COMP-330
Theory of Computation

Fall 2019 -- Prof. Claude Crépeau

Lec. 11 :
Pushdown Automata

DFA

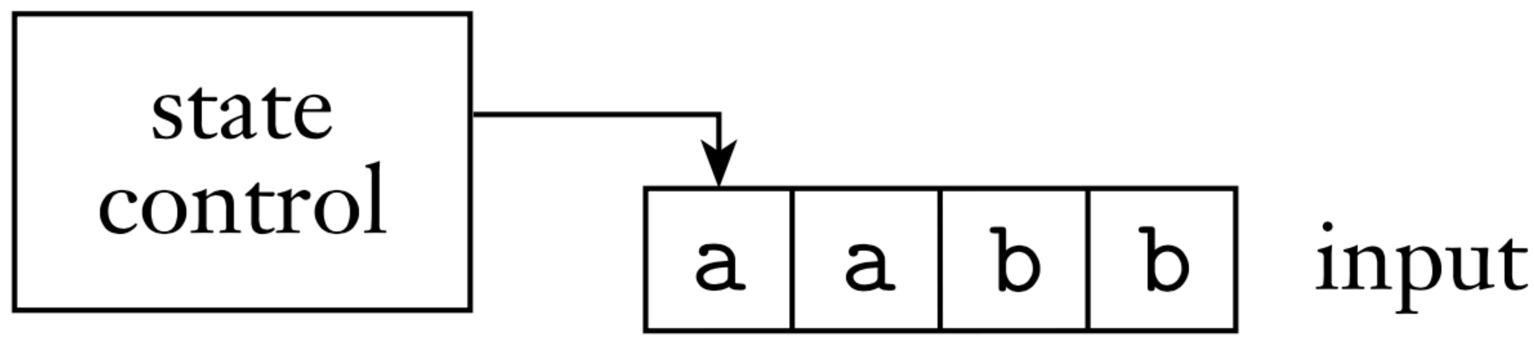


FIGURE 2.11
Schematic of a finite automaton

PDA

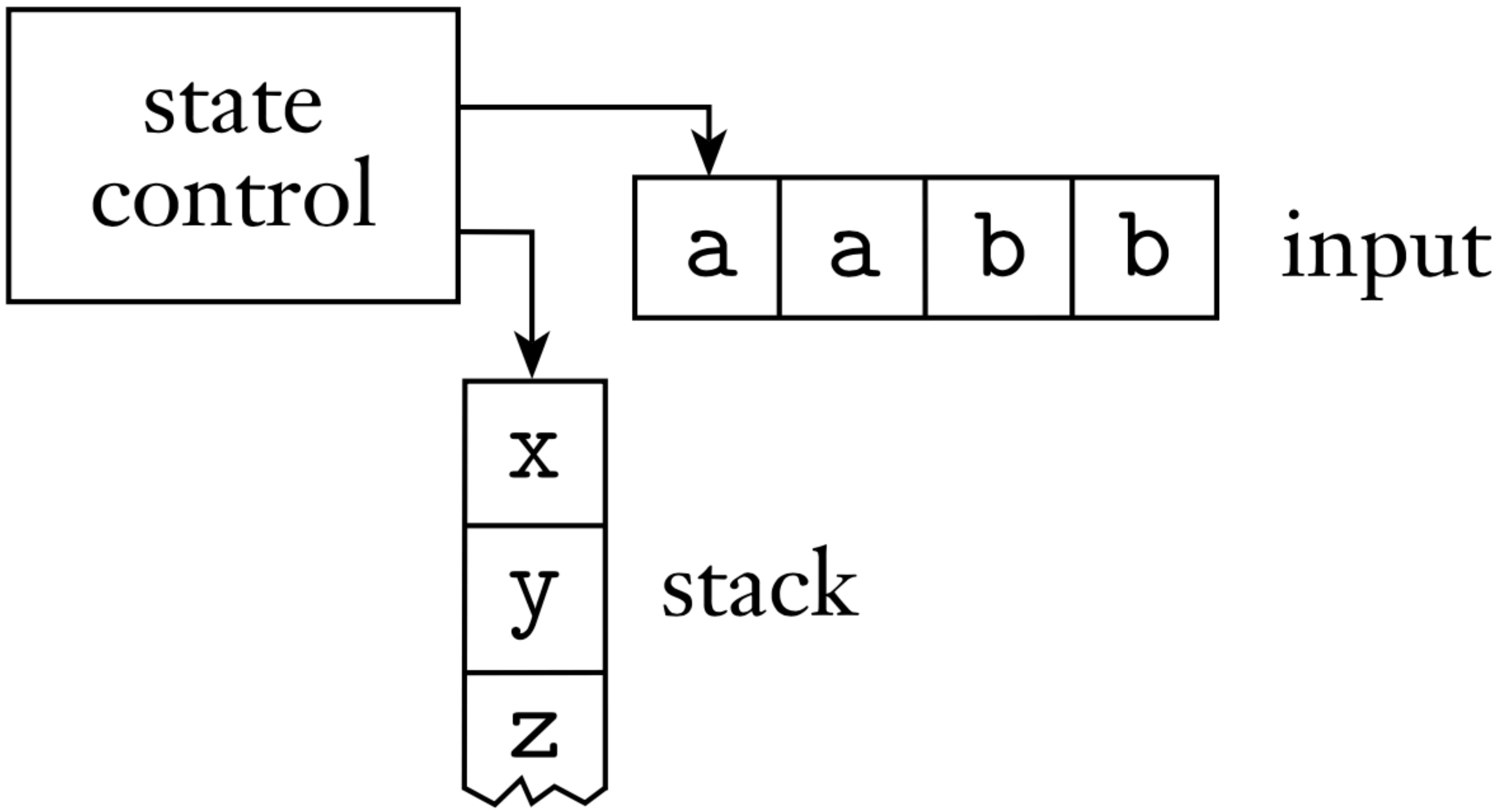
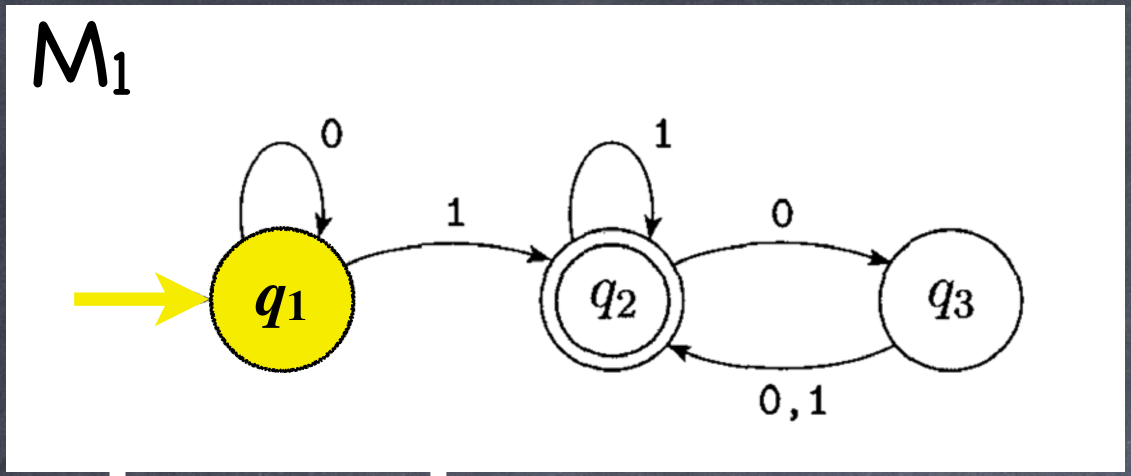
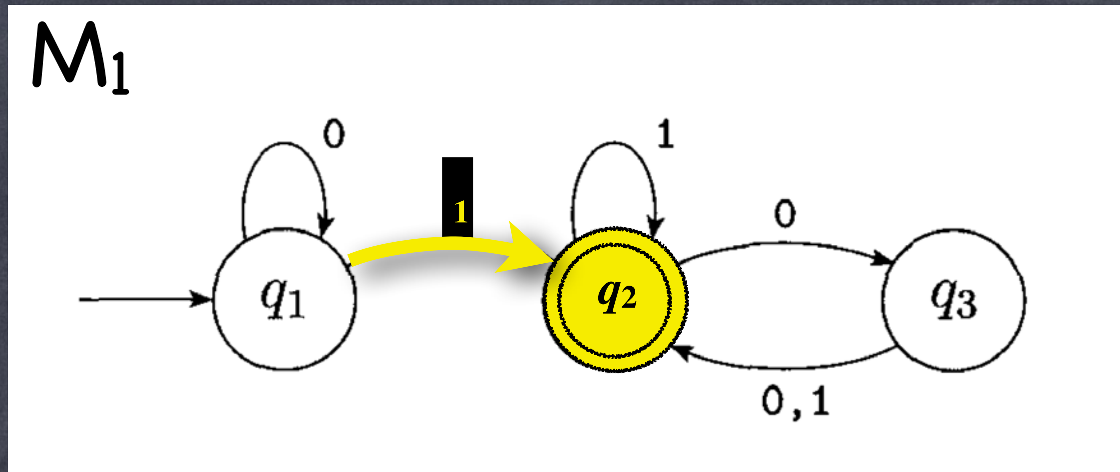


FIGURE 2.12
Schematic of a pushdown automaton



PDA = NFA+stack



PDA = NFA+stack

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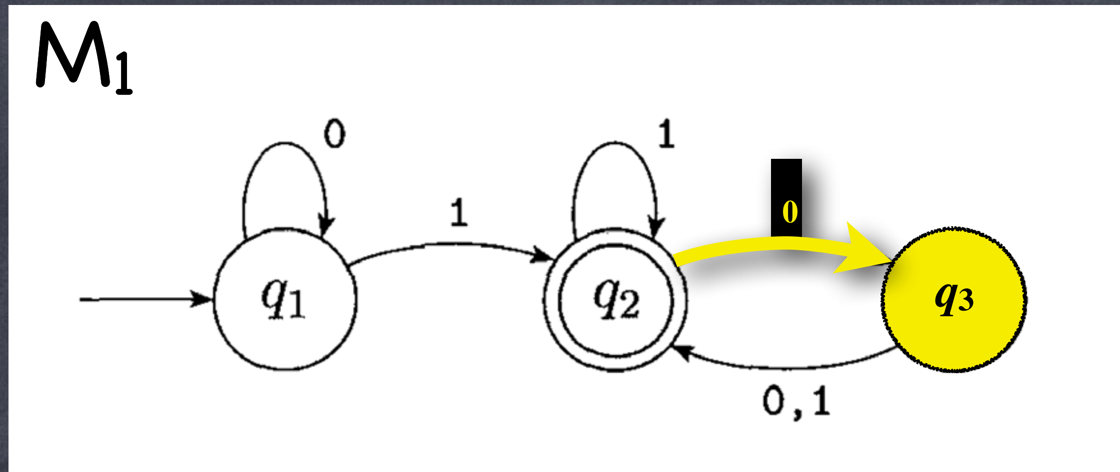

