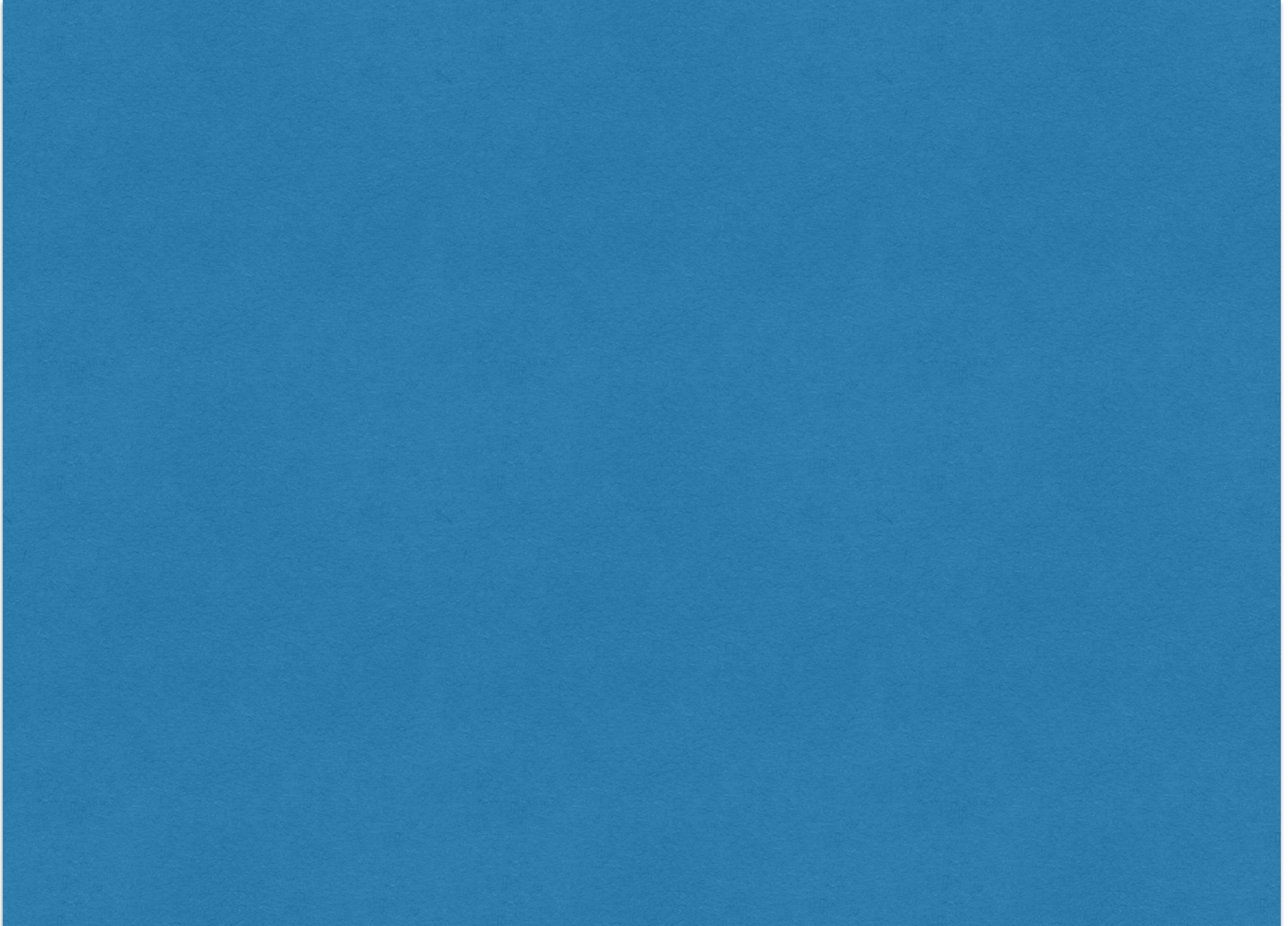


Winter 2016
COMP-250: Introduction
to Computer Science

Lecture 4, January 21, 2016

Fractional Numbers



Fractional Numbers

26.375

= (11010.)₂

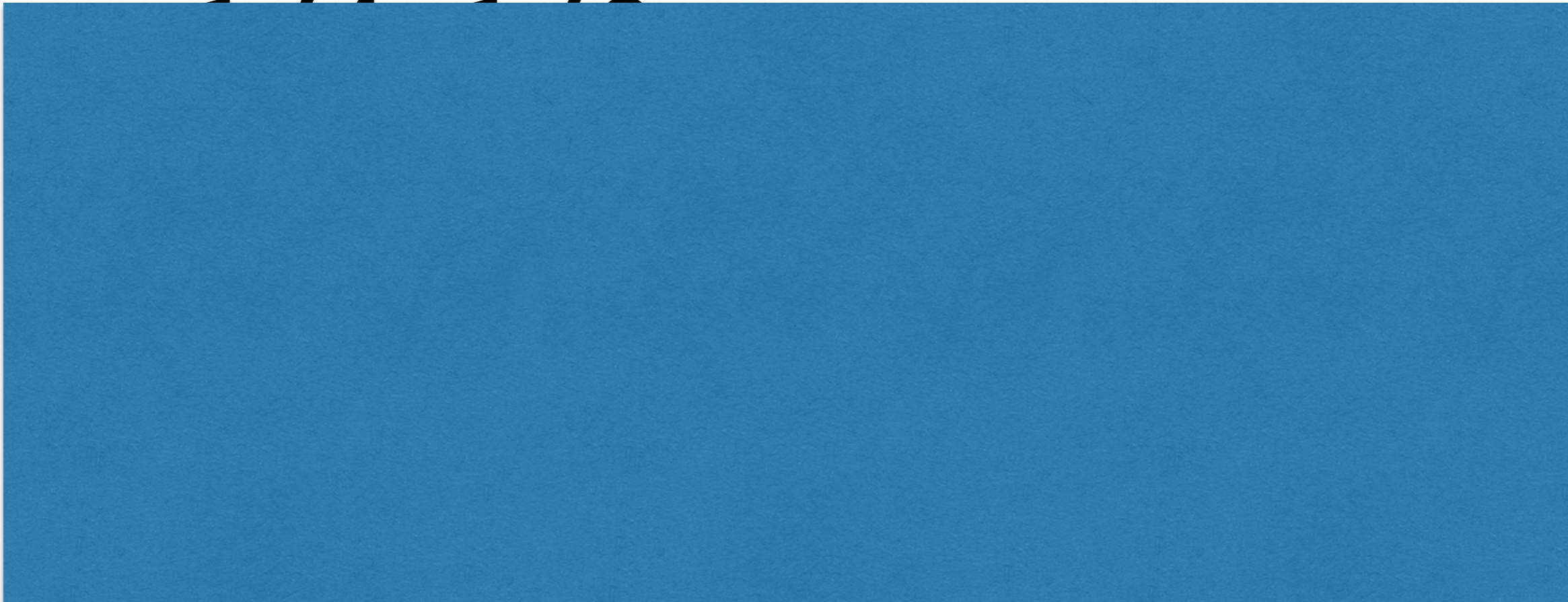
Fractional Numbers

26.375

= (11010.)₂

0.375

11010110



Fractional Numbers

26.375

= (11010.)₂

0.375

= 1/4 + 1/8

0.011

Fractional Numbers

26.375

$= (11010.\underline{\quad\quad})_2$

0.375

$= 1/4 + 1/8$

$= (0.011)_2$

Fractional Numbers

26.375

$= (11010.\underline{\hspace{2cm}})_2$

0.375

$= 1/4 + 1/8$

$= (0.011)_2$

26.375



Fractional Numbers

26.375

$= (11010.\underline{\quad})_2$

0.375

$= 1/4 + 1/8$

$= (0.011)_2$

26.375

$= (11010.011)_2$

Fractional Numbers

26.375

$= (11010.\underline{\quad})_2$

0.375

$= 1/4 + 1/8$

$= (0.011)_2$

26.375

$= (11010.011)_2$

Fractional Numbers

26.375

$= (11010.\underline{\quad})_2$

0.375

$= 1/4 + 1/8$

$= (0.011)_2$

26.375

$= (11010.011)_2$

Fractional Numbers

26.375

$$= 2 * 10^1$$

$$+ 6 * 10^0$$

$$+ 3 * 10^{-1}$$

$$+ 7 * 10^{-2}$$

$$+ 5 * 10^{-3}$$

0.375

$$= 3 * 10^{-1}$$

$$+ 7 * 10^{-2}$$

$$+ 5 * 10^{-3}$$

Fractional Numbers

$$0.375 * 8 / 8$$

$$= 3 / 8$$

$$= (11)_2 / (1000)_2$$

$$= (1.1)_2 / (100)_2$$

$$= (0.11)_2 / (10)_2$$

$$= (0.011)_2 / (1)_2$$

$$= (0.011)_2$$

Fractional Numbers

$$(1/3)_{10} = (0.\underline{3})_{10}$$

$$= (1/11)_2 = (0.\underline{01})_2$$

Fractional Numbers

$$1/5$$

$$= (0.2)_{10}$$

$$= (1/101)_2$$

$$\begin{array}{r} 1. \quad \quad \quad /101 \\ \underline{.101} \quad \quad \quad 0.\underline{0011} \\ .011 \\ \underline{.0101} \\ .0001 \end{array}$$

Check:

$$0.\underline{0011}$$

$$= 3 * \sum_{i \geq 1} 16^{-i}$$

$$= 3 * 1/15$$

$$= 1/5$$

$$= (0.\underline{0011})_2$$

Fractional Numbers

$$\begin{aligned}19.247 \\ &= 1 * 10^1 \\ &+ 9 * 10^0 \\ &+ 2 * 10^{-1} \\ &+ 4 * 10^{-2} \\ &+ 7 * 10^{-3}\end{aligned}$$

$$(19)_{10} = (10011)_2$$

$$\begin{aligned}0.247 \\ &= 2 * 10^{-1} \\ &+ 4 * 10^{-2} \\ &+ 7 * 10^{-3}\end{aligned}$$

Fractional Numbers

$$(0.247)_{10} \\ = (11110111/1111101000)_2$$

11110111.	/1111101000
<u>1111101.000</u>	0.0011...
1111010.000	
<u>111110.1000</u>	
11101.1	

...

