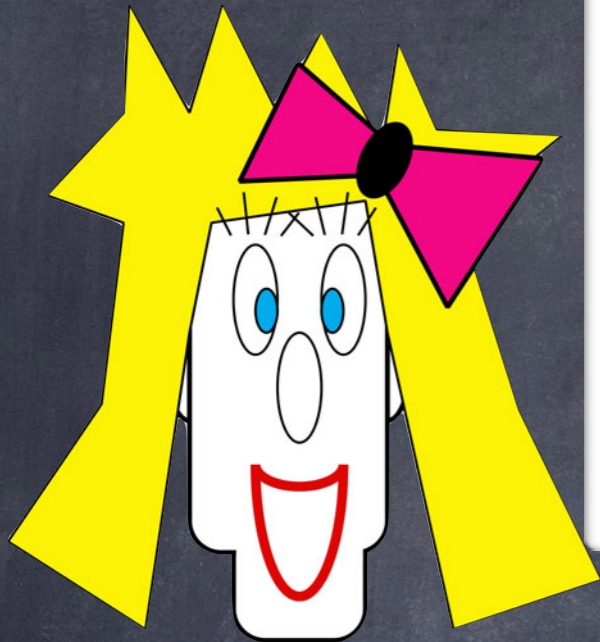


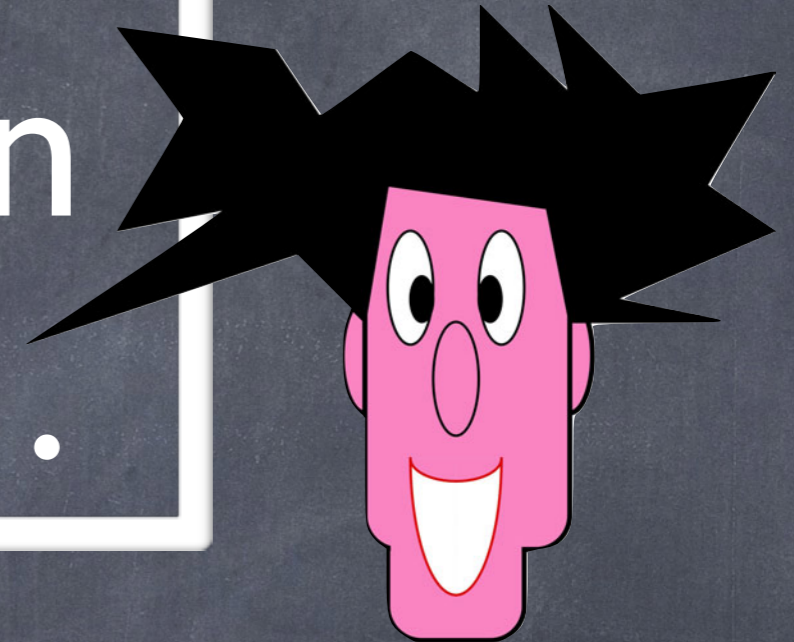
Winter 2016
COMP-250: Introduction
to Computer Science

Lecture 14, February 25, 2016

Alice and Bob's Adventures in Cryptoland...



Alice

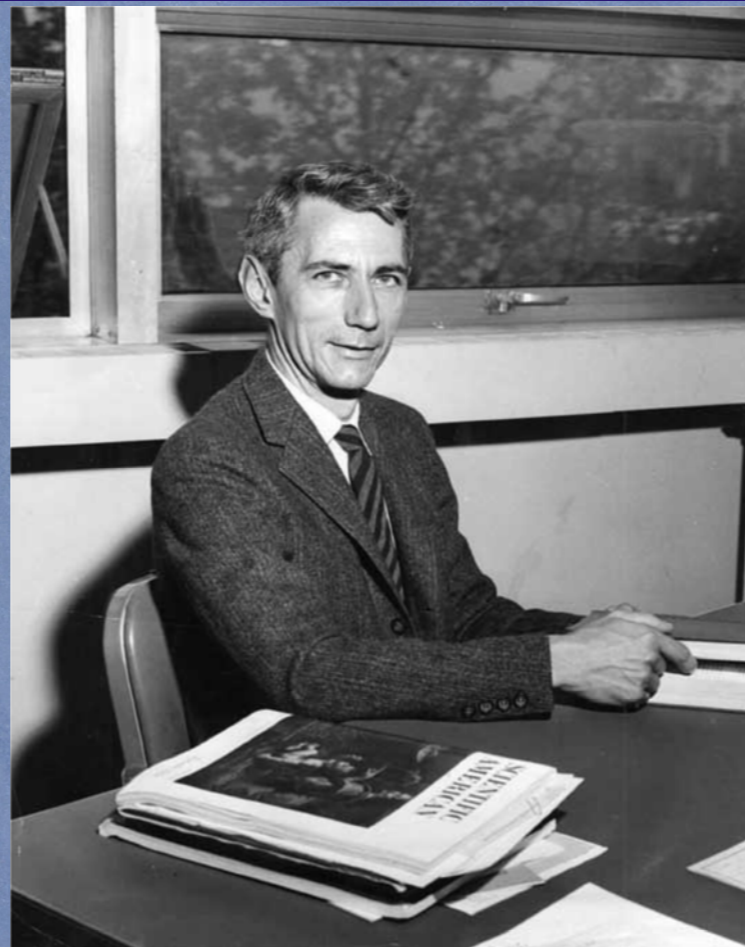


Bob



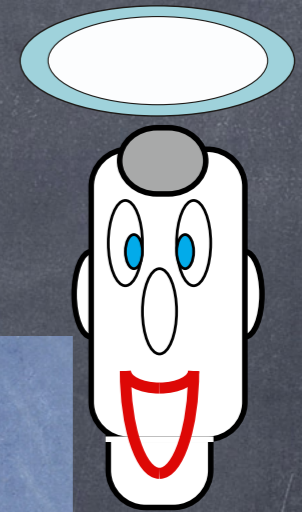
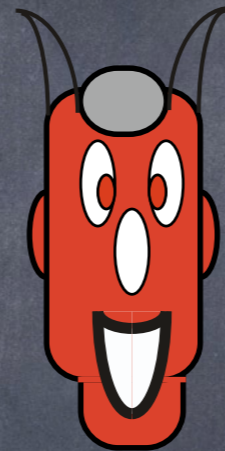
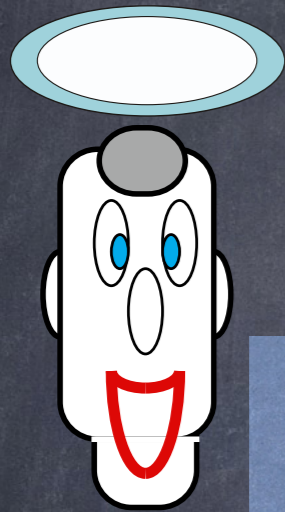
Information

Theoretical



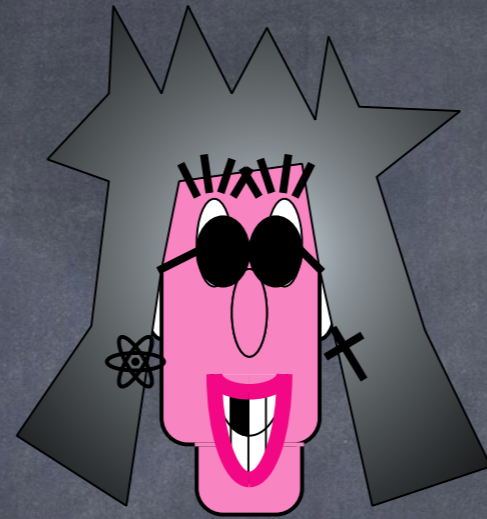
Cryptography

Information Theoretical Cryptography

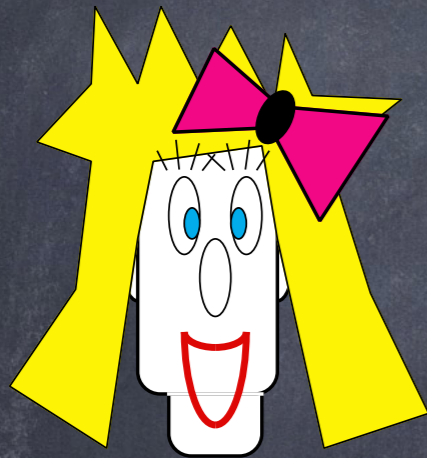


Key Distribution

Encryption

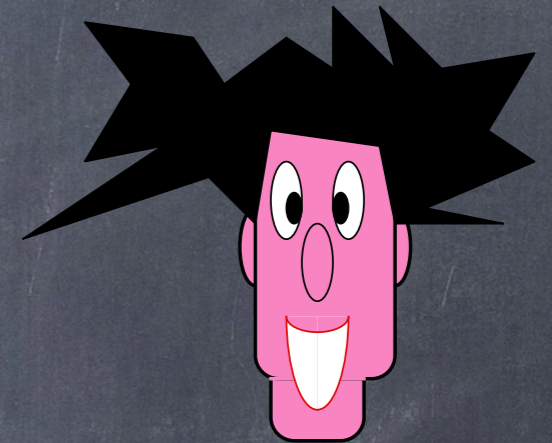


Resp-47

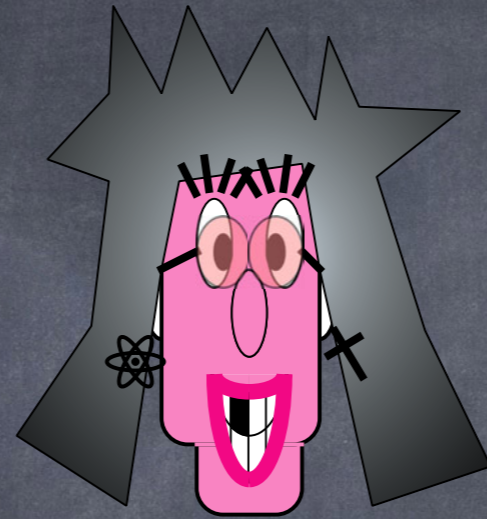


[Redacted]

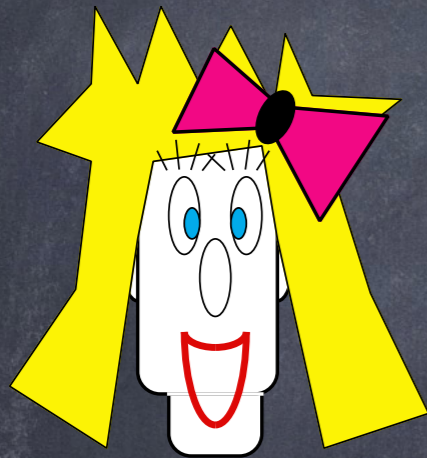
Will you marry me ?



Bob



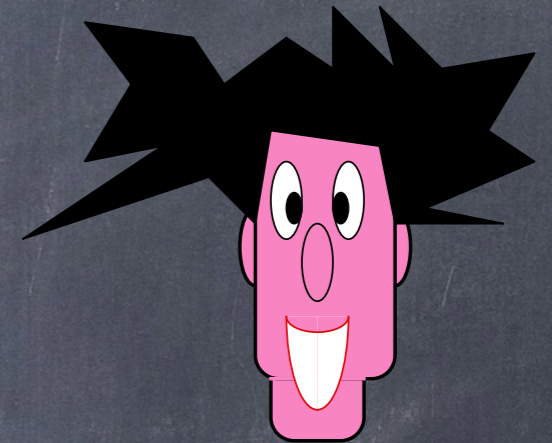
Resp-47



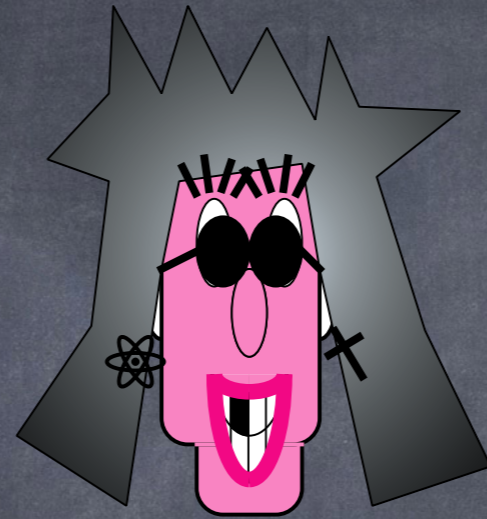
Alice

Will you marry me ?