PDA = NFA+stack

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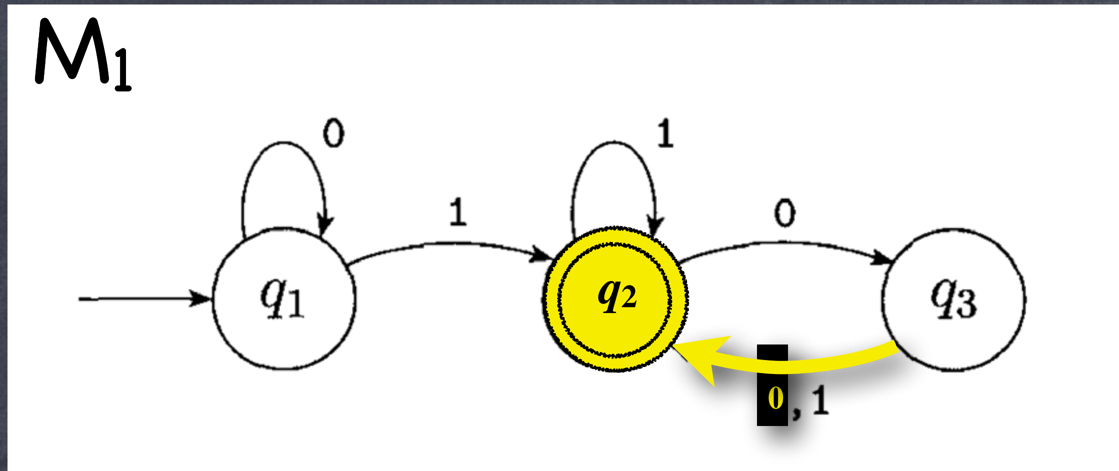
PDA = NFA+stack



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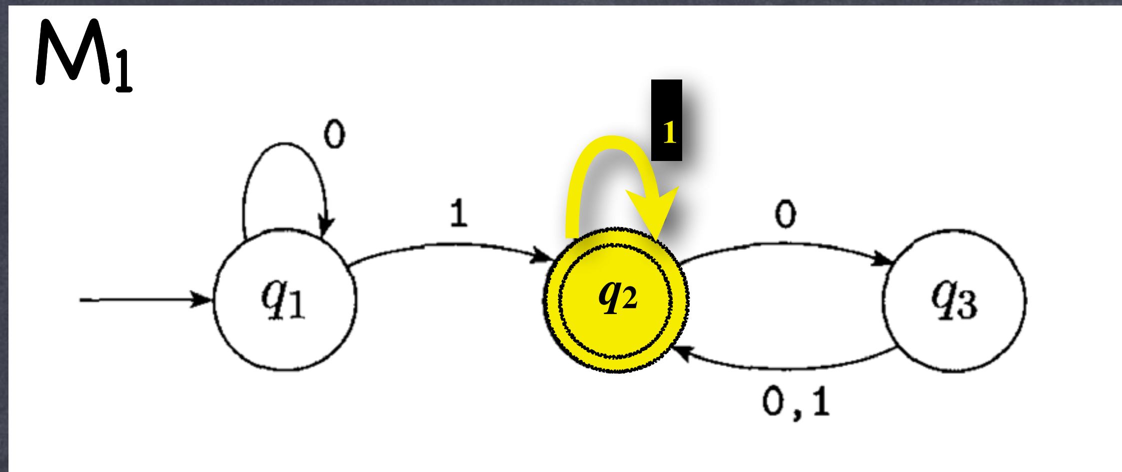

PDA = NFA+stack

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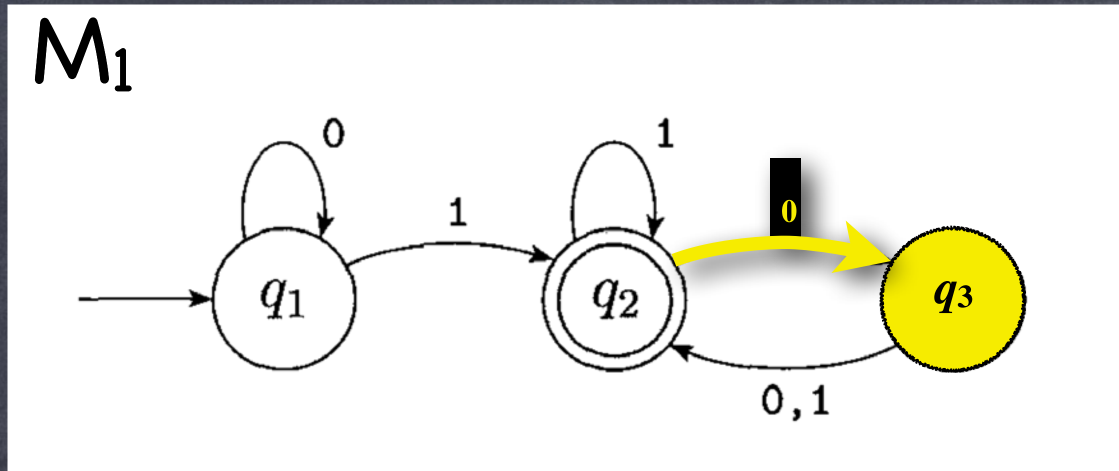

PDA = NFA+stack

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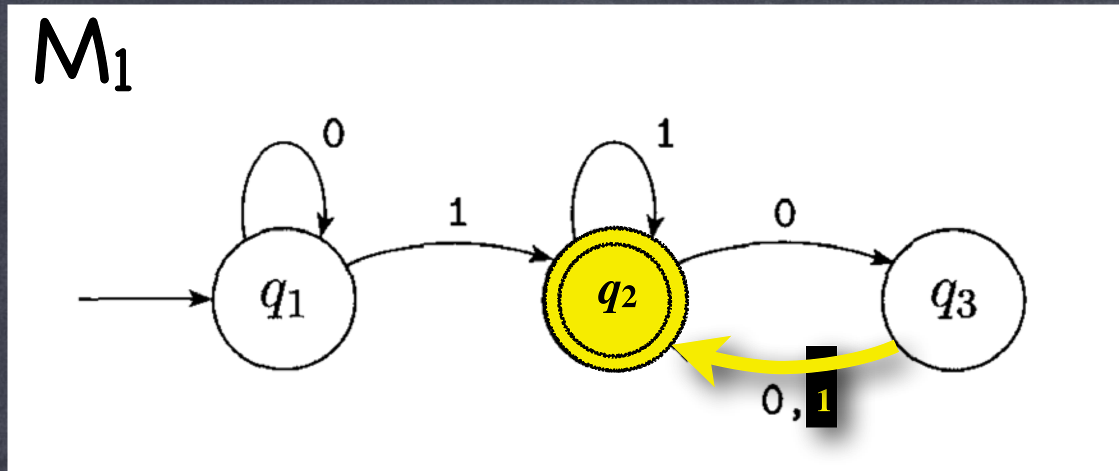

PDA = NFA+stack

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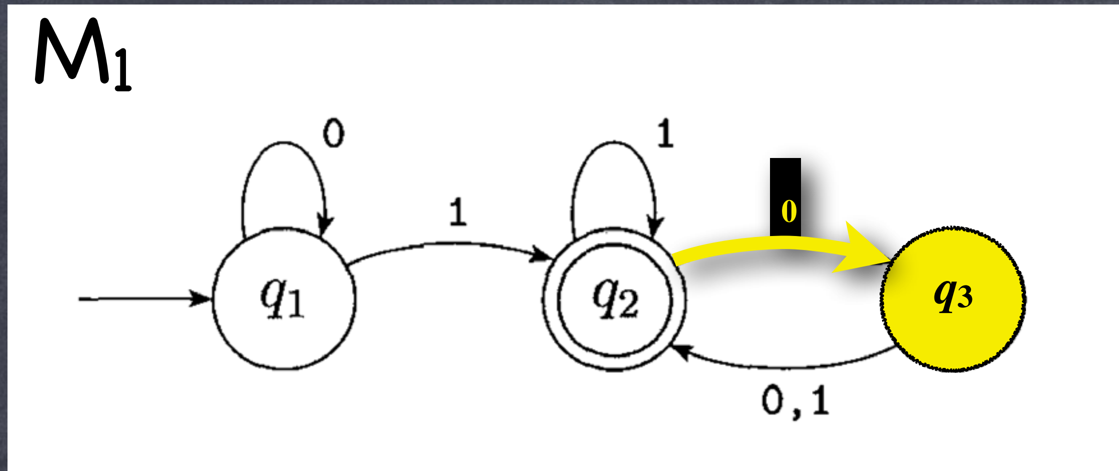

PDA = NFA+stack

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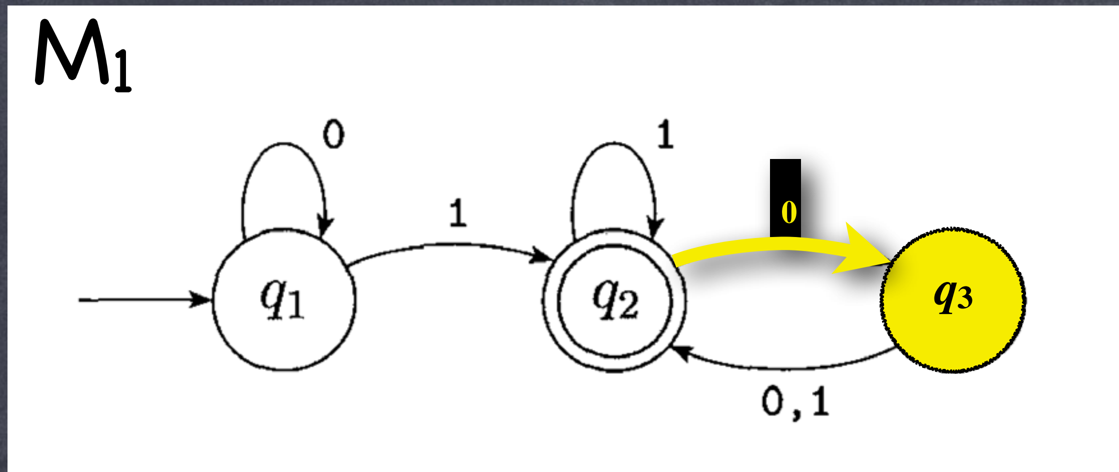
PDA = NFA+stack



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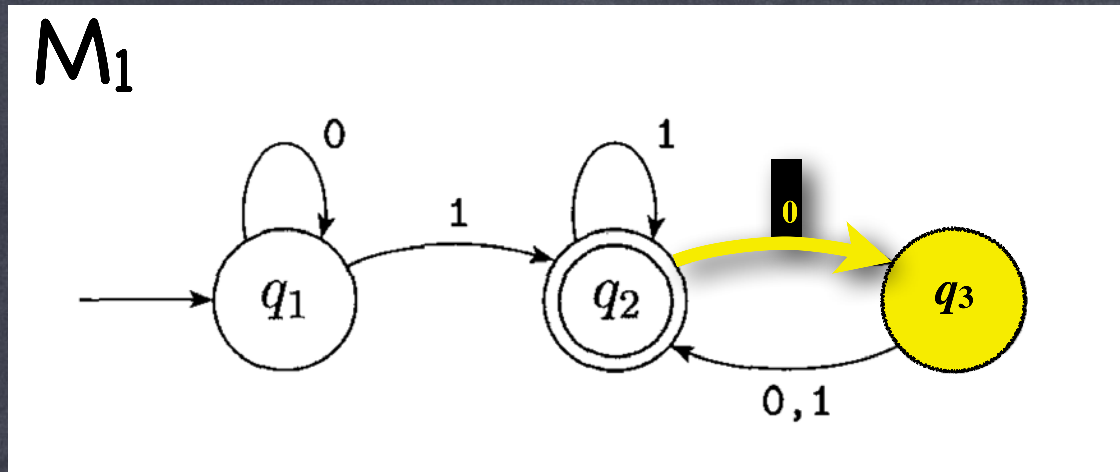

