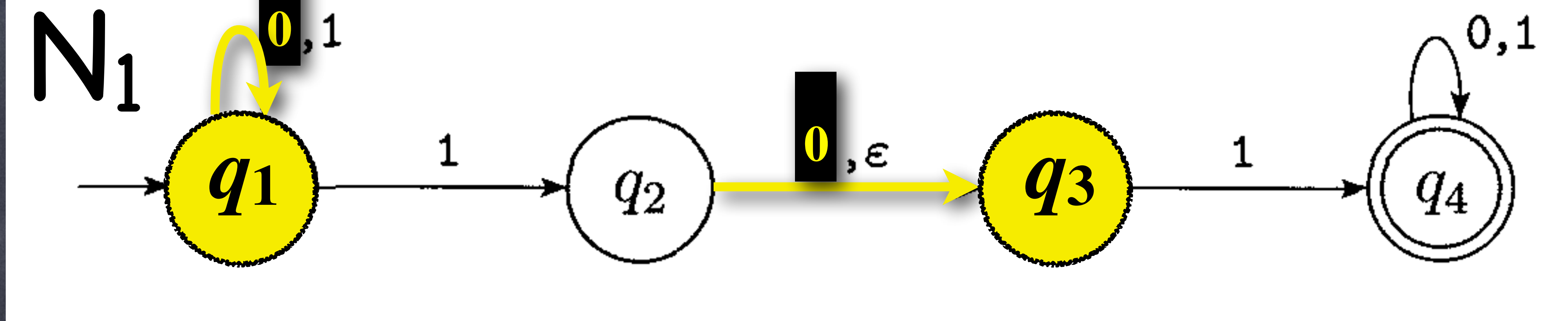

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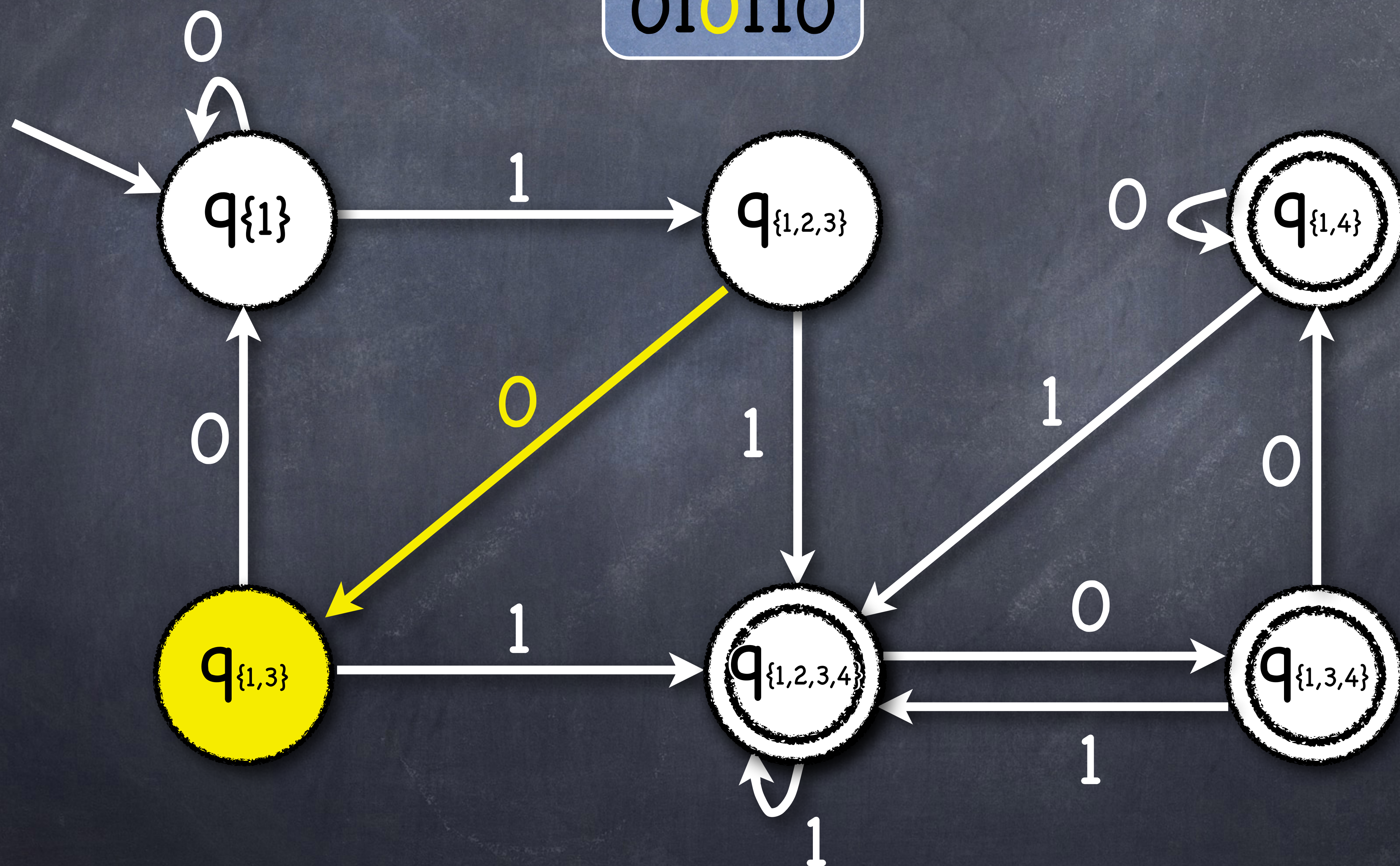

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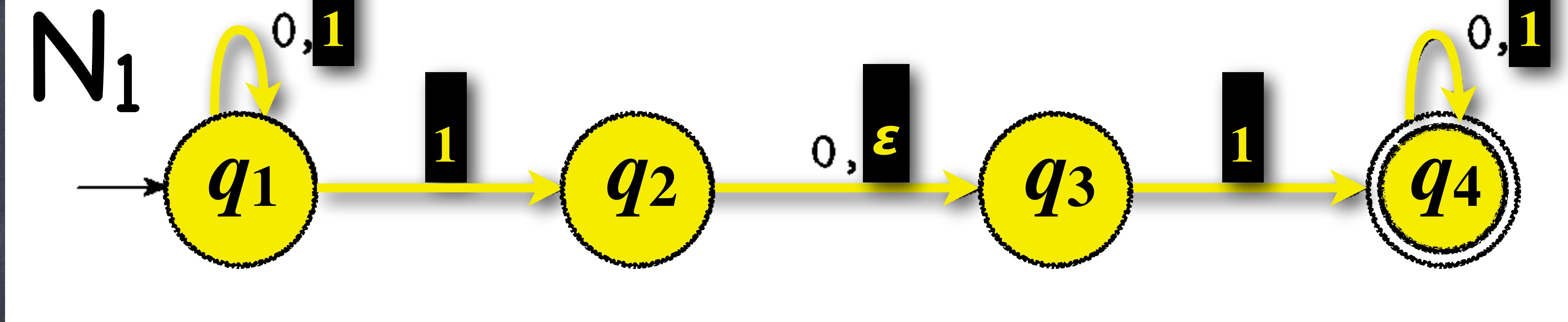
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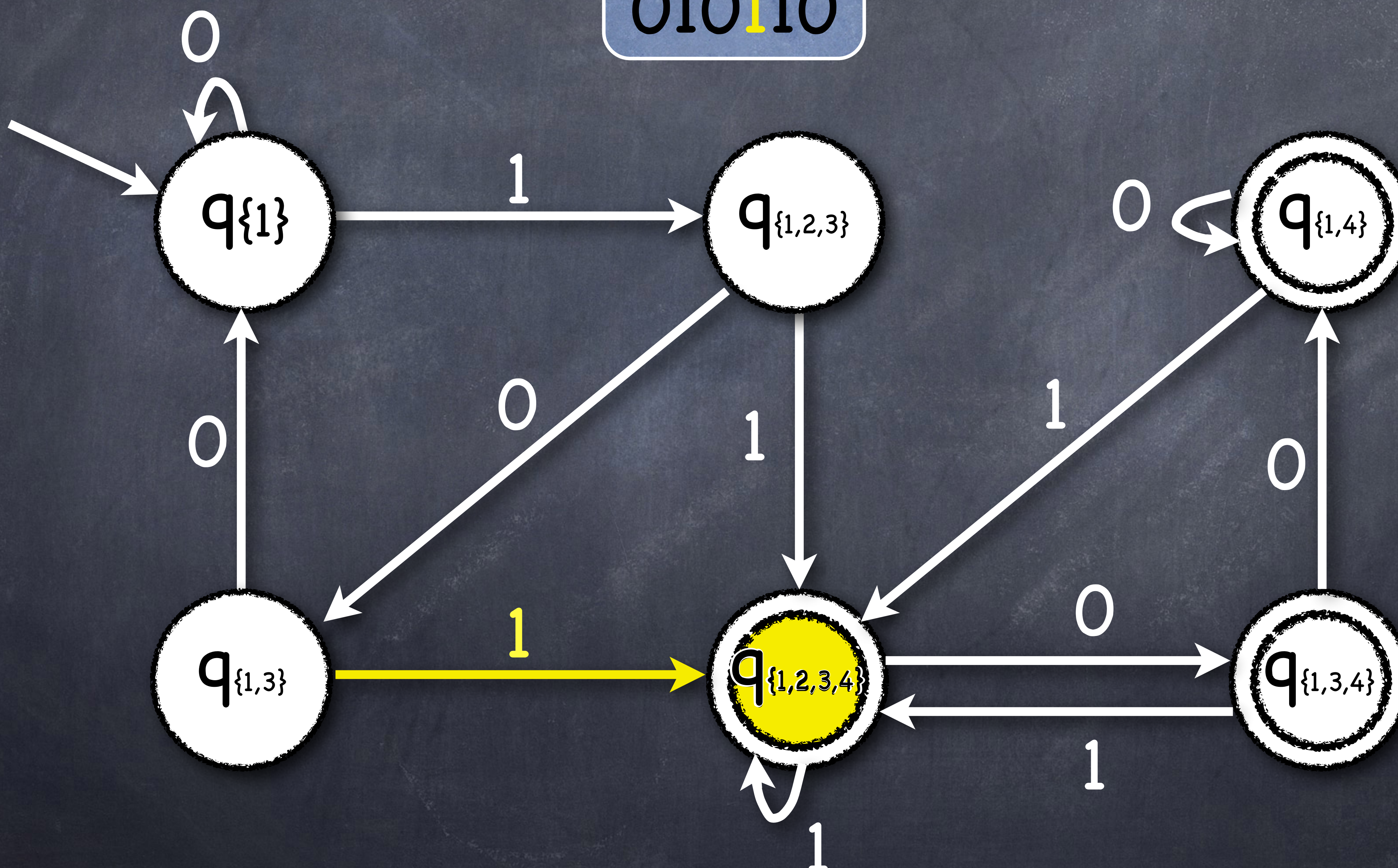
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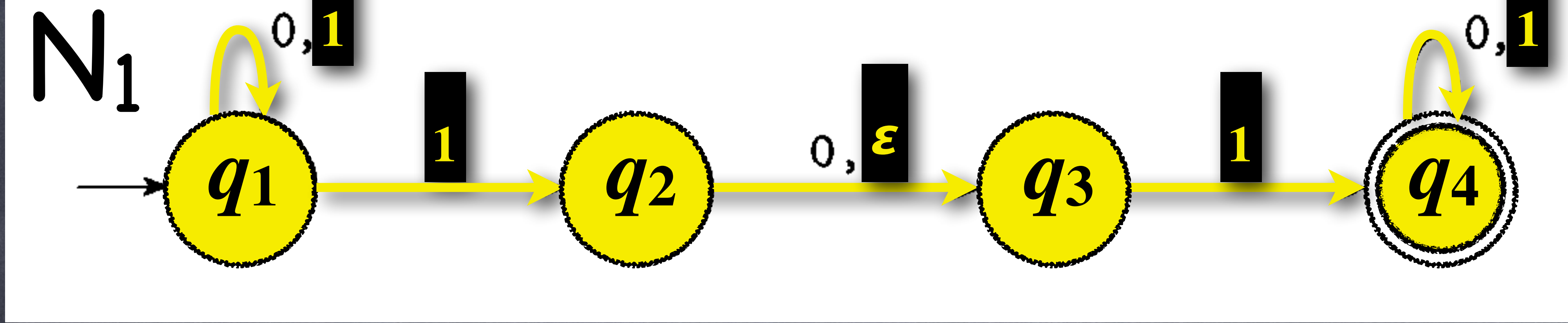
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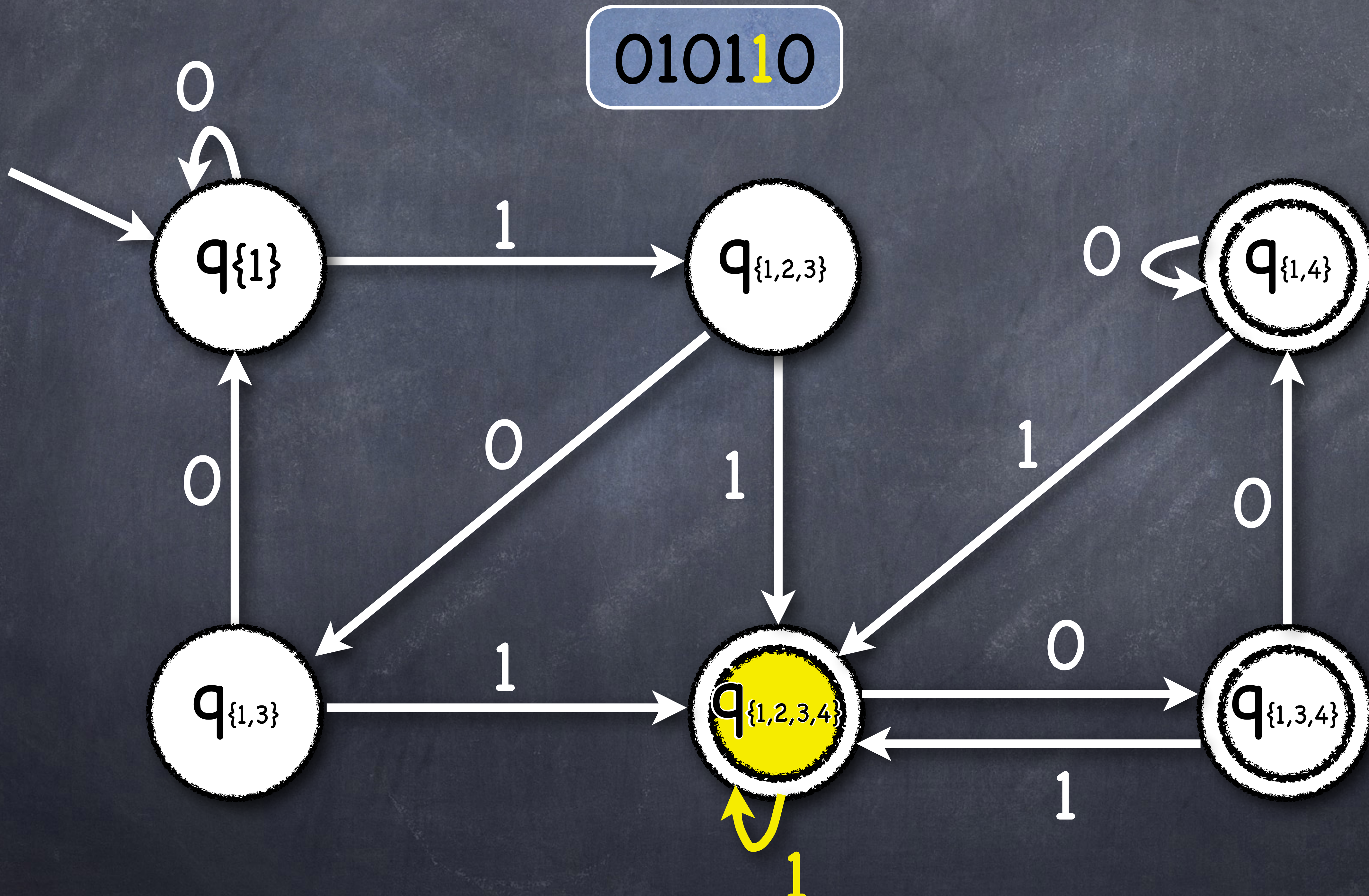
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q{1,3,4}
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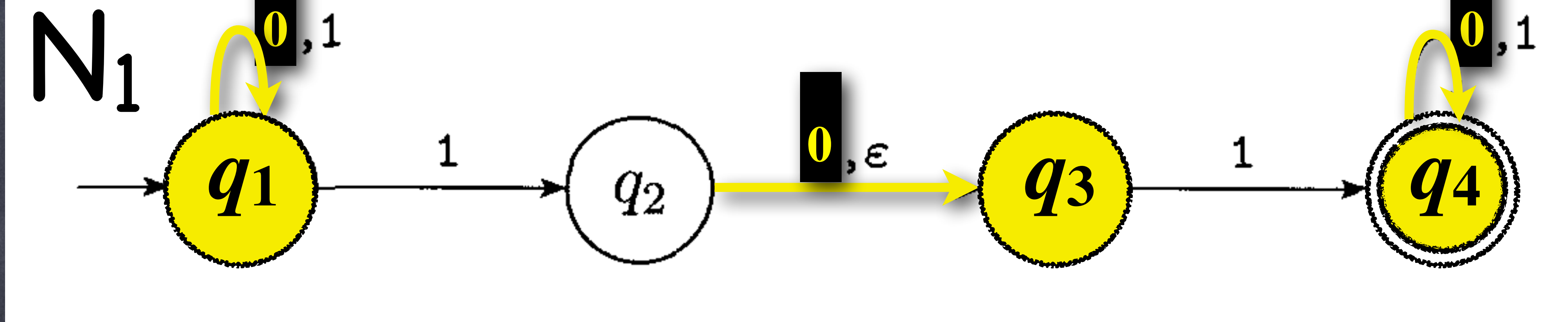

