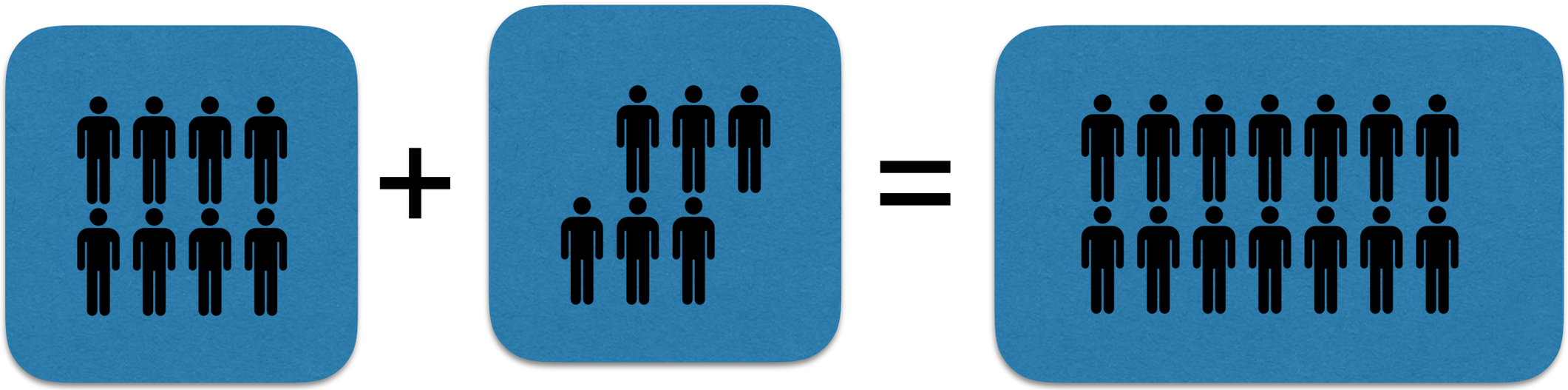
Winter 2016
COMP-250: Introduction to Computer Science

Lecture 2, January 14, 2016

Grade School Algorithms

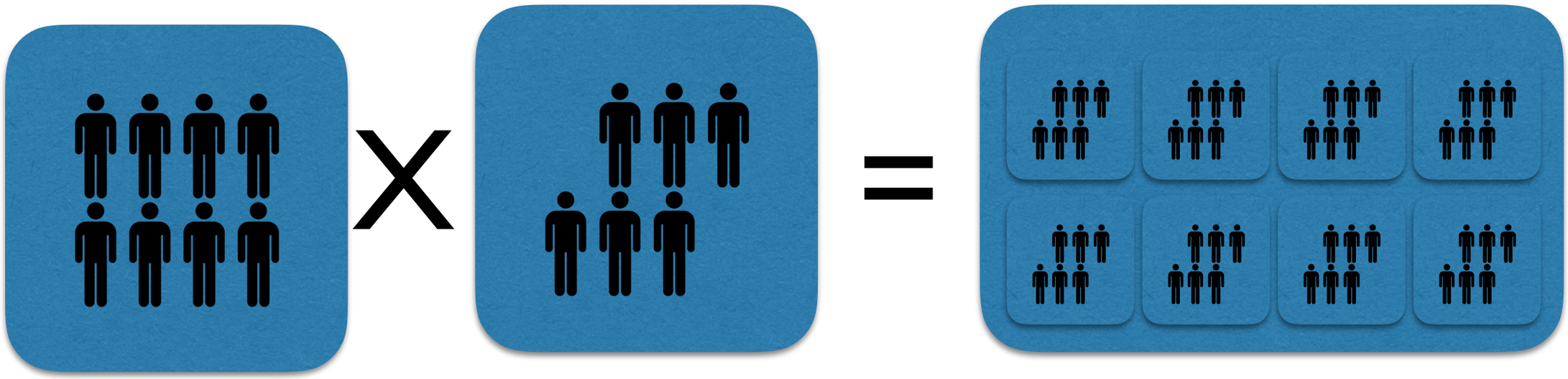
Grade School Algorithms



Representation quite inefficient

"+" easy to describe

Grade School Algorithms



Representation quite inefficient

"X" easy to describe

Inefficient Representation

1 million kids meditate for world peace
@ Phra Shammakaya temple Thailand

Roman Grade School

I	V	X	L
one	five	ten	fifty

C	D	M
one hundred	five hundred	one thousand

Representation very efficient

Roman Grade School

MMCCCXXIV

\+ MCMXXXVII

Representation very efficient

"+" complicated to describe

Roman Grade School

MMCCCXXIV

X MCMXXXVII

Representation very efficient

"X" complicated to describe

Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

$$\begin{array}{r} 2343 \\ + \quad 4519 \\ \hline \end{array}$$

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

$$\begin{array}{r} 2343 \\ +\ 4519 \\ \hline 12 \end{array}$$

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

$$\begin{array}{r} {\scriptstyle 1} \\ 2343 \\ +\ 4519 \\ \hline 2 \end{array}$$

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

$$\begin{array}{r} {\scriptstyle 0\ 1\ \ } \\ 2343 \\ +\ 4519 \\ \hline 62 \end{array}$$

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

$$\begin{array}{r} {\scriptstyle 0\,0\,1} \\ 2343 \\ +\quad 4519 \\ \hline 6862 \end{array}$$

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

$$\begin{array}{r} {\scriptstyle 0\ 0\ 0\ 1\ } \\ 2343 \\ +\ 4519 \\ \hline 6862 \end{array}$$

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

$$\begin{array}{r} {\scriptstyle 0\,0\,0\,1} \\ 2343 \\ +\ 4519 \\ \hline 06862 \end{array}$$

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

$$\begin{array}{r} {\scriptstyle 0\ 0\ 0\ 1\ } \\ 2343 \\ +\ 4519 \\ \hline 6862 \end{array}$$

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

---

$carry \leftarrow 0$
for $i \leftarrow 0$ **to** $N-1$ **do**
  $r[i] \leftarrow R[a[i],b[i],carry]$
  $carry \leftarrow L[a[i],b[i],carry]$
end for
$r[N] \leftarrow carry$

Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

$L[i,j,0]$

L	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	1
2	0	0	0	0	0	0	0	0	1	1
3	0	0	0	0	0	0	0	1	1	1
4	0	0	0	0	0	0	1	1	1	1
5	0	0	0	0	0	1	1	1	1	1
6	0	0	0	0	1	1	1	1	1	1
7	0	0	0	1	1	1	1	1	1	1
8	0	0	1	1	1	1	1	1	1	1
9	0	1	1	1	1	1	1	1	1	1

$R[i,j,0]$

R	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	0
2	2	3	4	5	6	7	8	9	0	1
3	3	4	5	6	7	8	9	0	1	2
4	4	5	6	7	8	9	0	1	2	3
5	5	6	7	8	9	0	1	2	3	4
6	6	7	8	9	0	1	2	3	4	5
7	7	8	9	0	1	2	3	4	5	6
8	8	9	0	1	2	3	4	5	6	7
9	9	0	1	2	3	4	5	6	7	8

Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

$L[i,j,0]$

L	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	1
2	0	0	0	0	0	0	0	0	1	1
3	0	0	0	0	0	0	0	1	1	1
4	0	0	0	0	0	0	1	1	1	1
5	0	0	0	0	0	1	1	1	1	1
6	0	0	0	0	1	1	1	1	1	1
7	0	0	0	1	1	1	1	1	1	1
8	0	0	1	1	1	1	1	1	1	1
9	0	1	1	1	1	1	1	1	1	1

$R[i,j,0]$

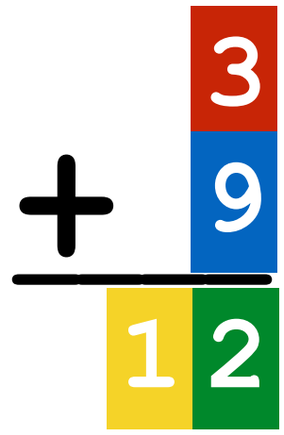
R	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	0
2	2	3	4	5	6	7	8	9	0	1
3	3	4	5	6	7	8	9	0	1	2
4	4	5	6	7	8	9	0	1	2	3
5	5	6	7	8	9	0	1	2	3	4
6	6	7	8	9	0	1	2	3	4	5
7	7	8	9	0	1	2	3	4	5	6
8	8	9	0	1	2	3	4	5	6	7
9	9	0	1	2	3	4	5	6	7	8

Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

$L[i,j,0]$

L	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	1
2	0	0	0	0	0	0	0	0	1	1
3	0	0	0	0	0	0	0	1	1	1
4	0	0	0	0	0	0	1	1	1	1
5	0	0	0	0	0	1	1	1	1	1
6	0	0	0	0	1	1	1	1	1	1
7	0	0	0	1	1	1	1	1	1	1
8	0	0	1	1	1	1	1	1	1	1
9	0	1	1	1	1	1	1	1	1	1



$R[i,j,0]$

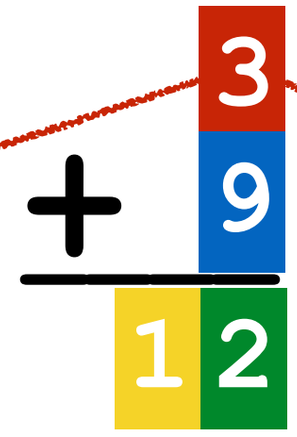
R	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	0
2	2	3	4	5	6	7	8	9	0	1
3	3	4	5	6	7	8	9	0	1	2
4	4	5	6	7	8	9	0	1	2	3
5	5	6	7	8	9	0	1	2	3	4
6	6	7	8	9	0	1	2	3	4	5
7	7	8	9	0	1	2	3	4	5	6
8	8	9	0	1	2	3	4	5	6	7
9	9	0	1	2	3	4	5	6	7	8

Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

$L[i,j,0]$

L	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	1
2	0	0	0	0	0	0	0	0	1	1
3	0	0	0	0	0	0	0	1	1	1
4	0	0	0	0	0	0	1	1	1	1
5	0	0	0	0	0	1	1	1	1	1
6	0	0	0	0	1	1	1	1	1	1
7	0	0	0	1	1	1	1	1	1	1
8	0	0	1	1	1	1	1	1	1	1
9	0	1	1	1	1	1	1	1	1	1