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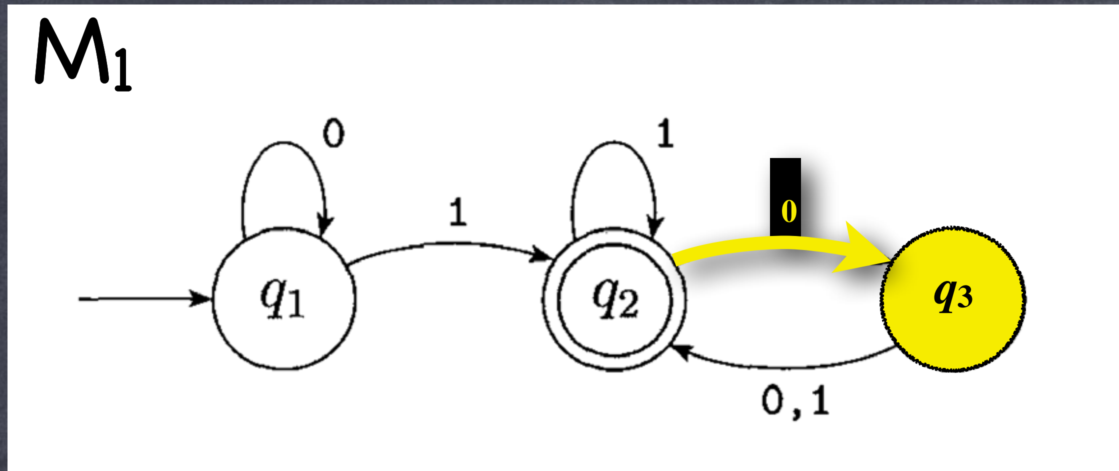
PDA = NFA+stack



PDA = NFA+stack

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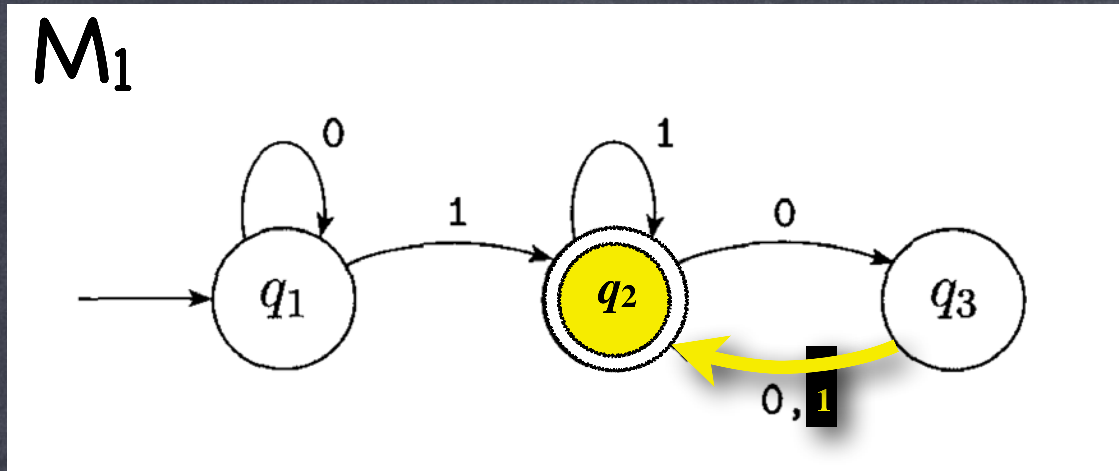

PDA = NFA+stack

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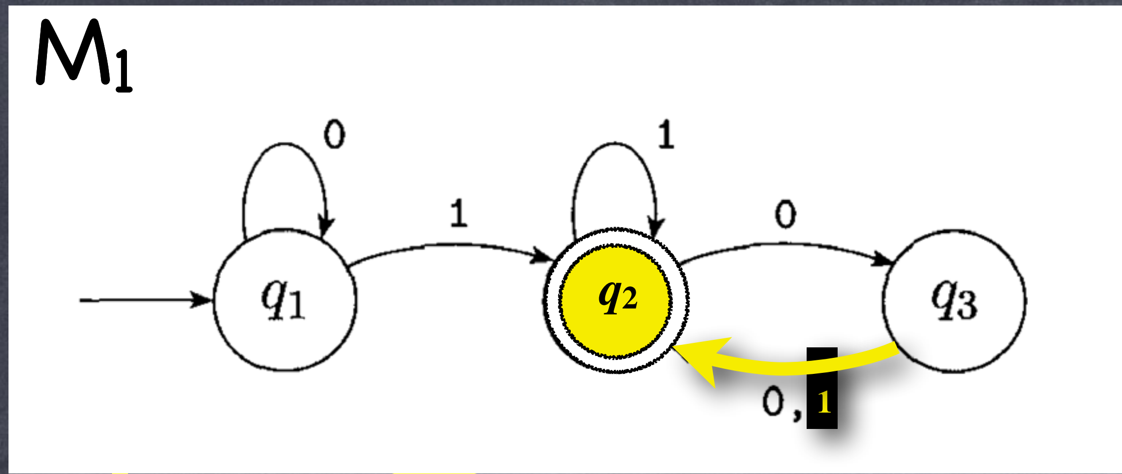

PDA =
NFA+stack

10010101

We must formalize the stack operations !!

Definition of PDA

- States
- Alphabets
- Transition function
- Start state
- Accept states

Definition of PDA

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Definition of PDA

- States

- Alphabets input: a,b,c,d
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Definition of PDA

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- Alphabets

input: a,b,c,d
STACK: A,B,C,D

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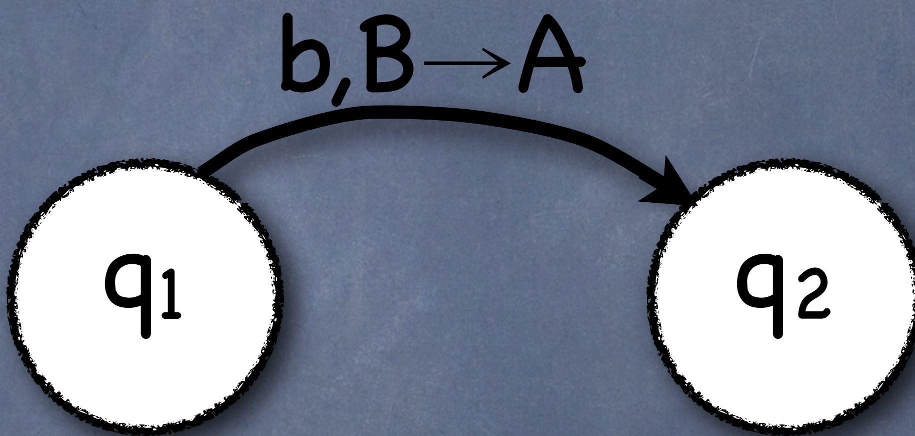

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- Accept states

Definition of PDA

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- Transition funct
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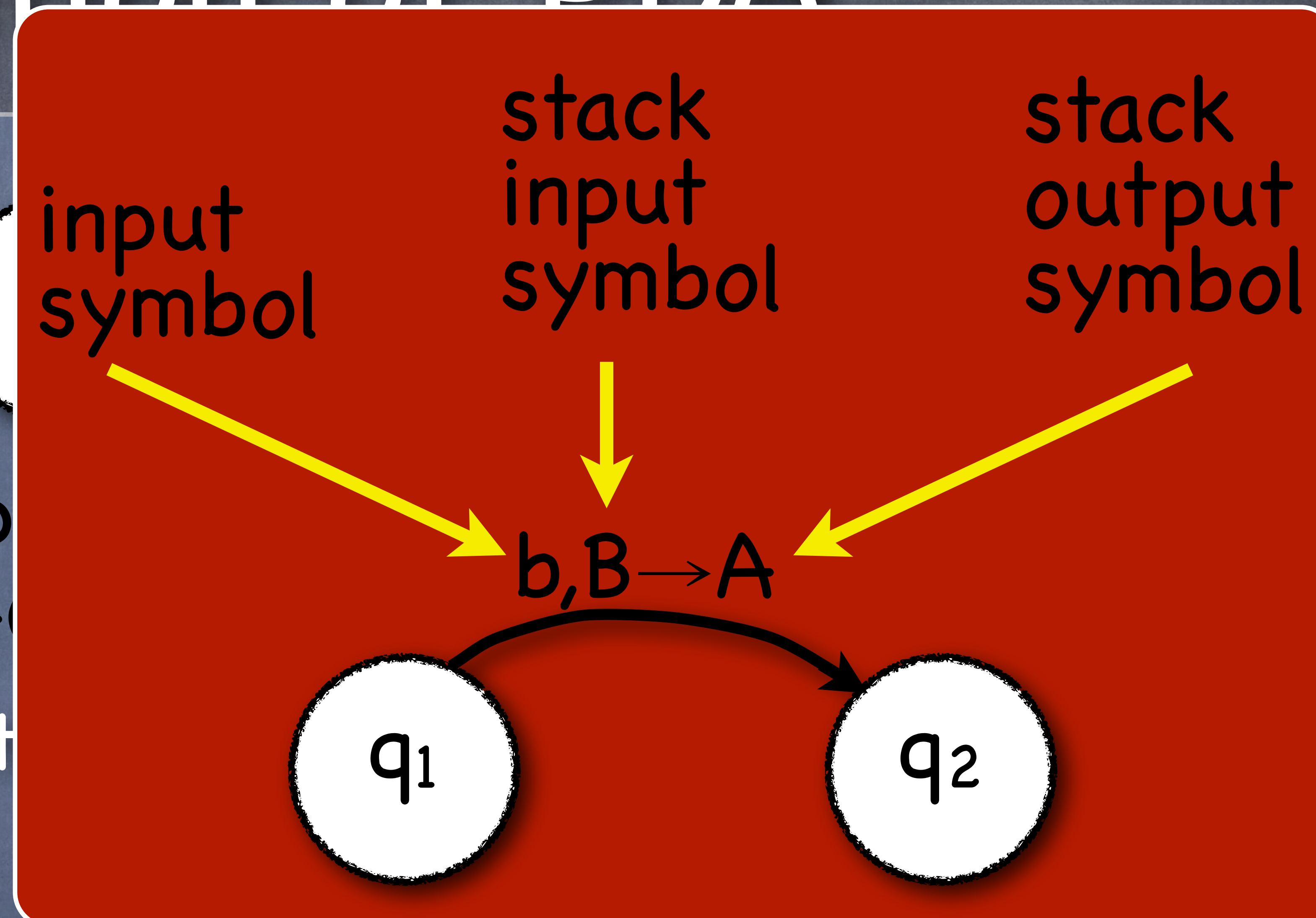