N1
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q4

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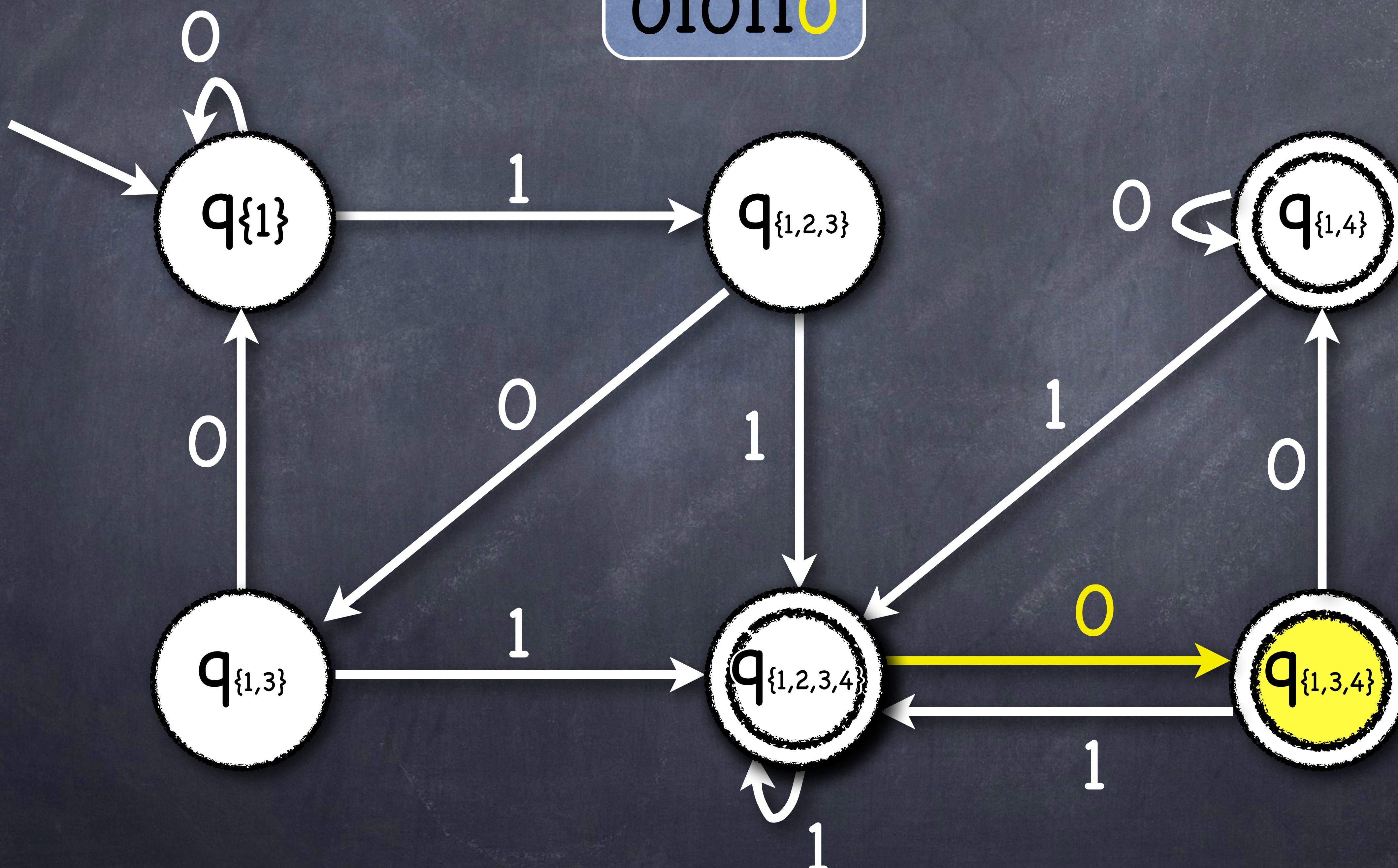

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q{1}
q{1,2,3}
q{1,4}
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q{1,2,3,4}
q{1,3,4}

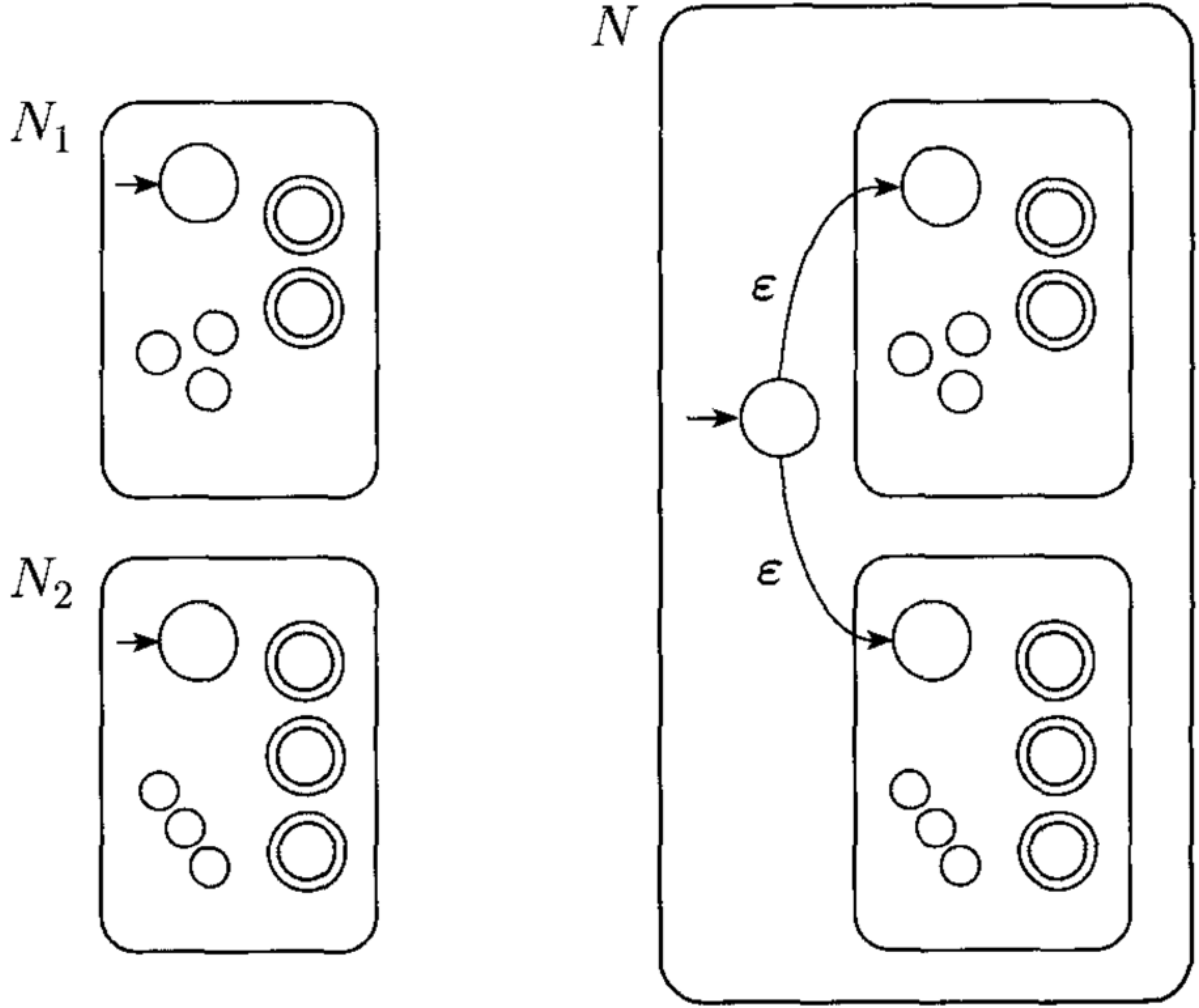
Regular Operations : Kleene's theorem (NFA)

Regular Operations : Kleene's theorem

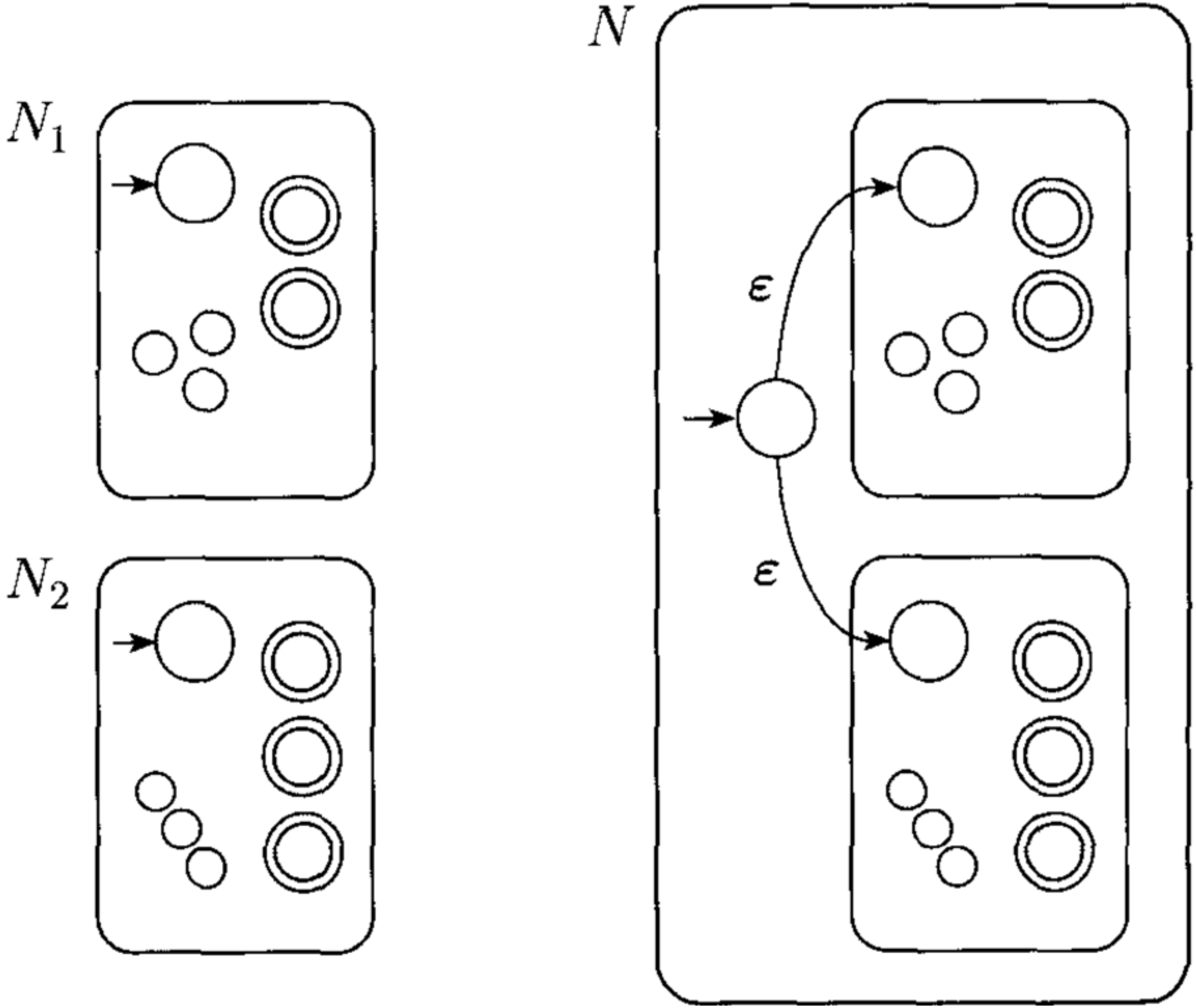
Regular Operations : Kleene's theorem

THEOREM 1.45 ....

The class of regular languages is closed under the union operation.

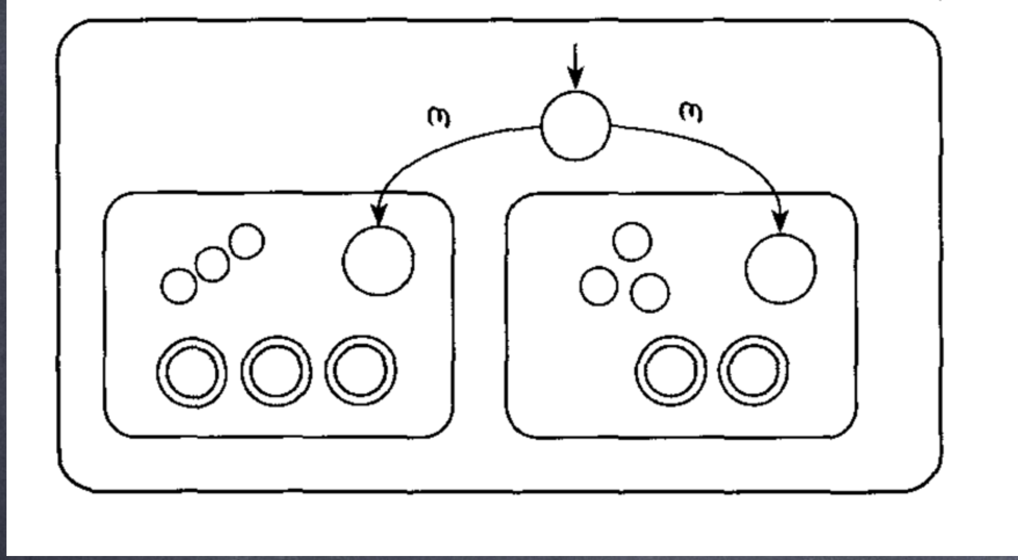


N_1
N_2
N
ε
ε

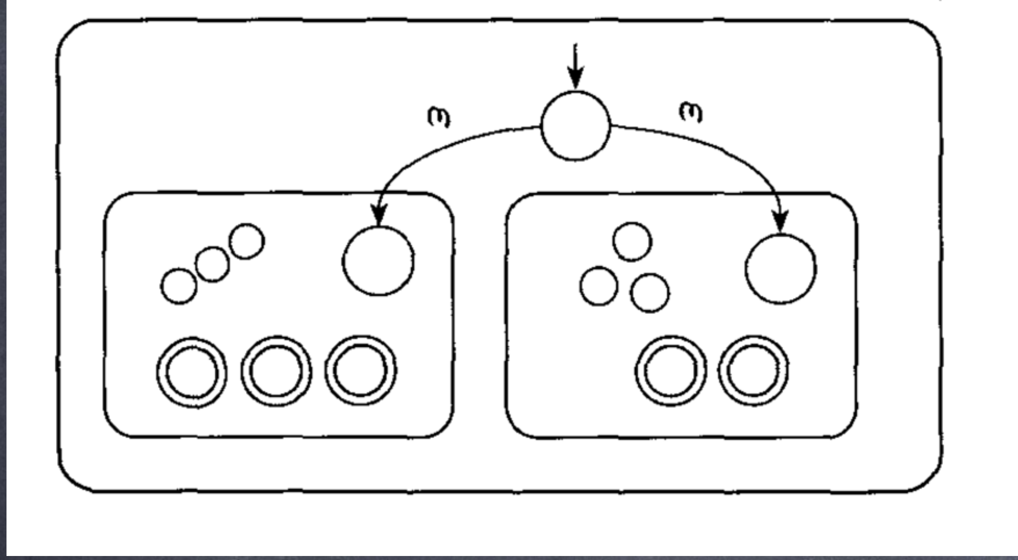


THEOREM 1.45

The class of regular languages is closed under the union operation.