Fractional Numbers

0.247



Fractional Numbers

0.247

= 0.247 * 64 / 64

Fractional Numbers

0.247

= 0.247 * 64 / 64

= 15.808 / 64

Fractional Numbers

0.247

$= 0.247 * 64 / 64$

$= 15.808 / 64$

$= (1111.\text{---})_2 / (1000000)_2$

Fractional Numbers

0.247

$= 0.247 * 64 / 64$

$= 15.808 / 64$

$= (1111.\text{————})_2 / (10000000)_2$

$= (0.001111\text{————})_2$

Fractional Numbers

0.247

$= 0.247 * 64 / 64$

$= 15.808 / 64$

$= (1111.\text{---})_2 / (10000000)_2$

$= (0.001111\text{---})_2$

$\approx (0.001111)_2$ lower bound

$\approx (0.01)_2$ upper bound

Fractional Numbers

$(19.247)_{10} > (10011.001111)_2$
lower bound

$(19.247)_{10} < (10011.01)_2$
upper bound

Fractional Numbers

0.247

Fractional Numbers

0.247

= 0.494 / 2

= 0.988 / 4

= 1.976 / 8

Fractional Numbers

0.247

=0.494/2

=0.988/4

=1.976/8

=3.952/16

=7.904/32

Fractional Numbers

0.247

=0.494/2

=0.988/4

=1.976/8

=3.952/16

=7.904/32

=15.808/64

Fractional Numbers

0.247

=0.494/2

=0.988/4

=1.976/8

=3.952/16

=7.904/32

=15.808/64

=31.616/128

=63.232/256

=126.464/512

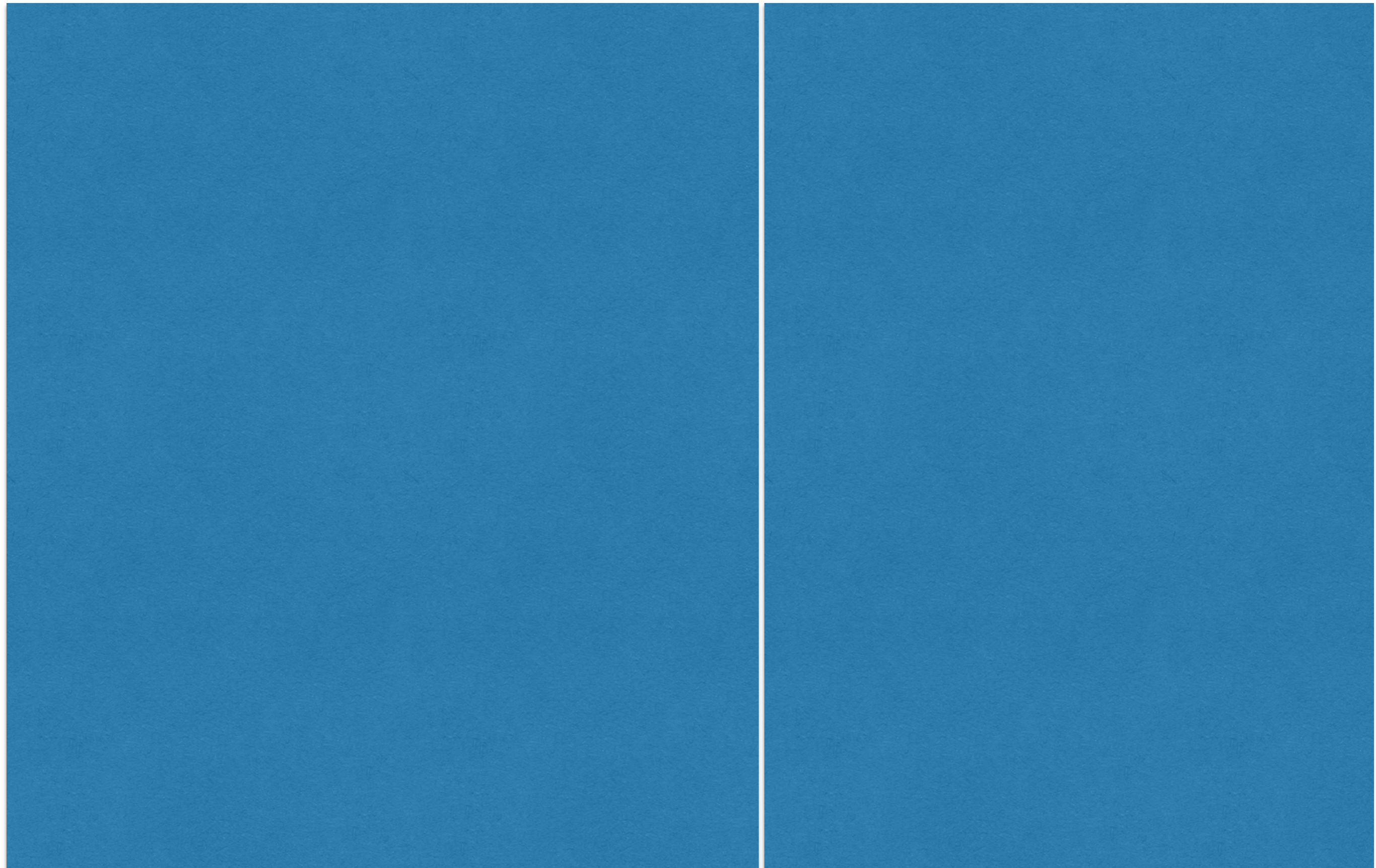
=252.928/1024

=505.856/2048

Fractional Numbers

0.247	$=1011.712/2^{12}$
$=0.494/2$	$=2023.424/2^{13}$
$=0.988/4$	$=4046.848/2^{14}$
$=1.976/8$	$=8093.696/2^{15}$
$=3.952/16$	$=16187.392/2^{16}$
$=7.904/32$	$=32374.784/2^{17}$
$=15.808/64$	$=64749.568/2^{18}$
$=31.616/128$	$=129499.136/2^{19}$
$=63.232/256$	$=258998.272/2^{20}$
$=126.464/512$	$=517996.544/2^{21}$
$=252.928/1024$	$=1035993.088/2^{22}$
$=505.856/2048$	$=2071986.176/2^{23}$