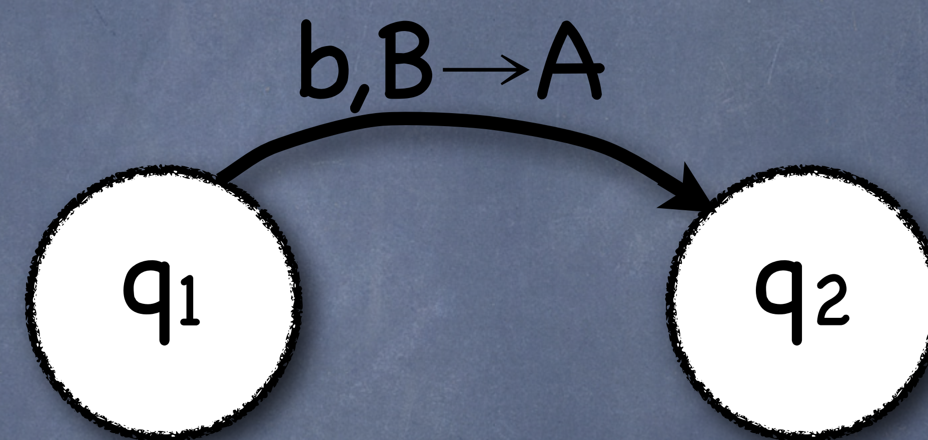
Definition of PDA

- States

- Alphabets

input: a,b,c,d
STACK: A,B,C,D

- Transition function



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Definition of PDA

- States

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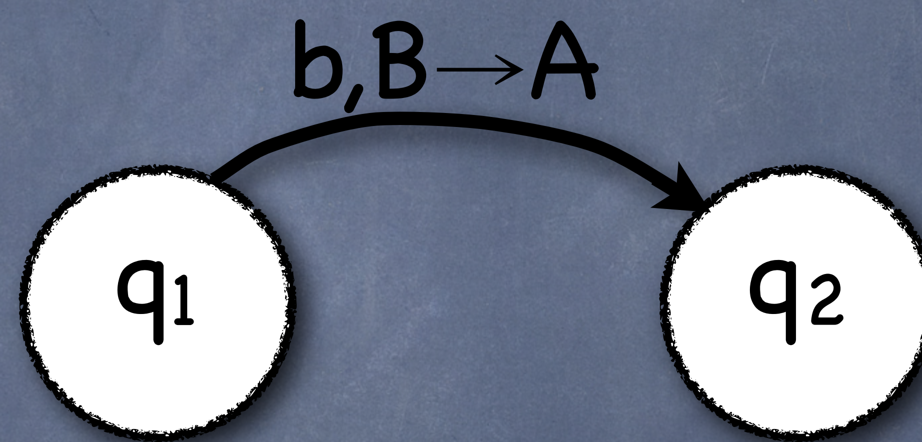
input: a,b,c,d
STACK: A,B,C,D



- Transition function

- Start state

- Accept states

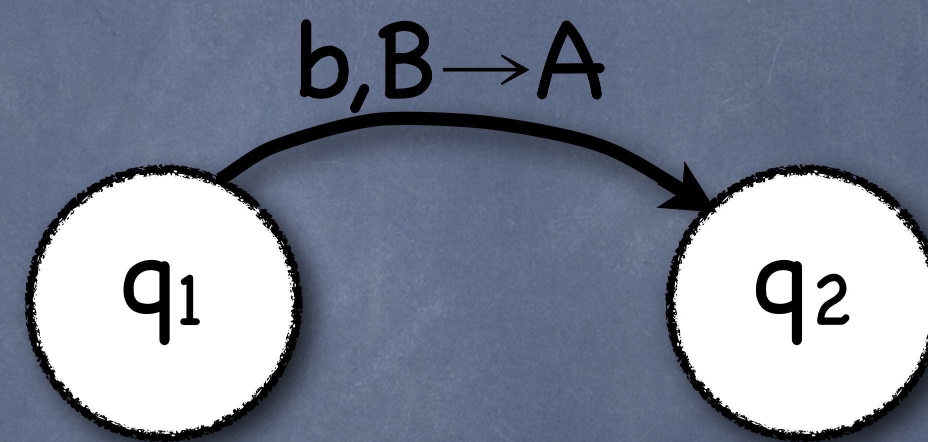
Definition of PDA

- States

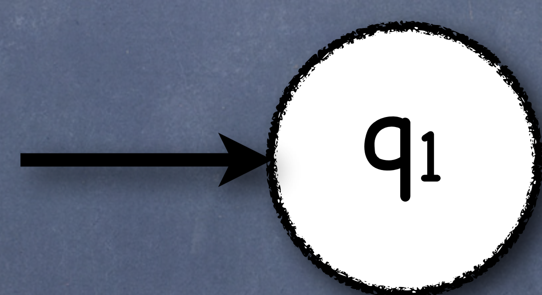
- Alphabets

input: a,b,c,d
STACK: A,B,C,D

- Transition function



- Start state



- Accept states

Definition of PDA

DEFINITION 2.13

A ***pushdown automaton*** is a 6-tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$, where Q, Σ, Γ, and F are all finite sets, and

1. Q is the set of states,
2. Σ is the input alphabet,
3. Γ is the stack alphabet,
4. $\delta \colon Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \longrightarrow \mathcal{P}(Q \times \Gamma_\varepsilon)$ is the transition function,
5. $q_0 \in Q$ is the start state, and
6. $F \subseteq Q$ is the set of accept states.

Definition of PDA

- Let $M = (Q,\Sigma,\Gamma,\delta,q_0,F)$ be a pushdown automaton and let $w=w_1w_2...w_n$ $(n\geq 0)$ be a string where each symbol $w_i \in \Sigma$.

- M **<u>accepts</u>** w if $\exists\, m\geq n$, $\exists\, r_0,r_1,\ldots,r_m \in Q$, $\exists\, s_0,s_1,\ldots,s_m \in \Gamma^*$ and $\exists\, y_1y_2...y_m = w$, with $y_i \in \Sigma_\varepsilon$ s.t.

  1. $r_0 = q_0$, $s_0 = \varepsilon$
  2. $r_{i+1},b \in \delta(r_i,y_{i+1},a)$ for $i = 0...m-1$, $s_i=at$, $s_{i+1}=bt$
  3. $r_m \in F$ for some $t \in \Gamma^*$, $a,b \in \Gamma_\varepsilon$

EXAMPLE 2.14

The following is the formal description of the PDA (next slide) that recognizes the language $\{0^n 1^n \mid n \geq 0\}$. Let M_1 be $(Q, \Sigma, \Gamma, \delta, q_1, F)$, where

$$Q = \{q_1, q_2, q_3, q_4\},$$

$$\Sigma = \{0,1\},$$

$$\Gamma = \{0, \$\},$$

$$F = \{q_1, q_4\}, \text{ and}$$

δ is given by the following table, wherein blank entries signify $\emptyset$.

Input:	0			1			ε		
Stack:	0	\$	ε	0	\$	ε	0	\$	ε
q_1									$\{(q_2, \$)\}$
q_2			$\{(q_2, 0)\}$	$\{(q_3, \varepsilon)\}$					
q_3				$\{(q_3, \varepsilon)\}$				$\{(q_4, \varepsilon)\}$	
q_4									

Examples of PDA

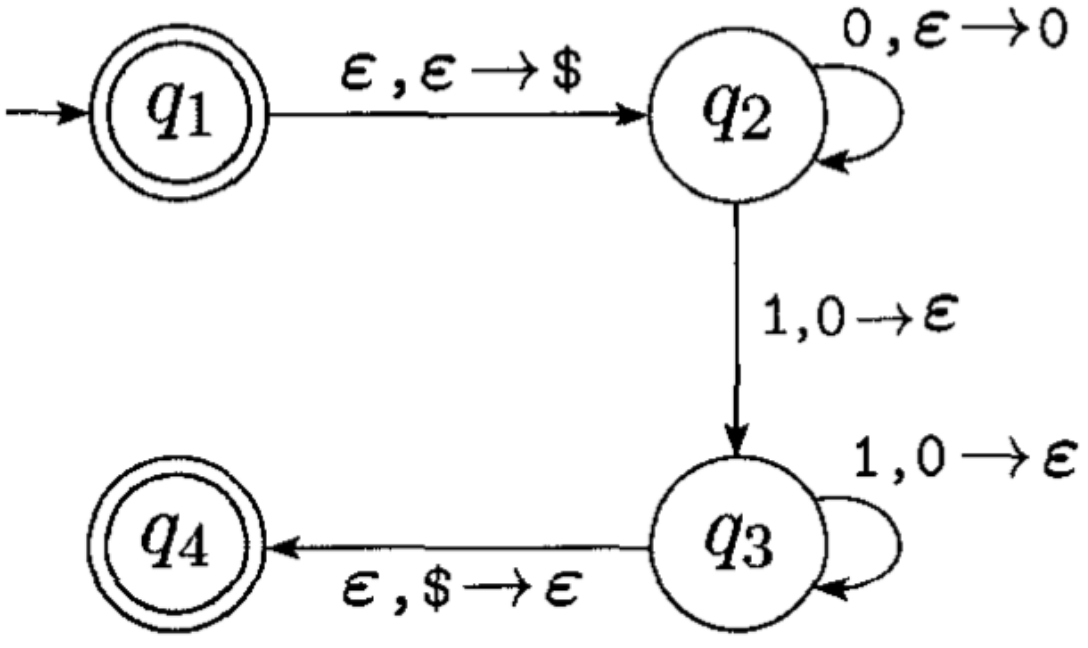
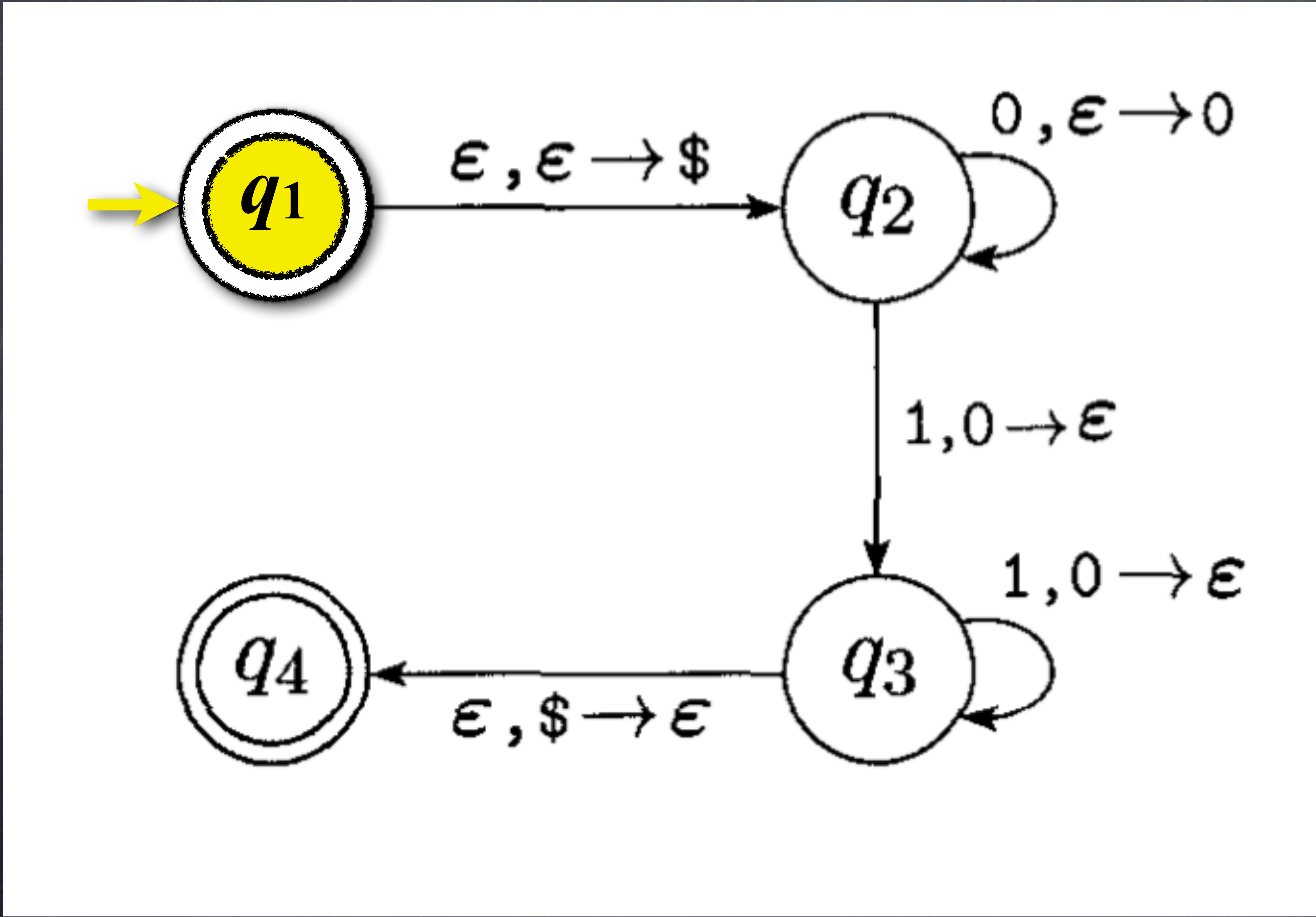
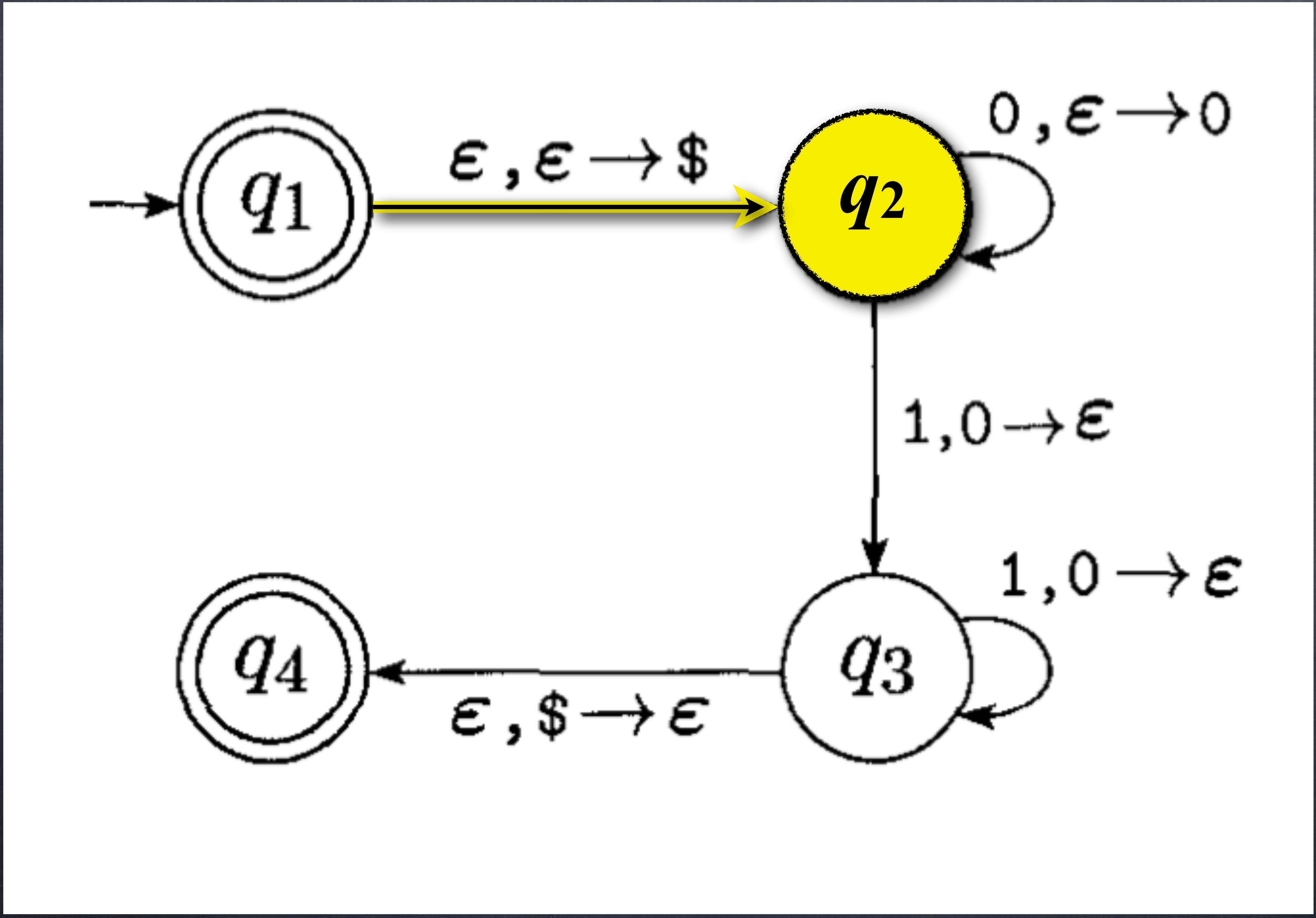