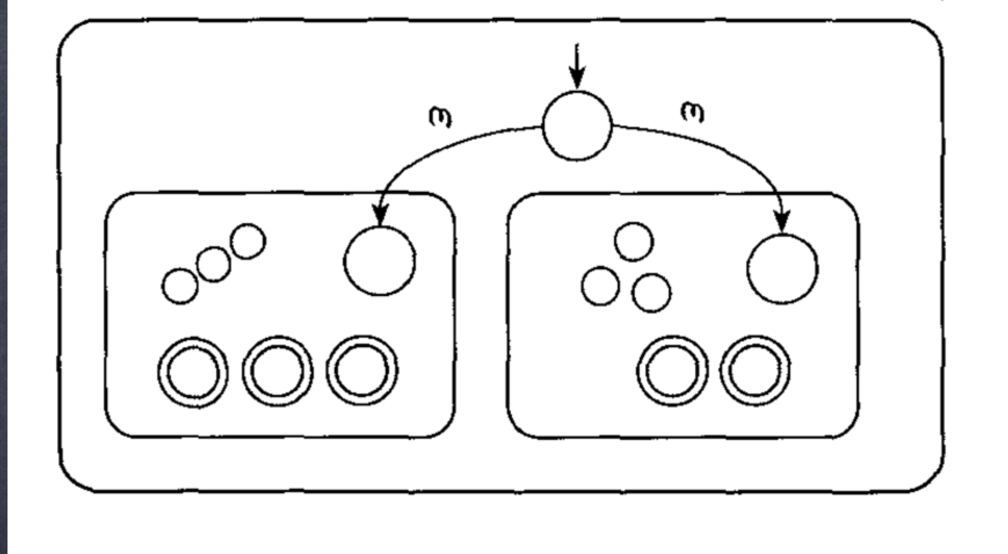


Kleene's theorem



Kleene's theorem

- Let $N_A=(Q_A,\Sigma,\delta_A,q_{0A},F_A)$ be a NFA accepting L_A and $N_B=(Q_B,\Sigma,\delta_B,q_{0B},F_B)$ be a NFA accepting L_B $(Q_A \cap Q_B=\emptyset)$.



Kleene's theorem

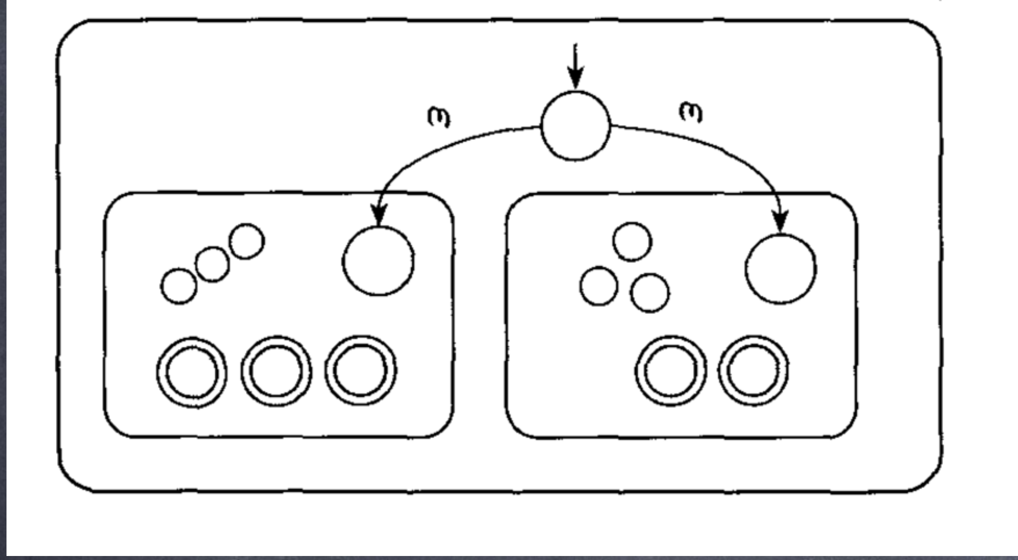
- Let $N_A=(Q_A,\Sigma,\delta_A,q_{0A},F_A)$ be a NFA accepting L_A and $N_B=(Q_B,\Sigma,\delta_B,q_{0B},F_B)$ be a NFA accepting L_B ($Q_A \cap Q_B=\varnothing$).

- Consider $N_U=(\{q_0\} \cup Q_A \cup Q_B, \Sigma, \delta_U, q_0, F_U)$ where

  $\delta_U(q_0,\varepsilon) = \{q_{0A},q_{0B}\}$, $\delta_U(q_0,a) = \varnothing$ for all $a \neq \varepsilon$,

  $\delta_U(q,a) = \delta_X(q,a)$ for all $q \in Q_X$, $X \in \{A,B\}$, and all a,

  $F_U = F_A \cup F_B$.



Kleene's theorem

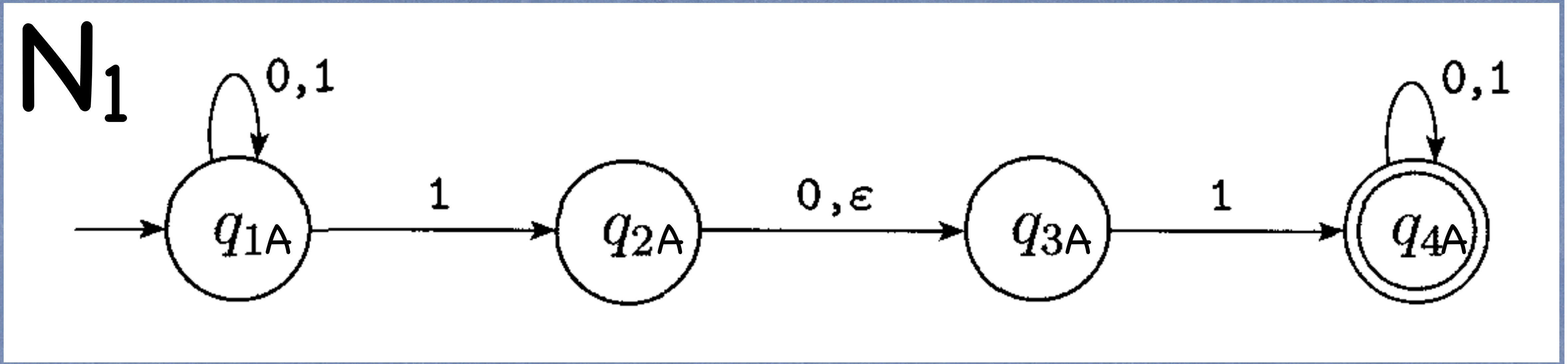
- Let $N_A=(Q_A,\Sigma,\delta_A,q_{0A},F_A)$ be a NFA accepting L_A and $N_B=(Q_B,\Sigma,\delta_B,q_{0B},F_B)$ be a NFA accepting L_B ($Q_A \cap Q_B=\emptyset$).
- Consider $N_U=(\{q_0\} \cup Q_A \cup Q_B, \Sigma, \delta_U, q_0, F_U)$ where

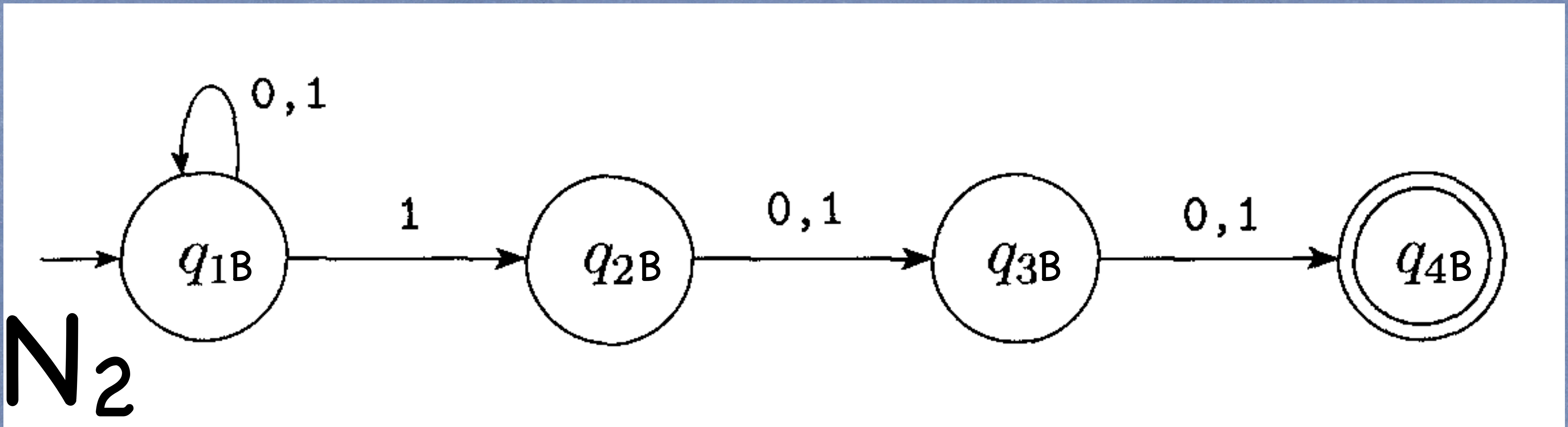
  $\delta_U(q_0,\varepsilon) = \{q_{0A},q_{0B}\}$, $\delta_U(q_0,a) = \emptyset$ for all $a \neq \varepsilon$,

  $\delta_U(q,a) = \delta_X(q,a)$ for all $q \in Q_X$, $X \in \{A,B\}$, and all a,

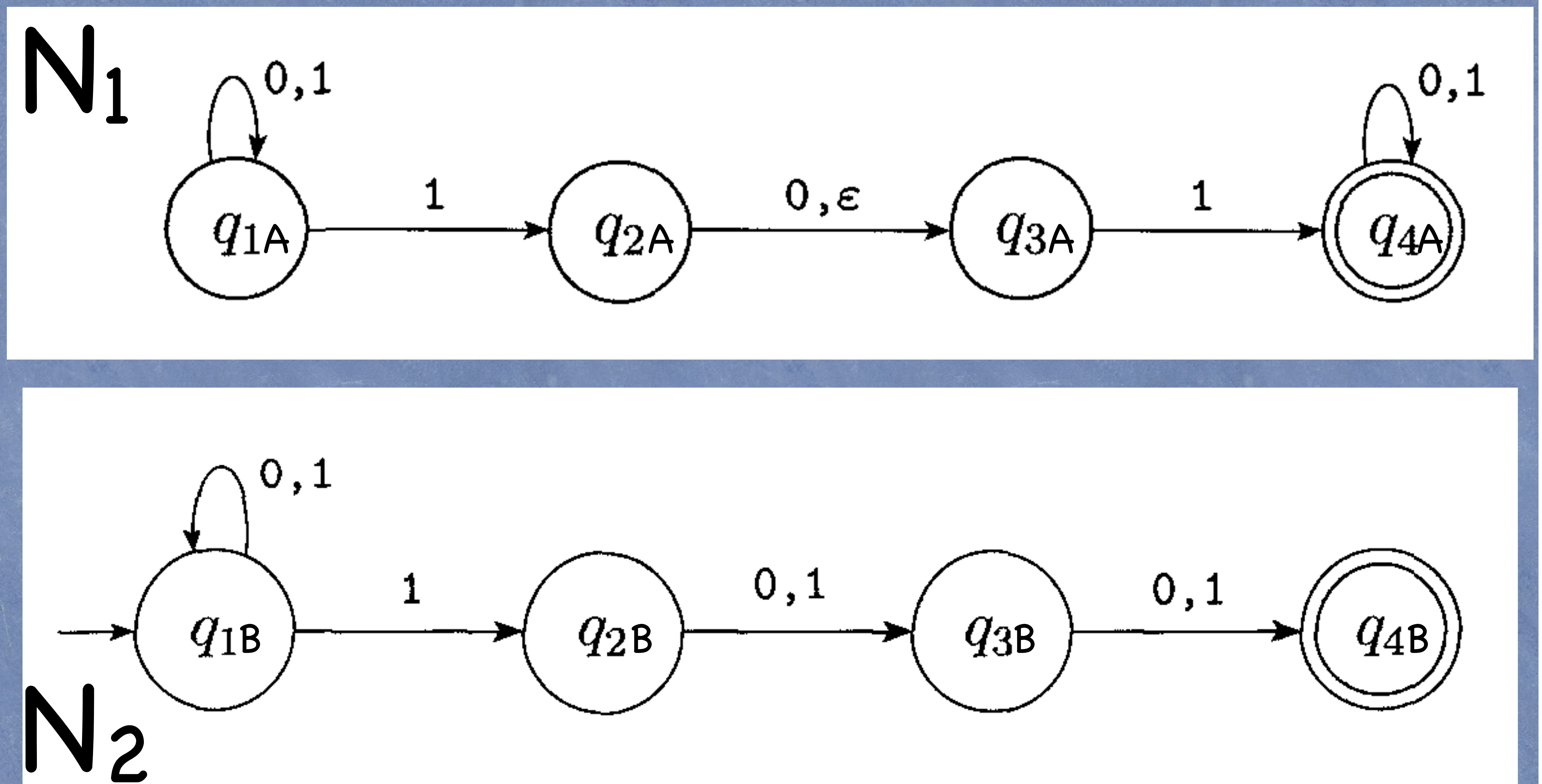
  $F_U = F_A \cup F_B$.
- $L_U = L_A \cup L_B$.