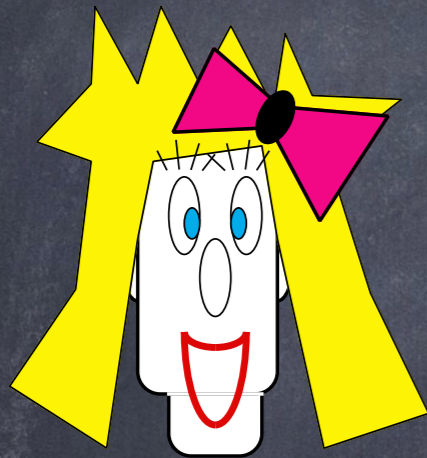
Divorce your wife first !



Bob

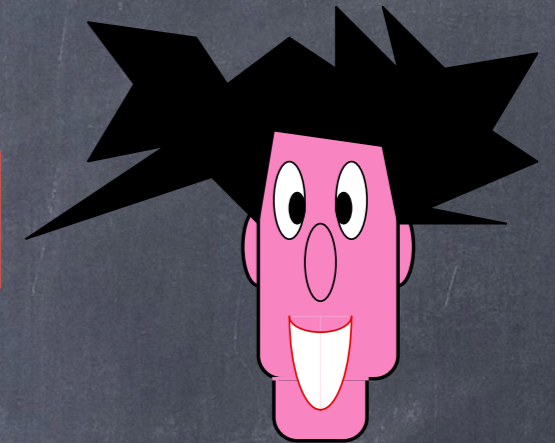


Resp-47



Alice

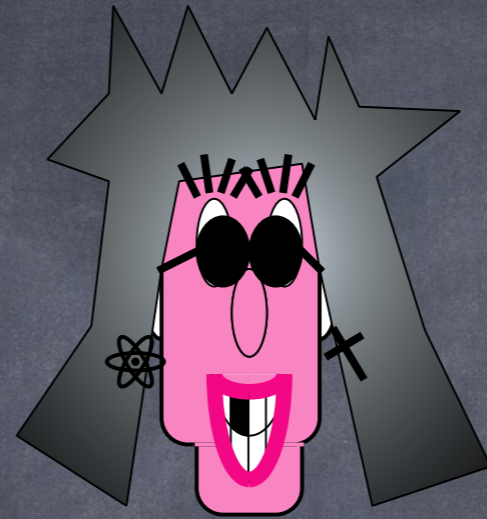
Will you marry me ?



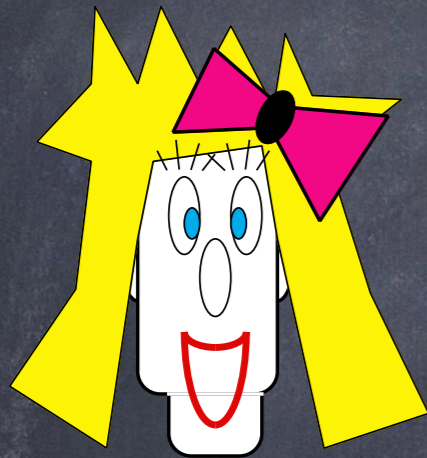
Bob

Divorce your wife first !

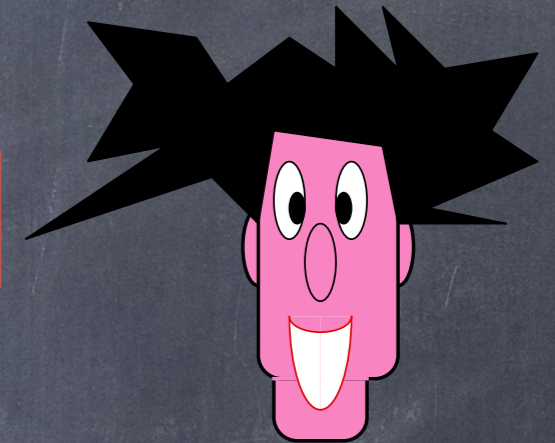
The papers are in the mail...



Resp-47



Alice



Bob

Will you marry me ?

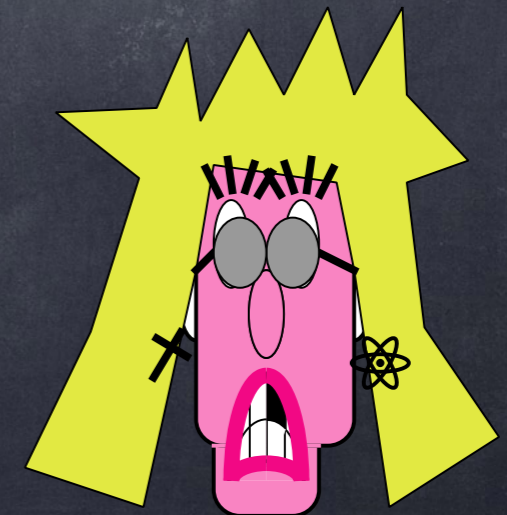
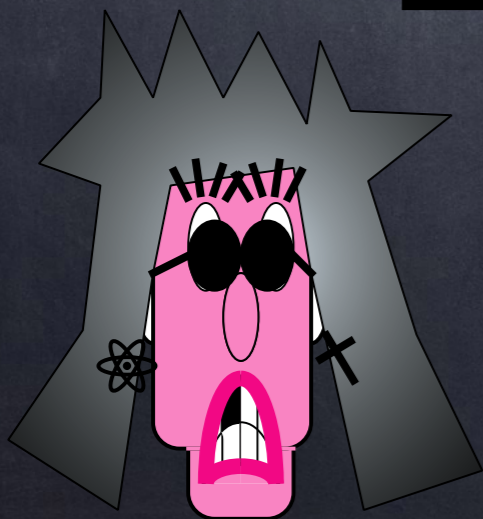
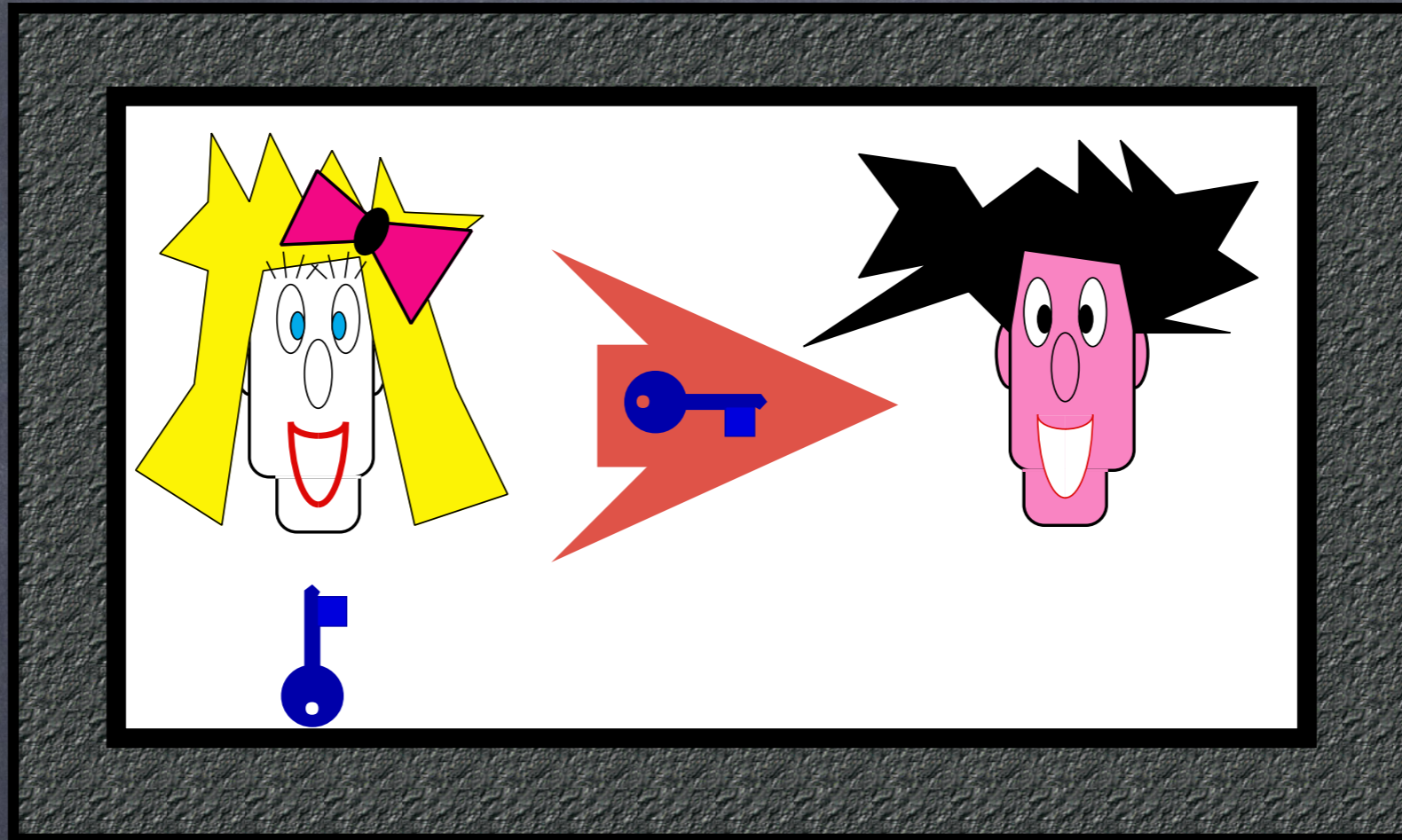
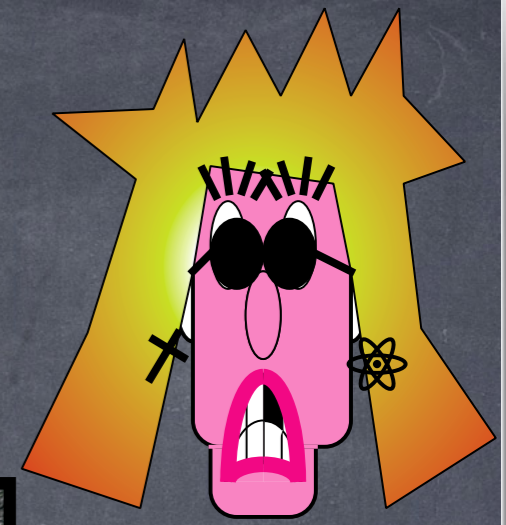
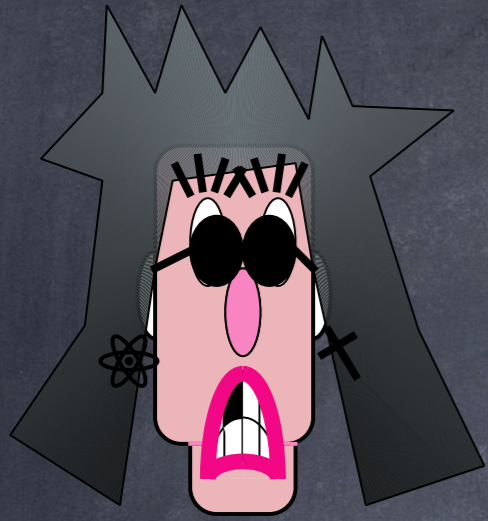
Divorce your wife first !

The papers are in the mail...

OK, I will !