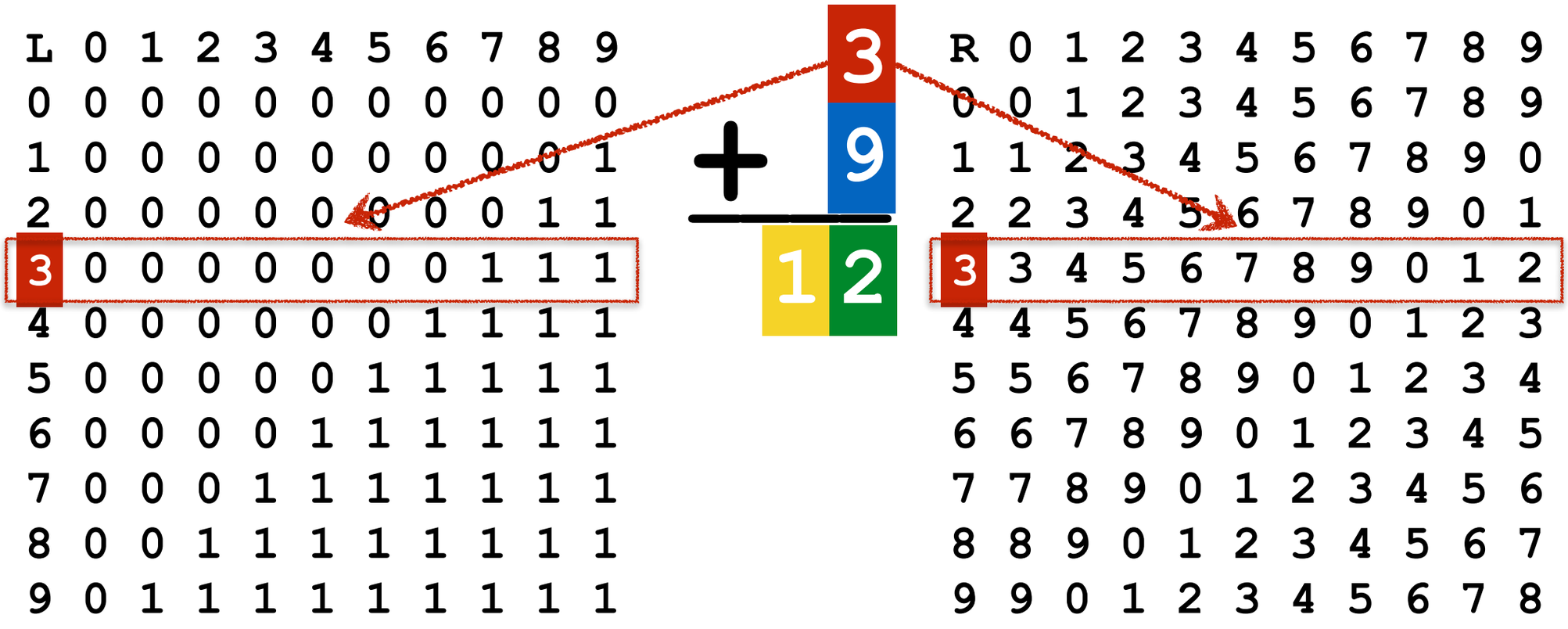
$R[i,j,0]$

R	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	0
2	2	3	4	5	6	7	8	9	0	1
3	3	4	5	6	7	8	9	0	1	2
4	4	5	6	7	8	9	0	1	2	3
5	5	6	7	8	9	0	1	2	3	4
6	6	7	8	9	0	1	2	3	4	5
7	7	8	9	0	1	2	3	4	5	6
8	8	9	0	1	2	3	4	5	6	7
9	9	0	1	2	3	4	5	6	7	8

Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

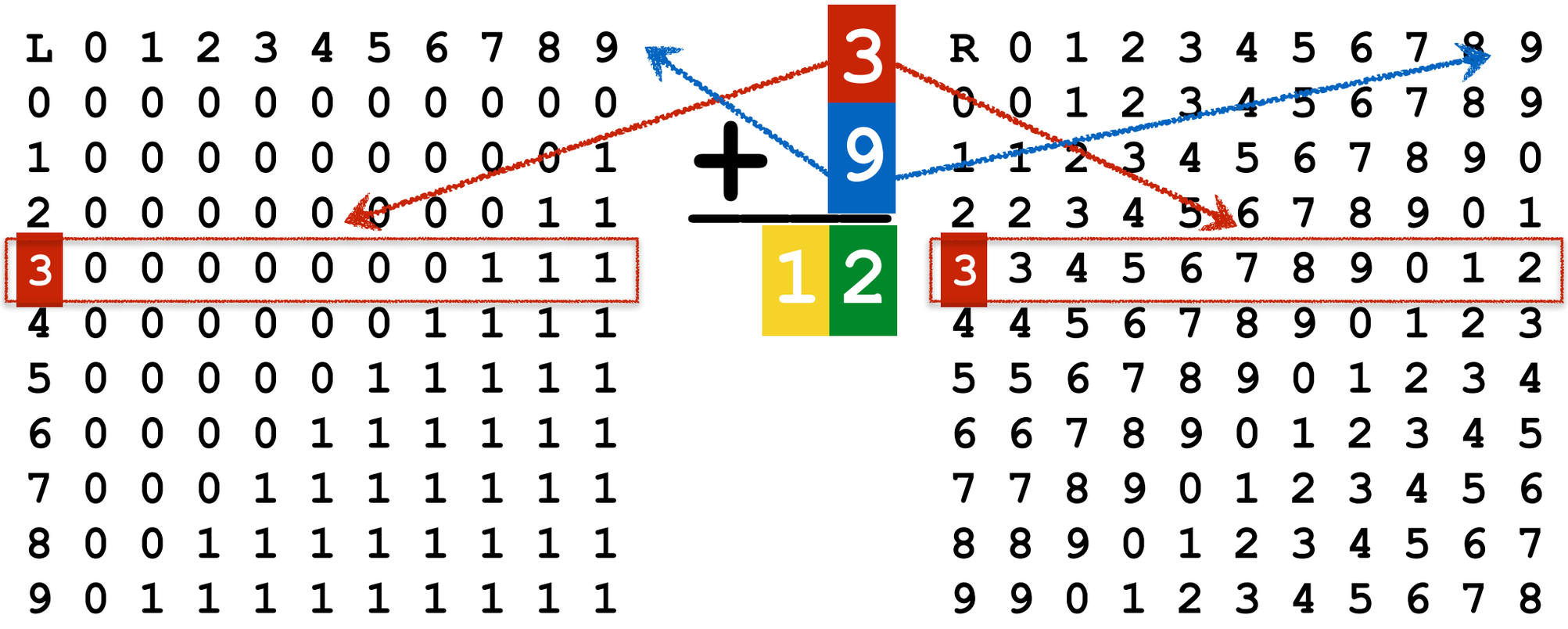
$L[i,j,0]$ $R[i,j,0]$



Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

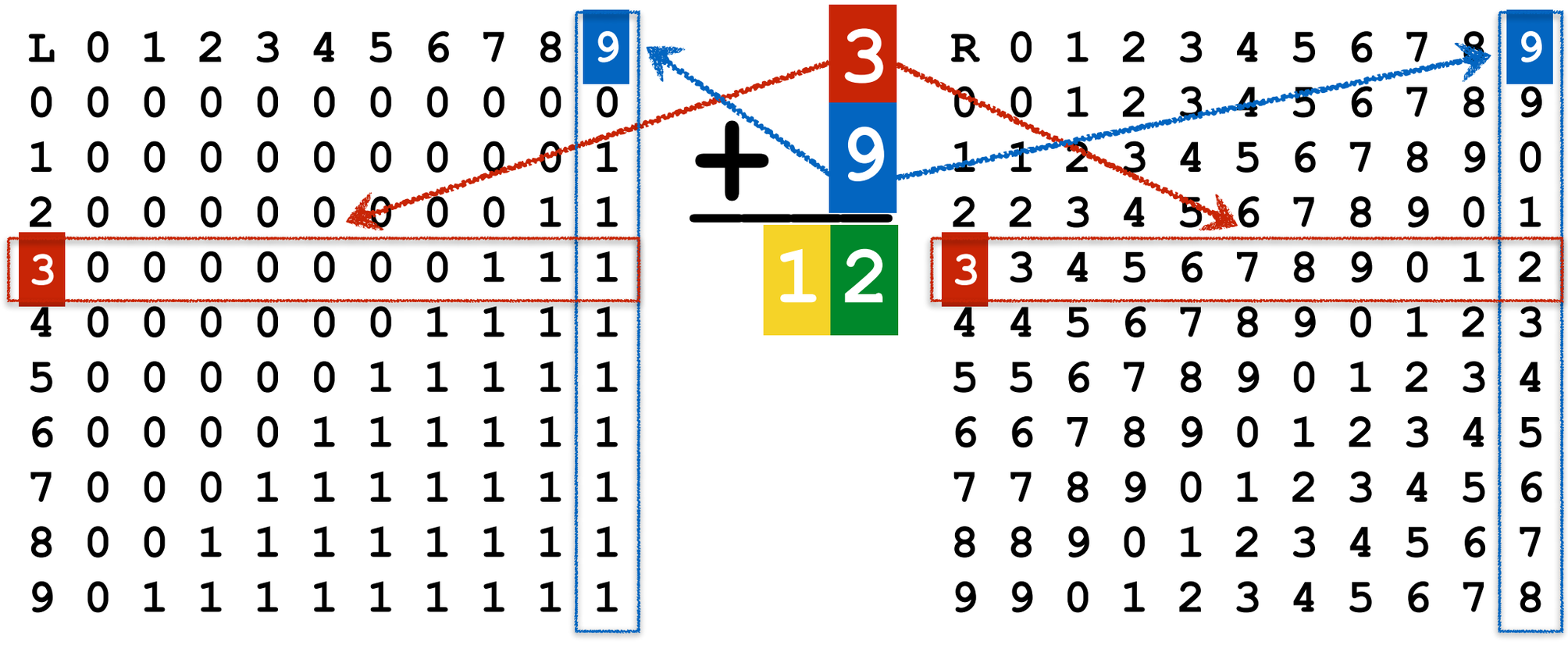
$L[i,j,0]$ $R[i,j,0]$



Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

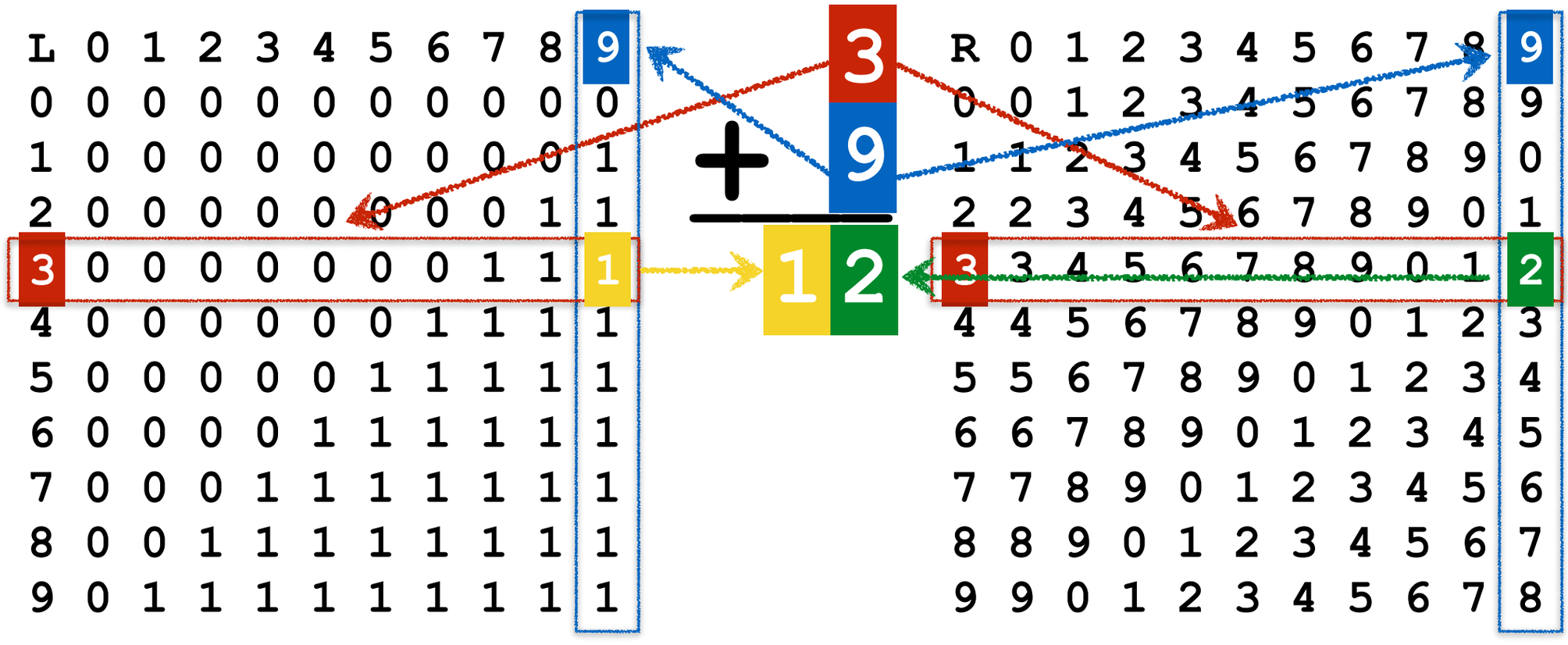
$L[i,j,0]$ $R[i,j,0]$



Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

$L[i,j,0]$ $R[i,j,0]$



Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

$L[i,j,1]$

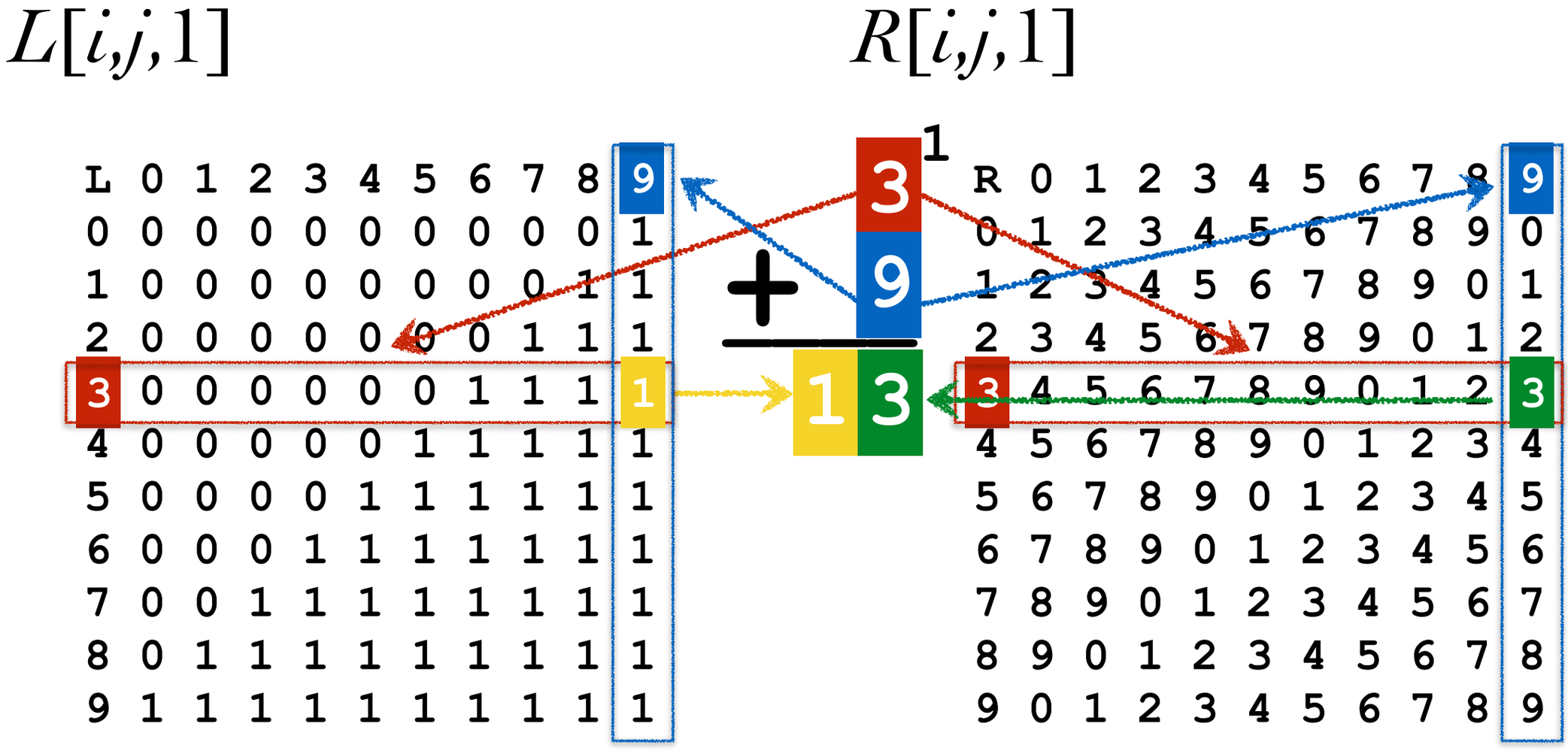
L	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	0	0	1	1
2	0	0	0	0	0	0	0	1	1	1
3	0	0	0	0	0	0	1	1	1	1
4	0	0	0	0	0	1	1	1	1	1
5	0	0	0	0	1	1	1	1	1	1
6	0	0	0	1	1	1	1	1	1	1
7	0	0	1	1	1	1	1	1	1	1
8	0	1	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	1	1

$R[i,j,1]$

R	0	1	2	3	4	5	6	7	8	9
0	1	2	3	4	5	6	7	8	9	0
1	2	3	4	5	6	7	8	9	0	1
2	3	4	5	6	7	8	9	0	1	2
3	4	5	6	7	8	9	0	1	2	3
4	5	6	7	8	9	0	1	2	3	4
5	6	7	8	9	0	1	2	3	4	5
6	7	8	9	0	1	2	3	4	5	6
7	8	9	0	1	2	3	4	5	6	7
8	9	0	1	2	3	4	5	6	7	8
9	0	1	2	3	4	5	6	7	8	9

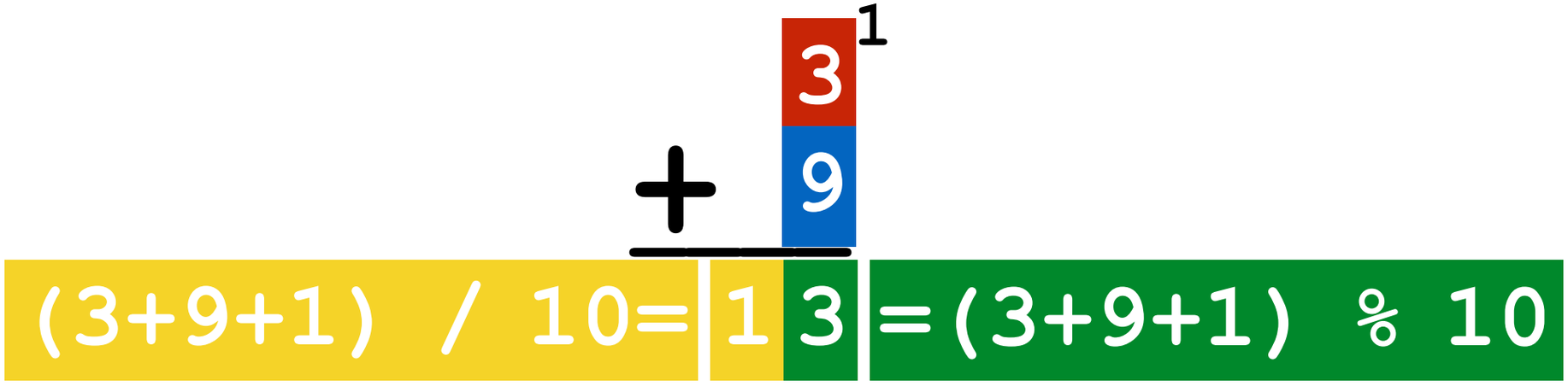
Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits



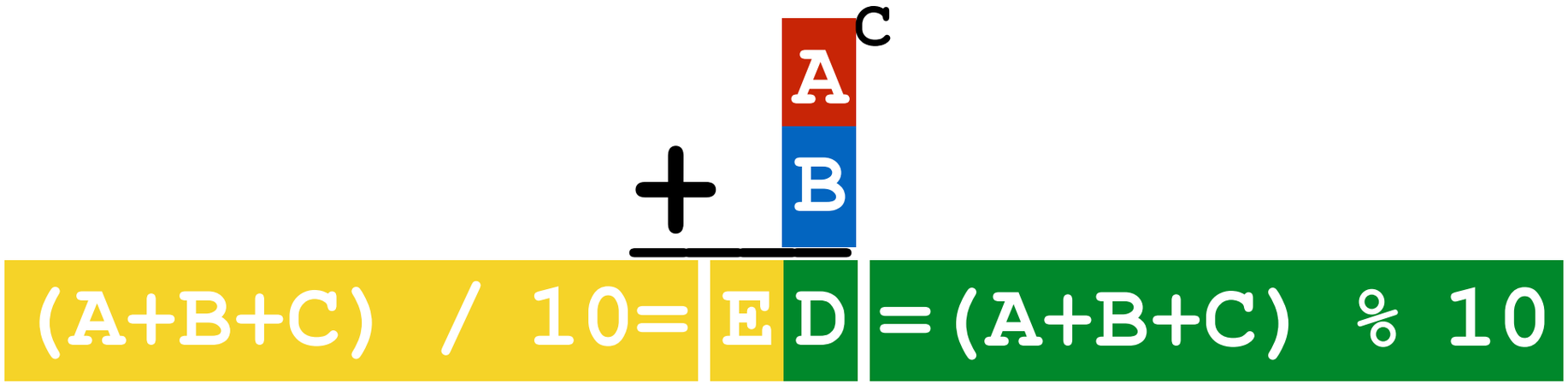
Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits



Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits



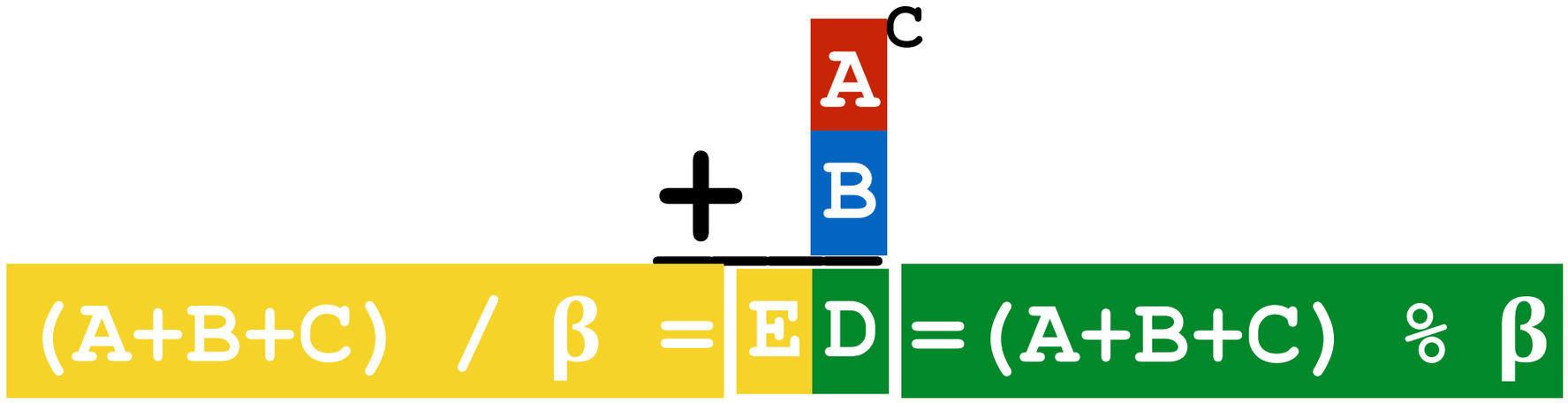
Grade School Algorithms

Algorithm 1 Addition (base 10): Add two N digit numbers a and b which are represented as arrays of digits

$carry = 0$
for $i = 0$ to $N - 1$ **do**
  $r[i] \leftarrow (a[i] + b[i] + carry)\ \%\ 10$
  $carry \leftarrow (a[i] + b[i] + carry)/10$
end for
$r[N] \leftarrow carry$

Grade School Algorithms

Algorithm 1 Addition (base **β**): Add two N **β-git** numbers a and b which are represented as arrays of **β-gits**



Grade School Algorithms

Algorithm 1 Addition (base **β**): Add two **β-git** numbers a and b which are represented as arrays of **β-gits**

$carry = 0$
for $i = 0$ to $N - 1$ **do**
  $r[i] \leftarrow (a[i] + b[i] + carry)$ % **β**
  $carry \leftarrow (a[i] + b[i] + carry)/$ **β**
end for
$r[N] \leftarrow carry$

Example: addition base 8

$$(1205)_8$$
$$+\ (736)_8$$
$$\_\_\_\_\_$$

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

Example: addition base 8

$$(1205)_8 + (736)_8 = (13)_8$$

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

Example: addition base 8

$$\begin{array}{r} {\scriptstyle 1} \\ (1205)_8 \\ +\ (736)_8 \\ \hline \\ (\ \ 3)_8 \end{array}$$

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

Example: addition base 8

$$\begin{array}{r} {\scriptstyle 0\ 1} \\ (1205)_8 \\ +\ (736)_8 \\ \hline (43)_8 \end{array}$$

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

Example: addition base 8

$$
\begin{array}{r}
{\scriptstyle 0\ 1\ \ \ } \\
(1205)_8 \\
+\ (736)_8 \\
\hline
(1143)_8
\end{array}
$$

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

Example: addition base 8

$$
\begin{array}{r}
{\scriptstyle 1\ 0\ 1} \\
(1205)_8 \\
+\ (736)_8 \\
\hline
(\ 143)_8
\end{array}
$$

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

Example: addition base 8

$$\begin{array}{r} {\scriptstyle 1\ 0\ 1} \\ (1205)_8 \\ +\ (736)_8 \\ \hline (2143)_8 \end{array}$$

+	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	10
2	2	3	4	5	6	7	10	11
3	3	4	5	6	7	10	11	12
4	4	5	6	7	10	11	12	13
5	5	6	7	10	11	12	13	14
6	6	7	10	11	12	13	14	15
7	7	10	11	12	13	14	15	16

Example: addition base 8

$$\begin{array}{r} {\scriptstyle 1\ 0\ 1\ \ } \\ (1205)_8 \\ +\ (736)_8 \\ \hline (2143)_8 \end{array}$$

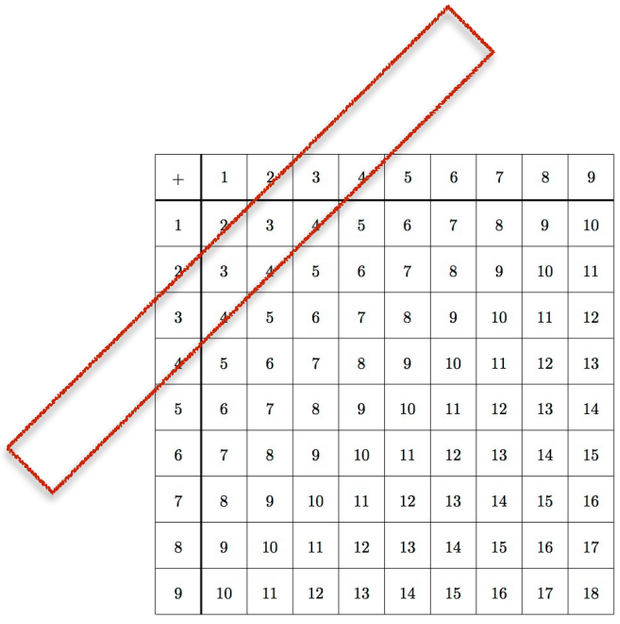
(1123 in base ten)

Example: addition base 8

$$
\begin{array}{r}
{\scriptstyle 1\ 0\ 1} \\
(1205)_8 \\
+\ (736)_8 \\
\hline
(2143)_8 \quad =(1123)_x
\end{array}
$$

Subtraction

$$\begin{array}{r} 6343 \\ -\ 4519 \\ \hline \end{array}$$



+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

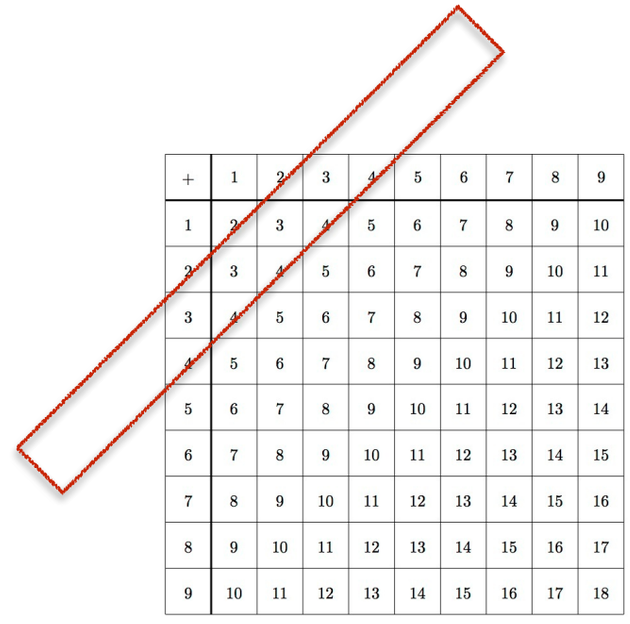
Grade School Algorithms

$$\begin{array}{r} {}^{3}{}_{1} \\ 6343 \\ -\ 4519 \\ \hline \end{array}$$

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Grade School Algorithms

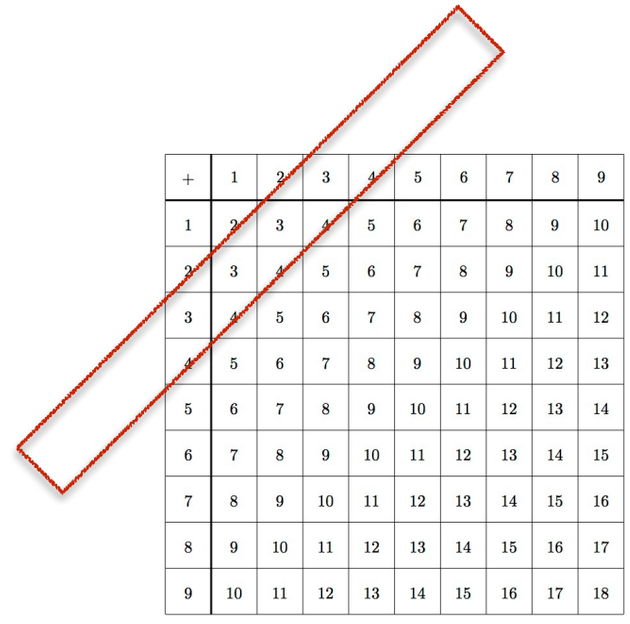
$$
\begin{array}{r}
{\scriptstyle 3\,1} \\
6343 \\
-\ 4519 \\
\hline
4
\end{array}
$$



+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Grade School Algorithms

$$
\begin{array}{r}
{\scriptstyle 3\,1} \\
6343 \\
-\ 4519 \\
\hline
24
\end{array}
$$



+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Grade School Algorithms

$$\begin{array}{r} {}^{5}\cancel{6}\,{}^{3}\cancel{4}3 \\ -\ 4519 \\ \hline 24 \end{array}$$