Example

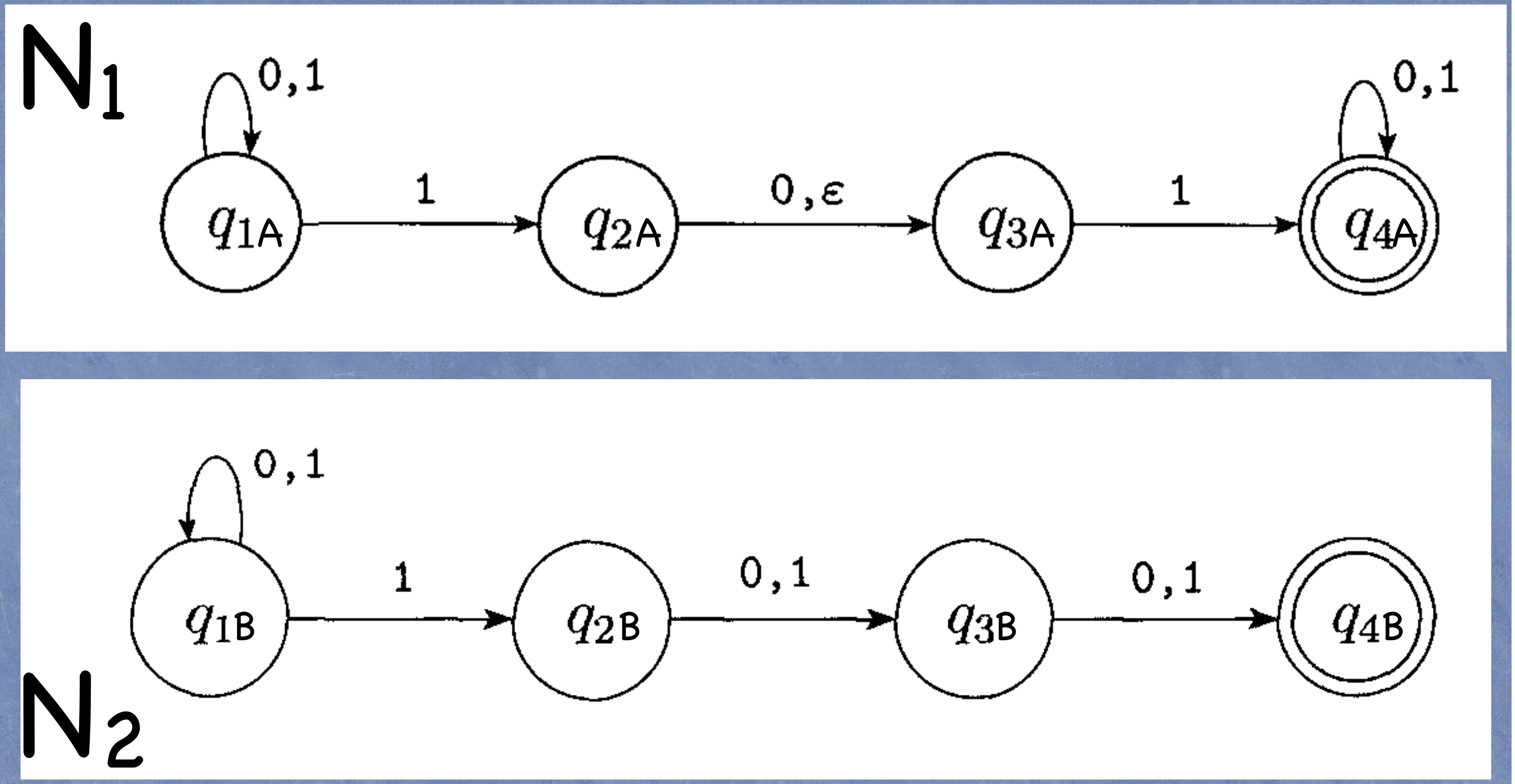




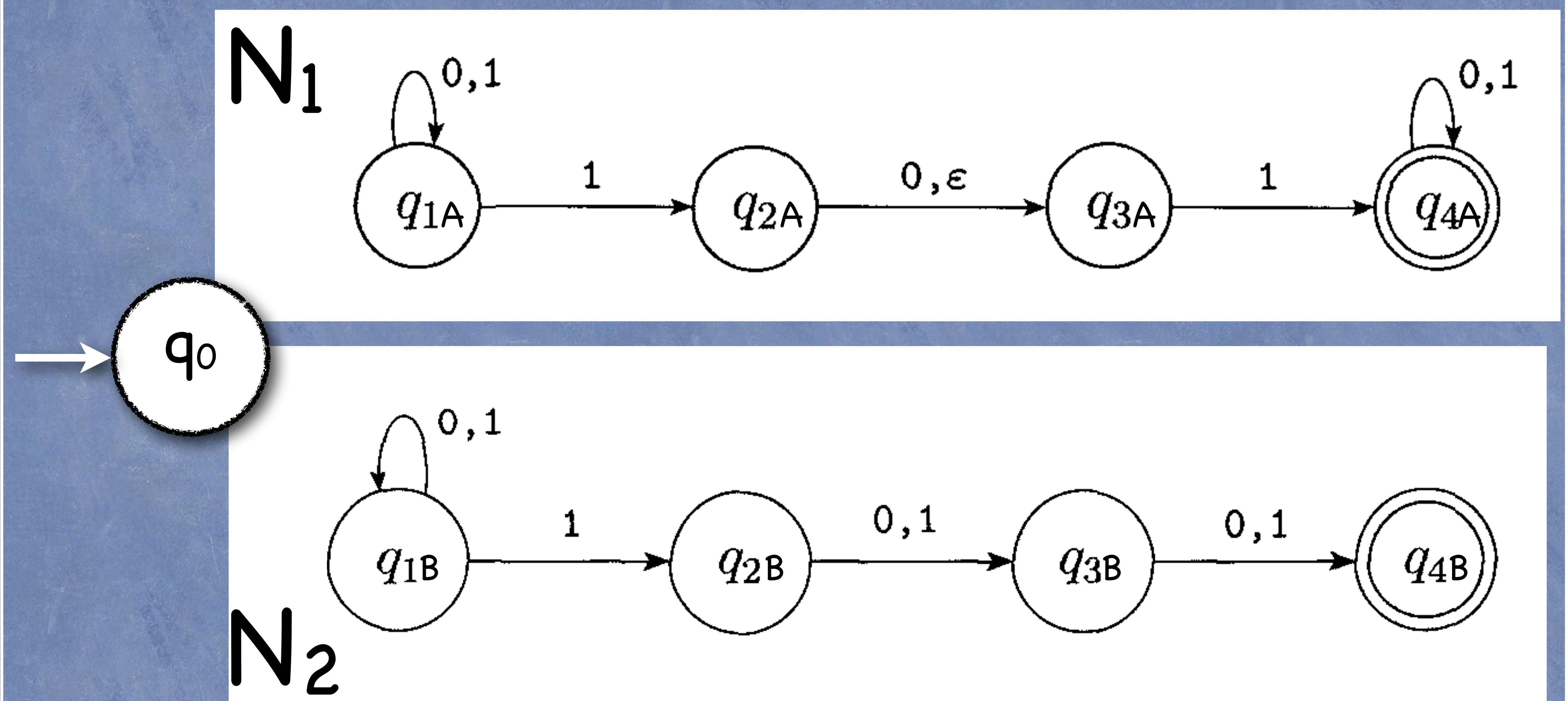
Example



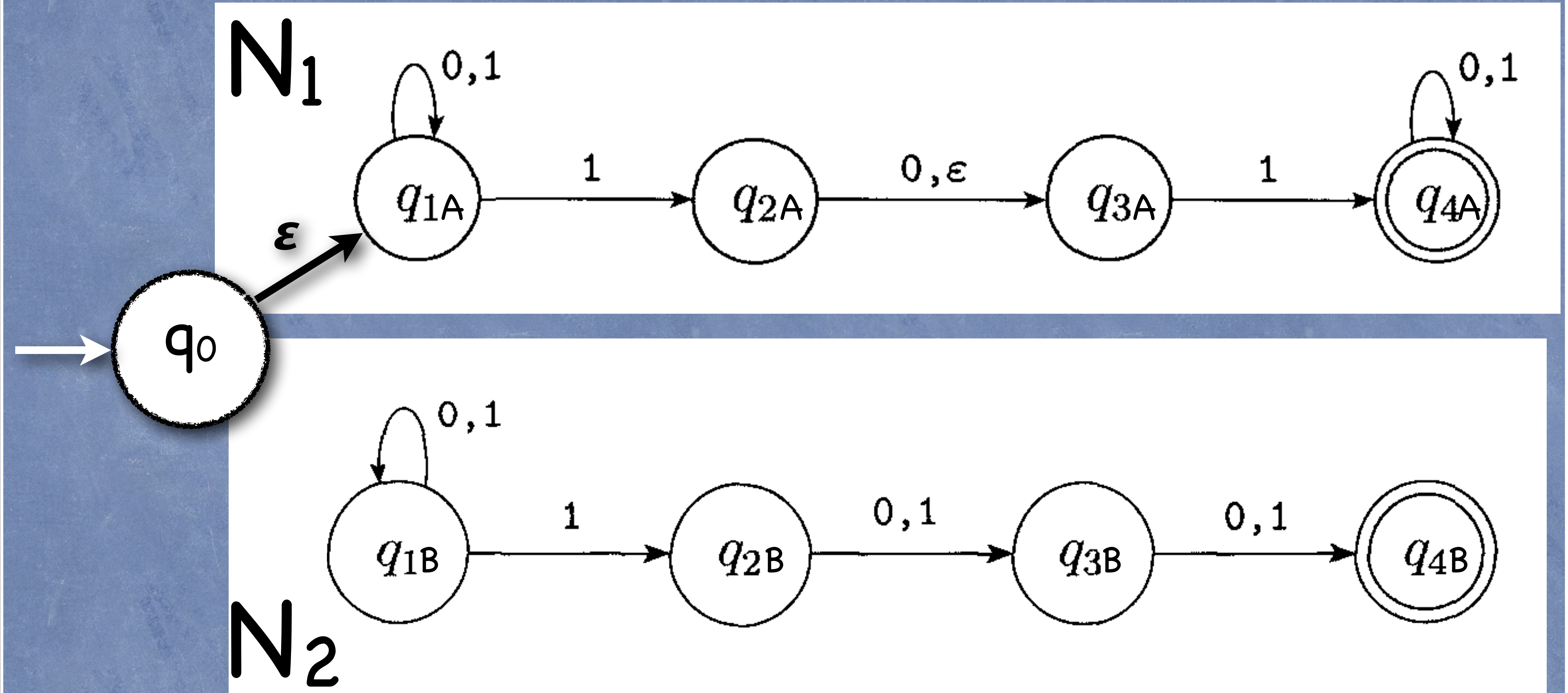
Example



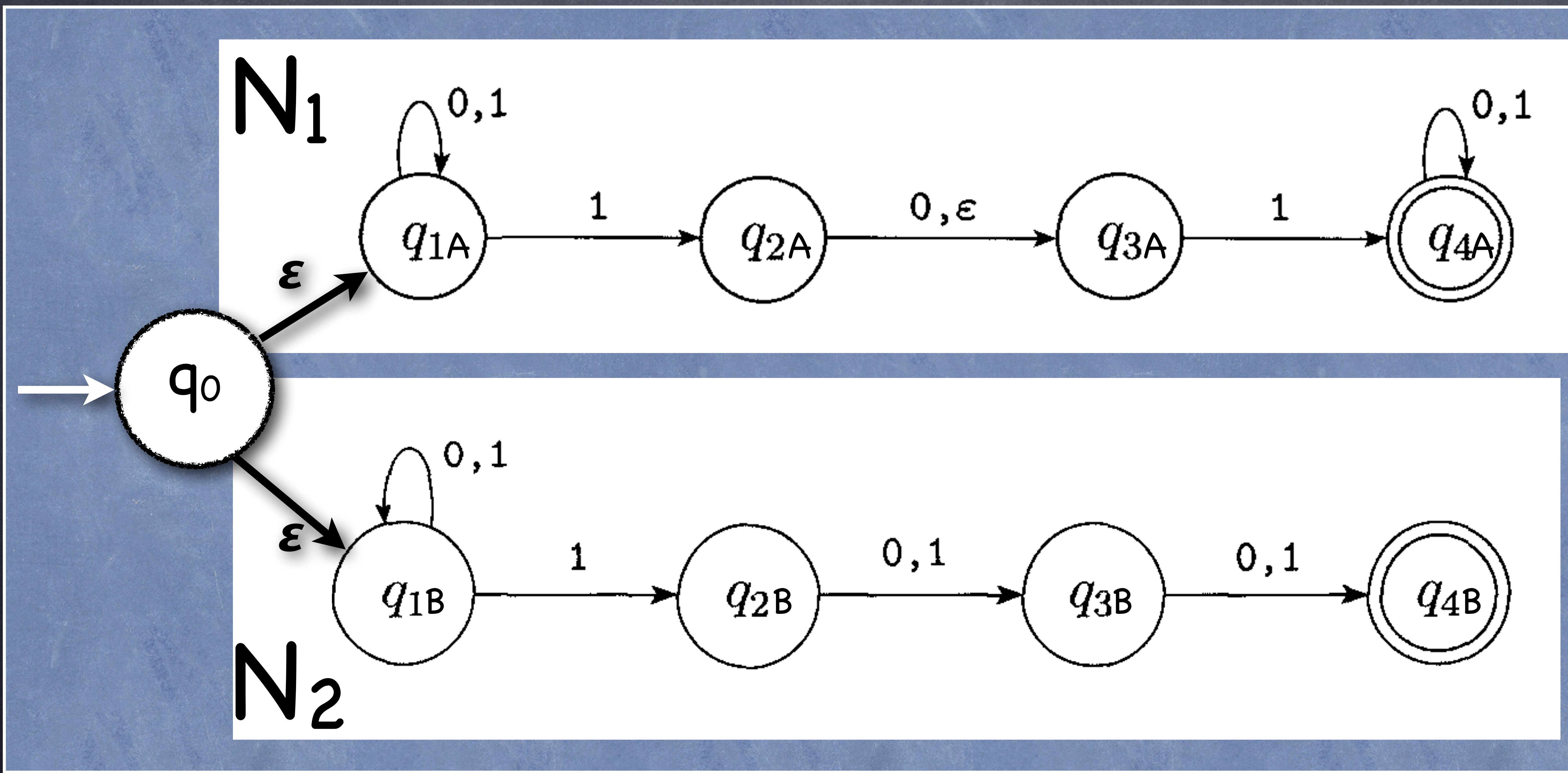
Example



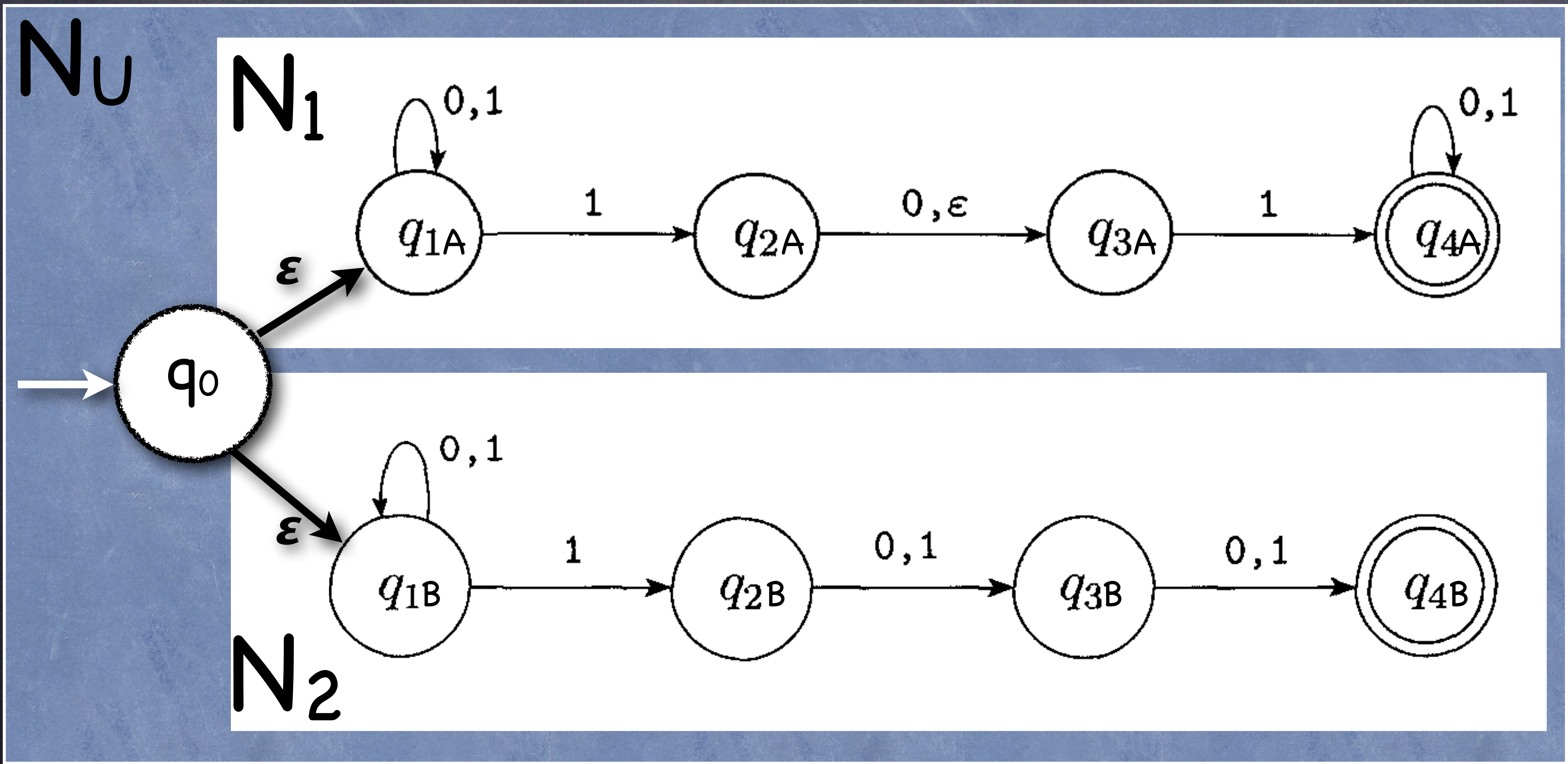
Example



Example



Example

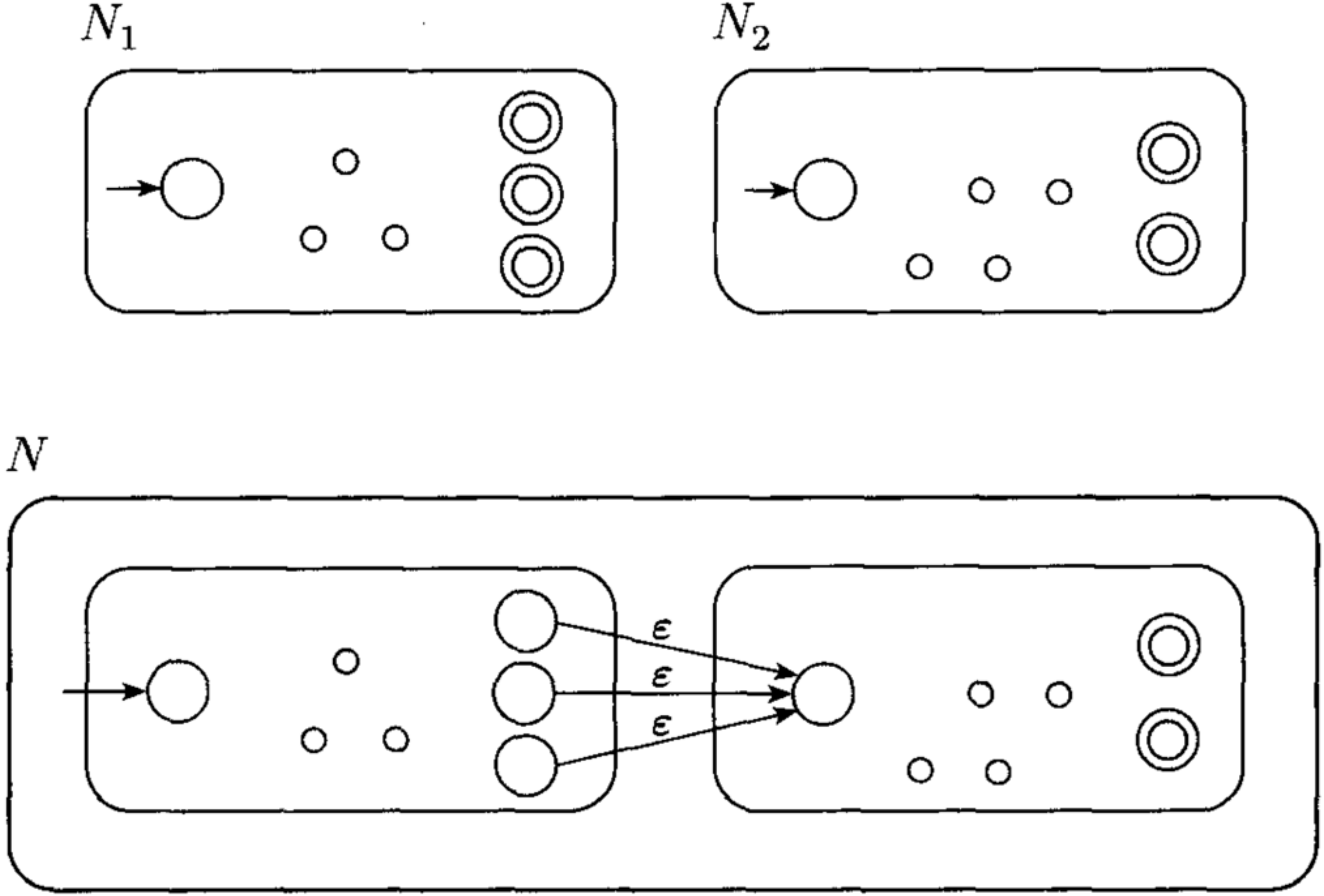


Regular Operations : Kleene’s theorem

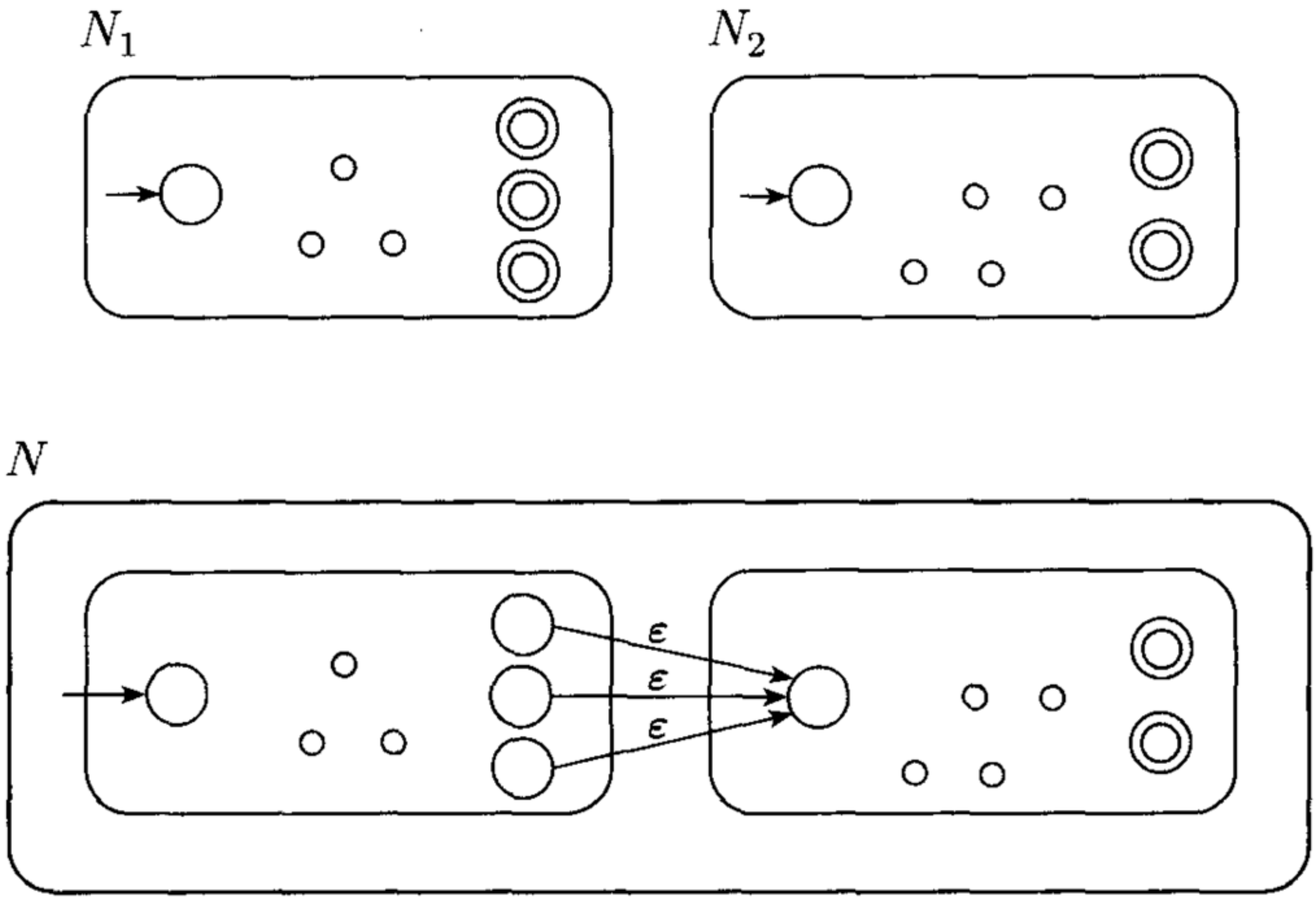
Regular Operations : Kleene's theorem

THEOREM 1.47 ....