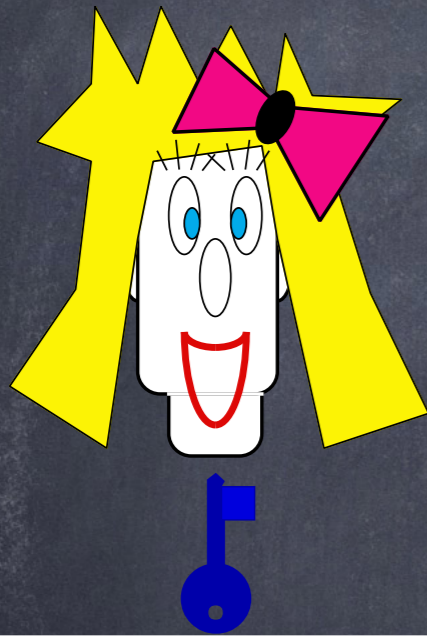
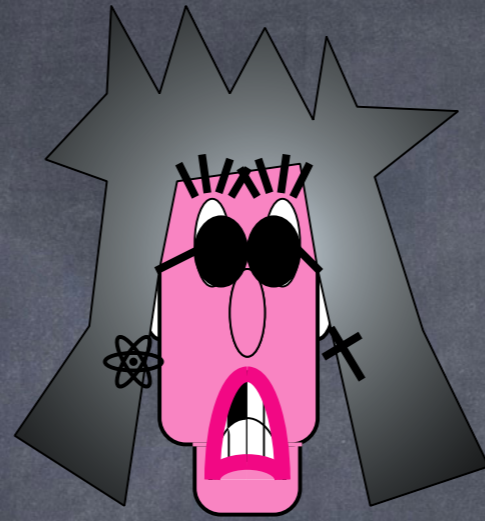
Key

Distribution

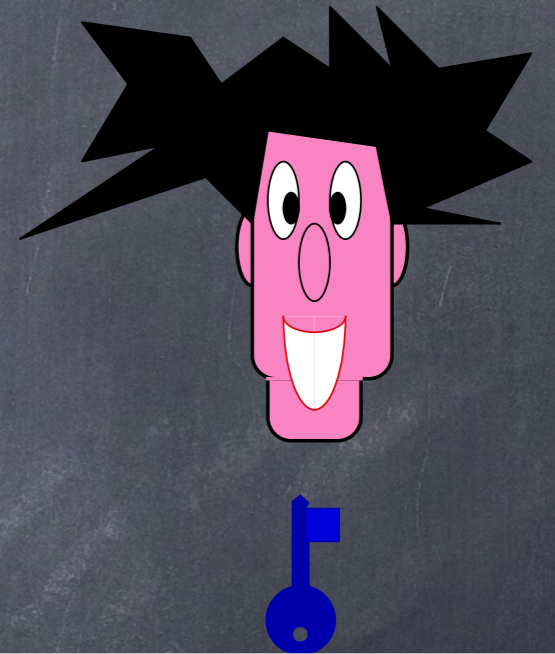


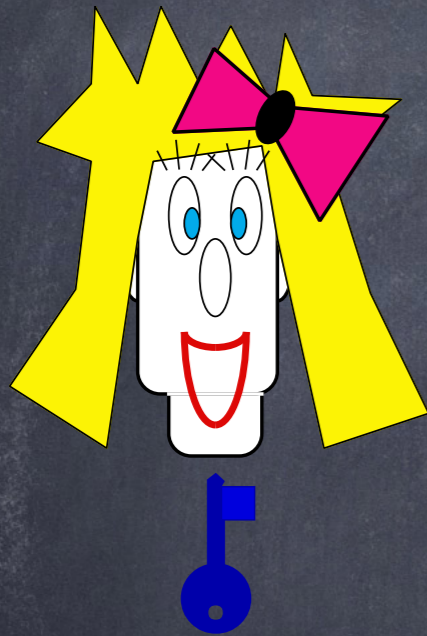
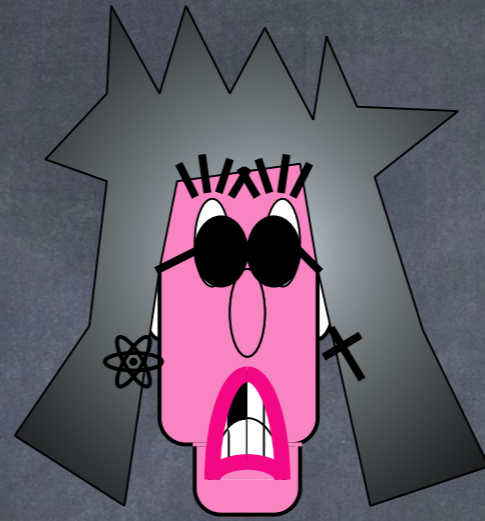


Encryption

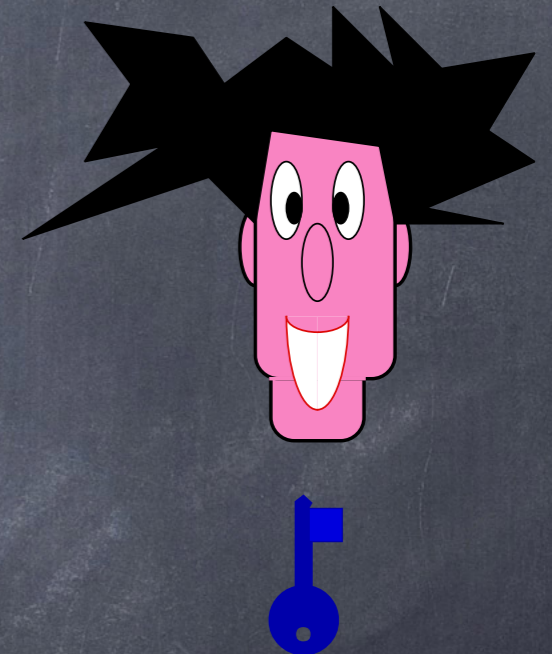


8RdewtU5qkLa\$es!T9@





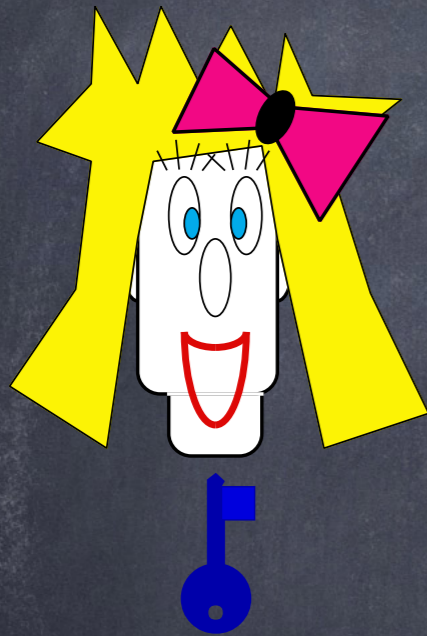
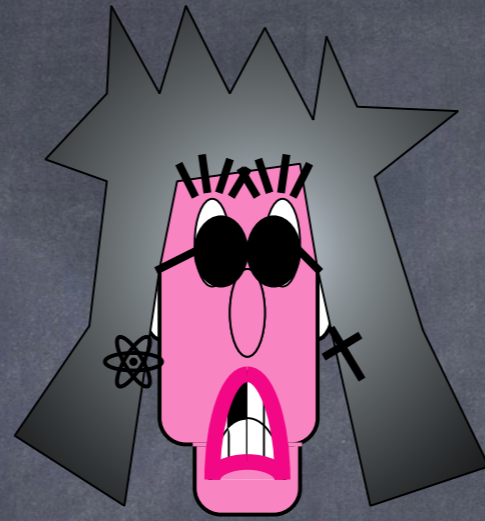
8RdewtU5qkLa\$es!T9@



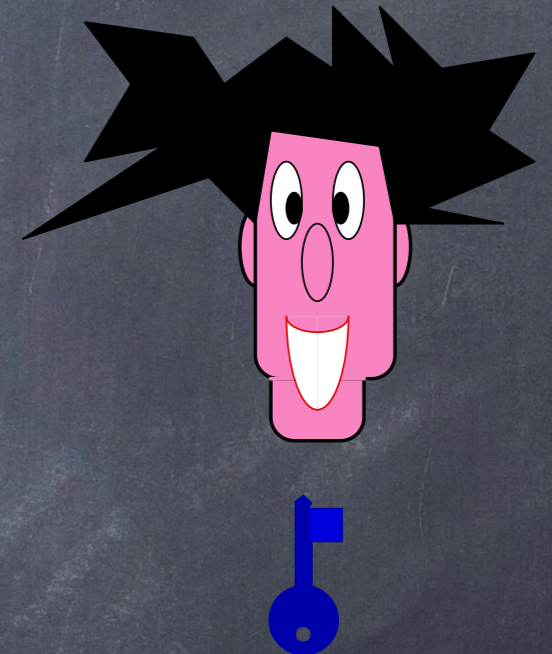
Encryption



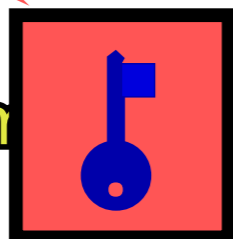
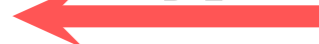
8RdewtU5qkLa\$  y me ?



8RdewtU5qkLa\$es!T9@

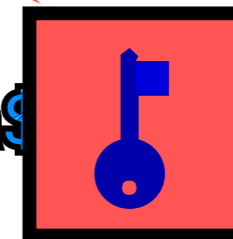


Decryption

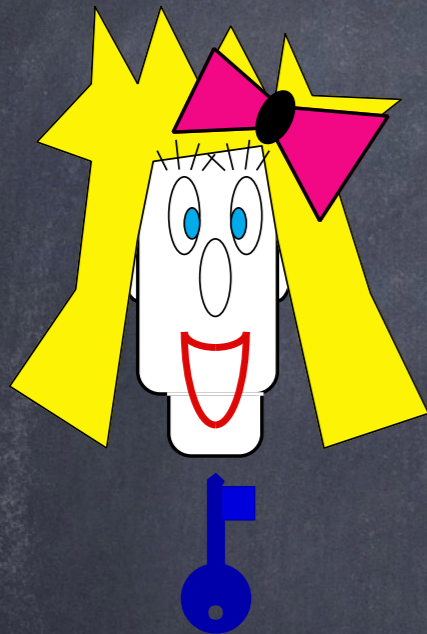
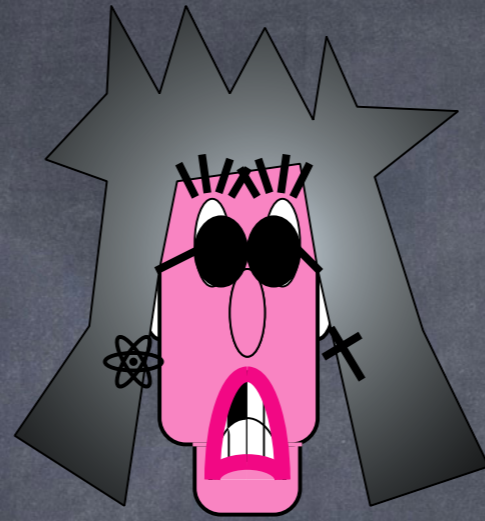


Will you marry me!T9@

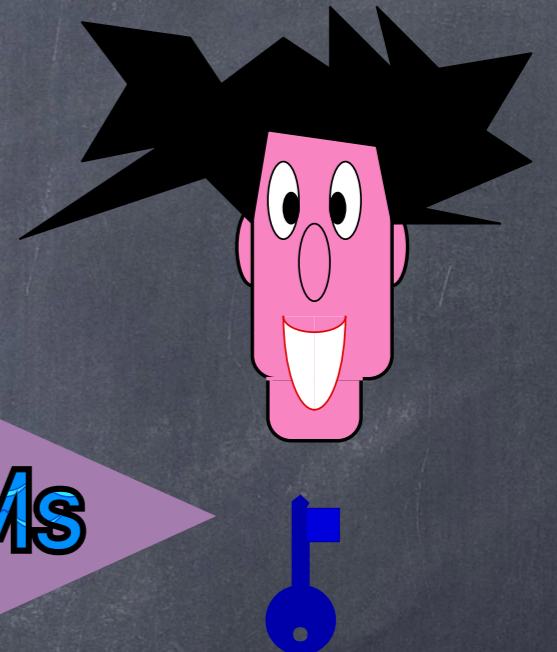
Encryption



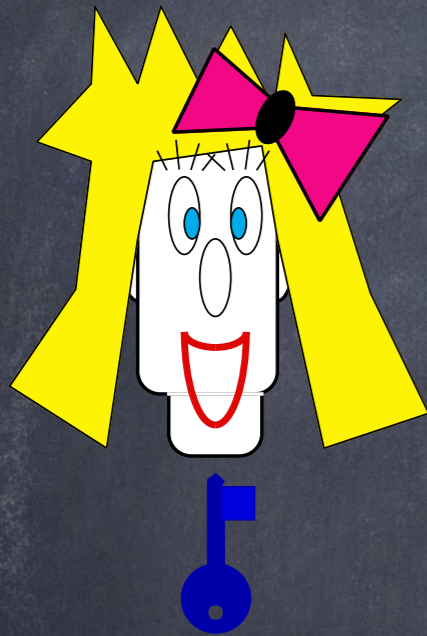
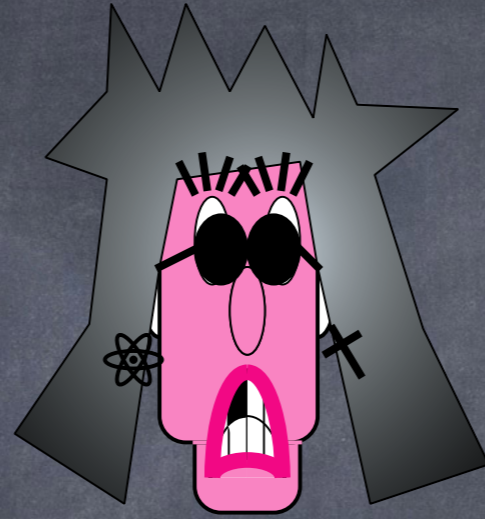
8RdewtU5qkLa\$es!T9@