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

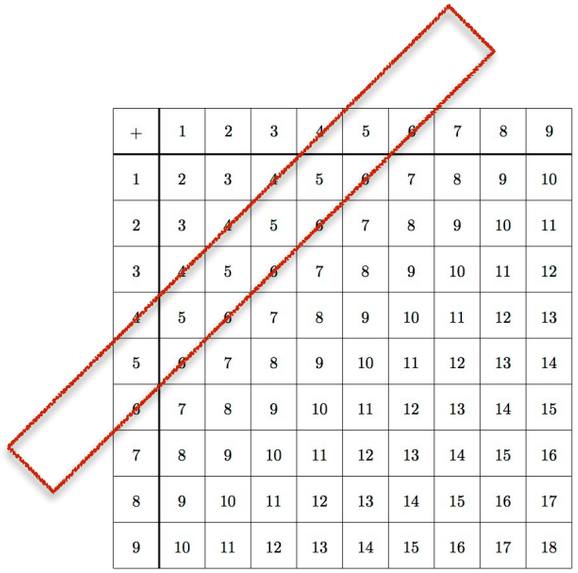
Grade School Algorithms

$$
\begin{array}{r}
{}^{5}\cancel{6}\,{}^{1}3\,{}^{1}\cancel{4}3 \\
-\;\;4519 \\
\hline
824
\end{array}
$$

+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Grade School Algorithms

$$
\begin{array}{r}
{}^{5}\cancel{6}\,{}^{3}\cancel{4}3 \\
-\ 4519 \\
\hline
1824
\end{array}
$$



+	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18

Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

Name: _____

Multiplication Table

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

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Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

```
   352
x    4
-------
```

Name: _____

Multiplication Table

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

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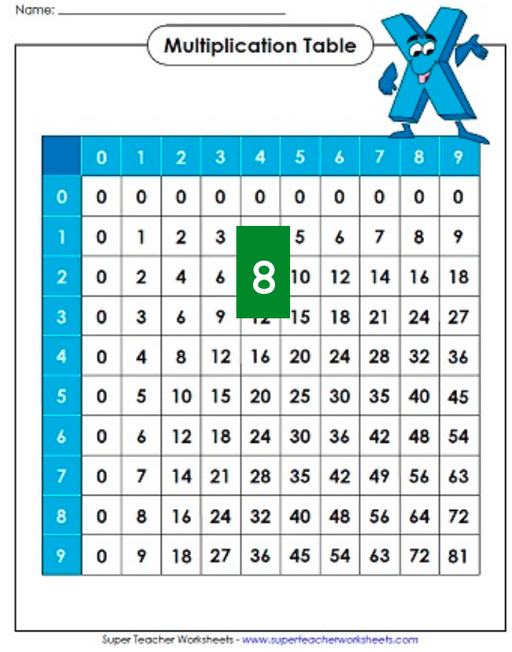
Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

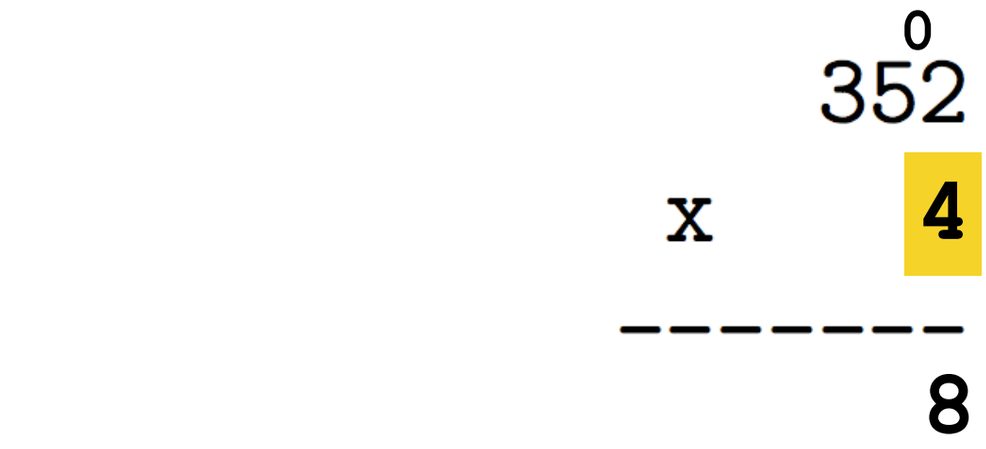


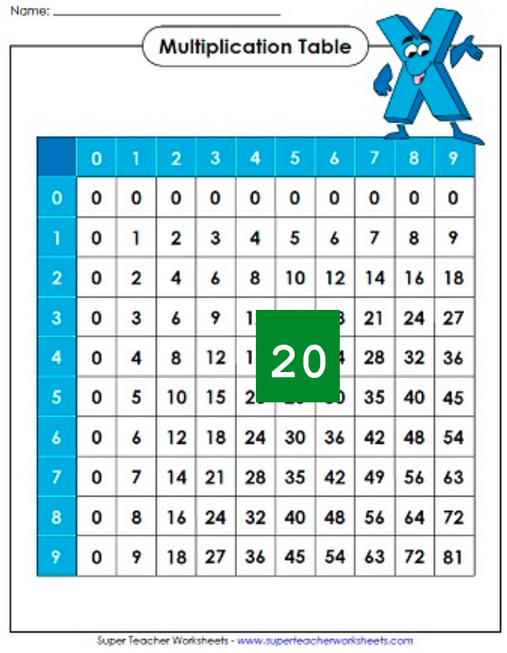
	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

$$\begin{array}{r} 352 \\ \times \quad 4 \\ \hline 8 \end{array}$$

Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

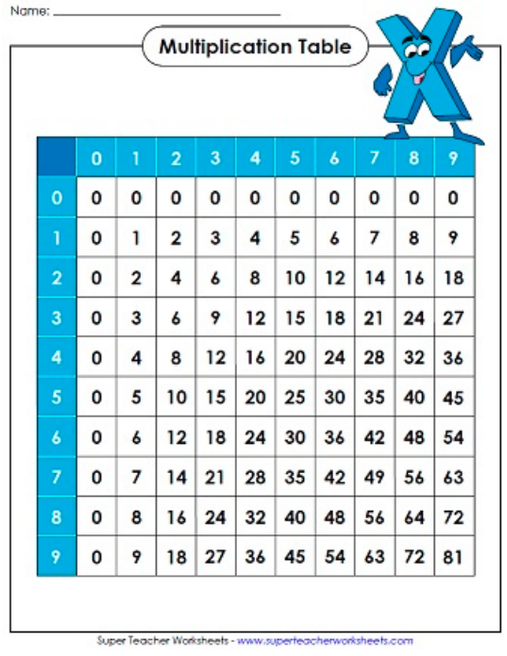




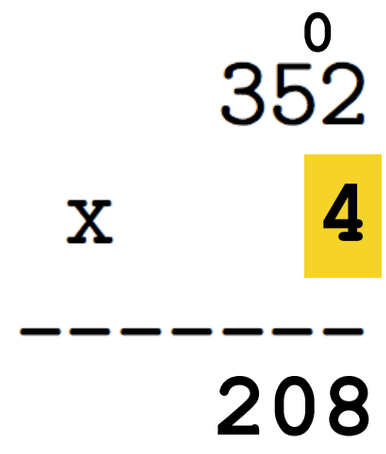
	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	1			21	24	27
4	0	4	8	12	1			28	32	36
5	0	5	10	15	2			35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

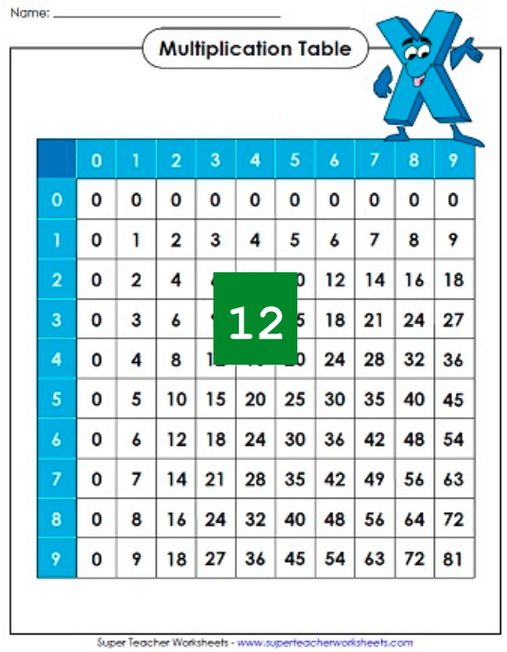


	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81



Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b



	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4				12	14	16	18
3	0	3	6				18	21	24	27
4	0	4	8				24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

```
  2 0
  352
x   4
-------
 1408
```

Name: _____

Multiplication Table

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

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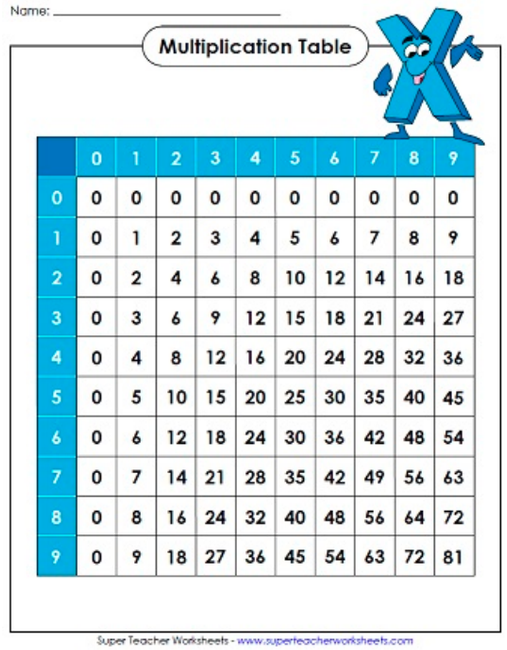
Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

```
   352
 x  964
-------
  1408
```


Name: _____

Multiplication Table

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

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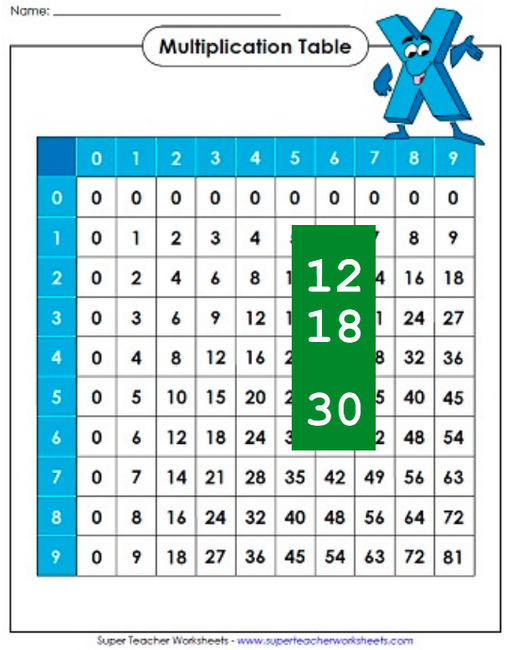
Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