Fractional Numbers



Fractional Numbers

$$=4143972.352/2^{24}$$

$$=8287944.704/2^{25}$$

$$=16575889.408/2^{26}$$

$$=33151778.816/2^{27}$$

$$=66303557.632/2^{28}$$

$$=132607115.264/2^{29}$$

$$=265214230.528/2^{30}$$

$$=530428461.056/2^{31}$$

$$=1060856922.112/2^{32}$$

$$=2121713844.224/2^{33}$$

$$=4243427688.448/2^{34}$$

$$=8486855376.896/2^{35}$$

Fractional Numbers

$$=4143972.352/2^{24}$$

$$=8287944.704/2^{25}$$

$$=16575889.408/2^{26}$$

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$$=1060856922.112/2^{32}$$

$$=2121713844.224/2^{33}$$

$$=4243427688.448/2^{34}$$

$$=8486855376.896/2^{35}$$

$$=————.792/2^{36}$$

$$=————.584/2^{37}$$

$$=————.168/2^{38}$$

$$=————.336/2^{39}$$

$$=————.672/2^{40}$$

$$=————.344/2^{41}$$

$$=————.688/2^{42}$$

$$=————.376/2^{43}$$

$$=————.752/2^{44}$$

$$=————.504/2^{45}$$

$$=————.008/2^{46}$$

$$=————.016/2^{47}$$

Fractional Numbers

Fractional Numbers

$$\text{=————} . 032 / 2^{48}$$

$$\text{=————} . 064 / 2^{49}$$

$$\text{=————} . 128 / 2^{50}$$

$$\text{=————} . 256 / 2^{51}$$

$$\text{=————} . 512 / 2^{52}$$

$$\text{=————} . 024 / 2^{53}$$

$$\text{=————} . 048 / 2^{54}$$

$$\text{=————} . 096 / 2^{55}$$

$$\text{=————} . 192 / 2^{56}$$

$$\text{=————} . 384 / 2^{57}$$

$$\text{=————} . 768 / 2^{58}$$

$$\text{=————} . 536 / 2^{59}$$

Fractional Numbers

$$\text{=————} . 032 / 2^{48}$$

$$\text{=————} . 064 / 2^{49}$$

$$\text{=————} . 128 / 2^{50}$$

$$\text{=————} . 256 / 2^{51}$$

$$\text{=————} . 512 / 2^{52}$$

$$\text{=————} . 024 / 2^{53}$$

$$\text{=————} . 048 / 2^{54}$$