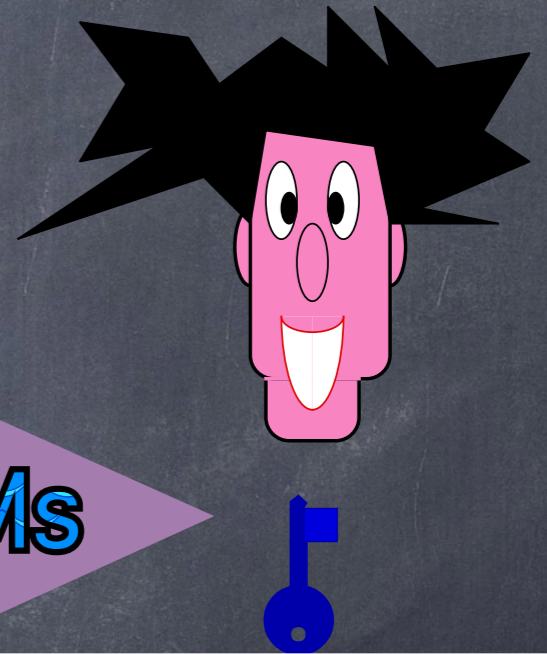
8RdewtU5qkLa\$es!T9@



I(D%eXhDqliykl#2cV7dEwnMs

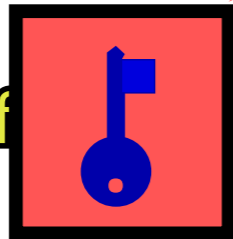


8RdewtU5qkLa\$es!T9@



I(D%eXhDqliykl#2cV7dEwnMs

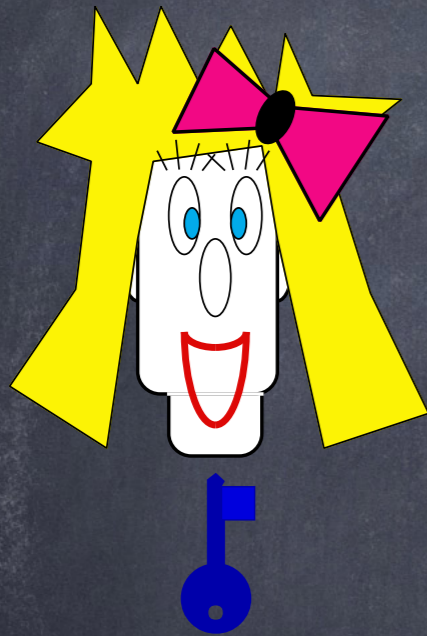
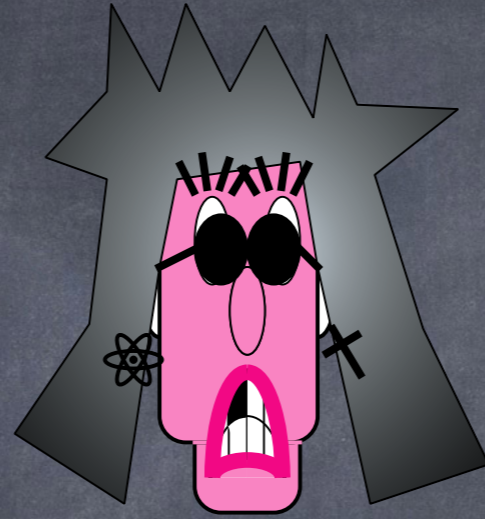
Encryption



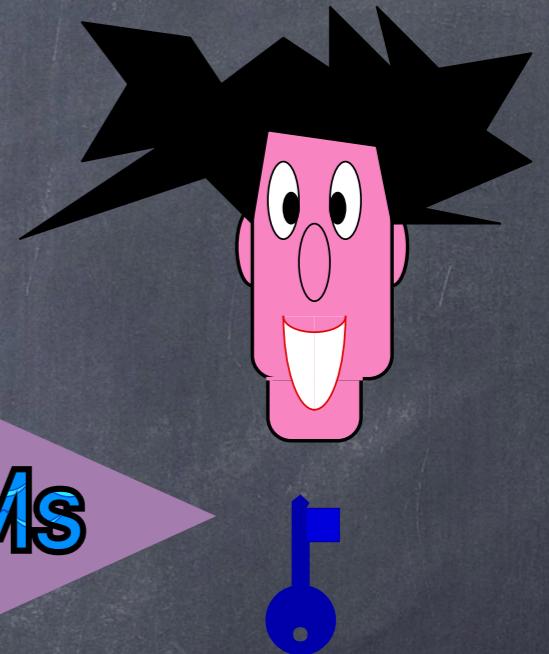
Divorce your wife

IcV7dEwnMs



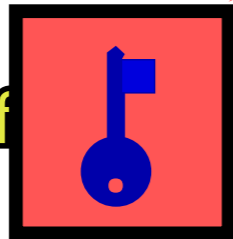


8RdewtU5qkLa\$es!T9@



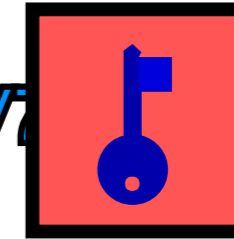
I(D%eXhDqliykl#2cV7dEwnMs

Encryption

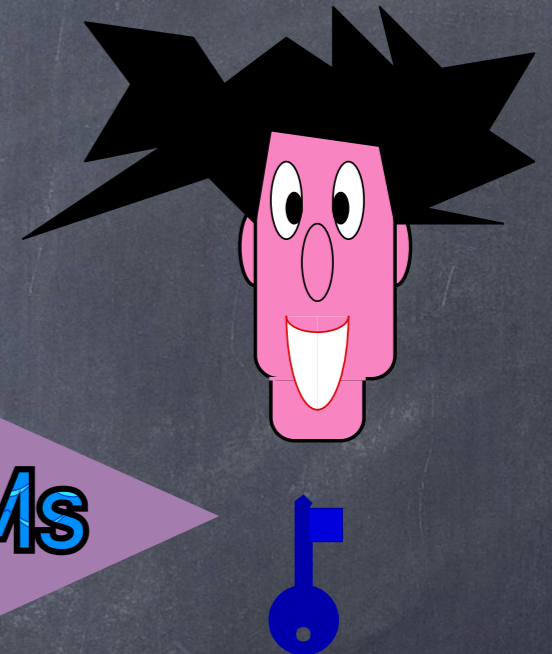
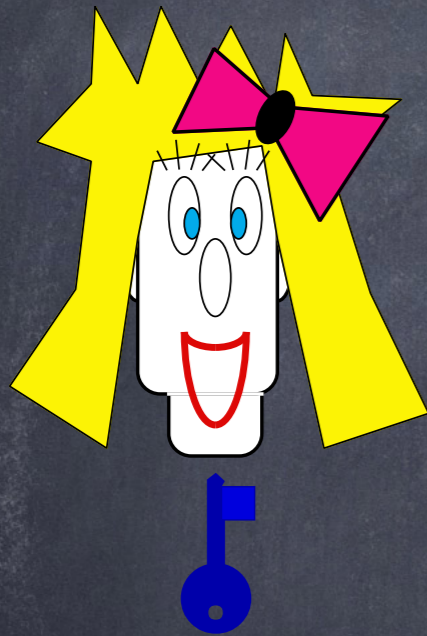
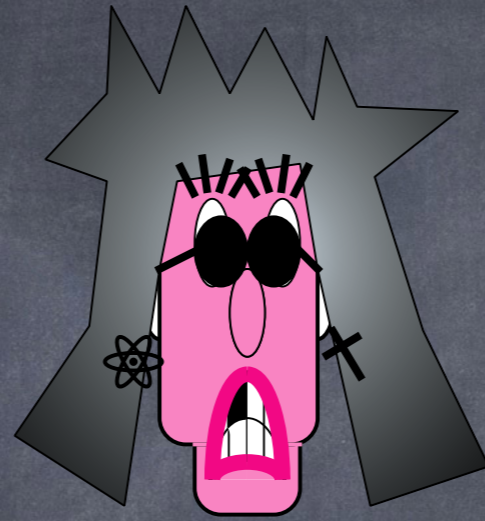


Divorce your wife &cV7dEwnMs

Decryption



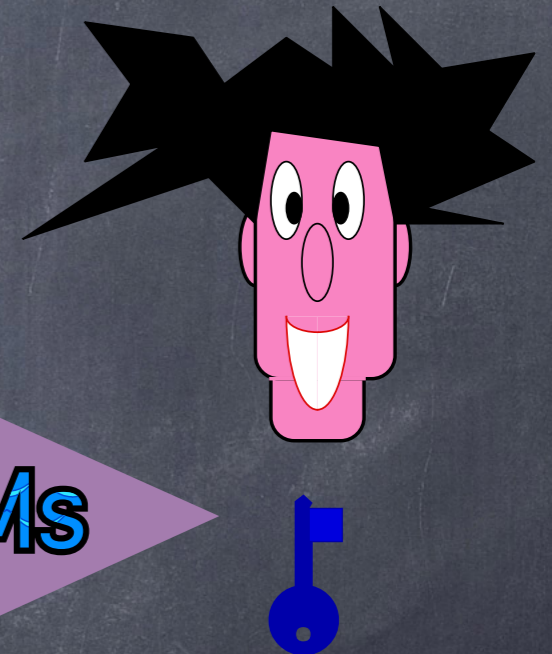
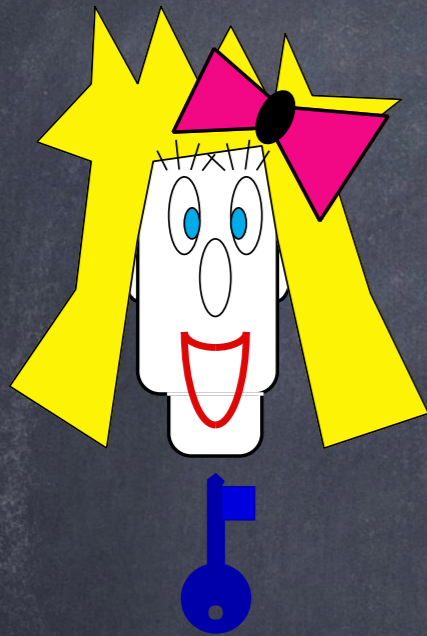
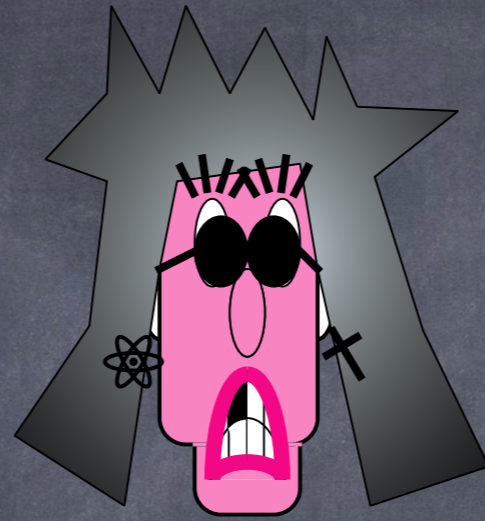
I(D%eXhDqliykl#2cV7ur wife first !