The class of regular languages is closed under the concatenation operation.

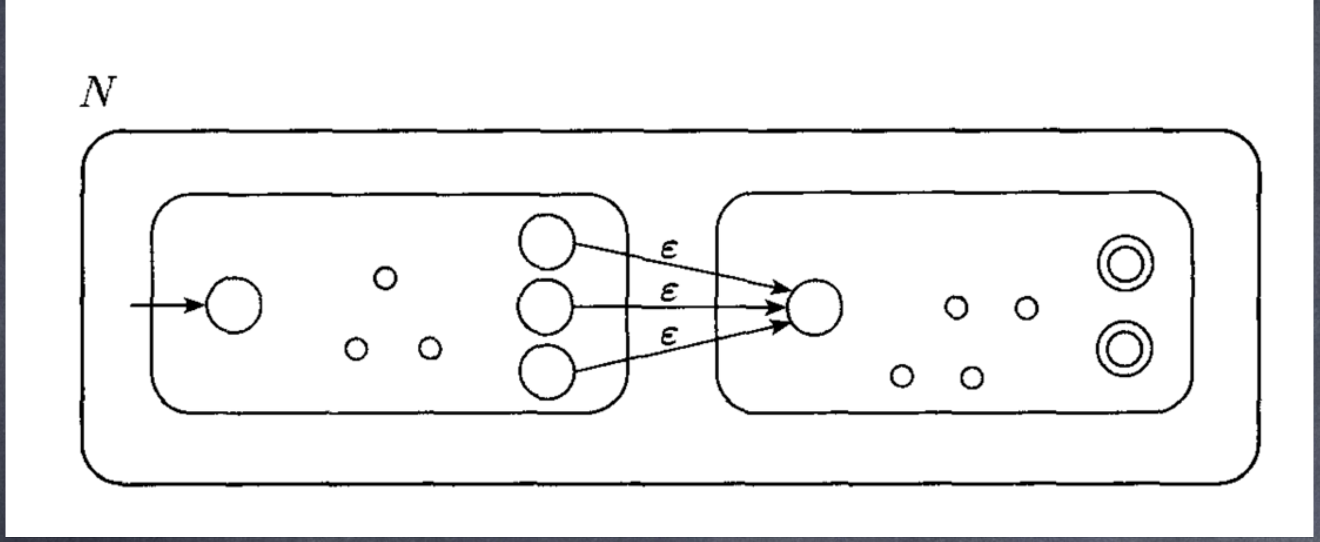


N_1
N_2
N
ε
ε
ε

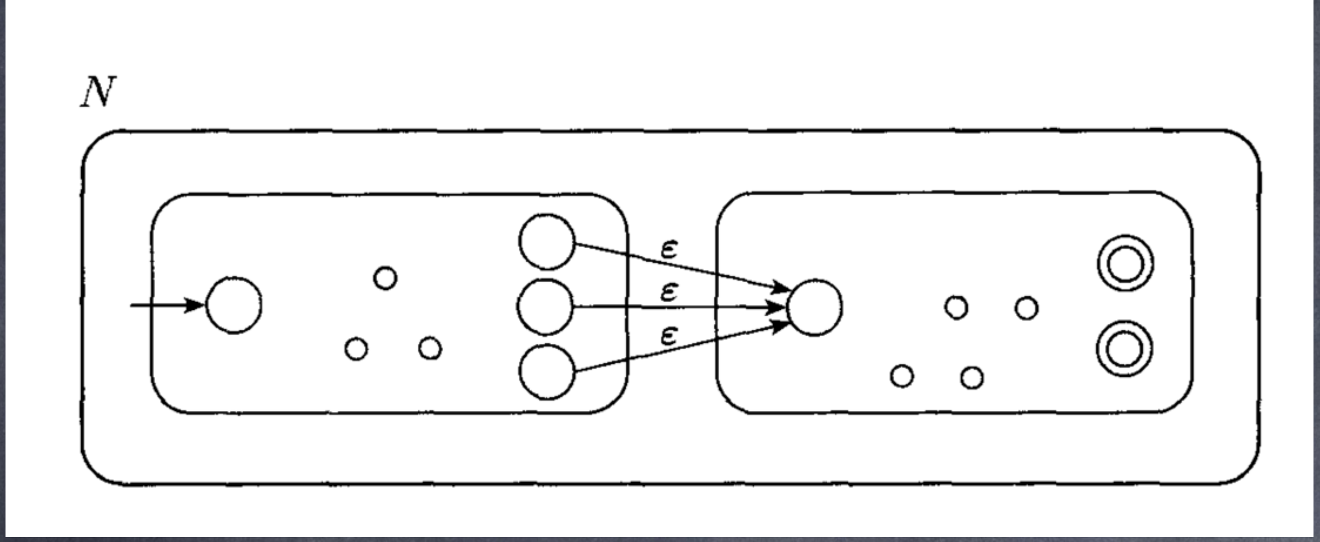


THEOREM 1.47

The class of regular languages is closed under the concatenation operation.

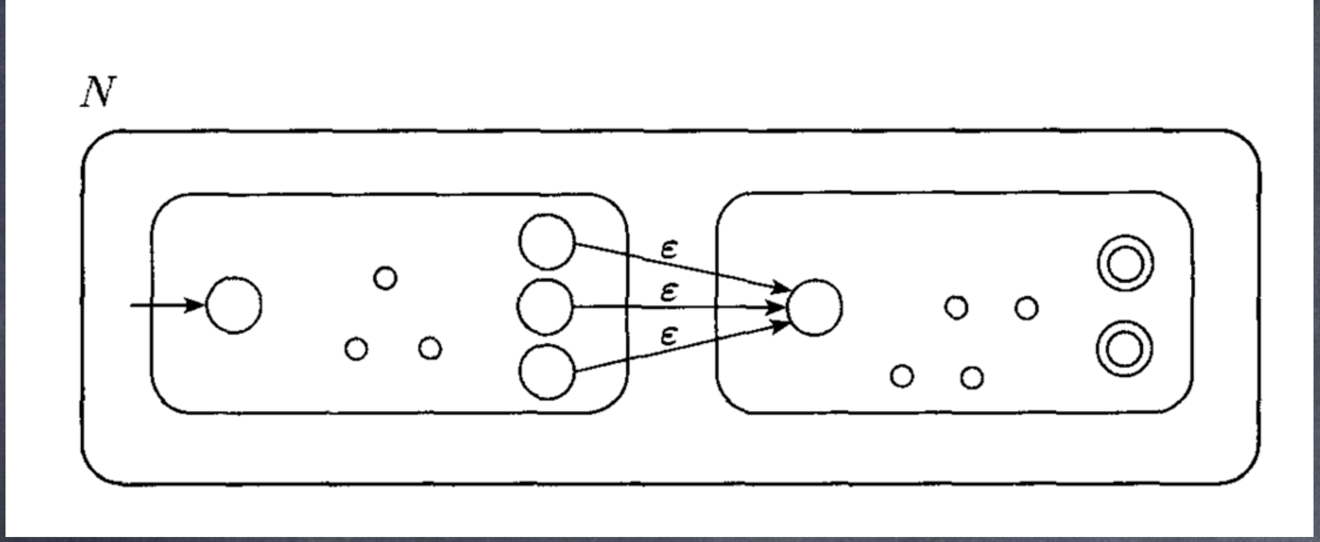


Kleene's theorem



Kleene's theorem

- Let $N_A=(Q_A,\Sigma,\delta_A,q_{0A},F_A)$ be a NFA accepting L_A and $N_B=(Q_B,\Sigma,\delta_B,q_{0B},F_B)$ be a NFA accepting L_B ($Q_A \cap Q_B=\varnothing$).