$$\text{=————} . 096 / 2^{55}$$

$$\text{=————} . 192 / 2^{56}$$

$$\text{=————} . 384 / 2^{57}$$

$$\text{=————} . 768 / 2^{58}$$

$$\text{=————} . 536 / 2^{59}$$

$$\text{=————} . 072 / 2^{60}$$

$$\text{=————} . 144 / 2^{61}$$

$$\text{=————} . 288 / 2^{62}$$

$$\text{=————} . 576 / 2^{63}$$

$$\text{=————} . 152 / 2^{64}$$

$$\text{=————} . 304 / 2^{65}$$

$$\text{=————} . 608 / 2^{66}$$

$$\text{=————} . 216 / 2^{67}$$

$$\text{=————} . 432 / 2^{68}$$

$$\text{=————} . 864 / 2^{69}$$

$$\text{=————} . 728 / 2^{70}$$

$$\text{=————} . 456 / 2^{71}$$

Fractional Numbers

$$\text{=————} . 032 / 2^{48}$$

$$\text{=————} . 064 / 2^{49}$$

$$\text{=————} . 128 / 2^{50}$$

$$\text{=————} . 256 / 2^{51}$$

$$\text{=————} . 512 / 2^{52}$$

$$\text{=————} . 024 / 2^{53}$$

$$\text{=————} . 048 / 2^{54}$$

$$\text{=————} . 096 / 2^{55}$$

$$\text{=————} . 192 / 2^{56}$$

$$\text{=————} . 384 / 2^{57}$$

$$\text{=————} . 768 / 2^{58}$$

$$\text{=————} . 536 / 2^{59}$$

$$\text{=————} . 072 / 2^{60}$$

$$\text{=————} . 144 / 2^{61}$$

$$\text{=————} . 288 / 2^{62}$$

$$\text{=————} . 576 / 2^{63}$$

$$\text{=————} . 152 / 2^{64}$$

$$\text{=————} . 304 / 2^{65}$$

$$\text{=————} . 608 / 2^{66}$$

$$\text{=————} . 216 / 2^{67}$$

$$\text{=————} . 432 / 2^{68}$$

$$\text{=————} . 864 / 2^{69}$$

$$\text{=————} . 728 / 2^{70}$$

$$\text{=————} . 456 / 2^{71}$$

$$\text{=————} . 912 / 2^{72}$$

$$\text{=————} . 824 / 2^{73}$$

$$\text{=————} . 648 / 2^{74}$$

$$\text{=————} . 296 / 2^{75}$$

$$\text{=————} . 592 / 2^{76}$$

$$\text{=————} . 184 / 2^{77}$$