8RdewtU5qkLa\$es!T9@

I(D%eXhDqliykl#2cV7dEwnMs

H&fs@tyHvFGhaOKpTrGbl.Z/rUih*



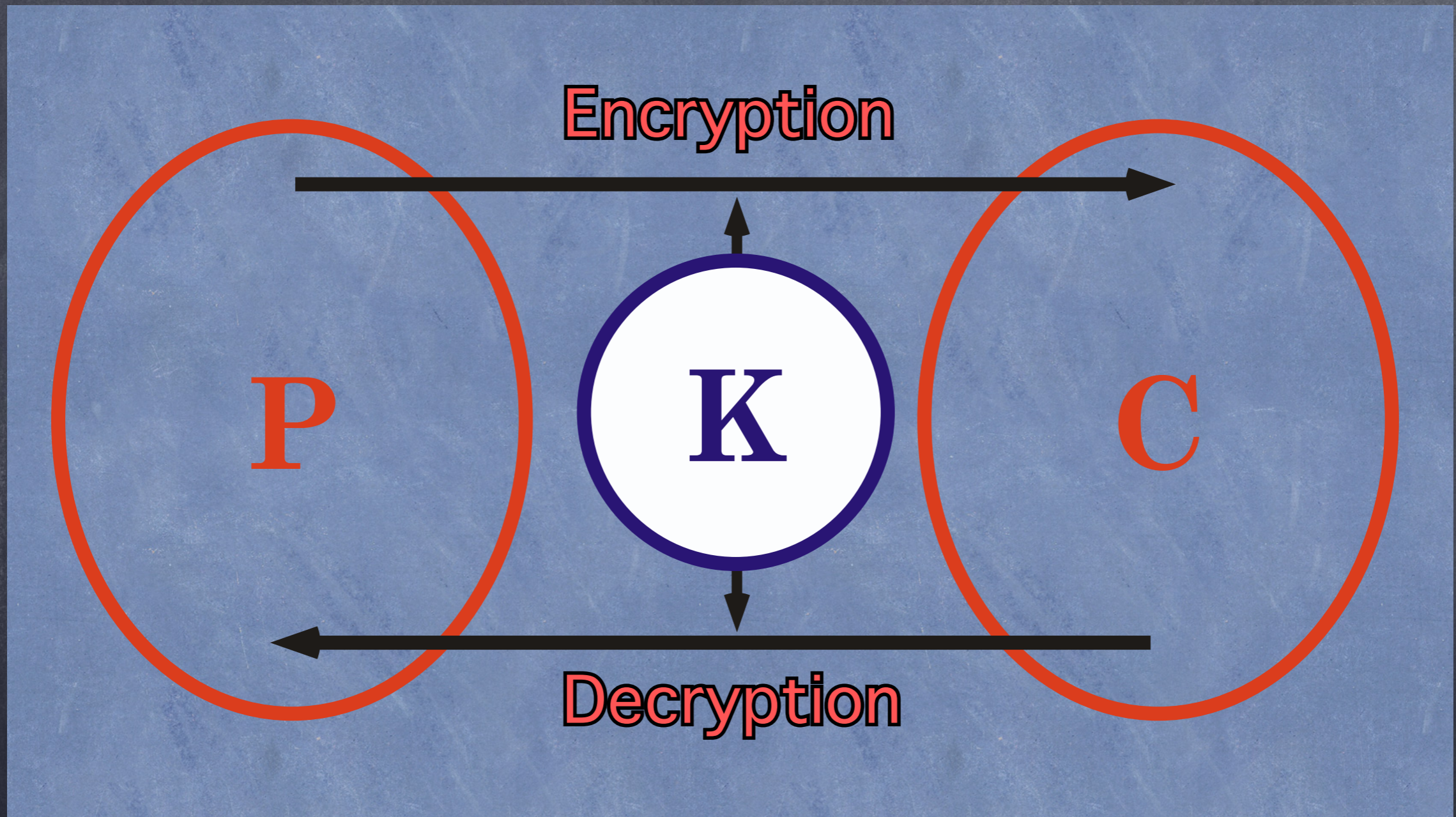
8RdewtU5qkLa\$es!T9@

I(D%eXhDqliykl#2cV7dEwnMs

H&fs@tyHvFGhaOKpTrGbl.Z/rUih*

B7B3tdsjUila

Symmetric Encryption



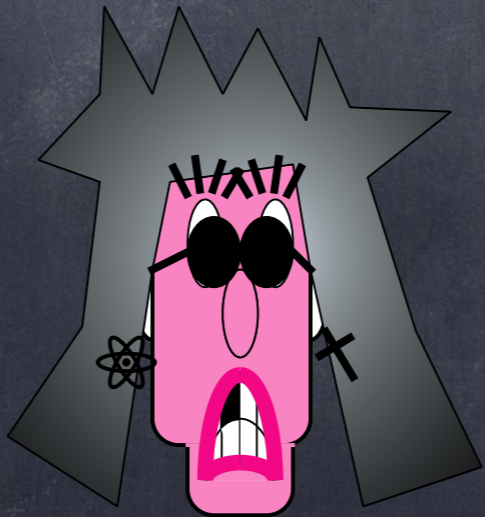
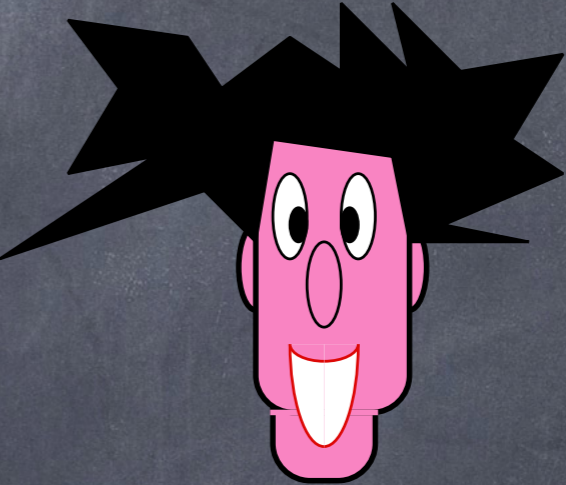
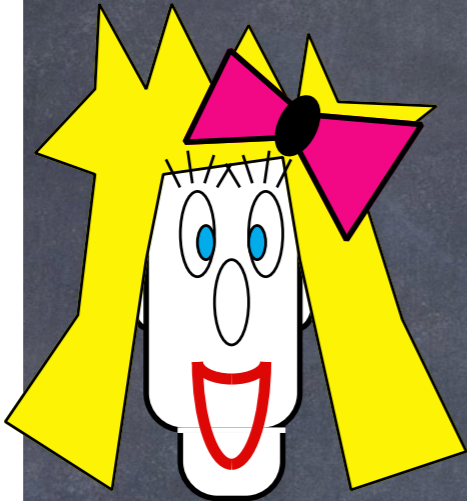
Information Theoretical Security

Symmetric Encryption



Caesar's Cipher

VERNAM's Cipher



m

1
0
1
0
0
1
0
0
1
1
1
1
1
0
0
1

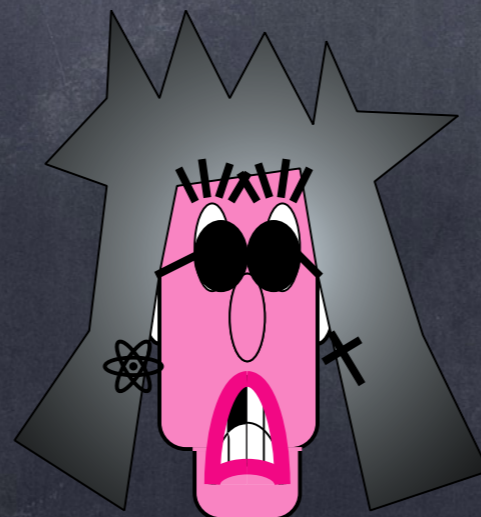
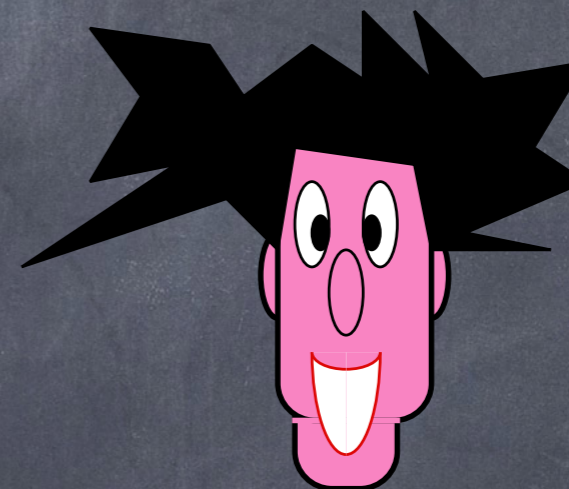
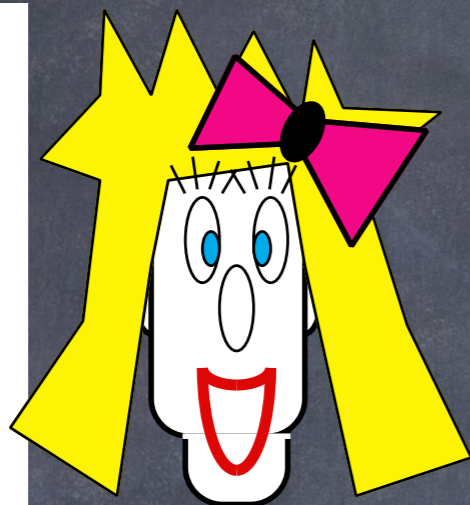
1910's

VERNAM's Cipher



$m \oplus k$

1	1
0	1
1	1
0	0
0	0
1	1
0	1
0	0
1	1
1	1
1	0
1	1
1	0
0	1
0	1
1	1



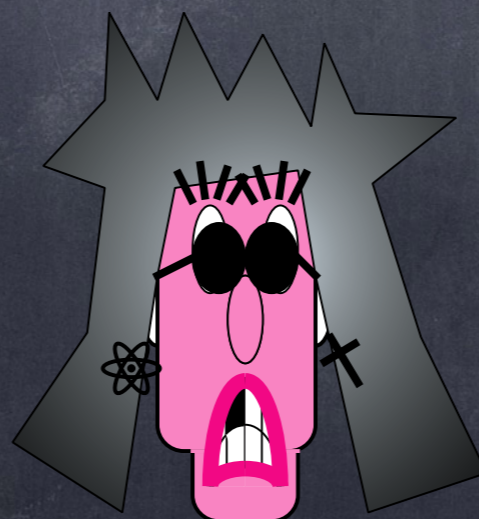
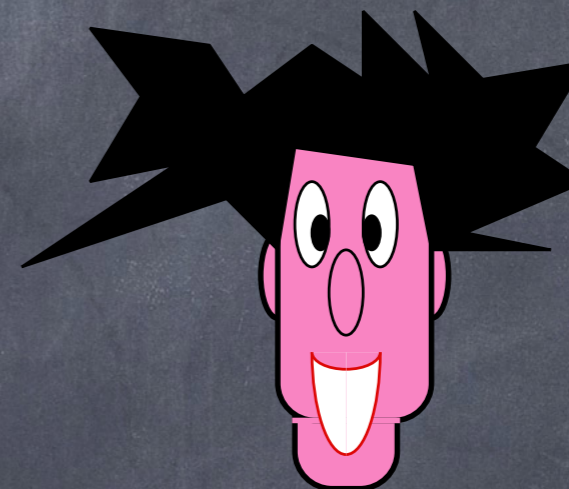
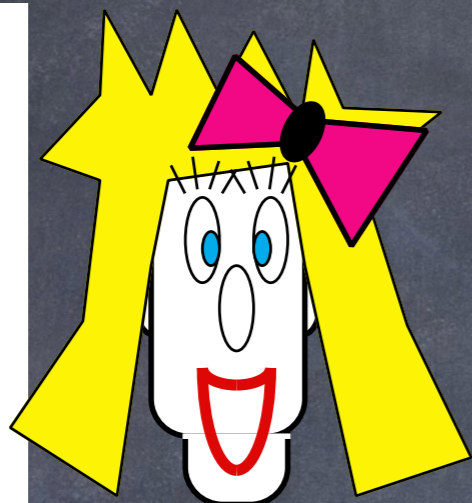
VERNAM's Cipher



$$m \oplus k = c$$

1	1	0
0	1	1
1	1	0
0	0	0
0	0	0
1	1	0
0	1	1
0	0	0
1	1	0
1	1	0
1	0	1
1	1	0
1	0	1
0	1	1
0	1	1
1	1	0