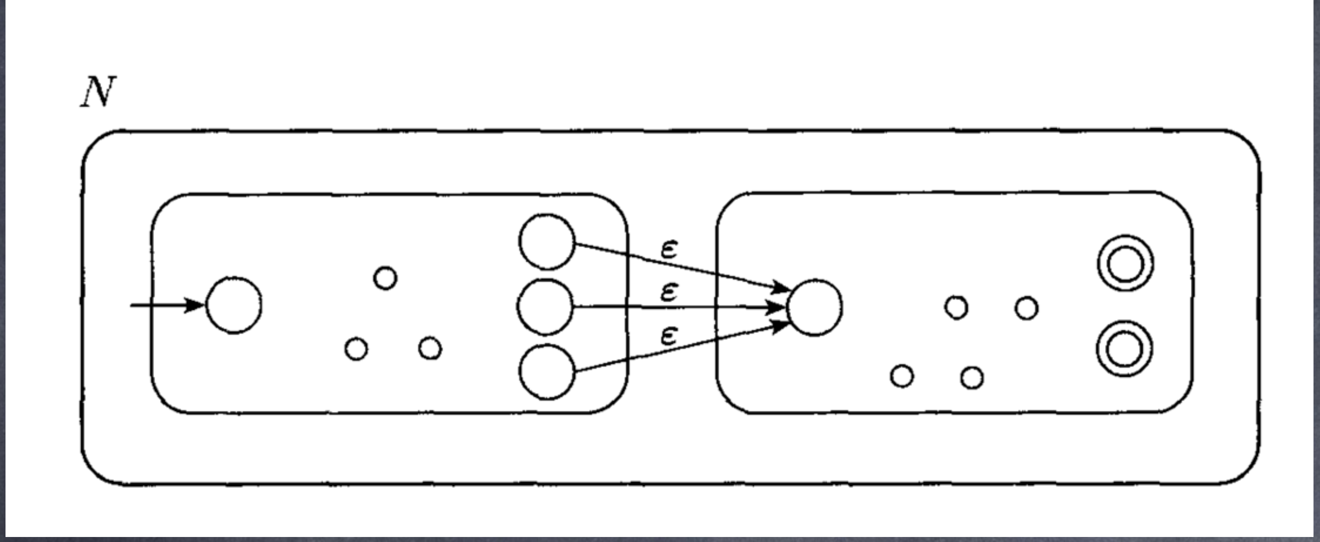


Kleene's theorem

- Let $N_A=(Q_A,\Sigma,\delta_A,q_{0A},F_A)$ be a NFA accepting L_A and $N_B=(Q_B,\Sigma,\delta_B,q_{0B},F_B)$ be a NFA accepting L_B ($Q_A \cap Q_B=\emptyset$).

- Consider $N_C=(Q_A \cup Q_B ,\Sigma,\delta_C,q_{0A},F_B)$ where

$$\delta_C(q,a) = \delta_B(q,a) \text{ for all } q\in Q_B, \text{ all } a,$$
$$\delta_C(q,a) = \delta_A(q,a) \text{ for all } q\in Q_A, \text{ all } a\neq\varepsilon,$$
$$\delta_C(q,\varepsilon) = \delta_A(q,\varepsilon) \text{ for all } q\in Q_A\setminus F_A,$$
$$\delta_C(q,\varepsilon) = \delta_A(q,\varepsilon)\cup\{q_{0B}\} \text{ for all } q\in F_A.$$



Kleene's theorem

- Let $N_A=(Q_A,\Sigma,\delta_A,q_{0A},F_A)$ be a NFA accepting L_A and $N_B=(Q_B,\Sigma,\delta_B,q_{0B},F_B)$ be a NFA accepting L_B ($Q_A \cap Q_B=\emptyset$).

- Consider $N_C=(Q_A \cup Q_B ,\Sigma,\delta_C,q_{0A},F_B)$ where

  $\delta_C(q,a) = \delta_B(q,a)$ for all $q \in Q_B$, all a,

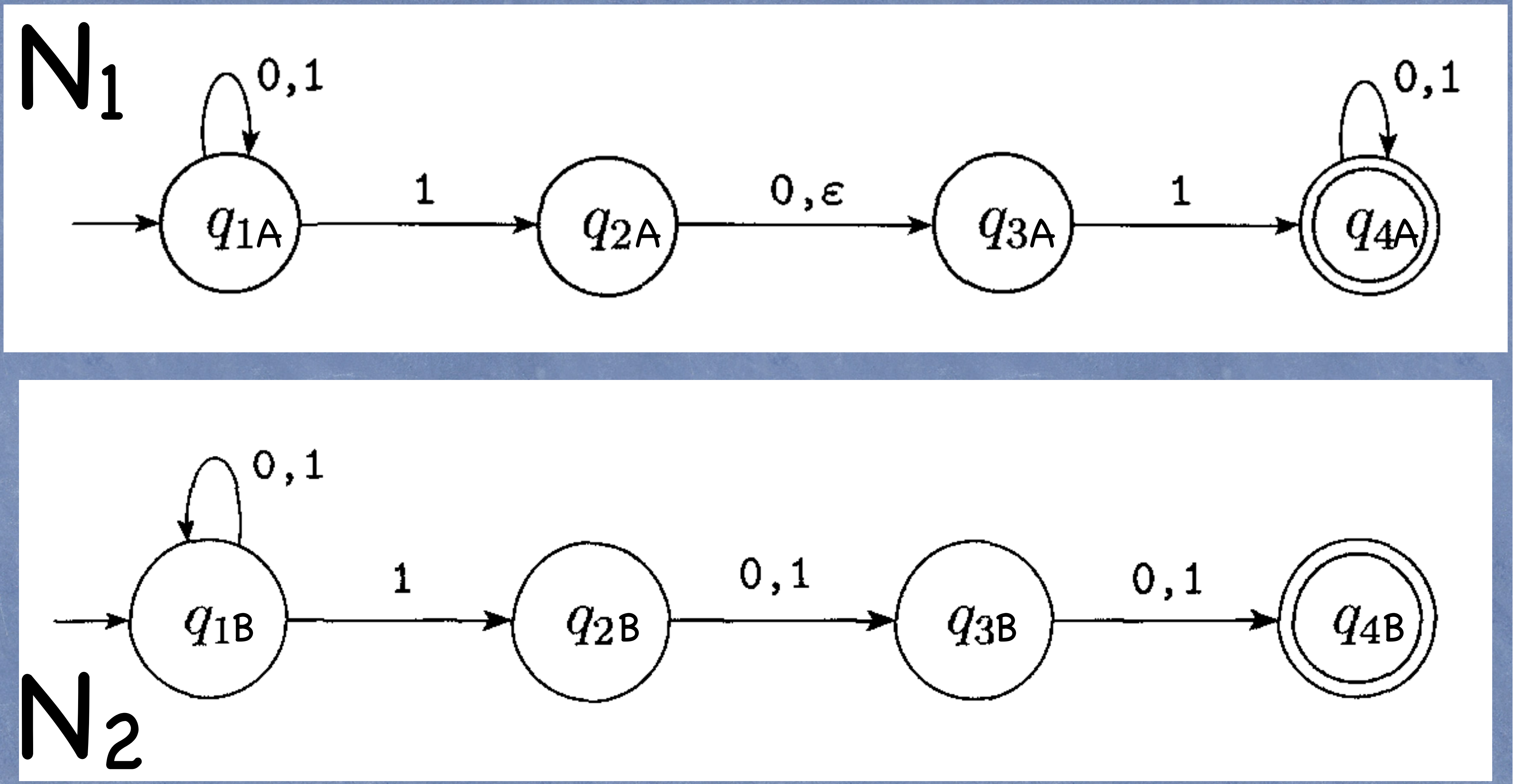
  $\delta_C(q,a) = \delta_A(q,a)$ for all $q \in Q_A$, all $a \neq \varepsilon$,

  $\delta_C(q,\varepsilon) = \delta_A(q,\varepsilon)$ for all $q \in Q_A \setminus F_A$,

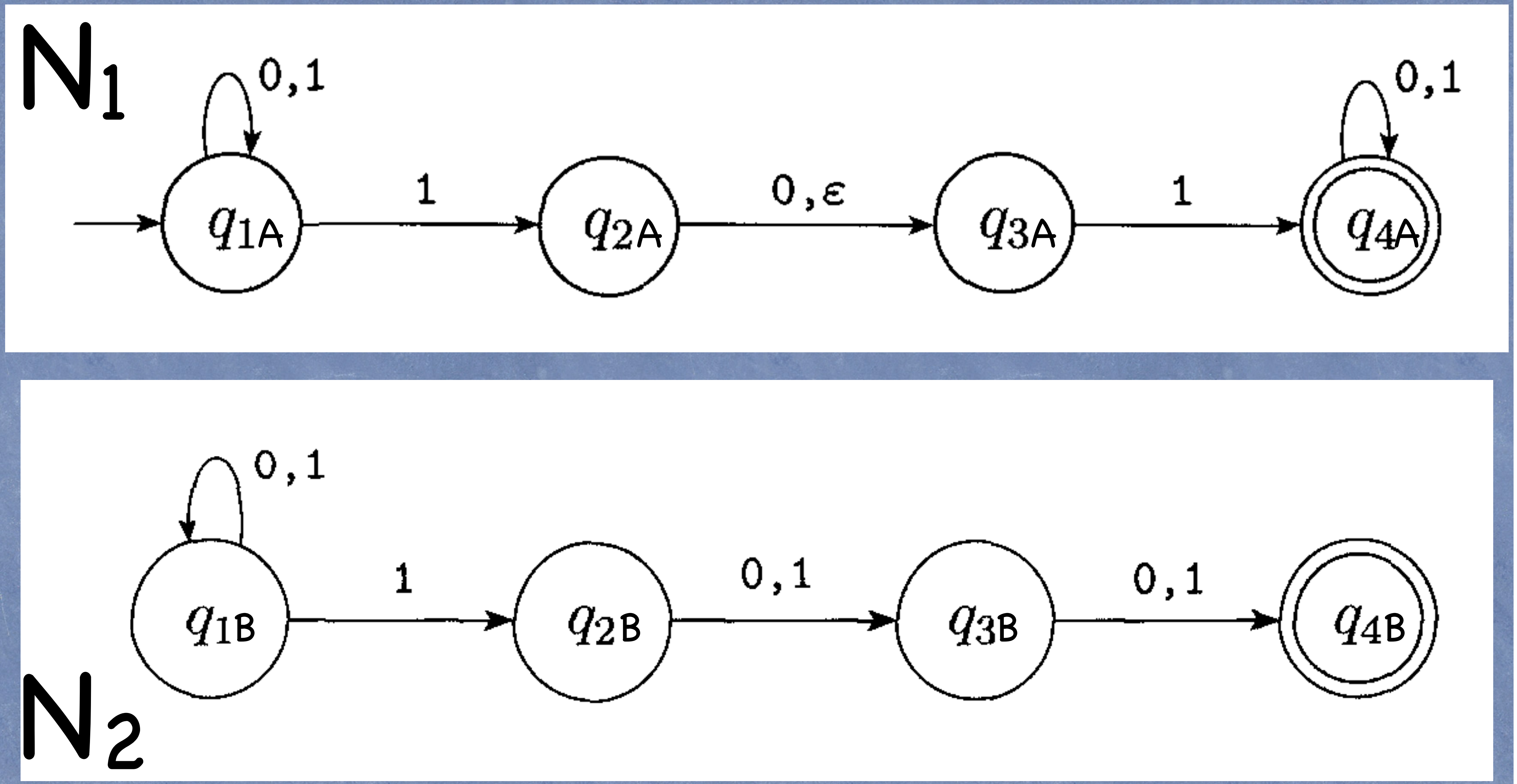
  $\delta_C(q,\varepsilon) = \delta_A(q,\varepsilon) \cup \{q_{0B}\}$ for all $q \in F_A$.

- $L_C = L_A \circ L_B$.

Example



Example



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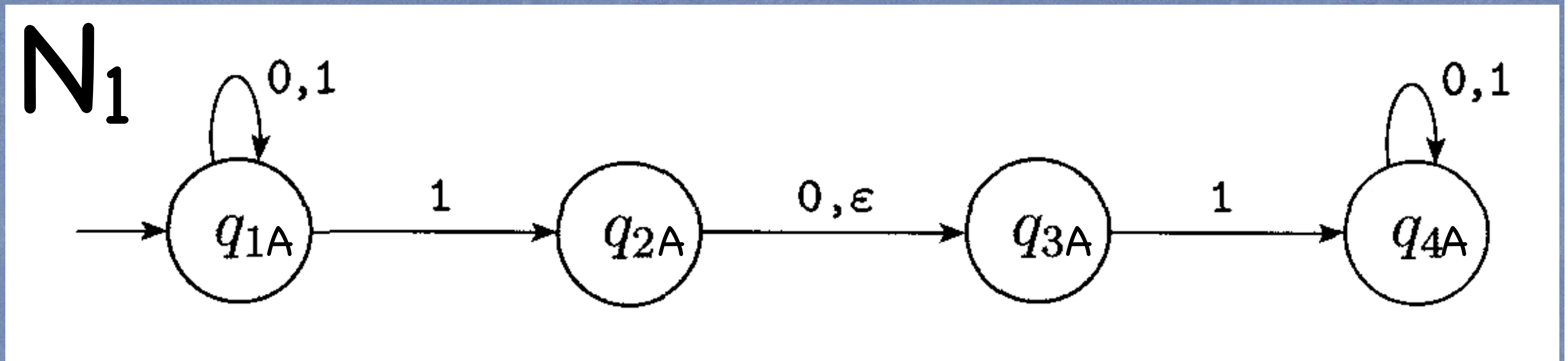