FIGURE 2.15
State diagram for the PDA M_1 that recognizes $\{0^n 1^n | n \geq 0\}$

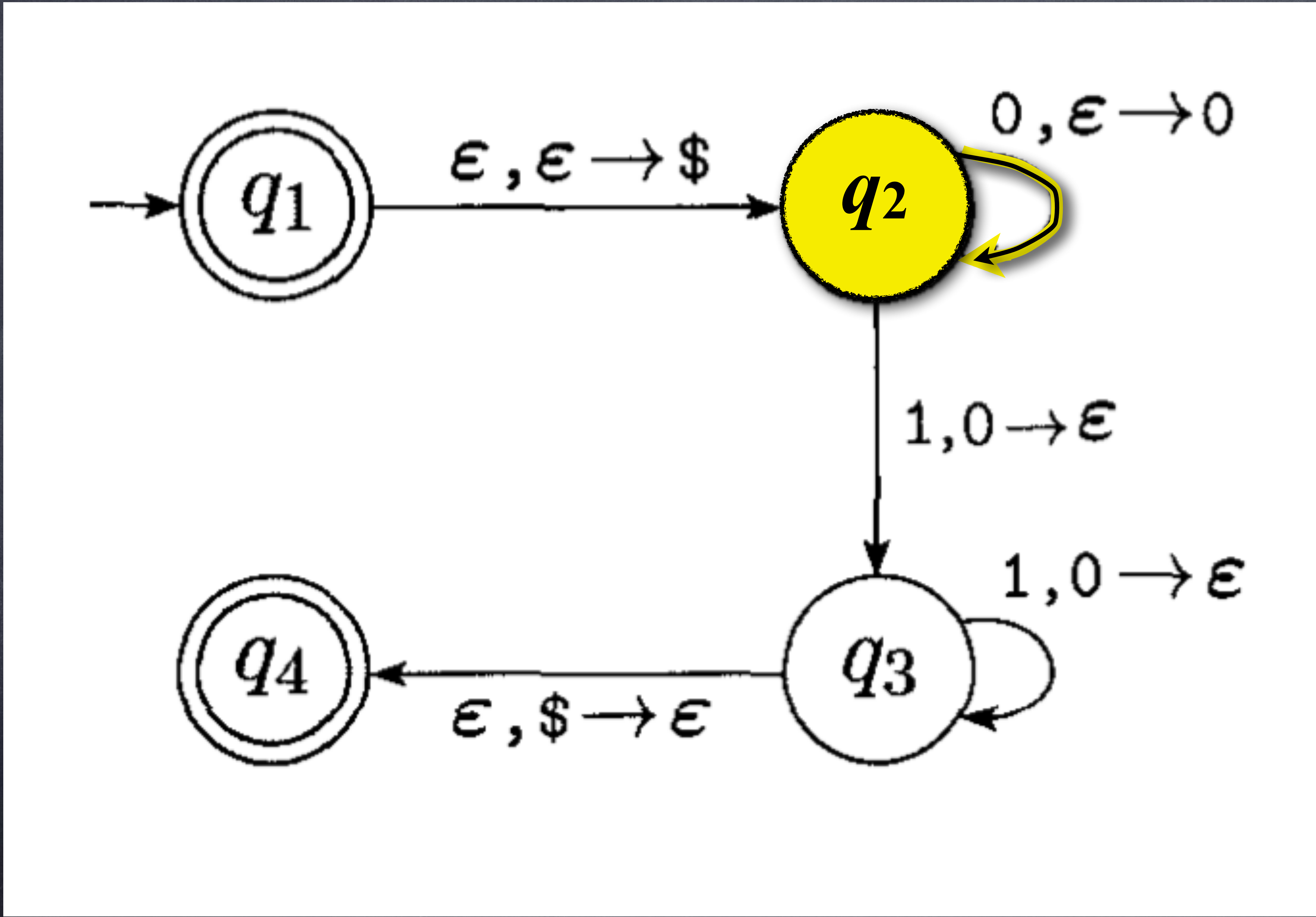
00001111 Stack:



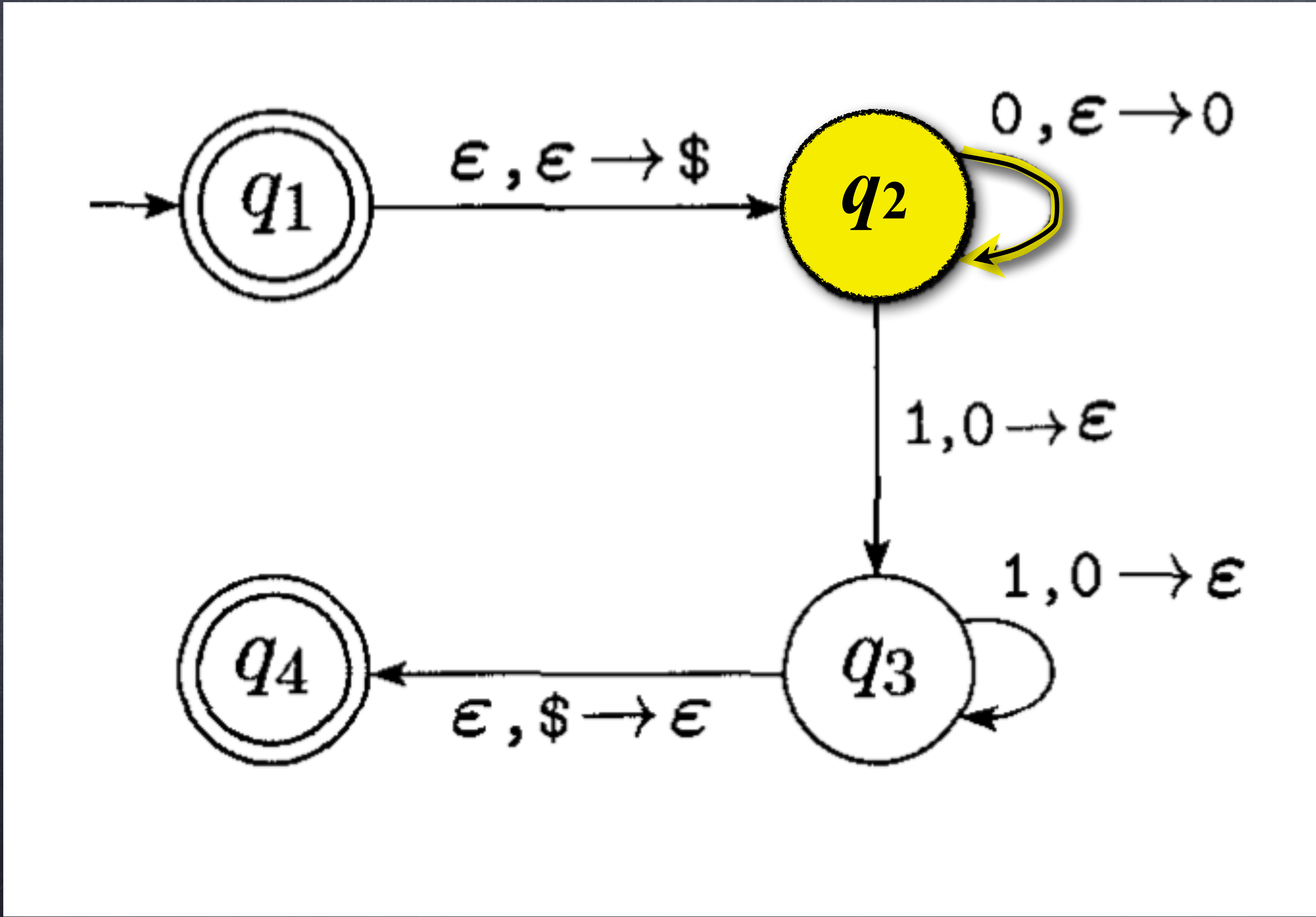
00001111 Stack: $



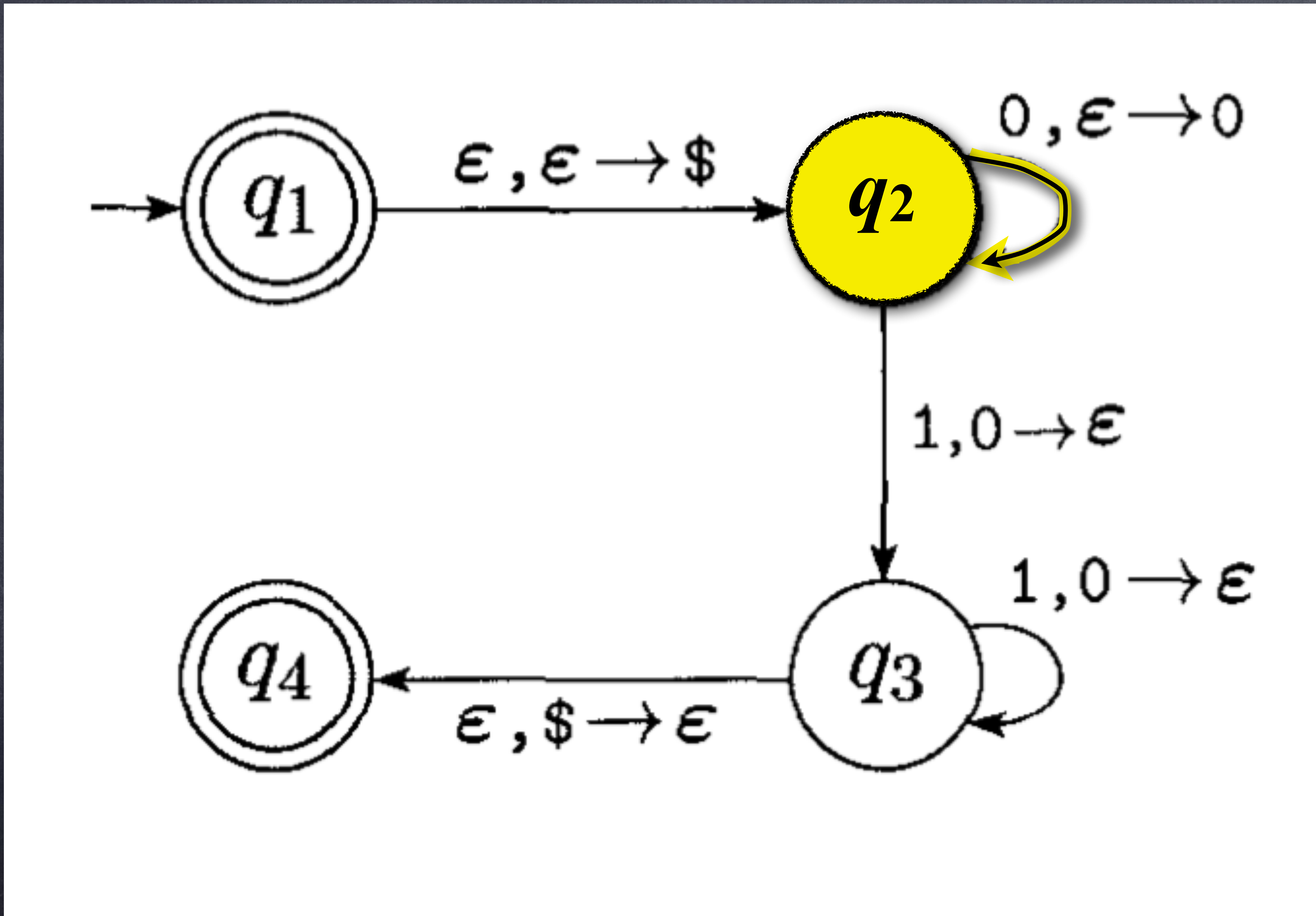
00001111 Stack: 0$



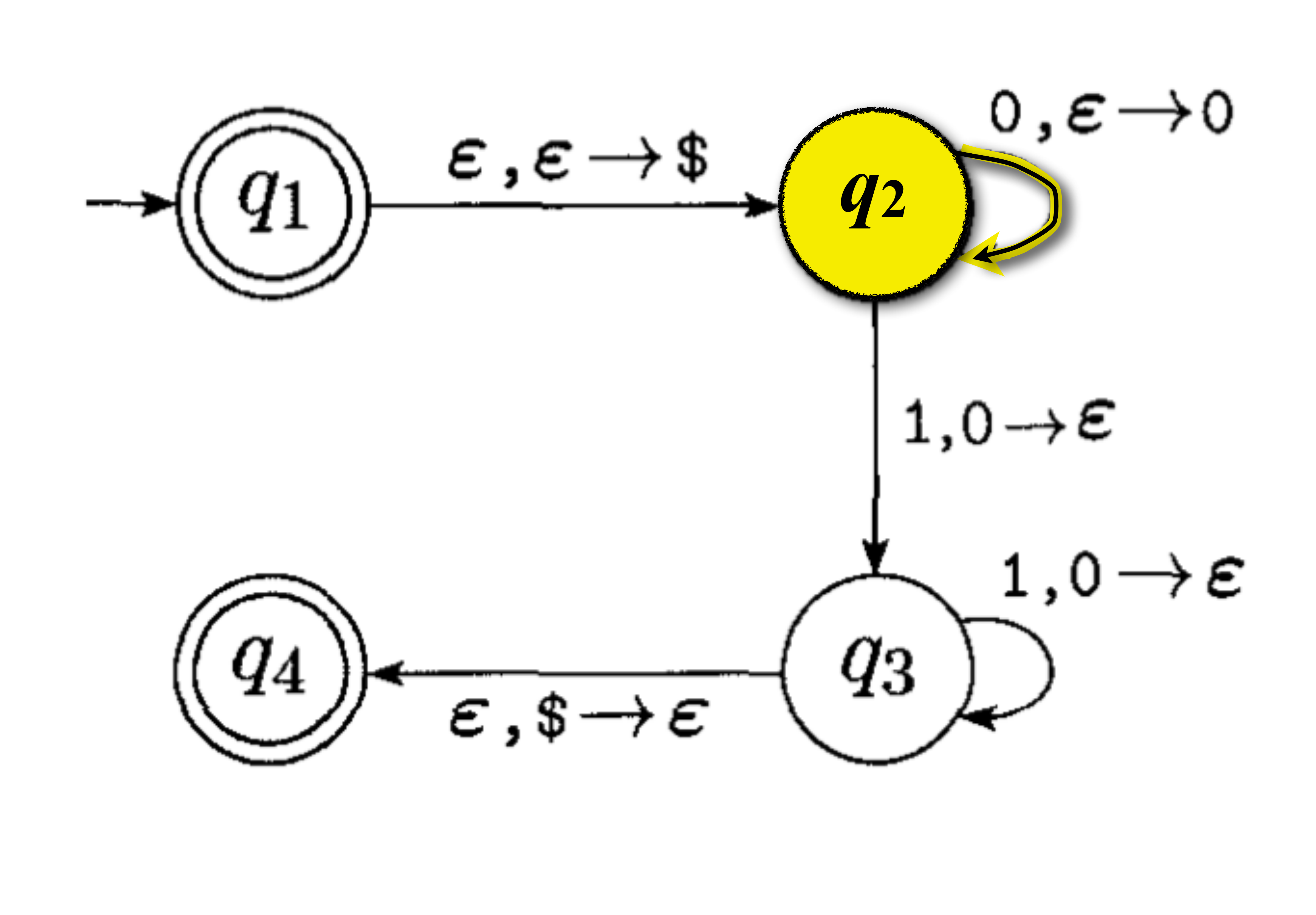
00001111 Stack: 00$



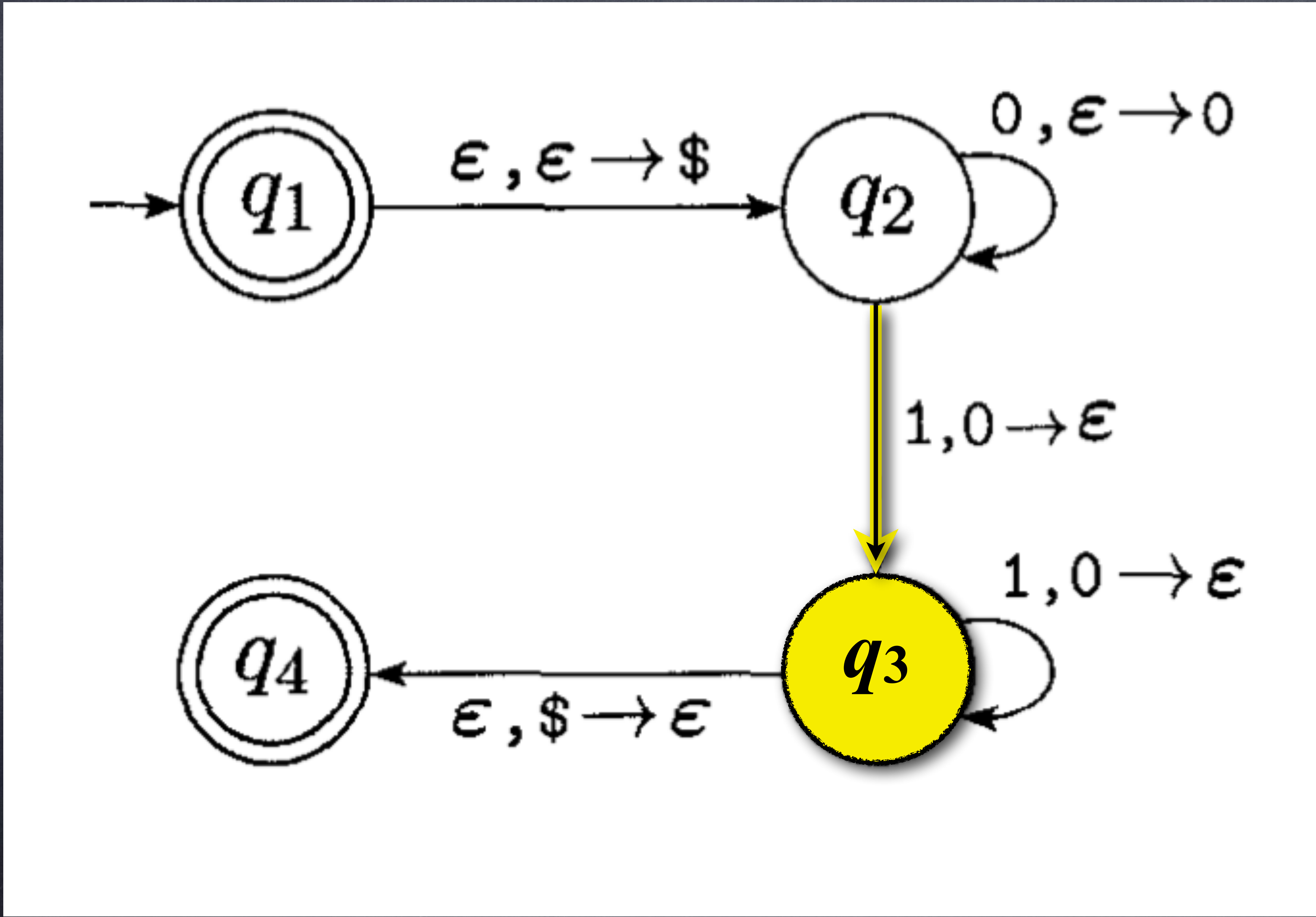
00001111 Stack: 000$



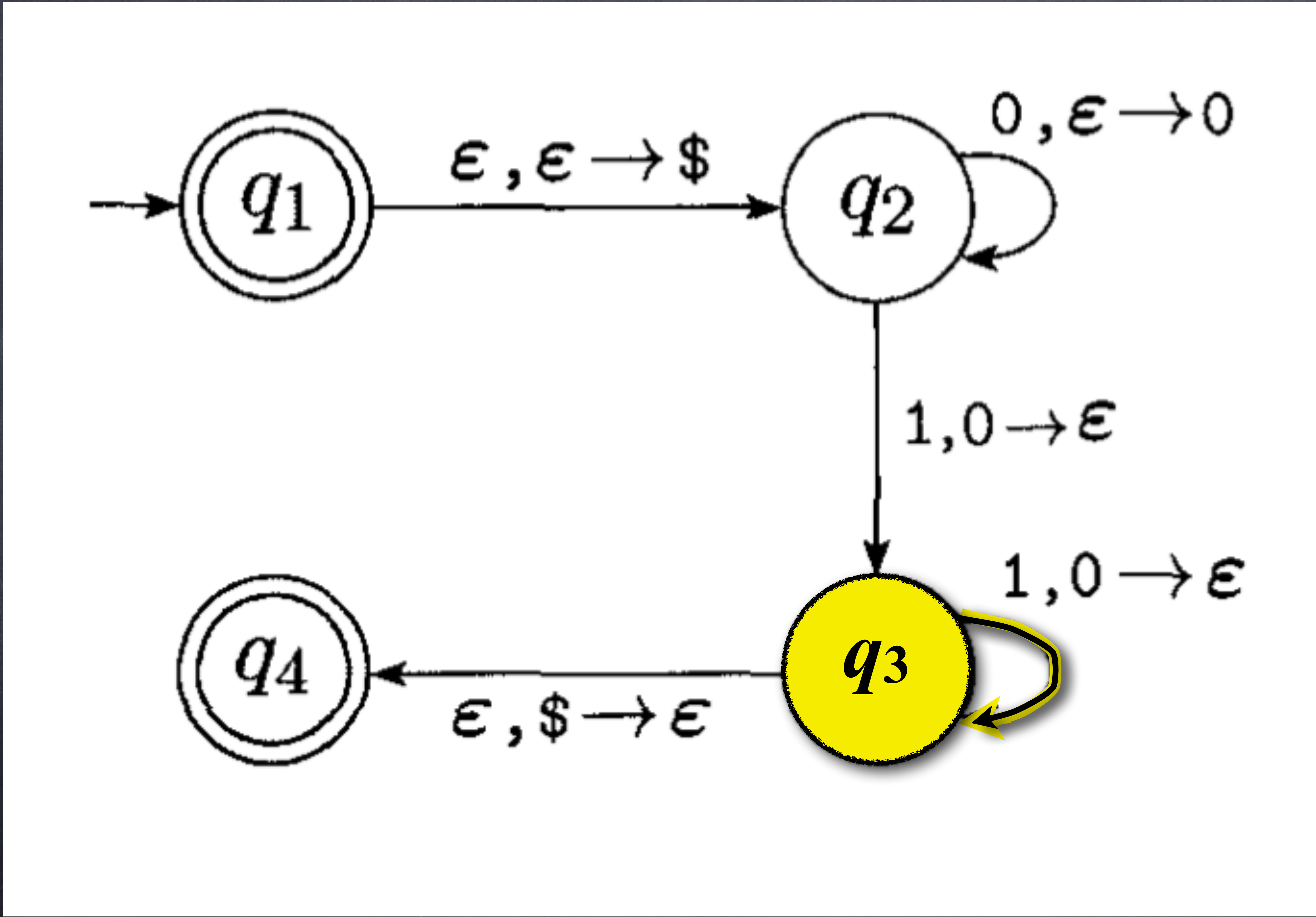
00001111 Stack: 0000$