$$\oplus =$$



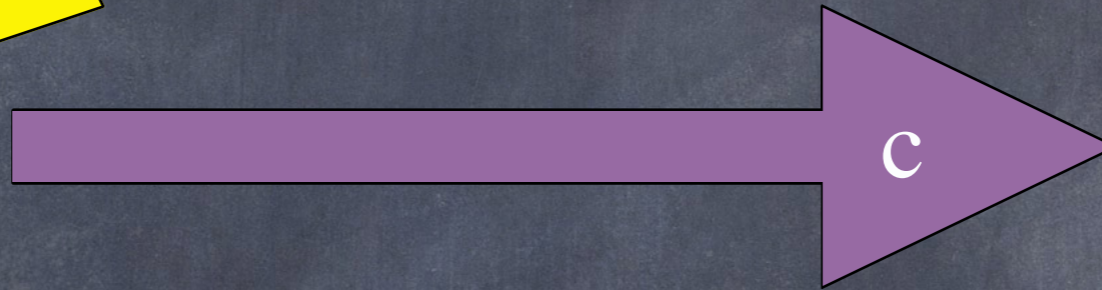
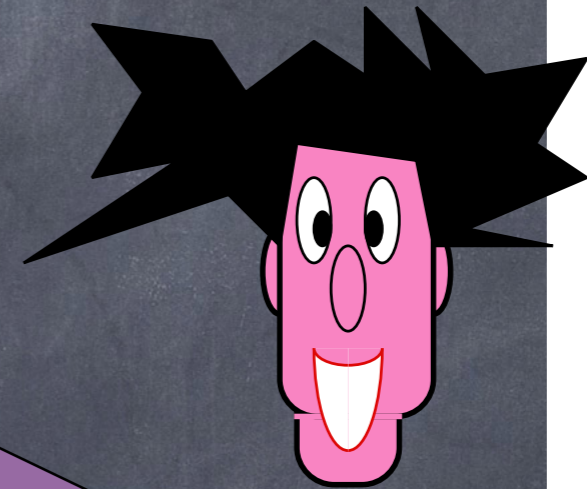
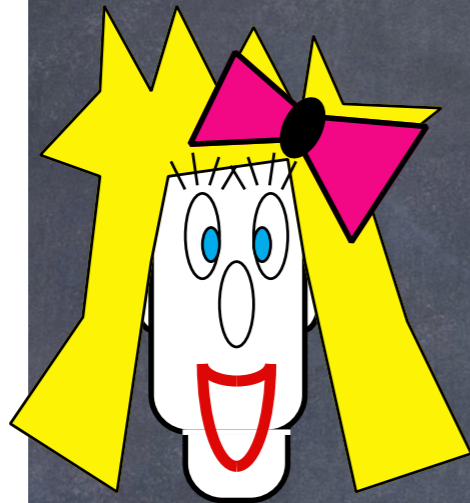
VERNAM's Cipher



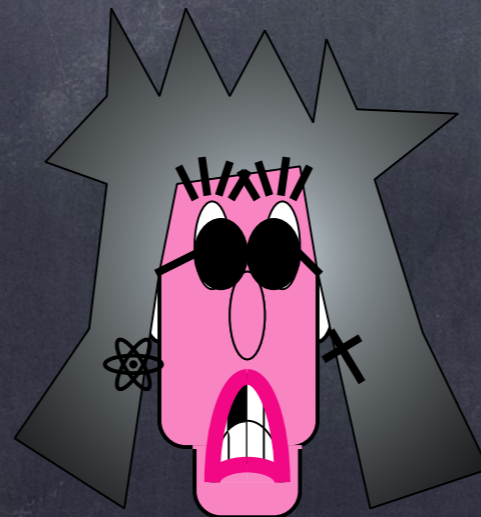
$$m \oplus k = c$$

1	1	0
0	1	1
1	1	0
0	0	0
0	0	0
1	1	0
0	1	1
0	0	0
1	1	0
1	1	0
1	0	1
1	1	0
1	0	1
0	1	1
0	1	1
1	1	0

$$\oplus =$$



c
0
1
0
0
0
0
0
0
1
0
0
0
1
0
1
0
1
1
1
0



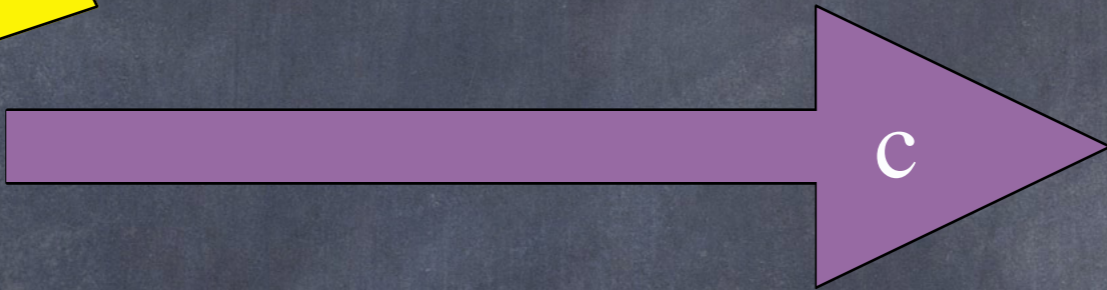
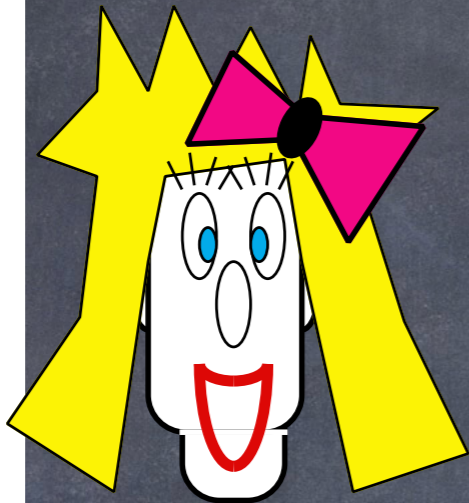
VERNAM's Cipher



$$m \oplus k = c$$

1	1	0
0	1	1
1	1	0
0	0	0
0	0	0
1	1	0
0	1	1
0	0	0
1	1	0
1	1	0
1	0	1
1	1	0
1	0	1
0	1	1
0	1	1
1	1	0

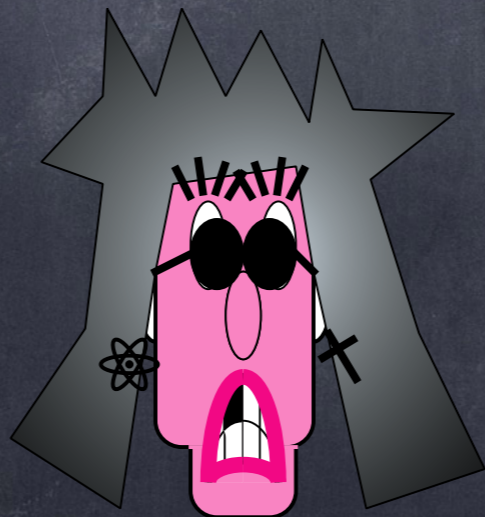
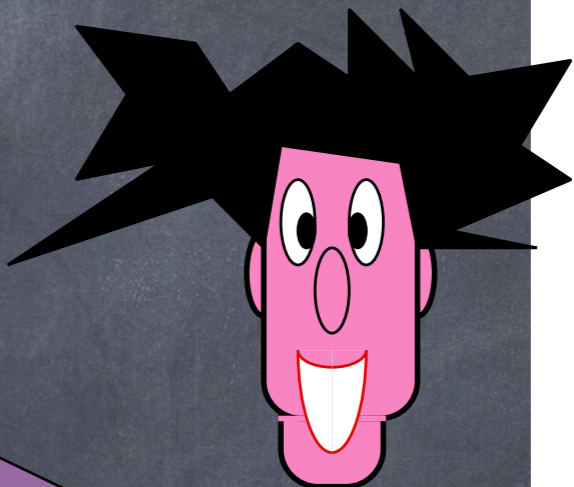
$$\oplus =$$



$$c \oplus k$$

0	1
1	1
0	1
0	0
0	0
0	1
1	1
0	0
0	1
0	1
1	0
0	1
1	0
1	1
1	1
0	1

$$\oplus =$$



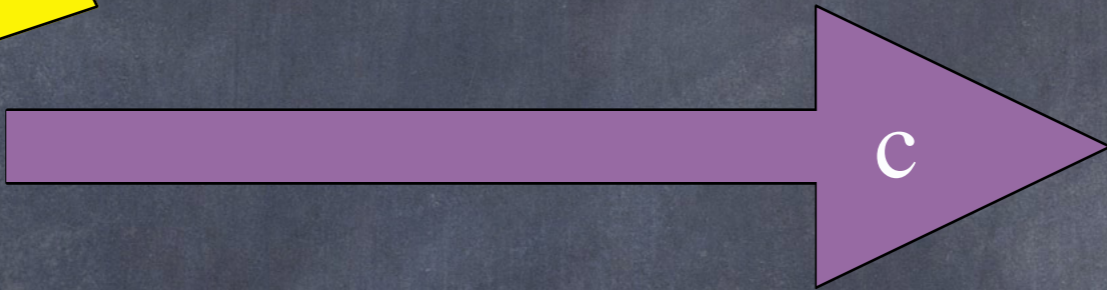
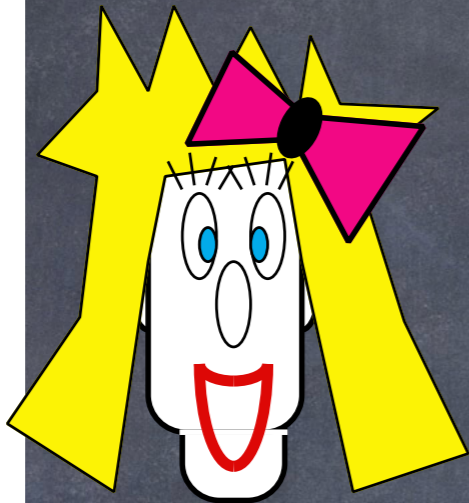
VERNAM's Cipher



$$m \oplus k = c$$

1	1	0
0	1	1
1	1	0
0	0	0
0	0	0
1	1	0
0	1	1
0	0	0
1	1	0
1	1	0
1	0	1
1	1	0
1	0	1
0	1	1
0	1	1
1	1	0

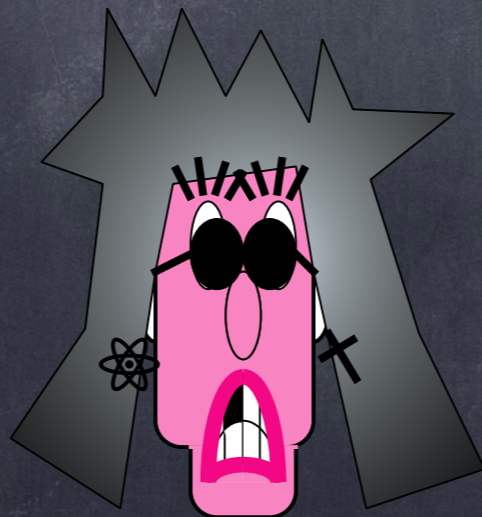
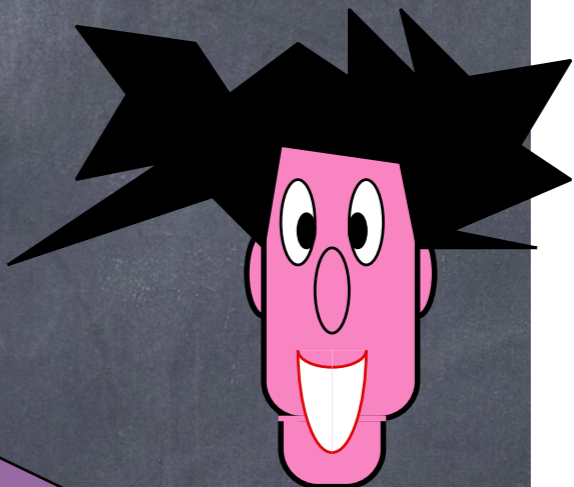
$$\oplus =$$