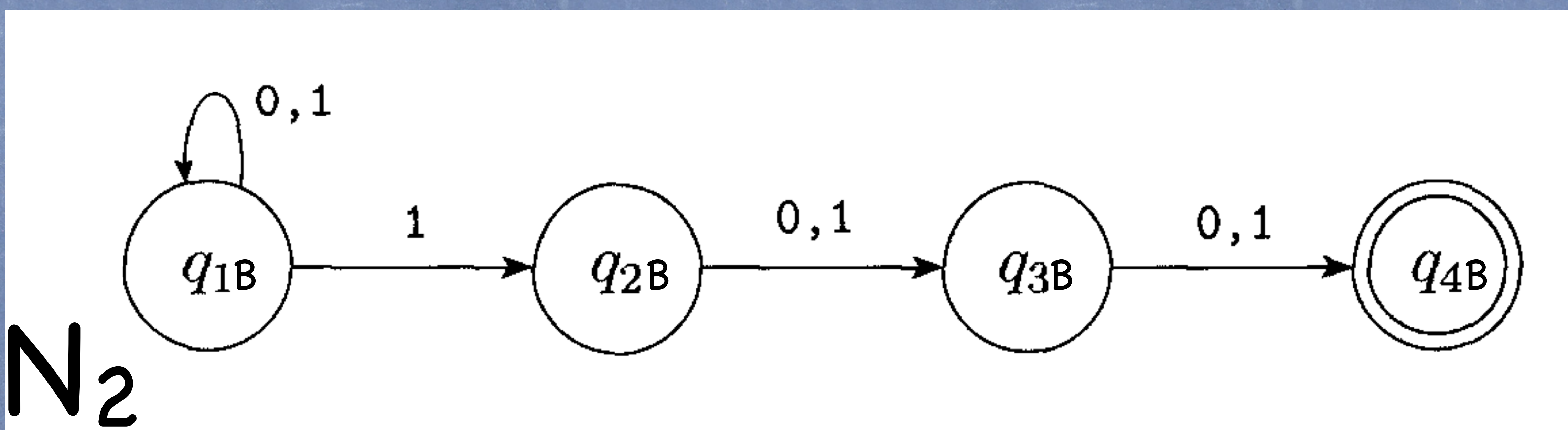

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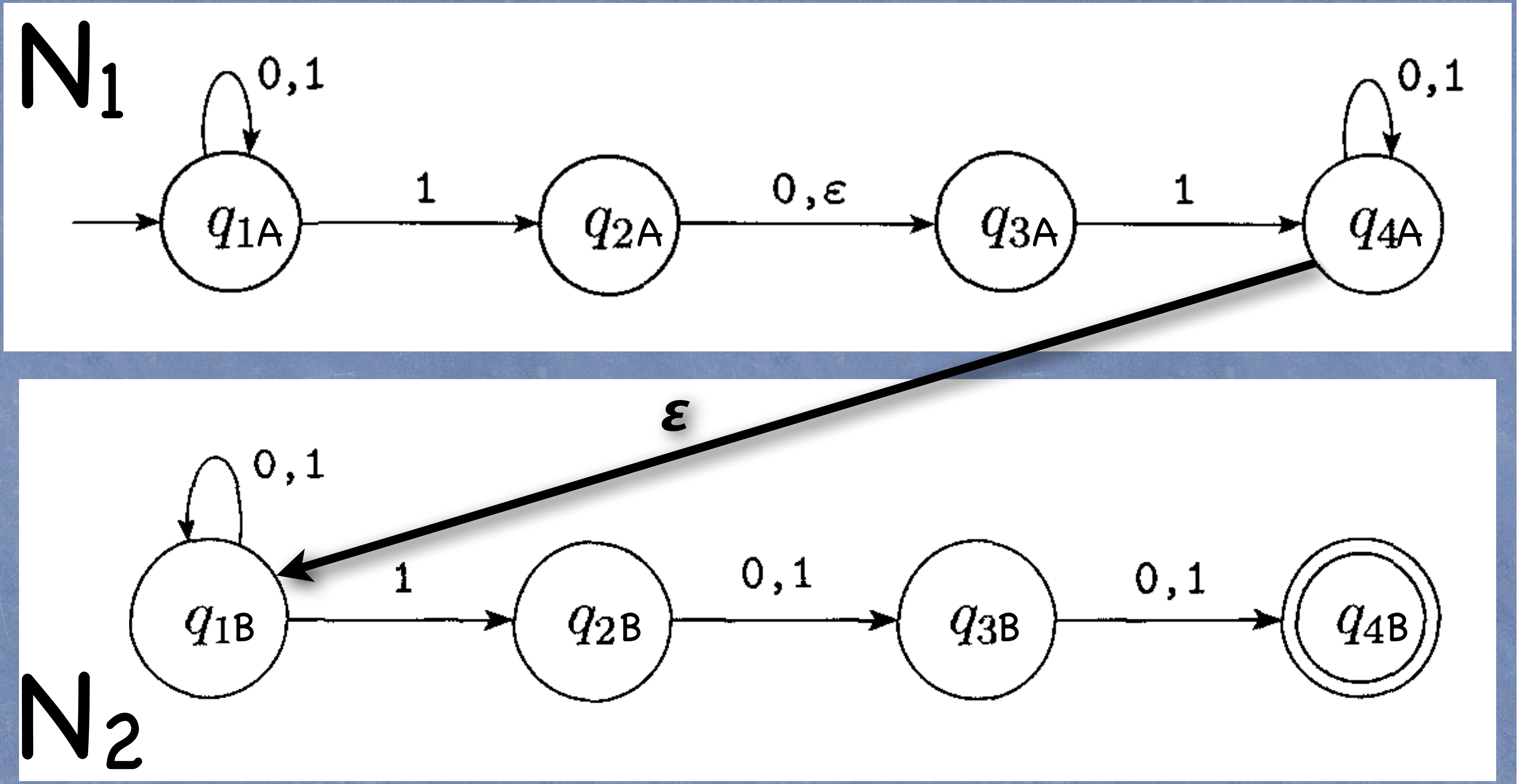
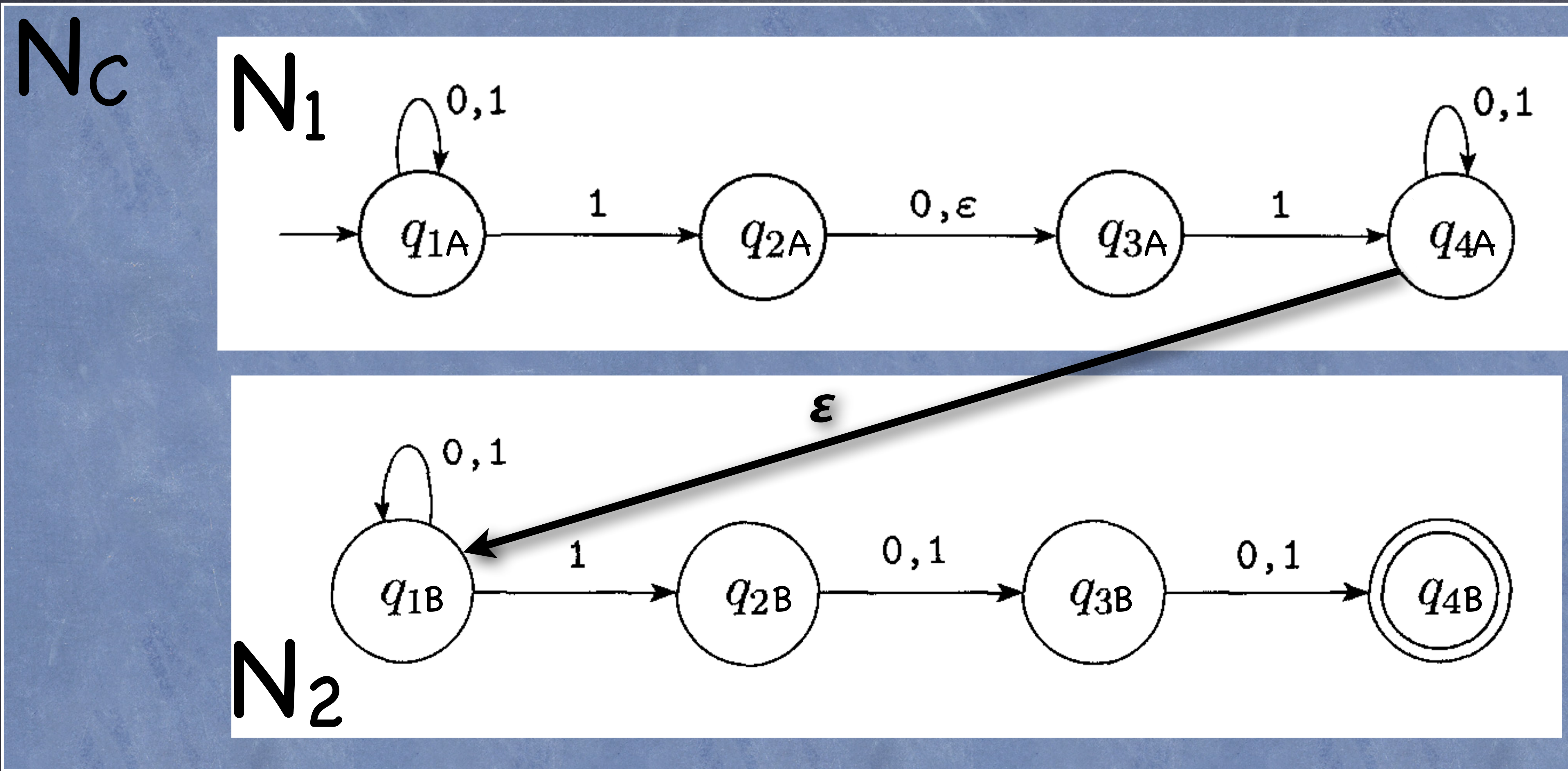


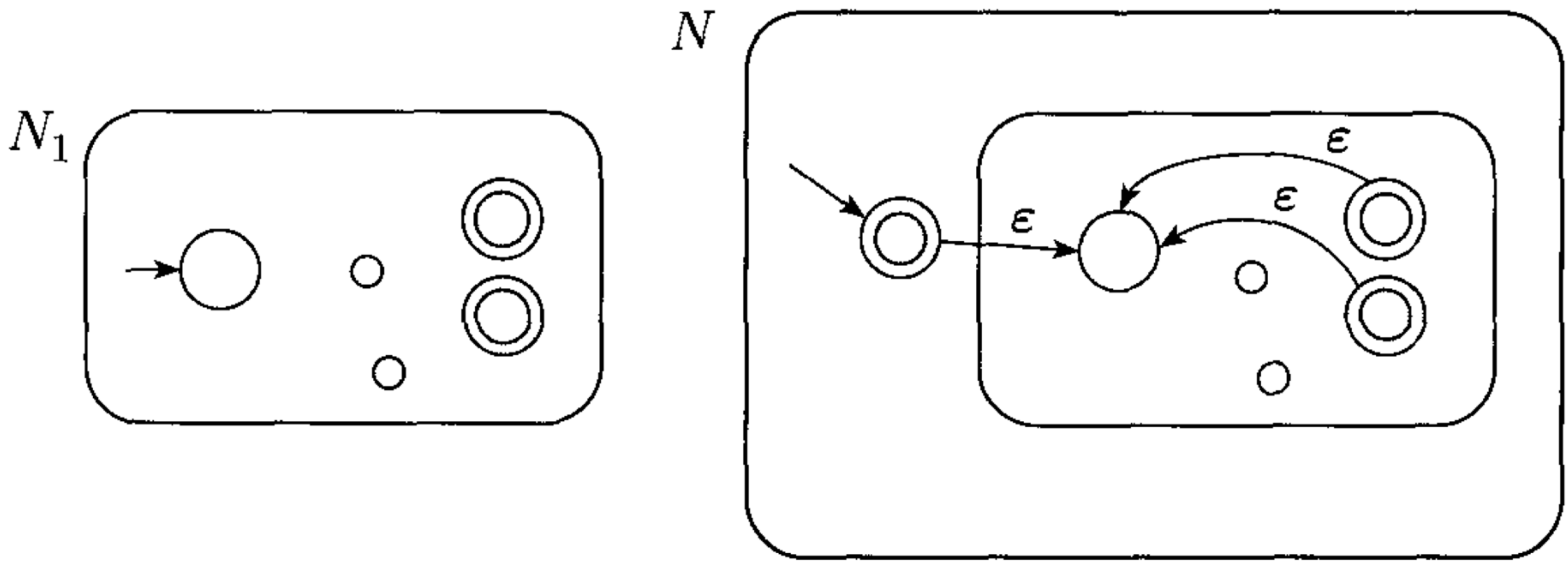
Example



Regular Operations : Kleene's theorem

THEOREM 1.49 ....

The class of regular languages is closed under the star operation.



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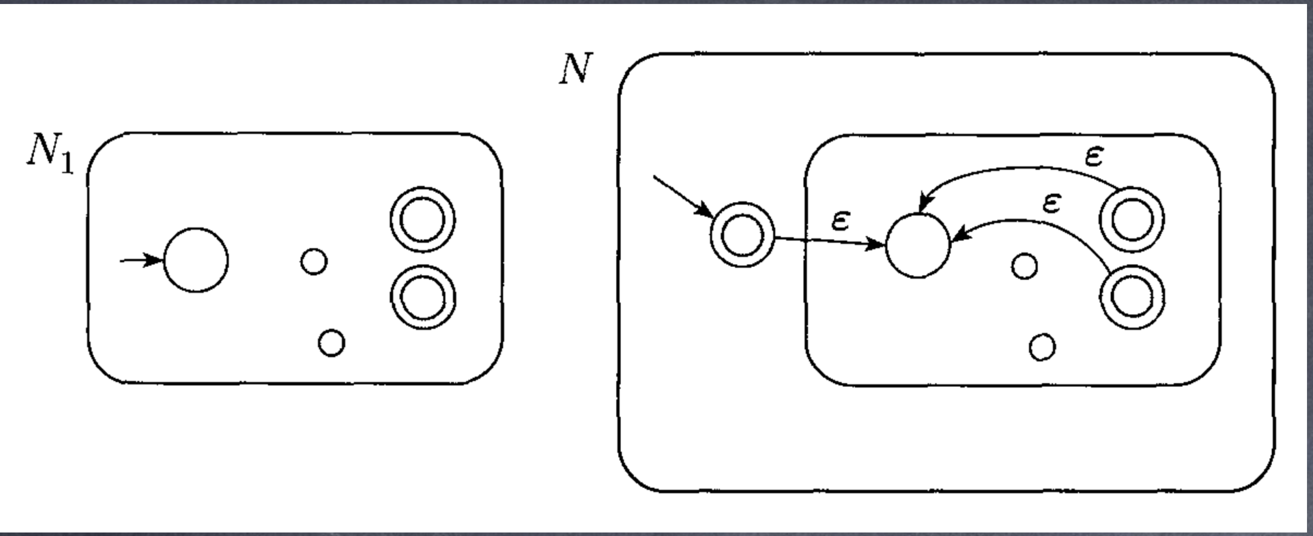
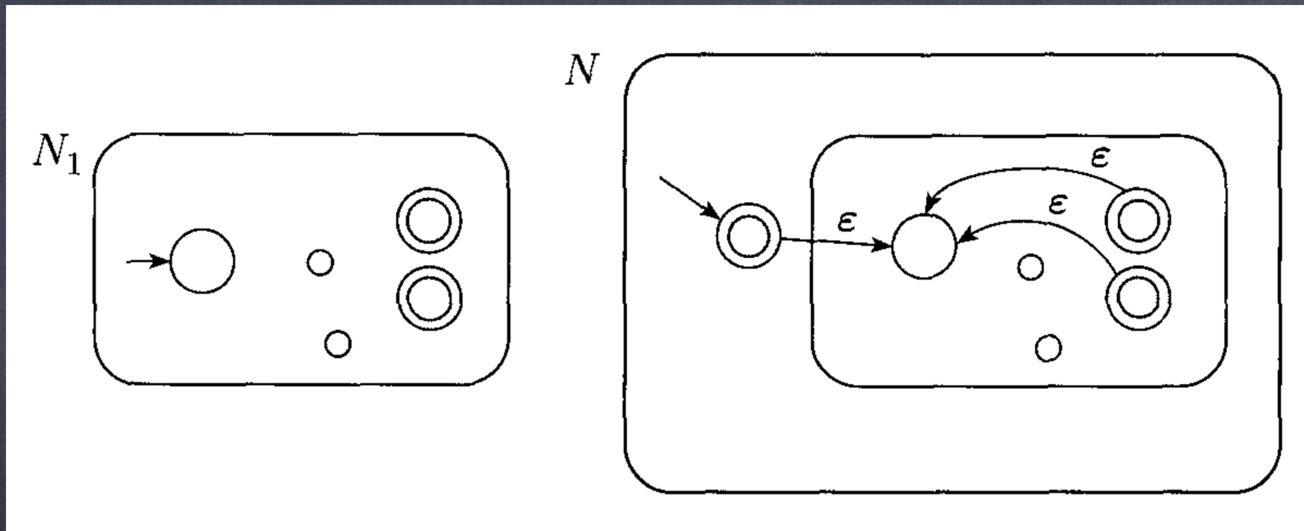
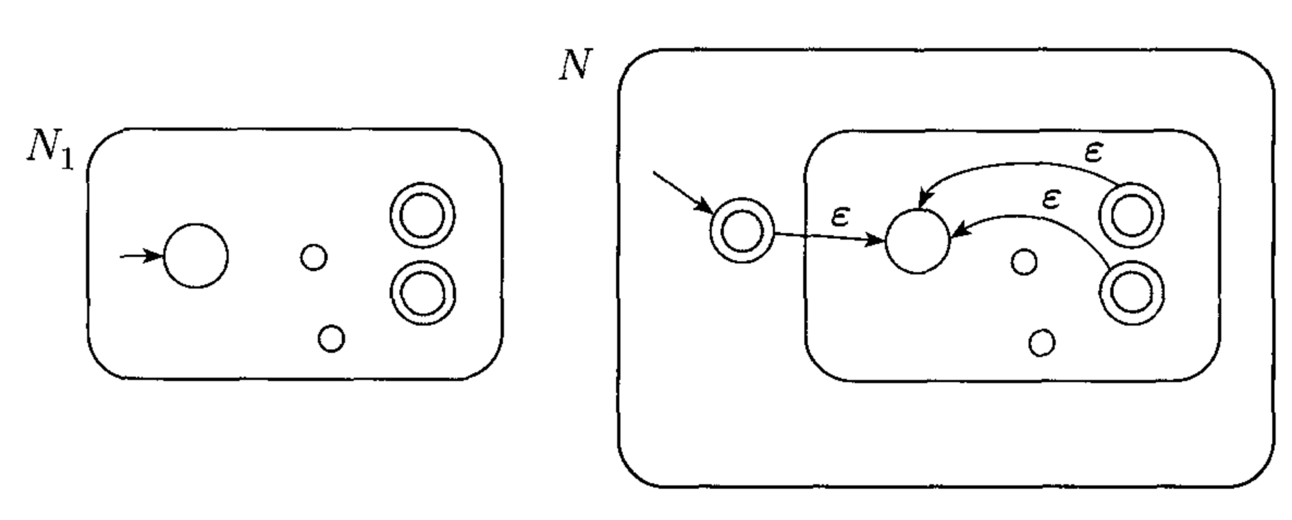


Kleene's theorem



Kleene's theorem

- Let $N_A=(Q_A,\Sigma,\delta_A,q_{0A},F_A)$ be a NFA accepting L_A.