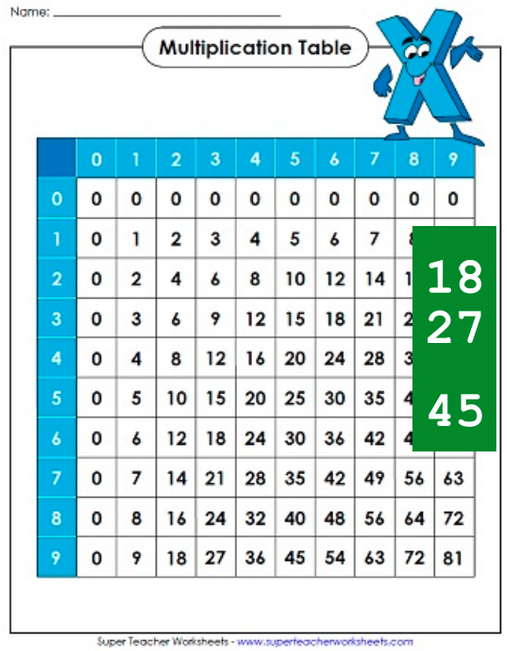
```
  352
x  964
-------
 1408
21120
```



Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

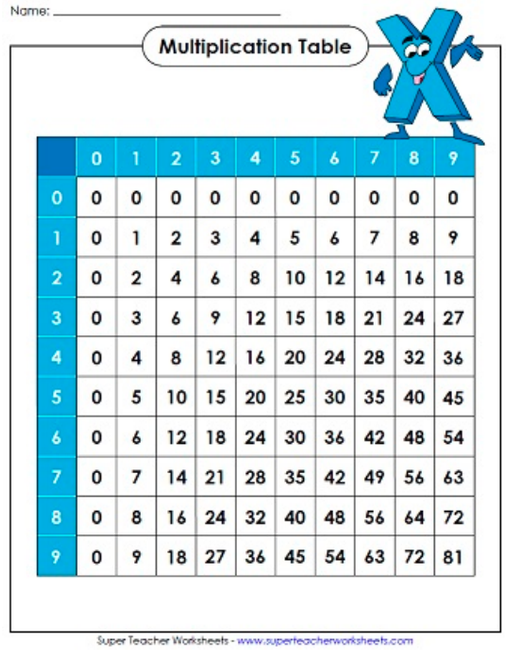
```
   352
x  964
-------
  1408
 21120
316800
```



Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

```
  352
x   964
-------
 1408
21120
316800
-------
339328
```


Name: _____

Multiplication Table

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

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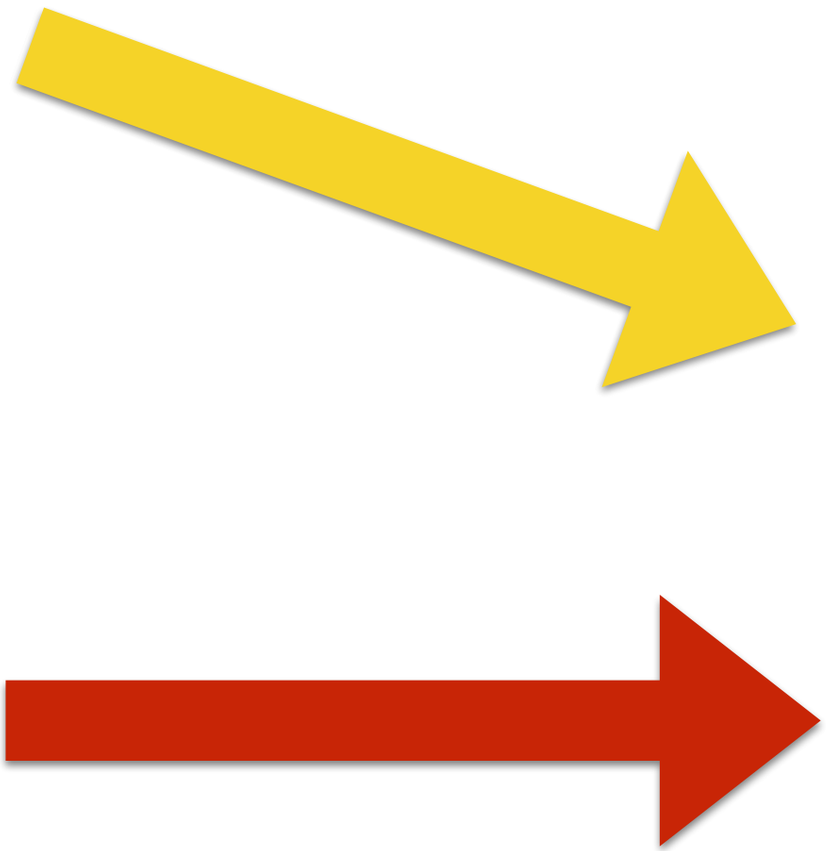
Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

```
for j = 0 to N − 1 do
    carry ← 0
    for i = 0 to N − 1 do
        prod ← (a[i] ∗ b[j] + carry)
        tmp[j][i + j] ← prod%10
        carry ← prod/10
    end for
    tmp[j][N + j] ← carry
end for
carry ← 0
for i = 0 to 2 ∗ N − 1 do
    sum ← carry
    for j = 0 to N − 1 do
        sum ← sum + tmp[j][i]
    end for
    r[i] ← sum%10
    carry ← sum/10
end for
r[2 ∗ N] ← carry
```



```
   352
x  964
------
  1408
 21120
316800
------
339328
```

Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

```
for j = 0 to N − 1 do
    carry ← 0
    for i = 0 to N − 1 do
        prod ← (a[i] ∗ b[j] + carry)
        tmp[j][i + j] ← prod%10
        carry ← prod/10
    end for
    tmp[j][N + j] ← carry
end for
carry ← 0
for i = 0 to 2 ∗ N − 1 do
    sum ← carry
    for j = 0 to N − 1 do
        sum ← sum + tmp[j][i]
    end for
    r[i] ← sum%10
    carry ← sum/10
end for
r[2 ∗ N] ← carry
```

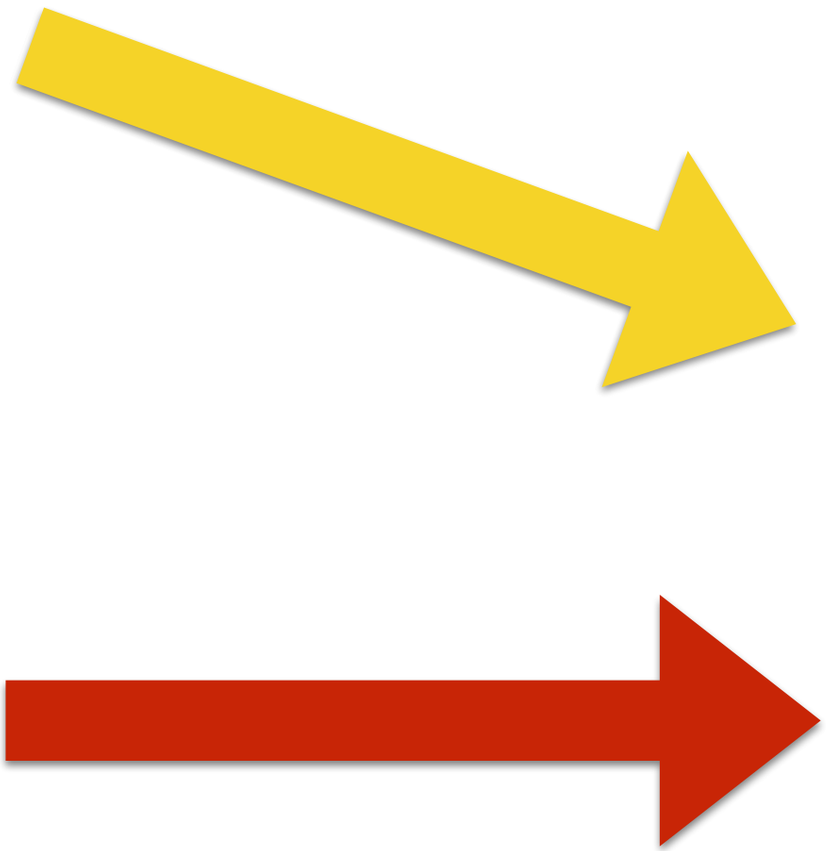
$a[i]$

```
   352
x  964
------
  1408
 21120
316800
------
339328
```

Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

```
for j = 0 to N − 1 do
    carry ← 0
    for i = 0 to N − 1 do
        prod ← (a[i] ∗ b[j] + carry)
        tmp[j][i + j] ← prod%10
        carry ← prod/10
    end for
    tmp[j][N + j] ← carry
end for
carry ← 0
for i = 0 to 2 ∗ N − 1 do
    sum ← carry
    for j = 0 to N − 1 do
        sum ← sum + tmp[j][i]
    end for
    r[i] ← sum%10
    carry ← sum/10
end for
r[2 ∗ N] ← carry
```



$a[i]$

```
   352
x  964
-------
  1408
 21120
316800
-------
339328
```

Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

```
for j = 0 to N − 1 do
    carry ← 0
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        prod ← (a[i] ∗ b[j] + carry)
        tmp[j][i + j] ← prod%10
        carry ← prod/10
    end for
    tmp[j][N + j] ← carry
end for
carry ← 0
for i = 0 to 2 ∗ N − 1 do
    sum ← carry
    for j = 0 to N − 1 do
        sum ← sum + tmp[j][i]
    end for
    r[i] ← sum%10
    carry ← sum/10
end for
r[2 ∗ N] ← carry
```
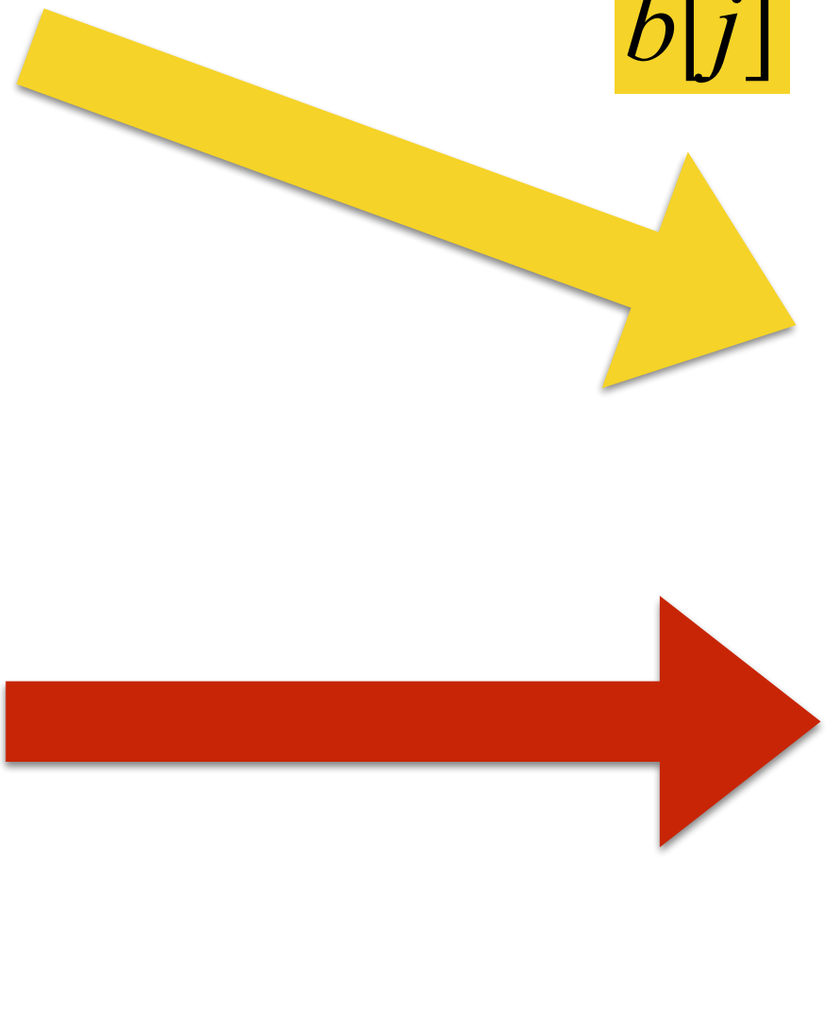