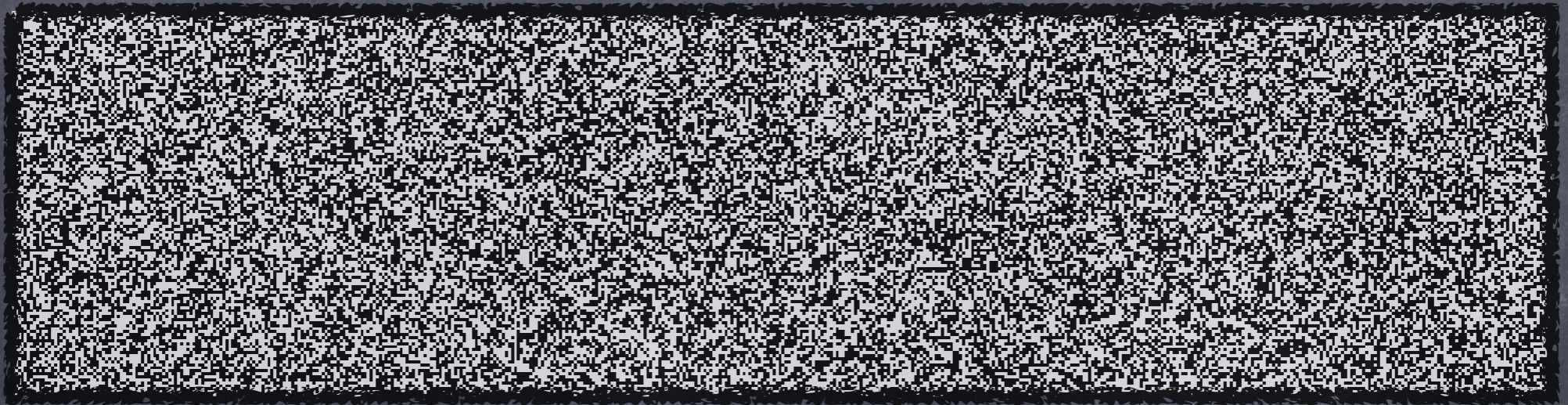


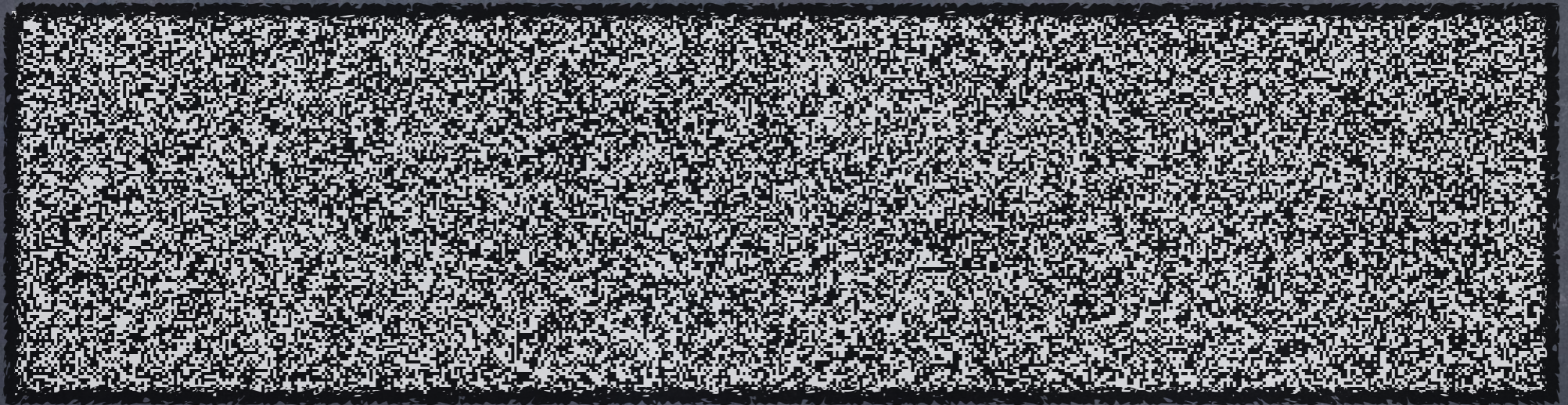
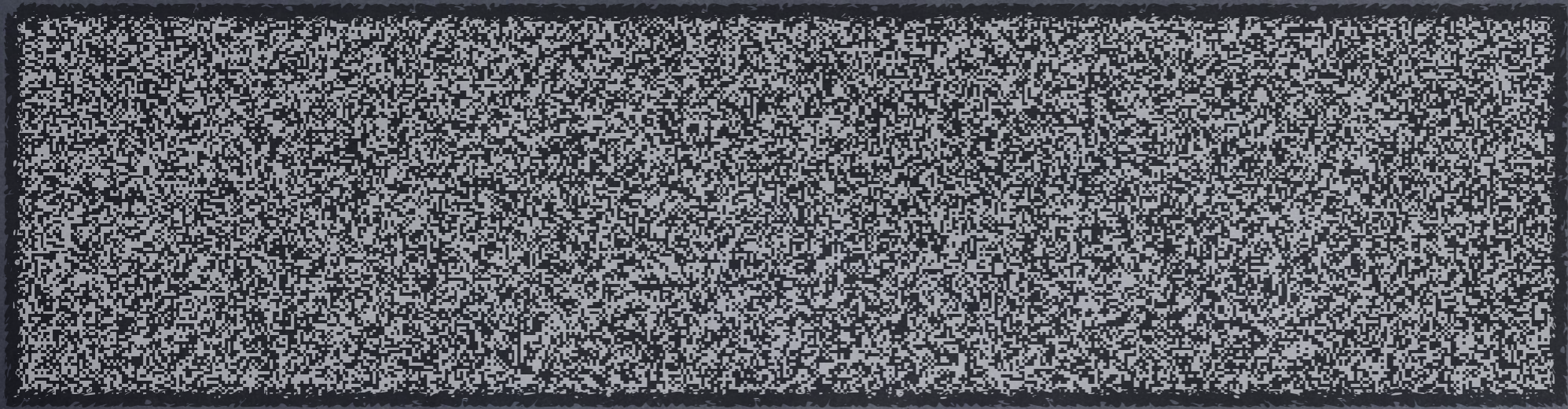
$$c \oplus k = m$$

0	1	1
1	1	0
0	1	1
0	0	0
0	0	0
0	1	1
1	1	0
0	0	0
0	1	1
0	1	1
0	1	1
1	0	1
0	1	1
1	0	1
1	1	0
1	1	0
0	1	1

$$\oplus =$$

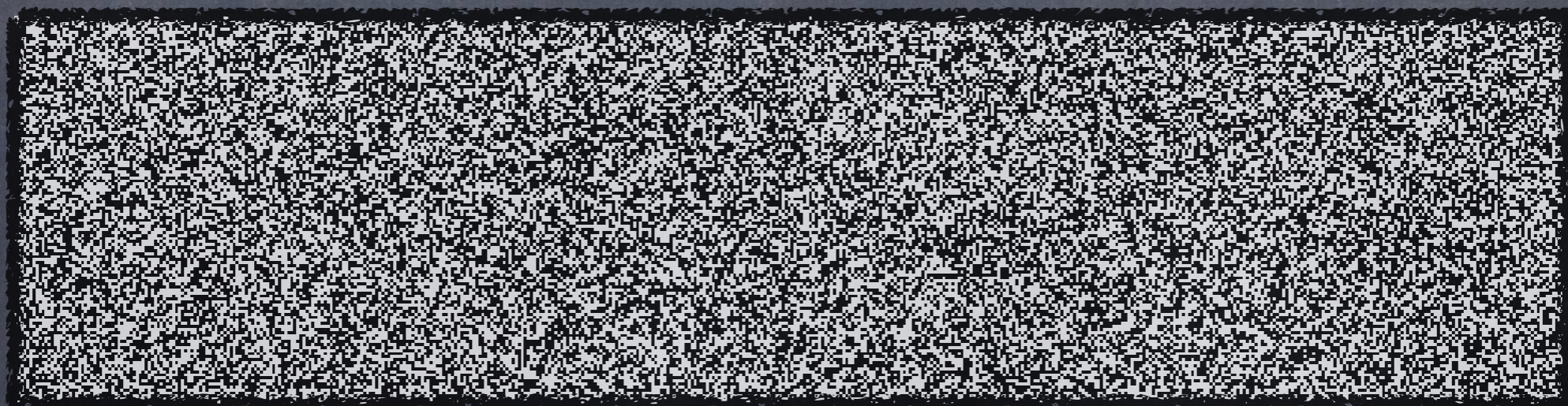
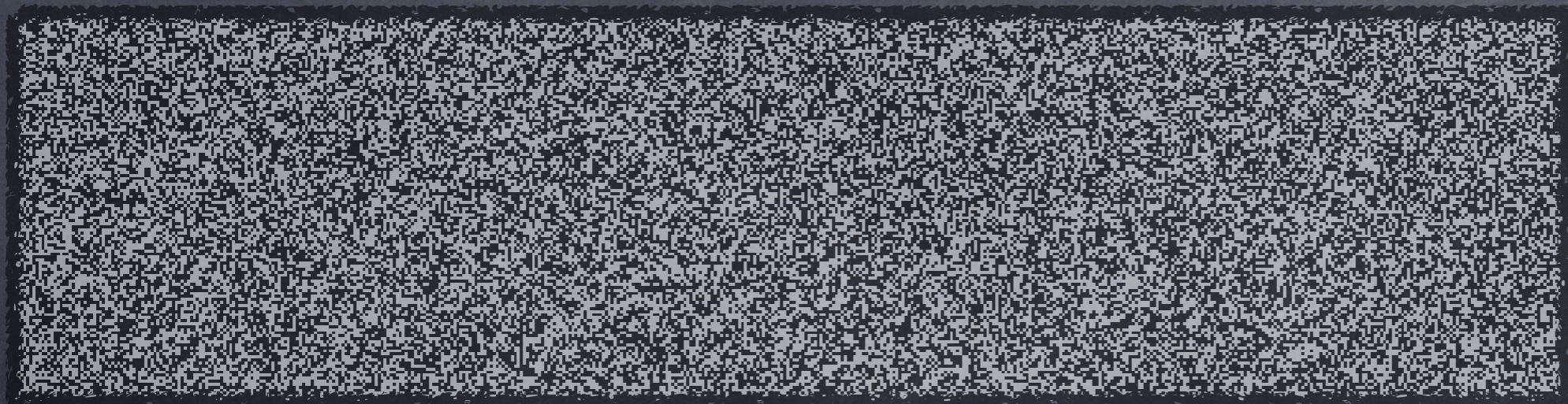