$$\text{=————} . 368 / 2^{78}$$

$$\text{=————} . 736 / 2^{79}$$

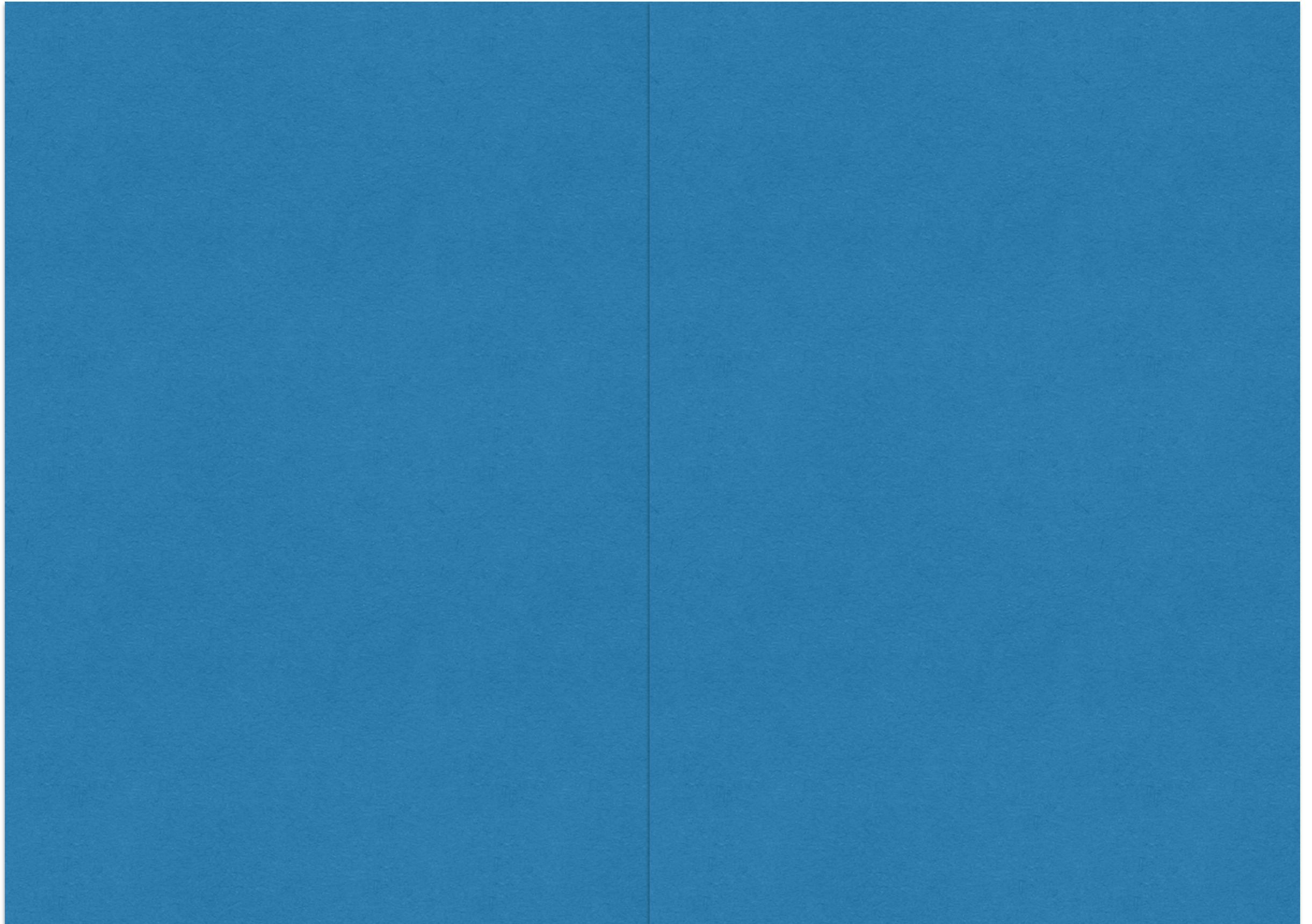
$$\text{=————} . 472 / 2^{80}$$

$$\text{=————} . 944 / 2^{81}$$

$$\text{=————} . 888 / 2^{82}$$

$$\text{=————} . 776 / 2^{83}$$

Fractional Numbers



Fractional Numbers

$$\text{=————} . 552 / 2^{84}$$

$$\text{=————} . 104 / 2^{85}$$

$$\text{=————} . 208 / 2^{86}$$

$$\text{=————} . 416 / 2^{87}$$

$$\text{=————} . 832 / 2^{88}$$

$$\text{=————} . 664 / 2^{89}$$

$$\text{=————} . 328 / 2^{90}$$

$$\text{=————} . 656 / 2^{91}$$

$$\text{=————} . 312 / 2^{92}$$

$$\text{=————} . 624 / 2^{93}$$

$$\text{=————} . 248 / 2^{94}$$

$$\text{=————} . 496 / 2^{95}$$

Fractional Numbers

$$= \text{————} . 552 / 2^{84}$$

$$= \text{————} . 104 / 2^{85}$$

$$= \text{————} . 208 / 2^{86}$$

$$= \text{————} . 416 / 2^{87}$$

$$= \text{————} . 832 / 2^{88}$$

$$= \text{————} . 664 / 2^{89}$$

$$= \text{————} . 328 / 2^{90}$$

$$= \text{————} . 656 / 2^{91}$$

$$= \text{————} . 312 / 2^{92}$$

$$= \text{————} . 624 / 2^{93}$$

$$= \text{————} . 248 / 2^{94}$$

$$= \text{————} . 496 / 2^{95}$$

$$= \text{————} . 992 / 2^{96}$$

$$= \text{————} . 984 / 2^{97}$$

$$= \text{————} . 968 / 2^{98}$$

$$= \text{————} . 936 / 2^{99}$$

$$= \text{————} . 872 / 2^{100}$$

$$= \text{————} . 744 / 2^{101}$$

$$= \text{————} . 488 / 2^{102}$$

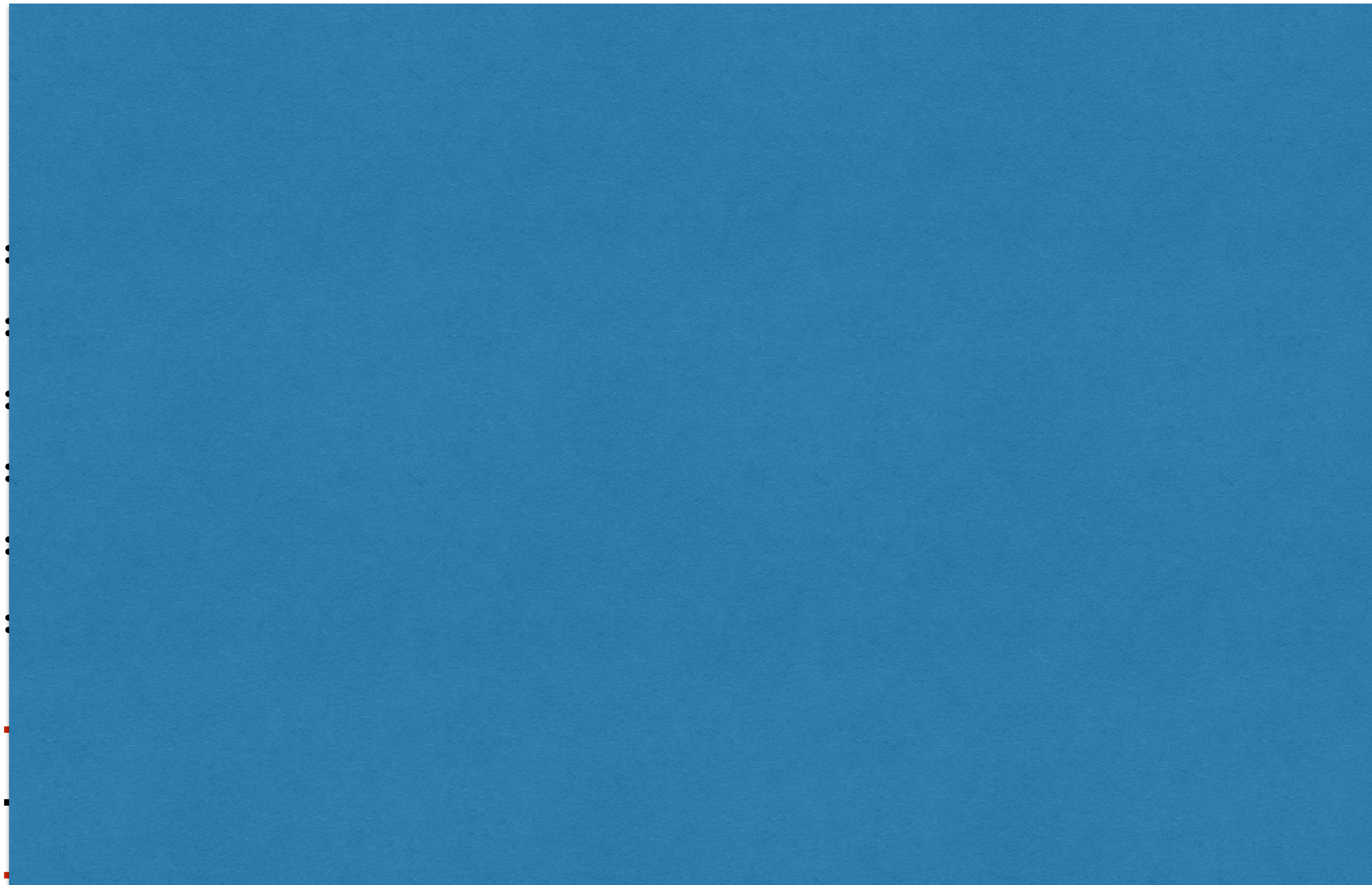
$$= \text{—W—} . 976 / 2^{103}$$

$$= 2\ 504\ 877\ 586\ 050$$

$$981\ 297\ 357\ 485$$

$$533\ 822 . 976 / 2^{103}$$

$$0.247 = (11110111/1111101000)_2$$
$$= 1.976/8$$