Kleene's theorem

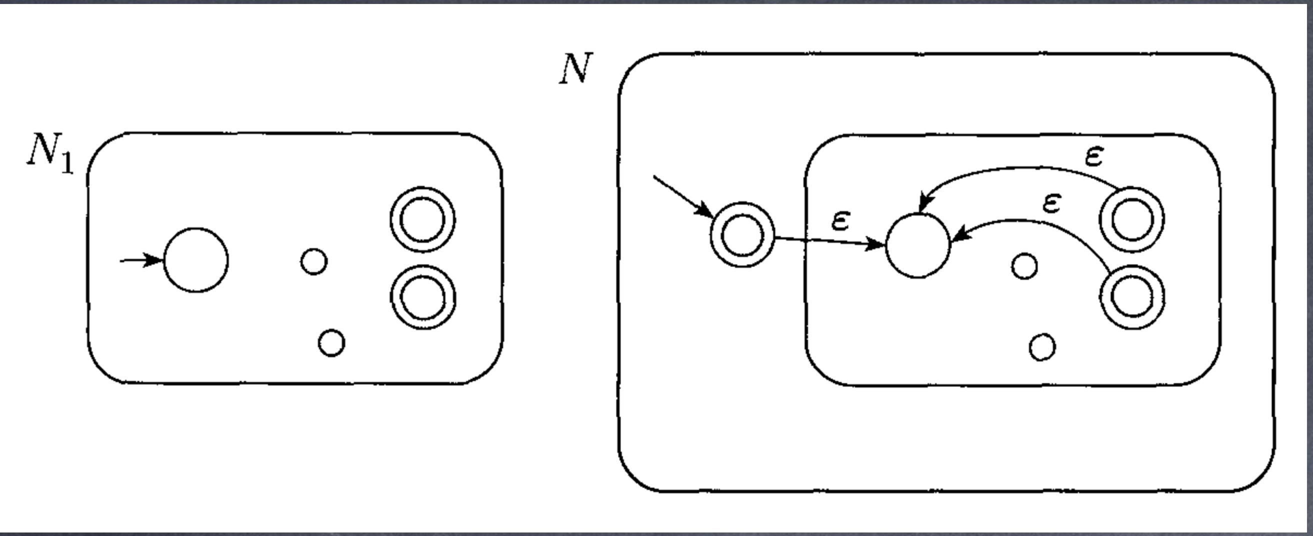
- Let $N_A=(Q_A,\Sigma,\delta_A,q_{0A},F_A)$ be a NFA accepting L_A.
- Consider $N_S=(\ Q_A \cup \{q_0\}\ ,\Sigma,\delta_S,q_0,F_A \cup \{q_0\})$ where

  $\delta_S(q_0,\varepsilon) = q_{0A}$, and $\delta_S(q_0,a) = \varnothing$ for all $a\neq\varepsilon$,

  $\delta_S(q,a) = \delta_A(q,a)$ for all $q\in Q_A \backslash F_A$, all a,

  $\delta_s(q,\varepsilon) = \delta_A(q,\varepsilon)\cup\{q_{0A}\}$ for all $q\in F_A$,

  $\delta_s(q,a) = \delta_A(q,a)$ for all $q\in F_A$, all $a\neq\varepsilon$.



Kleene's theorem

- Let $N_A=(Q_A,\Sigma,\delta_A,q_{0A},F_A)$ be a NFA accepting L_A.
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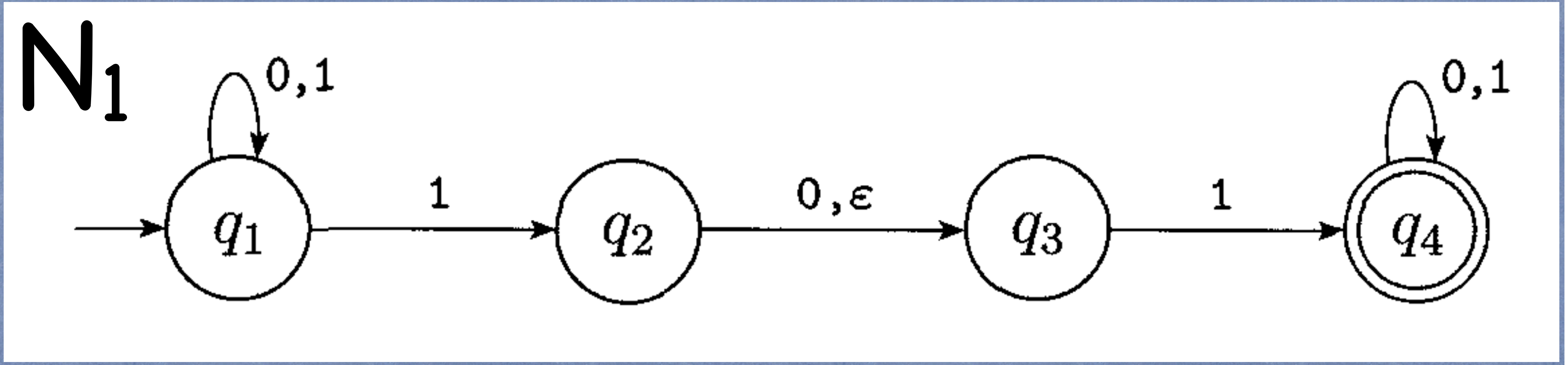
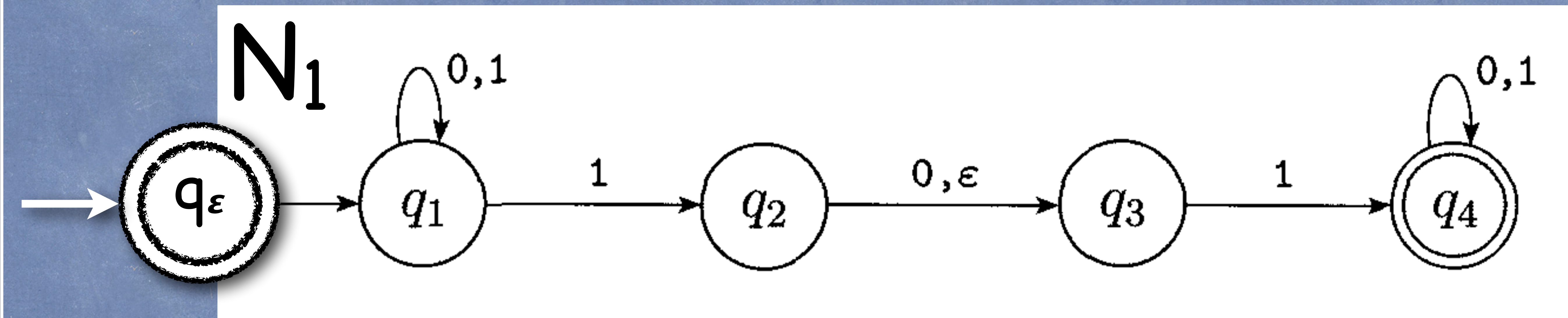
  $\delta_s(q,\varepsilon) = \delta_A(q,\varepsilon) \cup \{q_{0A}\}$ for all $q \in F_A$,

  $\delta_s(q,a) = \delta_A(q,a)$ for all $q \in F_A$, all $a \neq \varepsilon$.
- $L_S = (L_A)^*$.

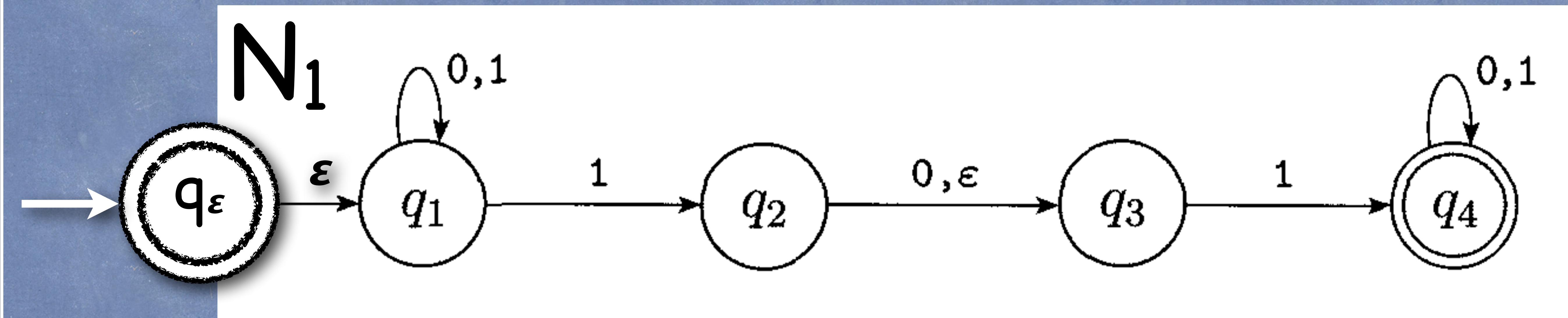
Example



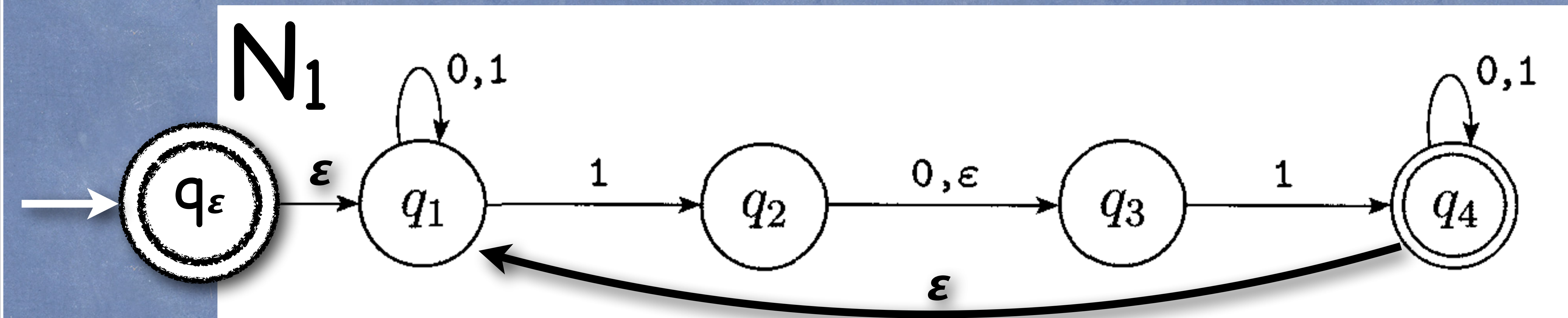
Example



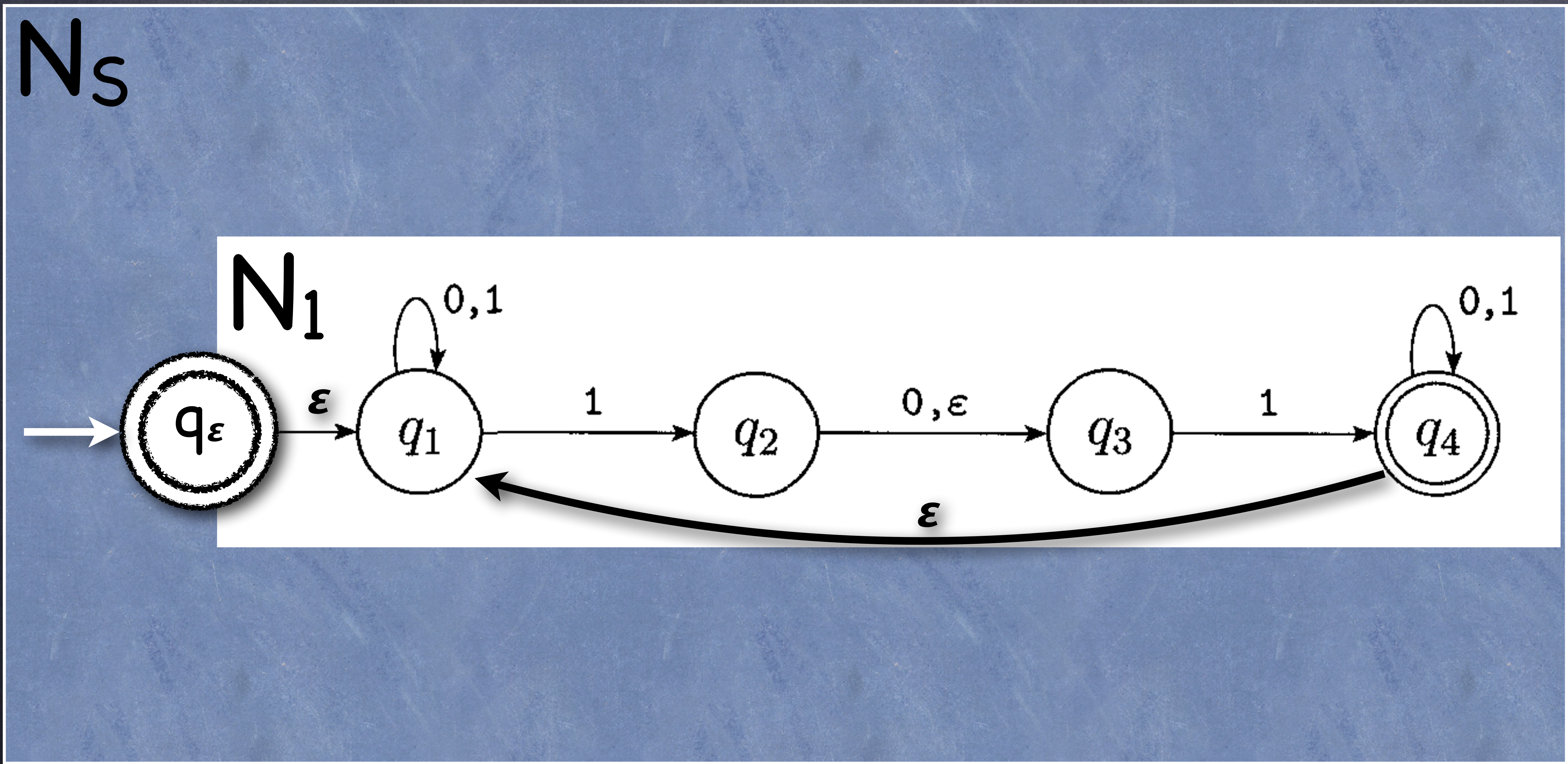
Example



Example



Example



COMP-330
Theory of Computation

Fall 2019 -- Prof. Claude Crépeau

Lec. 5 : NFA-DFA equivalence