$a[i]$		352
$b[j]$	x	964

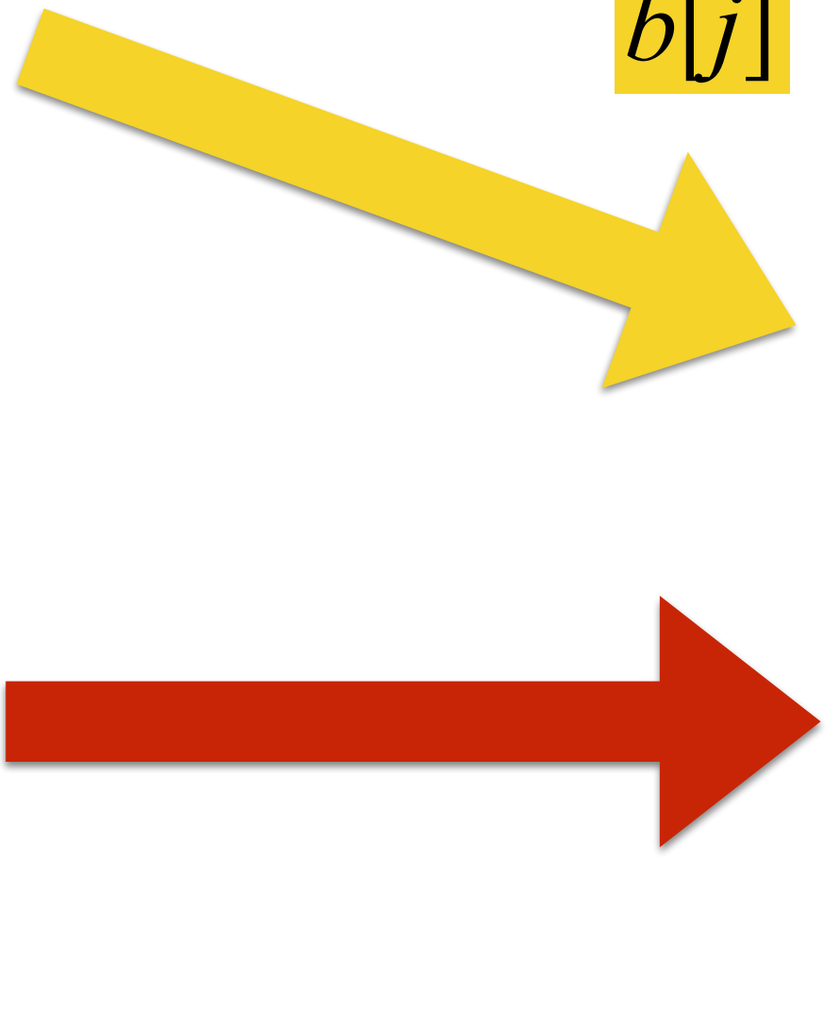
		1408
		21120
		316800

		339328

Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

```
for j = 0 to N − 1 do
    carry ← 0
    for i = 0 to N − 1 do
        prod ← (a[i] ∗ b[j] + carry)
        tmp[j][i + j] ← prod%10
        carry ← prod/10
    end for
    tmp[j][N + j] ← carry
end for
carry ← 0
for i = 0 to 2 ∗ N − 1 do
    sum ← carry
    for j = 0 to N − 1 do
        sum ← sum + tmp[j][i]
    end for
    r[i] ← sum%10
    carry ← sum/10
end for
r[2 ∗ N] ← carry
```



$a[i]$		352
$b[j]$	x	964

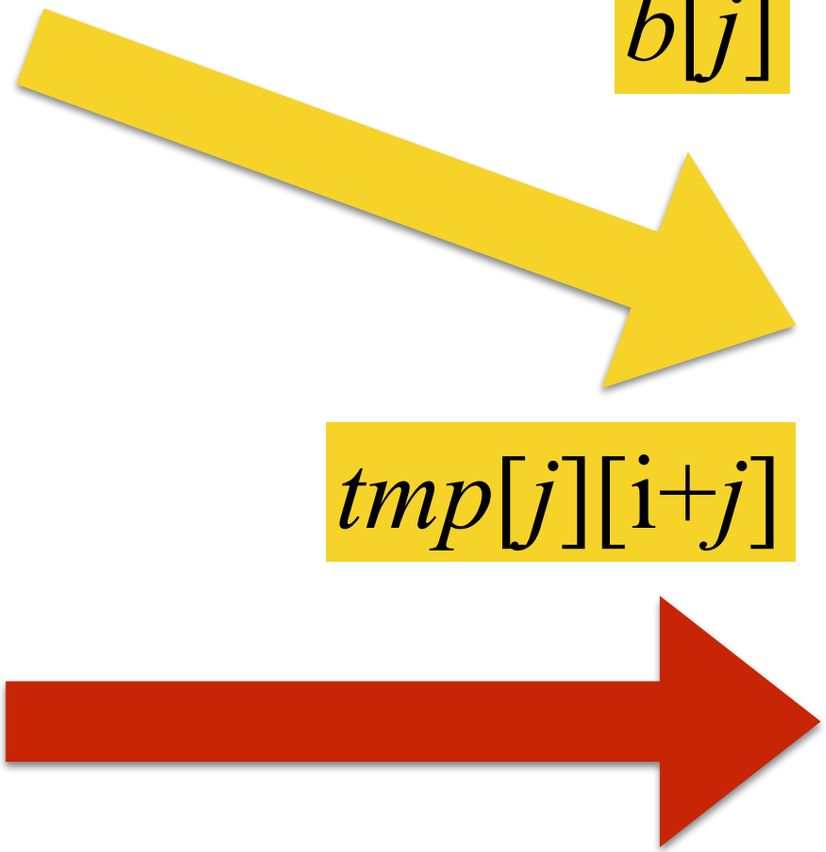
		1408
		21120
		316800

		339328

Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

```
for j = 0 to N − 1 do
    carry ← 0
    for i = 0 to N − 1 do
        prod ← (a[i] ∗ b[j] + carry)
        tmp[j][i + j] ← prod%10
        carry ← prod/10
    end for
    tmp[j][N + j] ← carry
end for
carry ← 0
for i = 0 to 2 ∗ N − 1 do
    sum ← carry
    for j = 0 to N − 1 do
        sum ← sum + tmp[j][i]
    end for
    r[i] ← sum%10
    carry ← sum/10
end for
r[2 ∗ N] ← carry
```

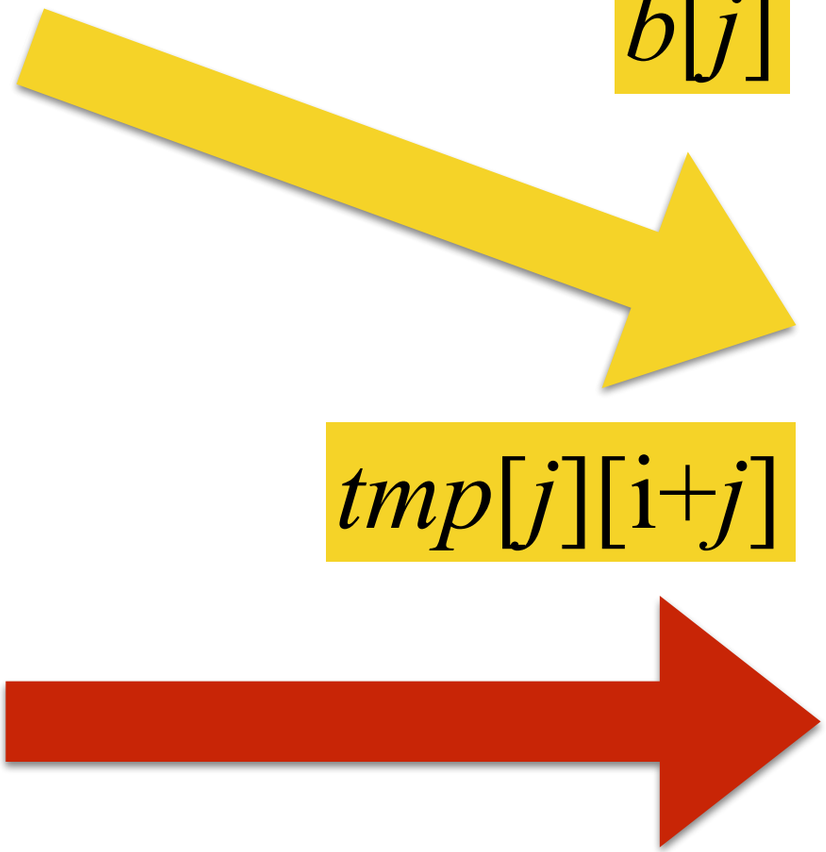



```
  352
x 964
-------
 1408
21120
316800
-------
339328
```

Grade School Algorithms

Algorithm 2 Multiplication (base 10) of two numbers a and b

```
for j = 0 to N − 1 do
    carry ← 0
    for i = 0 to N − 1 do
        prod ← (a[i] ∗ b[j] + carry)
        tmp[j][i + j] ← prod%10
        carry ← prod/10
    end for
    tmp[j][N + j] ← carry
end for
carry ← 0
for i = 0 to 2 ∗ N − 1 do
    sum ← carry
    for j = 0 to N − 1 do
        sum ← sum + tmp[j][i]
    end for
    r[i] ← sum%10
    carry ← sum/10
end for
r[2 ∗ N] ← carry
```