$$0.247 = (11110111/1111101000)_2$$
$$= 1.976/8$$

$$0.976 = 8 * 0.247 - 1$$

$$0.247 = (11110111/1111101000)_2$$
$$= 1.976/8$$

$$0.976 = 8 * 0.247 - 1$$

$$= -W - .976/2^{103}$$

$$0.247 = (11110111/1111101000)_2$$
$$= 1.976/8$$

$$0.976 = 8 * 0.247 - 1$$

$$= -W - .976 / 2^{103}$$

$$= (-W + 8 * 0.247 - 1) / 2^{103}$$

$$0.247 = (11110111/1111101000)_2$$
$$= 1.976/8$$

$$0.976 = 8 * 0.247 - 1$$

$$= -W - .976 / 2^{103}$$

$$= (-W + 8 * 0.247 - 1) / 2^{103}$$

$$= (-W - 1) / 2^{103} + 0.247 / 2^{100}$$

$$0.247 = (11110111/1111101000)_2$$

$$= 1.976/8$$

$$0.976 = 8 * 0.247 - 1$$

$$= -W - .976 / 2^{103}$$

$$= (-W + 8 * 0.247 - 1) / 2^{103}$$

$$= (-W - 1) / 2^{103} + 0.247 / 2^{100}$$

$$= (-W - 1) / 2^{103} + (-W - 1) / 2^{203} + 0.247 / 2^{200}$$

$$= (-W - 1) / 8 \sum_{i \geq 1} 2^{-100 * i}$$

$$= (0.001\underline{11111001110110110010}$$

$$\underline{001011010000111001010110000001}$$

$$\underline{000001100010010011011101001011}$$

$$\underline{11000110101001111110})_2$$

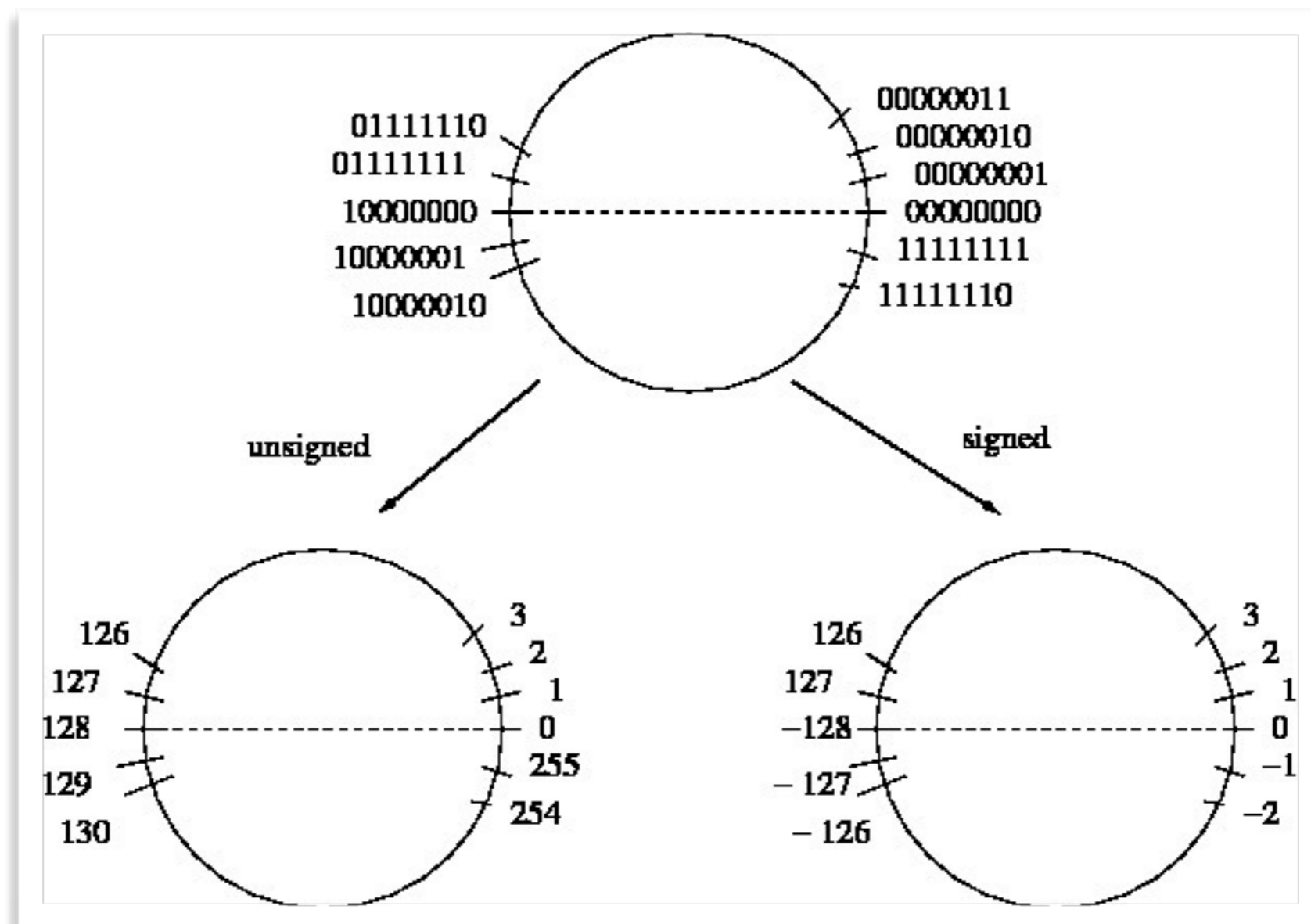
Fractional Numbers

$$(19.247)_{10} =$$

$$\begin{aligned} & (10011.001\underline{11111001110110110010} \\ & \underline{001011010000111001010110000001} \\ & \underline{000001100010010011011101001011} \\ & \underline{11000110101001111110})_2 \end{aligned}$$

More Binary Representation

More Binary Representation



Representation

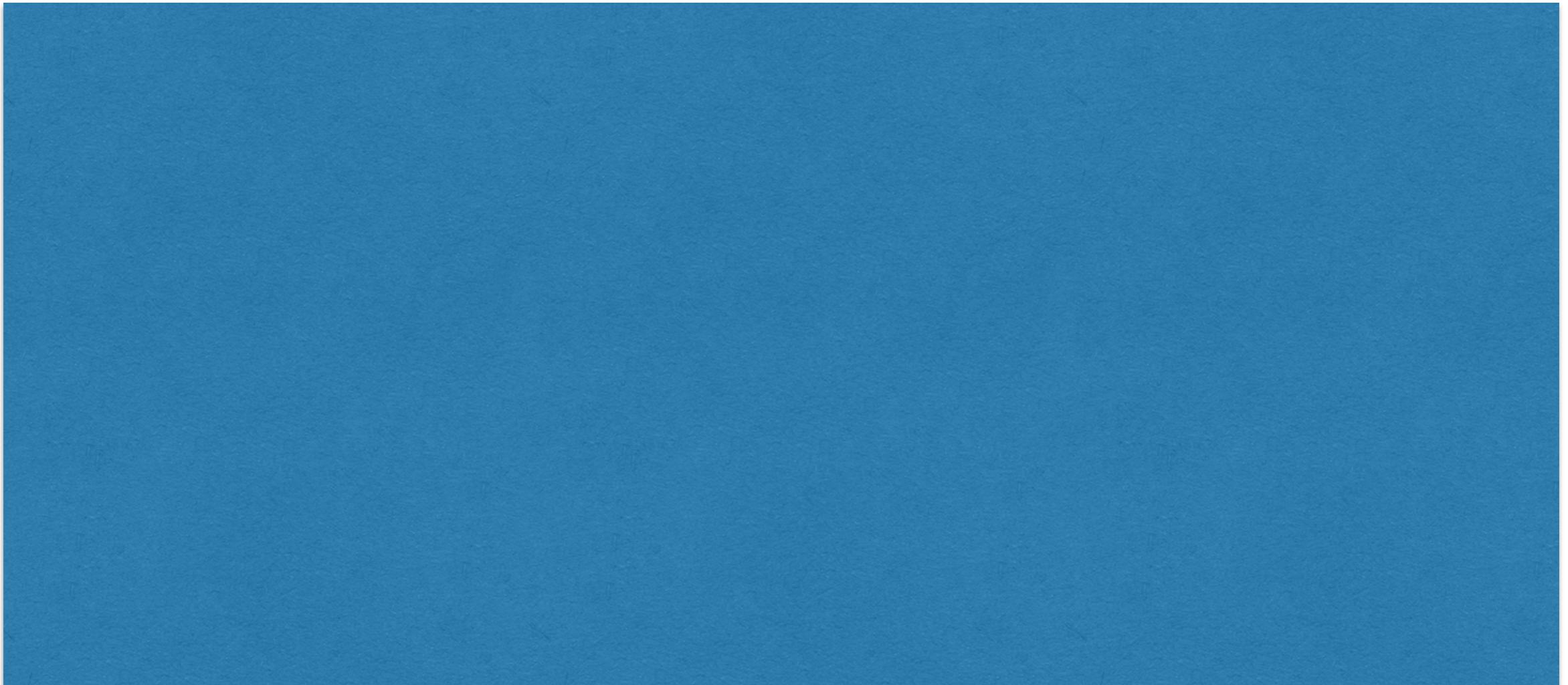
<u>binary</u>	<u>signed</u>	<u>unsigned</u>
00000000	0	0
00000001	1	1
:	:	:
01111111	127	127
10000000	-128	128
10000001	-127	129
:	:	:
11111111	-1	255