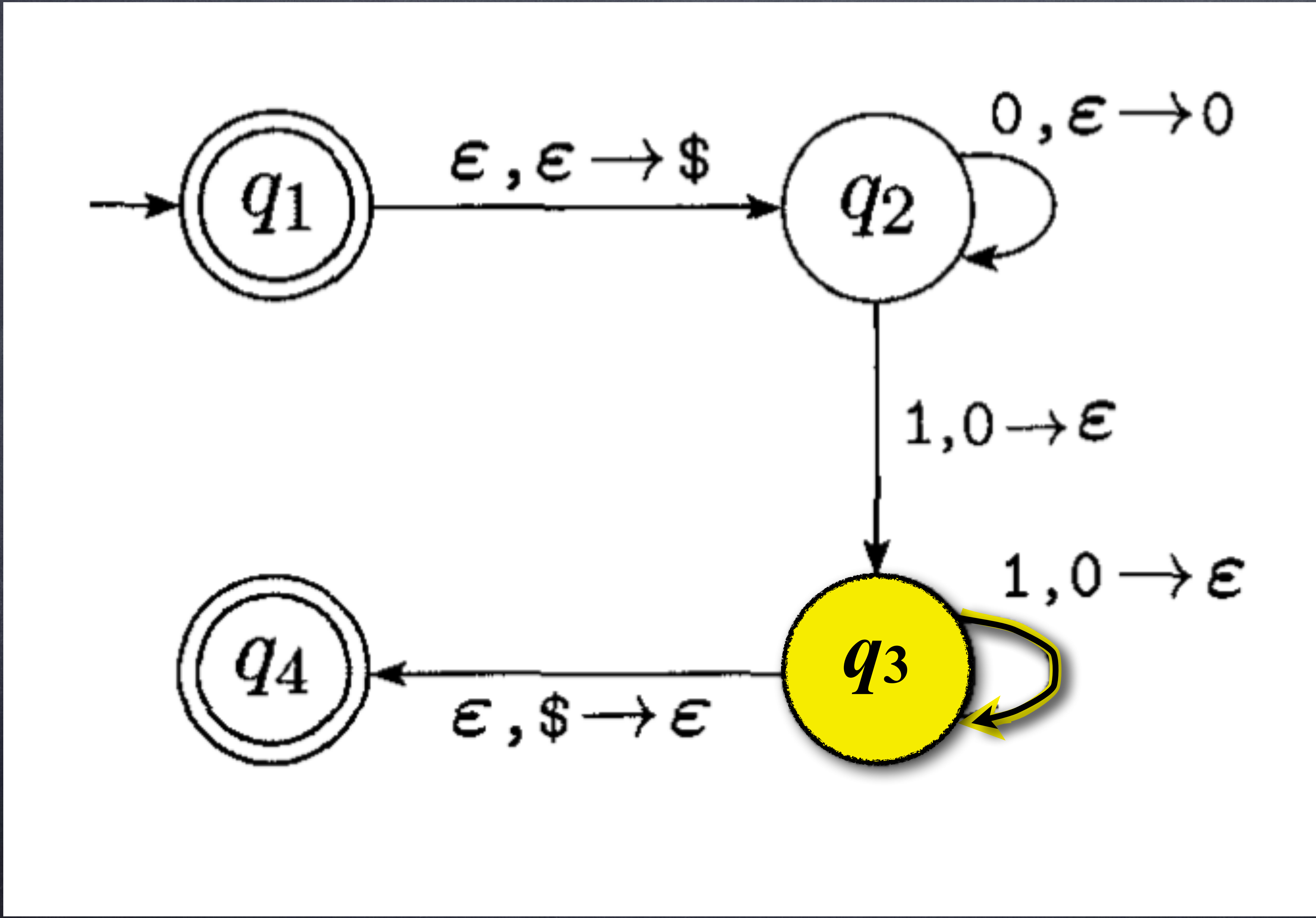
00001111 Stack: 000$



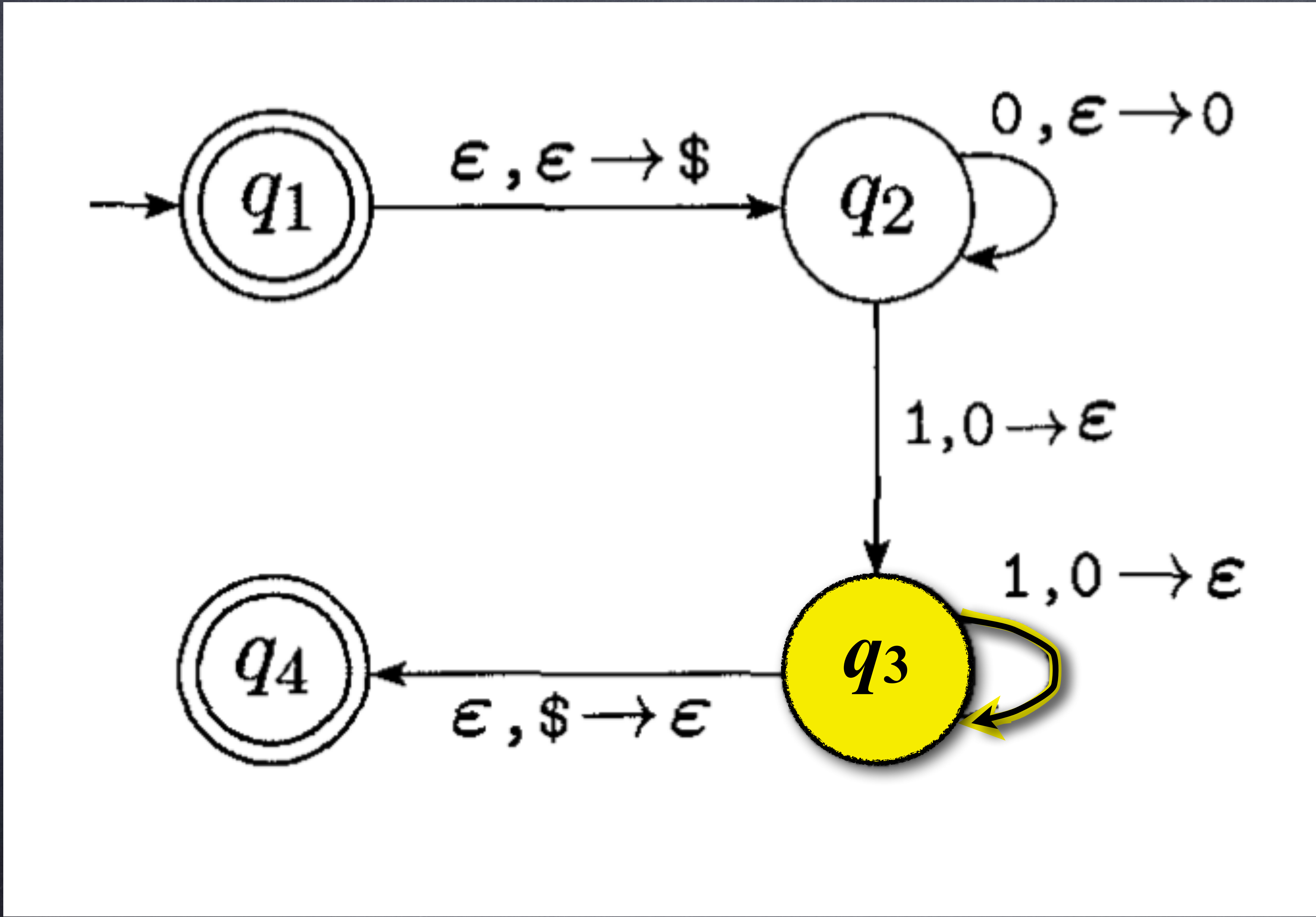
00001111 Stack: 00$



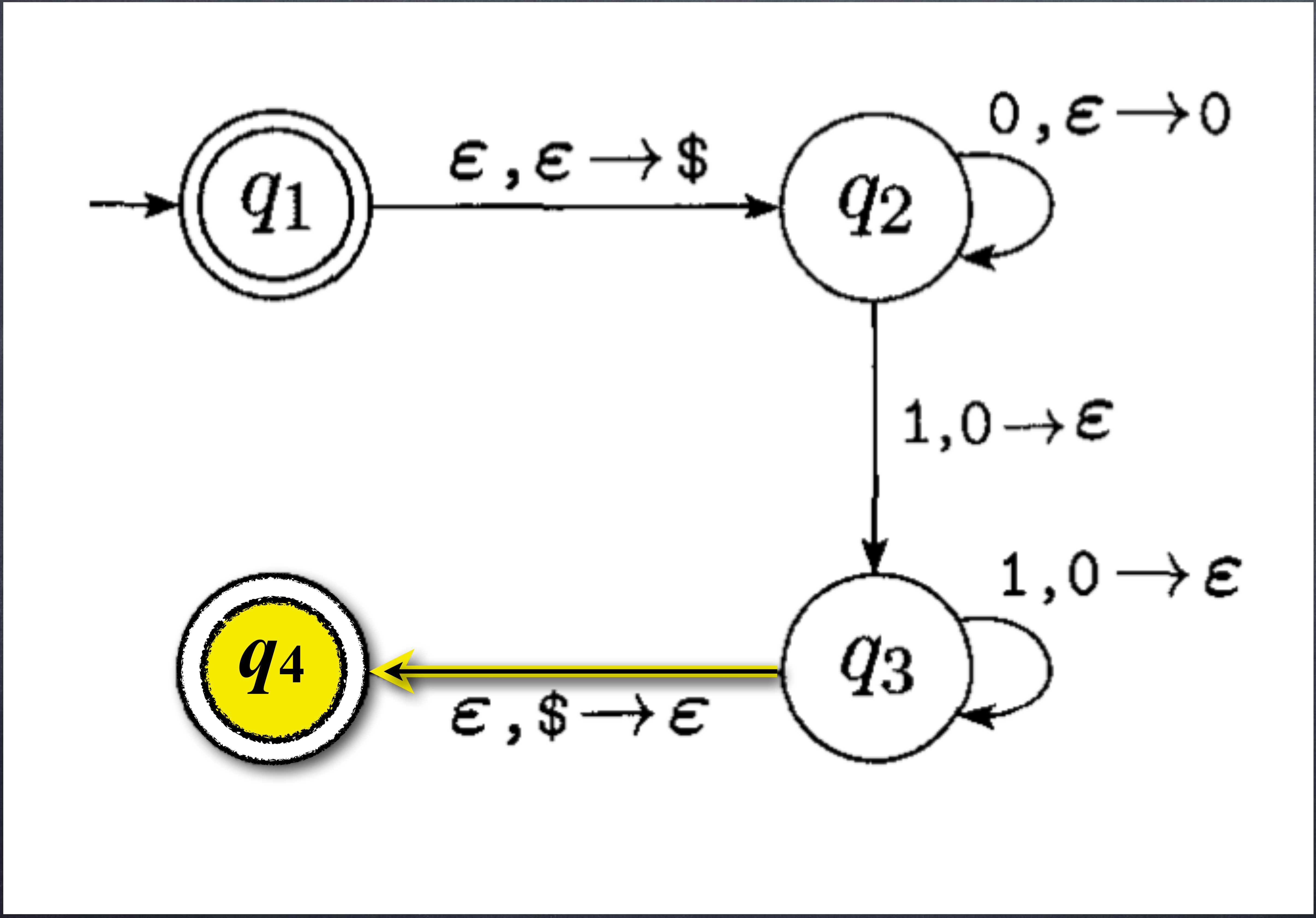
00001111 Stack: 0$



00001111 Stack: $



00001111 Stack:



Examples of PDA

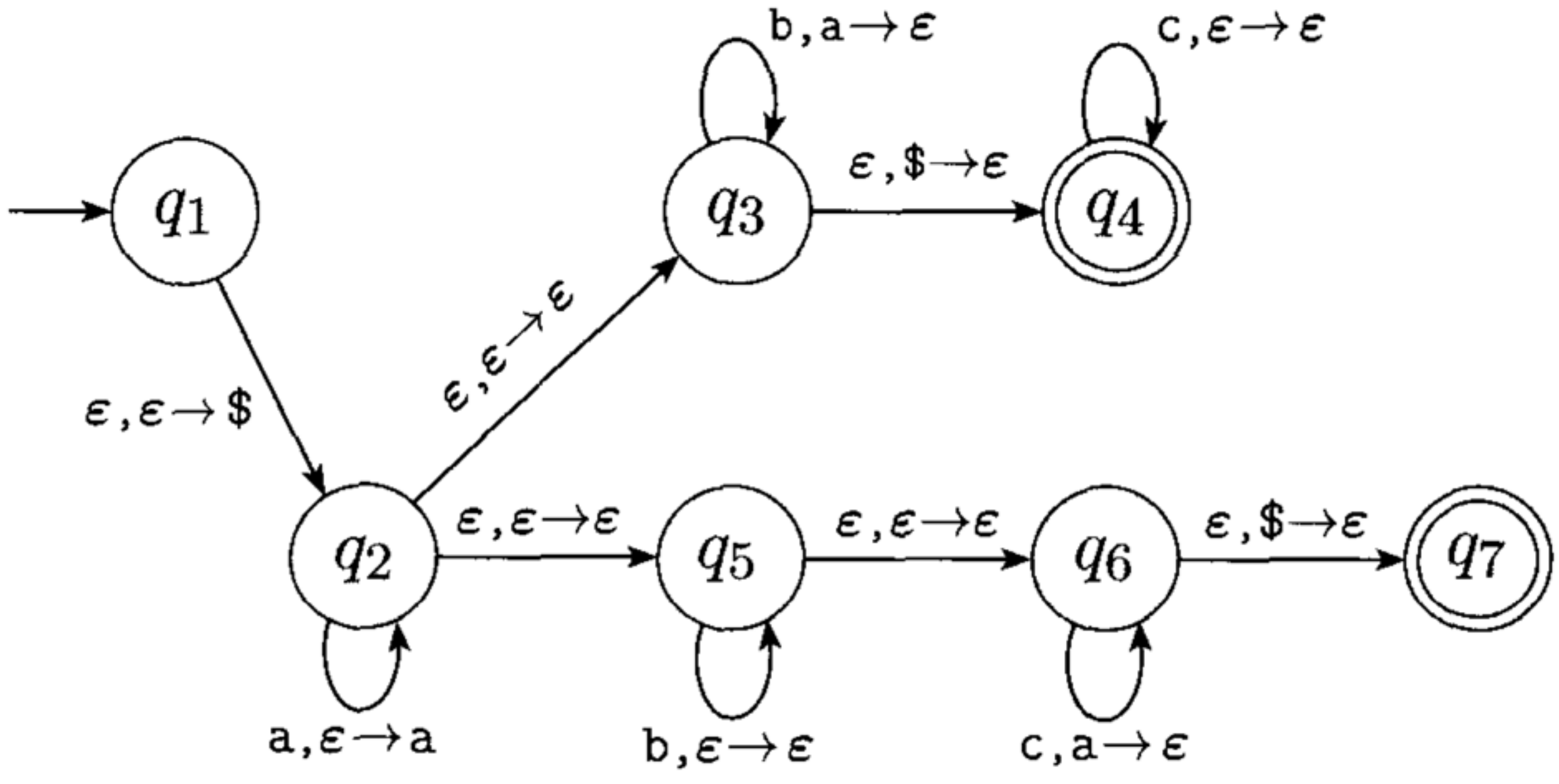


FIGURE 2.17