Multiplication

$$
\begin{array}{l}
\textbf{for } j = 0 \text{ to } N-1 \textbf{ do} \\
\quad carry \leftarrow 0 \\
\quad \textbf{for } i = 0 \text{ to } N-1 \textbf{ do} \\
\quad\quad prod \leftarrow (a[i] * b[j] + carry) \\
\quad\quad tmp[j][i+j] \leftarrow prod\%10 \\
\quad\quad carry \leftarrow prod/10 \\
\quad \textbf{end for} \\
\quad tmp[j][N+j] \leftarrow carry \\
\textbf{end for}
\end{array}
$$

Multiplication

$carry \leftarrow 0$
for $i = 0$ to $2 * N - 1$ **do**
    $sum \leftarrow carry$
    for $j = 0$ to $N - 1$ **do**
        $sum \leftarrow sum + tmp[j][i]$
    end for
    $r[i] \leftarrow sum\%10$
    $carry \leftarrow sum/10$
end for
$r[2 * N] \leftarrow carry$

Multiplication

Algorithm 2 Multiplication (base β) of two numbers a and b

```
for j = 0 to N − 1 do
    carry ← 0
    for i = 0 to N − 1 do
        prod ← (a[i] ∗ b[j] + carry)
        tmp[j][i + j] ← prod% β
        carry ← prod/ β
    end for
    tmp[j][N + j] ← carry
end for
```

```
carry ← 0
for i = 0 to 2 ∗ N − 1 do
    sum ← carry
    for j = 0 to N − 1 do
        sum ← sum + tmp[j][i]
    end for
    r[i] ← sum% β
    carry ← sum/ β
end for
r[2 ∗ N] ← carry
```

Multiplication base 8

$$(1205)_8$$
$$\times\ (736)_8$$
$$\_\_\_\_\_$$
$$(7436)_8$$
$$(36170)_8$$
$$(1064300)_8$$
$$\_\_\_\_\_$$
$$(1132126)_8 = (308310)_x$$

Long Division

723 | 41672542996

Name: _____

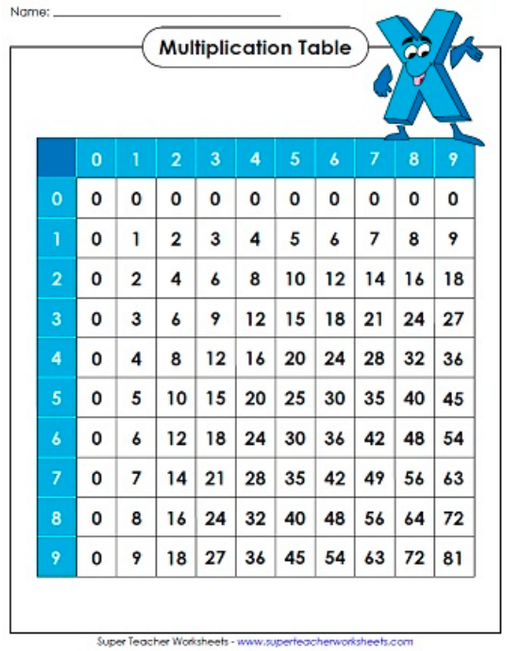
Multiplication Table

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Grade School Algorithms

5 . . .

723 | 41672542996



Name: _____

Multiplication Table

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

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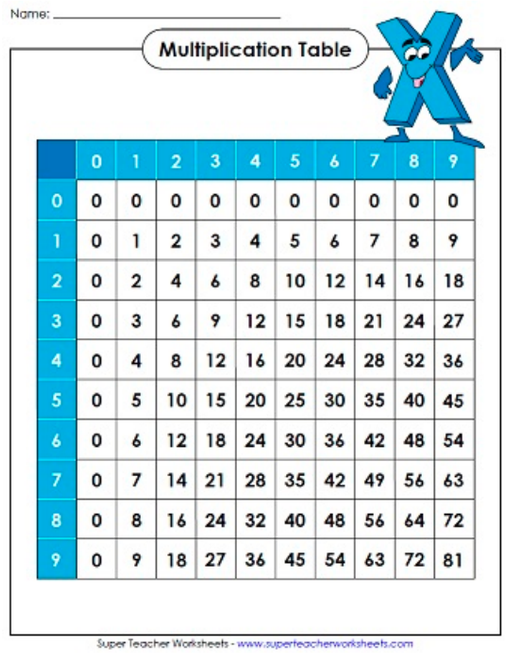
Grade School Algorithms

5 . . .

723 | 41672542996

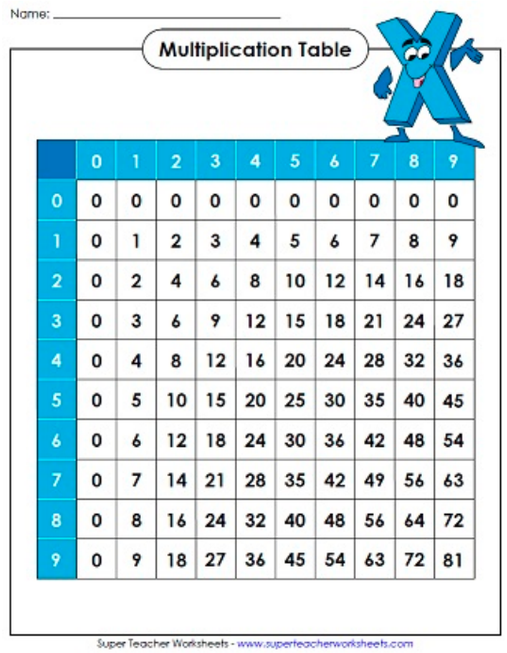
3615



Grade School Algorithms

5 . . .

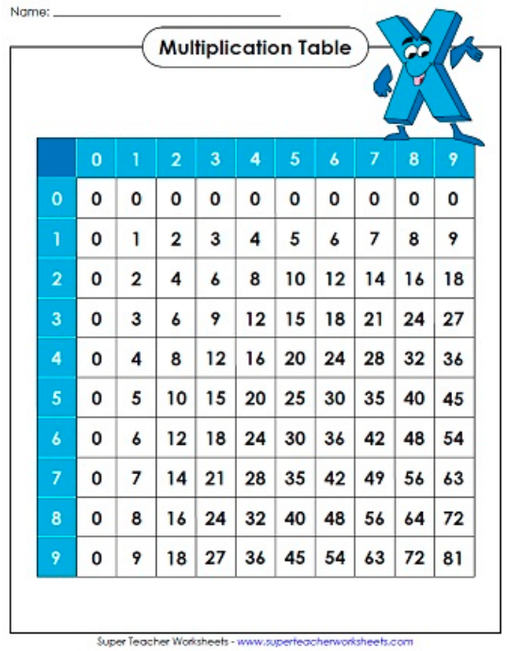
```
  ----------------
723 | 41672542996
      3615
      ----
       552  . . .
```



Grade School Algorithms

5 . . .

723 | 5522542996



Name: _____

Multiplication Table

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

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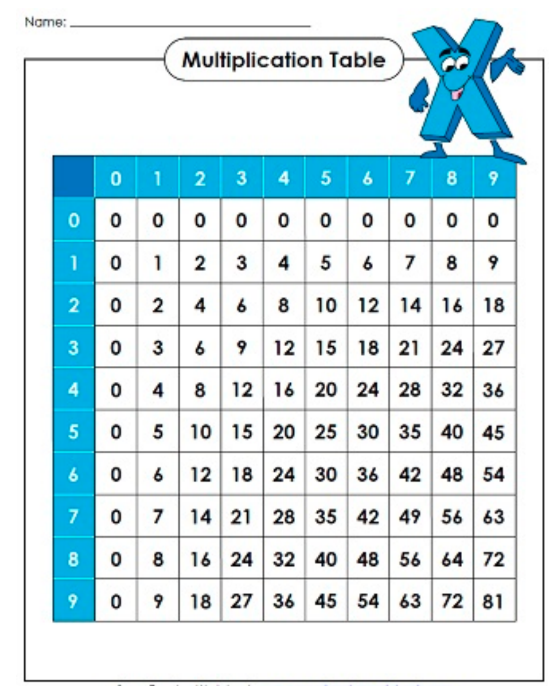
Grade School Algorithms

```
             57...
        ______________
723 |     5522542996
          5061
          ____
           461 ...
```


Name:

Multiplication Table

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

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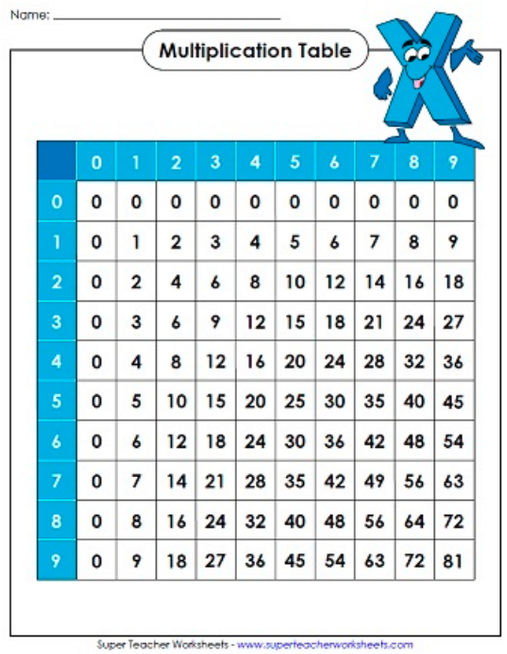
Grade School Algorithms

57638372

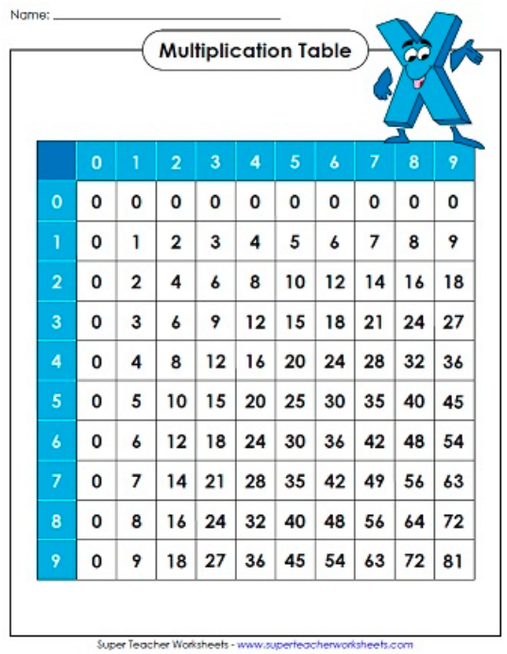
723 | 50



Grade School Algorithms

57638372

723 | 50


Name: _____

Multiplication Table

	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

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41672542996 / 723 = 57638372

41672542996 % 723 = 50

Winter 2016
COMP-250: Introduction to Computer Science

Lecture 2, January 14, 2016