Representation

<u>binary</u>	<u>signed</u>	<u>unsigned</u>
00000000000000000000	0	0
00000000000000000001	1	1
:	:	:
0000000001111111	127	127
0000000010000000	128	128
0000000010000001	129	129
:	:	:
0111111111111111	$2^{15} - 1$	$2^{15} - 1$
1000000000000000	-2^{15}	2^{15}
1000000000000001	$-2^{15} + 1$	$2^{15} + 1$
:	:	:
1111111101111111	-129	$2^{16} - 129$
1111111110000000	-128	$2^{16} - 128$
1111111110000001	-127	$2^{16} - 127$
:	:	:
1111111111111111	-1	$2^{16} - 1$

Fun with Java !!!

```
for (short s = 32767; s < 32768; s++)  
    System.out.println(s);
```



Fun with Java !!!

```
for (short s = 32767; s < 32768; s++)  
    System.out.println(s);
```

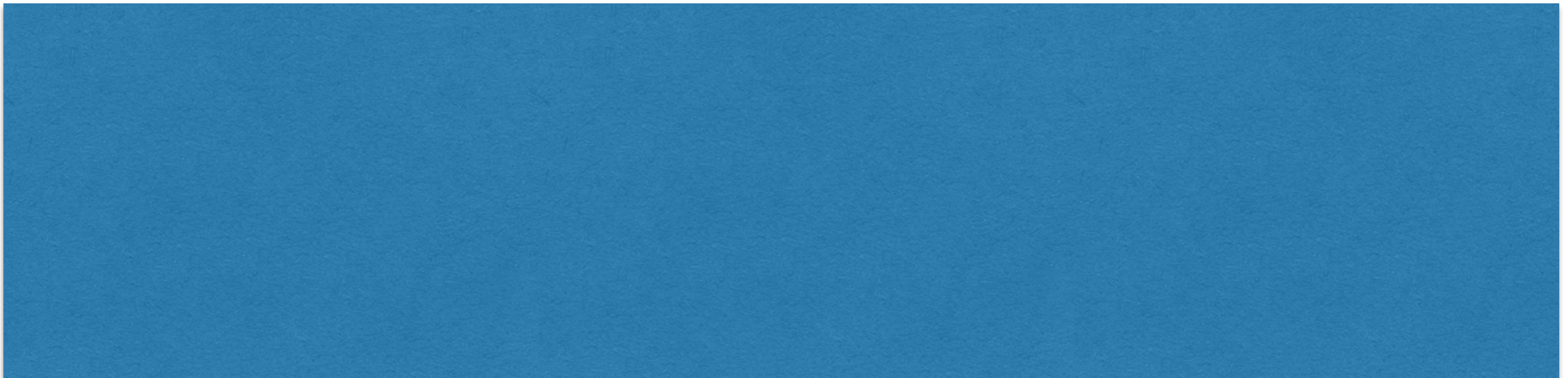
32767



Fun with Java !!!

```
for (short s = 32767; s < 32768; s++)  
    System.out.println(s);
```

```
32767  
-32768  
-32767
```



Fun with Java !!!

```
for (short s = 32767; s < 32768; s++)  
    System.out.println(s);
```

32767

-32768

-32767

...

-1

0

1

Fun with Java !!!

```
for (short s = 32767; s < 32768; s++)  
    System.out.println(s);
```

32767

-32768

-32767

...

-1

0

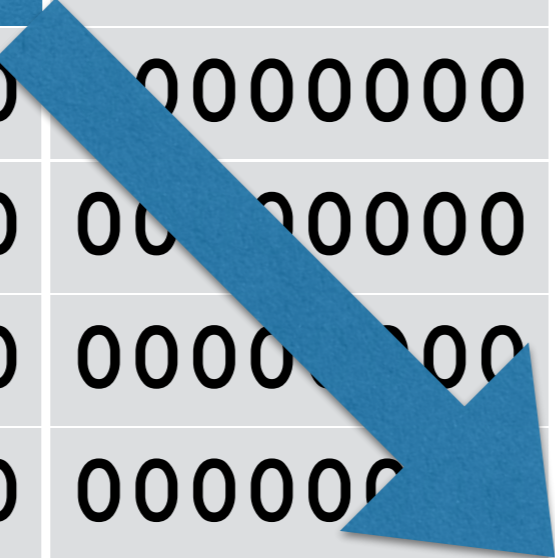
1

A Byte

10100110

An address

00010110	000000000	000000000	000000000
000000000	000000000	000000000	000000000
000000000	000000000	000000000	000000000
000000000	000000000	000000000	000000000
000000000	000000000	000000000	000000000
000000000	000000000	10100110	000000000
000000000	000000000	000000000	000000000
000000000	000000000	000000000	000000000
000000000	000000000	000000000	000000000
000000000	000000000	000000000	000000000
000000000	000000000	000000000	000000000
000000000	000000000	000000000	000000000




A (32-bit) address

00000000	00000000	00000000	00010110
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	10100110	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000



A (64-bit) address

00000000	00000000	00000000	00000000
00000000	00000000	00000000	00010110
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	10100110	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000



Storage sizes

- 2^{10} bytes = 1 kilobyte (1 KB) $\approx 10^3$ bytes (one thousand)
- 2^{20} bytes = 1 megabyte (1 MB) $\approx 10^6$ bytes (one million)
- 2^{30} bytes = 1 gigabyte (1 GB) $\approx 10^9$ bytes (one billion)
- 2^{40} bytes = 1 terabyte (1 TB) $\approx 10^{12}$ bytes (one trillion)
- 2^{50} bytes = 1 petabyte (1 PB) $\approx 10^{15}$ bytes
- 2^{60} bytes = 1 exabyte (1 EB) $\approx 10^{18}$ bytes