State diagram for PDA M_2 that recognizes
$\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j \text{ or } i = k\}$

Examples of PDA

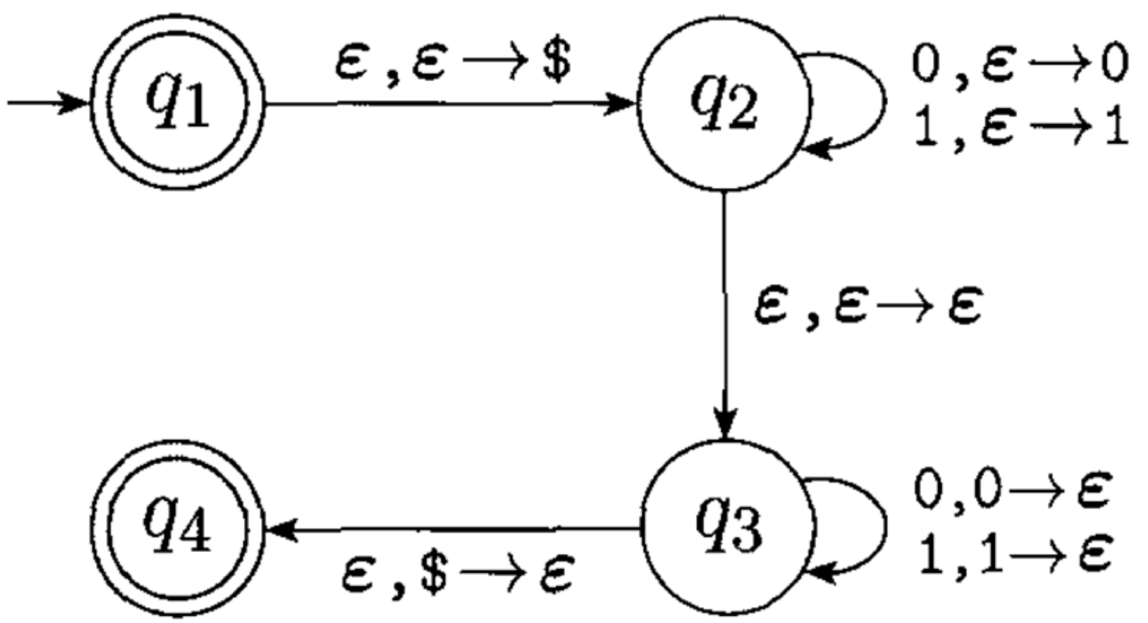


FIGURE 2.19
State diagram for the PDA M_3 that recognizes $\{ww^{\mathcal{R}} \mid w \in \{0,1\}^*\}$

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