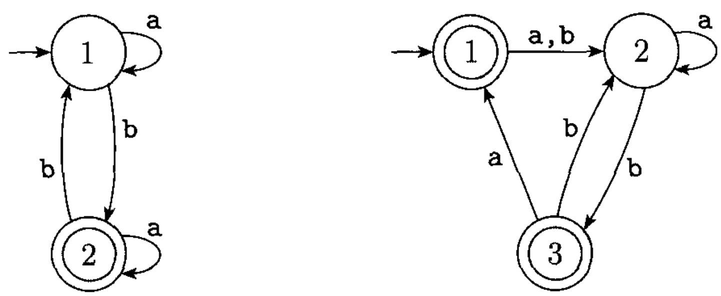
COMP 330A 2017, Assignment 2
Due Thursday, October 19th 23:59

[16%] **1.21** Use the procedure described in Lemma 1.60 to convert the following finite automata to regular expressions.



(a) (b)

[10%] **1.22** In certain programming languages, comments appear between delimiters such as `/#` and `#/`. Let C be the language of all valid delimited comment strings. A member of C must begin with `/#` and end with `#/` but have no intervening `#/`. For simplicity, we'll say that the comments themselves are written with only the symbols a and b; hence the alphabet of C is $\Sigma = \{\mathtt{a}, \mathtt{b}, /, \#\}$.

a. Give a DFA that recognizes C.

b. Give a regular expression that generates C.

[10%] **1.47** Let $\Sigma = \{1, \#\}$ and let

$$Y = \{w \mid w = x_1\#x_2\#\cdots\#x_k \text{ for } k \geq 0, \text{ each } x_i \in 1^*, \text{ and } x_i \neq x_j \text{ for } i \neq j\}.$$

Prove that Y is not regular.

IN EITHER 1.47 (above) or 1.53 (below), AT YOUR CHOOSING, YOU MUST USE THE MYHILL-NERODE THEOREM TO PROVE NON-REGULARITY.

[10%] **1.53** Let $\Sigma = \{0, 1, +, =\}$ and

$$ADD = \{x{=}y{+}z \mid x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}.$$

Show that ADD is not regular.

[12%]

1.54 Consider the language $F = \{a^ib^jc^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$.

a. Show that F is not regular.

b. Show that F acts like a regular language in the pumping lemma. In other words, give a pumping length p and demonstrate that F satisfies the three conditions of the pumping lemma for this value of p.

c. Explain why parts (a) and (b) do not contradict the pumping lemma.

[12%]

1.60 Let $\Sigma = \{a, b\}$. For each $k \geq 1$, let C_k be the language consisting of all strings that contain an a exactly k places from the right-hand end. Thus $C_k = \Sigma^* a \Sigma^{k-1}$. Describe an NFA with $k + 1$ states that recognizes C_k, both in terms of a state diagram and a formal description.

[8%]

1.61 Consider the languages C_k defined in Problem 1.60. Prove that for each k, no DFA can recognize C_k with fewer than 2^k states.

[12%]

1.64 Let N be an NFA with k states that recognizes some language A.

a. Show that, if A is nonempty, A contains some string of length at most k.

b. Show that, by giving an example, that part (a) is not necessarily true if you replace both A's by $\overline{A}$.

c. Show that, if $\overline{A}$ is nonempty, $\overline{A}$ contains some string of length at most 2^k.

d. Show that the bound given in part (b) is nearly tight; that is, for each k, demonstrate an NFA recognizing a language A_k where $\overline{A_k}$ is nonempty and where $\overline{A_k}$'s shortest member strings are of length exponential in k. Come as close to the bound in (b) as you can.

1.999 Consider the (decimal) languages defined below. For each one, either give a regular expression for its elements or prove the language is non-regular:

[10%]

In all examples, a number cannot start with a 0 (unless it *is* 0 itself)

a) $\mathbf{L_a}$ = { w | as an integer w is a multiple of 25}.

b) $\mathbf{L_b}$ = { w | as an integer w is a multiple of 10}.

c) $\mathbf{L_c}$ = { w | as an integer w is a power of 10}.

d) $\mathbf{L_d}$ = { w | as an integer w is a multiple of 2}.

e) $\mathbf{L_e}$ = { w | as an integer w is such that the sum of its digits is a multiple of 2}.

f) $\mathbf{L_f}$ = { w | as an integer w is a power of 2}.

g) $\mathbf{L_g}$ = { w | w is an integer} (with $\Sigma = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, - \}$).

h) $\mathbf{L_h}$ = { w | w is the decimal representation of a rational number}.

(with $\Sigma = \{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, -, ., [,] \}$)

Examples of such strings are 0.[3] representing the number 0.3333… = 1/3 and -23.15[24] representing the number -23.1524242424… = -76403/3300.