**Big
Data**

Java Primitive Types

Boolean	00000000	00000000	00000000	00000000
Byte	00000000	00000000	00000000	00000000
Char	00000000	00000000	00000000	00000000
Short	00000000	00000000	00000000	00000000
Int	00000000	00000000	00000000	00000000
Long	00000000	10100110	00000000	00000000
	00000000	00000000	00000000	00000000
Float	00000000	00000000	00000000	00000000
Double	00000000	00000000	00000000	00000000
	00000000	00000000	00000000	00000000

(32-bit) addresses

00000000	00000000	00000000	00010010
00000000	00000000	00000000	00101010
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	10100110	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000

The image shows a 15x4 grid of 32-bit addresses. The top two rows are highlighted in blue and red respectively. A large red arrow points from the top row to the bottom row, specifically to the address 00000000 00000000 00000000 00000000. A blue arrow points from the top row to the bit 10100110 in the third column of the fifth row. The address 10100110 00000000 00000000 00000000 is highlighted in yellow.

Java Reference Types

32-Bit

Address

00000000 00000000 00000000 00000000

Address

00000000 10100110 00000000 00000000

00000000 00000000 00000000 00000000

64-Bit

a[0]=166;

a :

00000000	00000000	00000000	00010010
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	10100110	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000
00000000	00000000	00000000	00000000



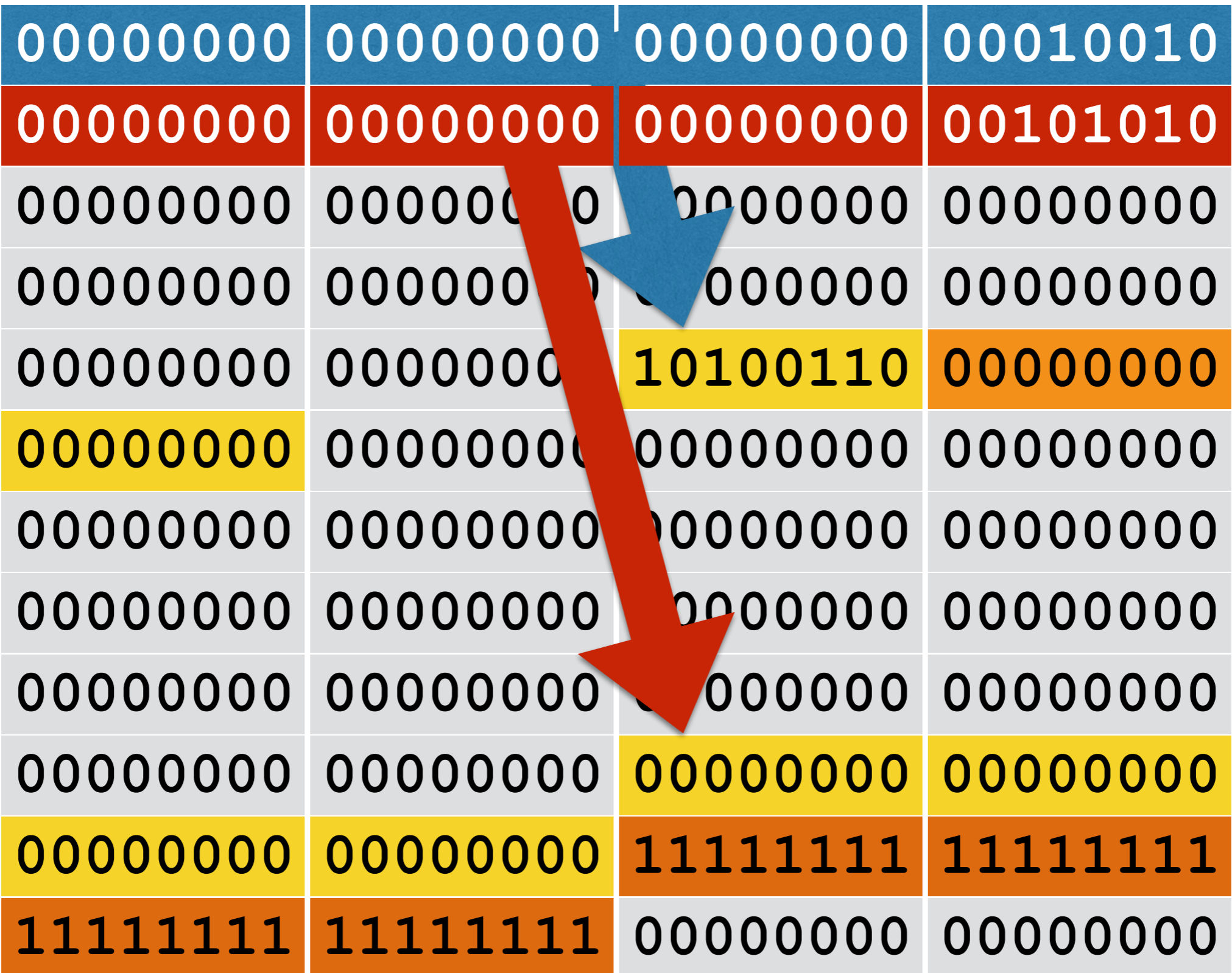
int[] b;

a:	00000000	00000000	00000000	00010010
b:	00000000	00000000	00000000	00000000
	00000000	00000000	00000000	00000000
	00000000	00000000	00000000	00000000
	00000000	00000000	10100110	00000000
	00000000	00000000	00000000	00000000
	00000000	00000000	00000000	00000000
	00000000	00000000	00000000	00000000
	00000000	00000000	00000000	00000000
	00000000	00000000	00000000	00000000
	00000000	00000000	00000000	00000000
	00000000	00000000	00000000	00000000
	00000000	00000000	00000000	00000000



`b[l] = -l;`

a:	00000000	00000000	00000000	00010010
b:	00000000	00000000	00000000	00101010
	00000000	00000000	00000000	00000000
	00000000	00000000	00000000	00000000
	00000000	00000000	10100110	00000000
	00000000	00000000	00000000	00000000
	00000000	00000000	00000000	00000000
	00000000	00000000	00000000	00000000
	00000000	00000000	00000000	00000000
	00000000	00000000	00000000	00000000
	00000000	00000000	11111111	11111111
	11111111	11111111	00000000	00000000



Winter 2016
COMP-250: Introduction
to Computer Science

Lecture 4, January 21, 2016