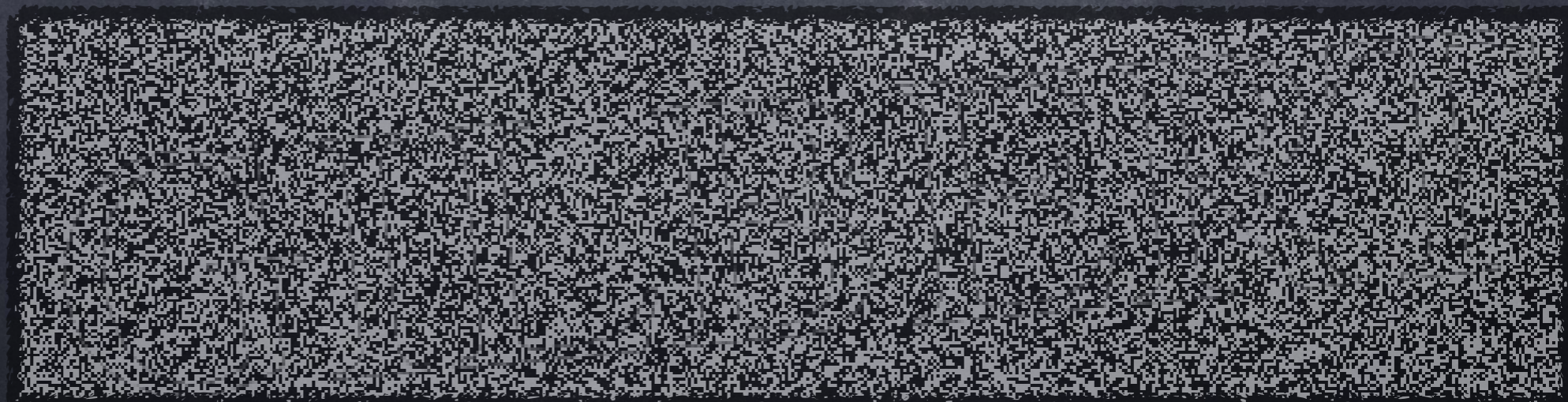
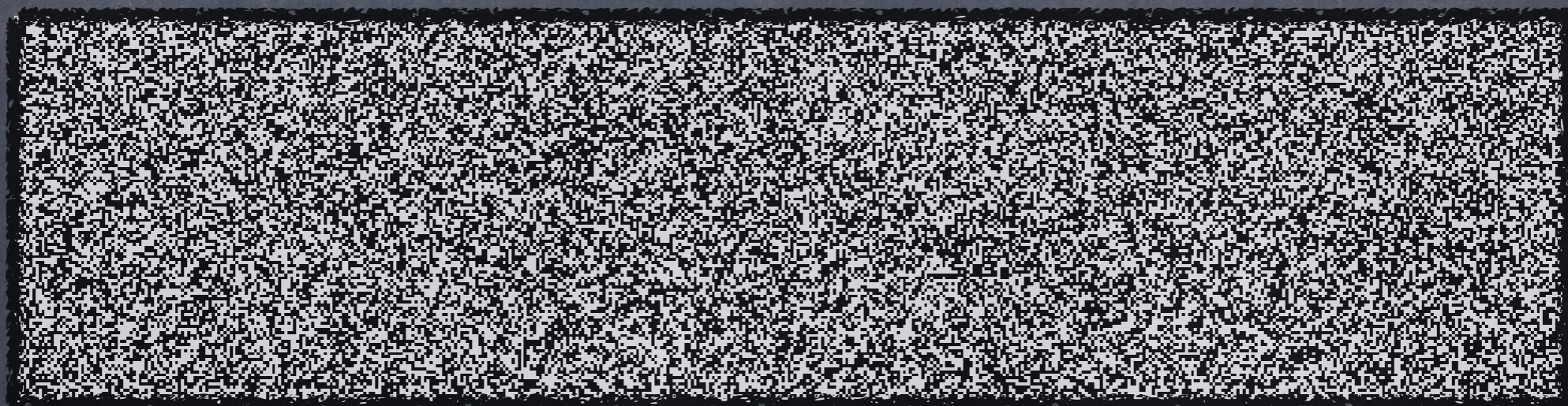
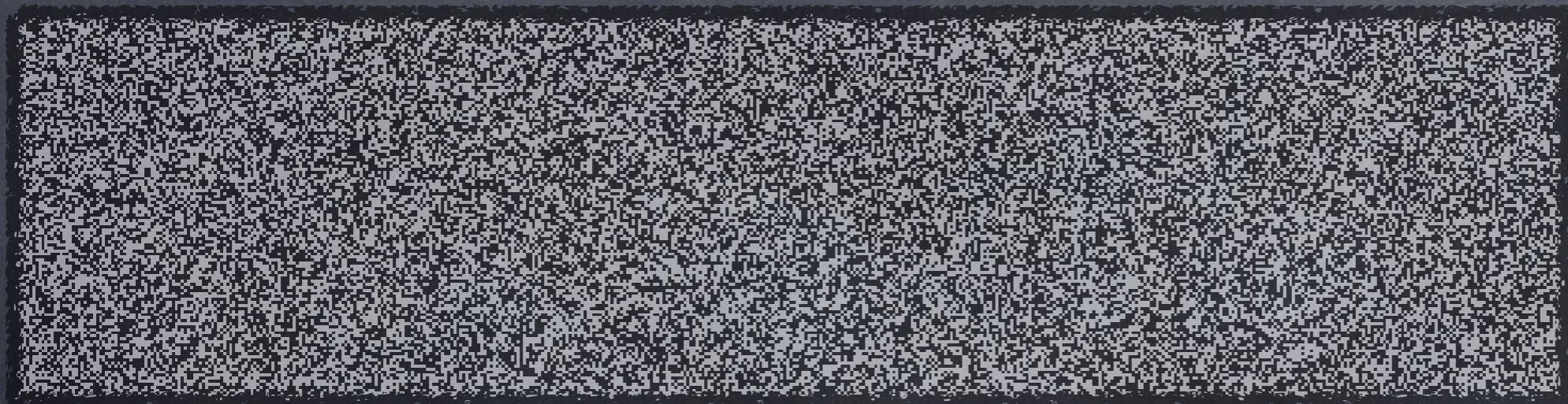


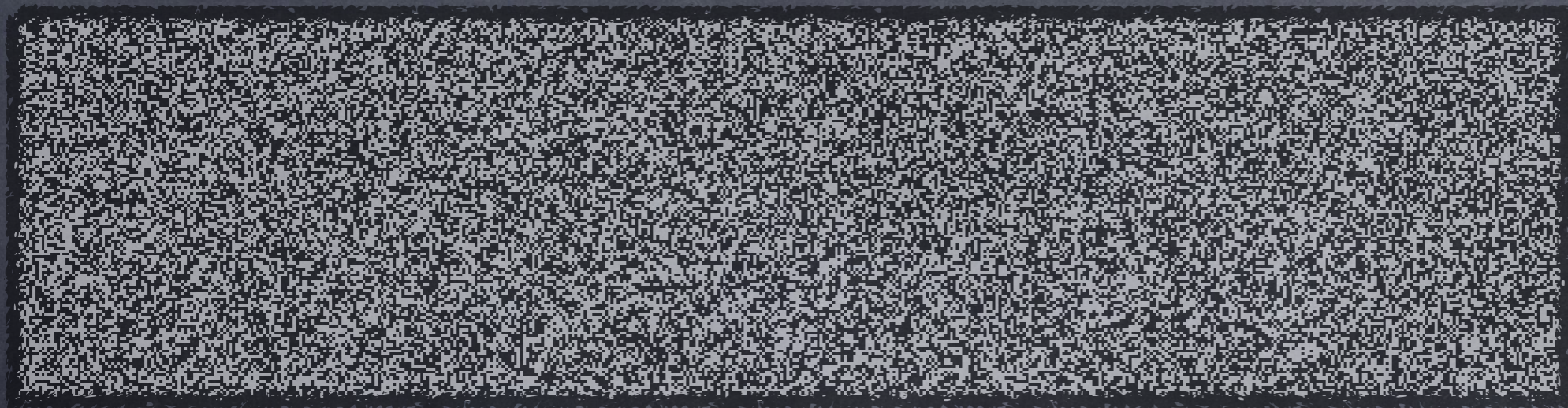


VERNAM



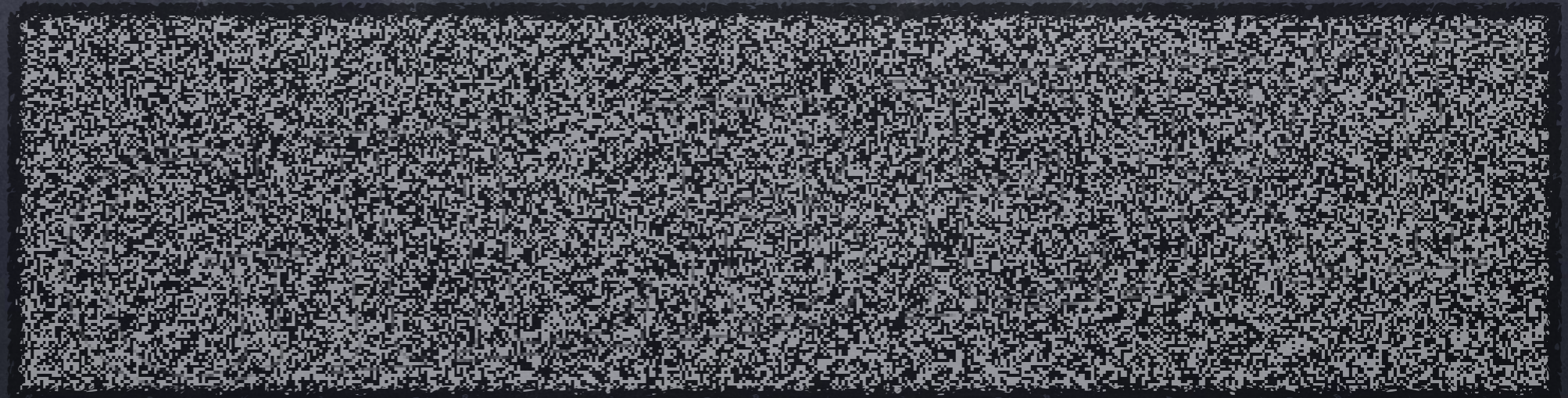
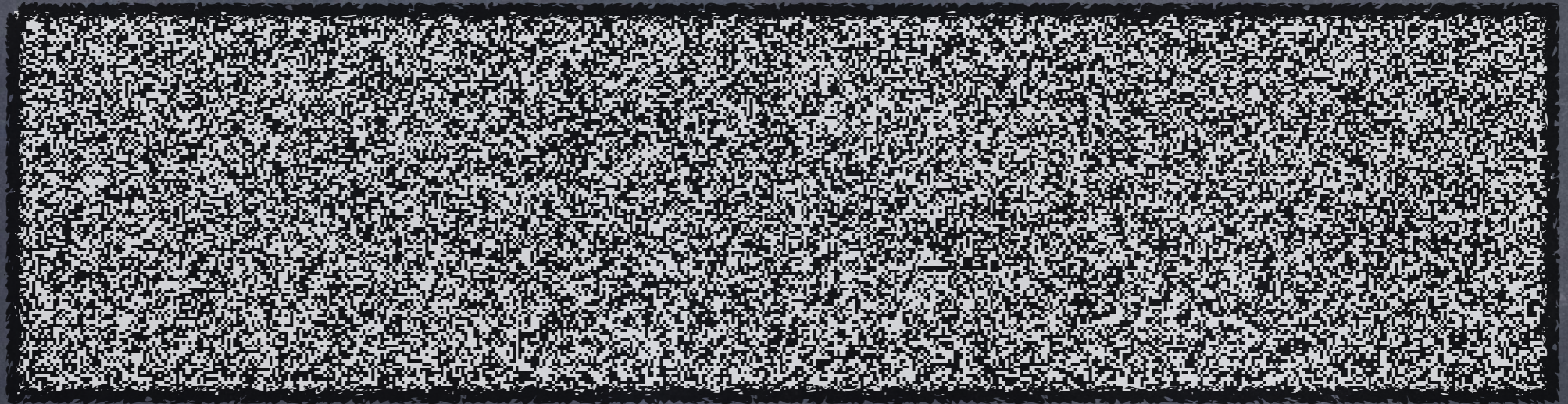
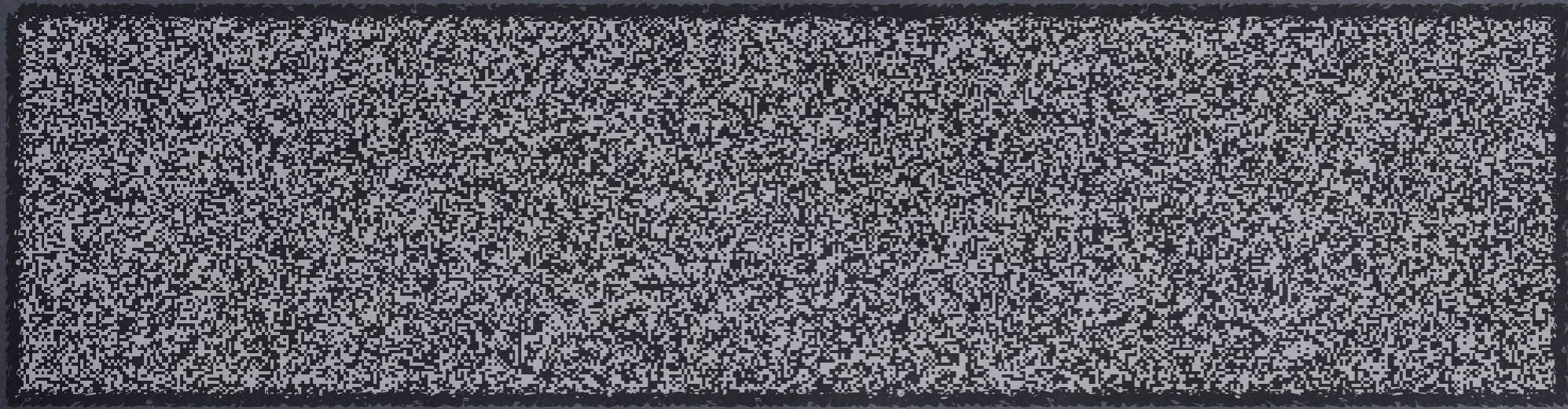


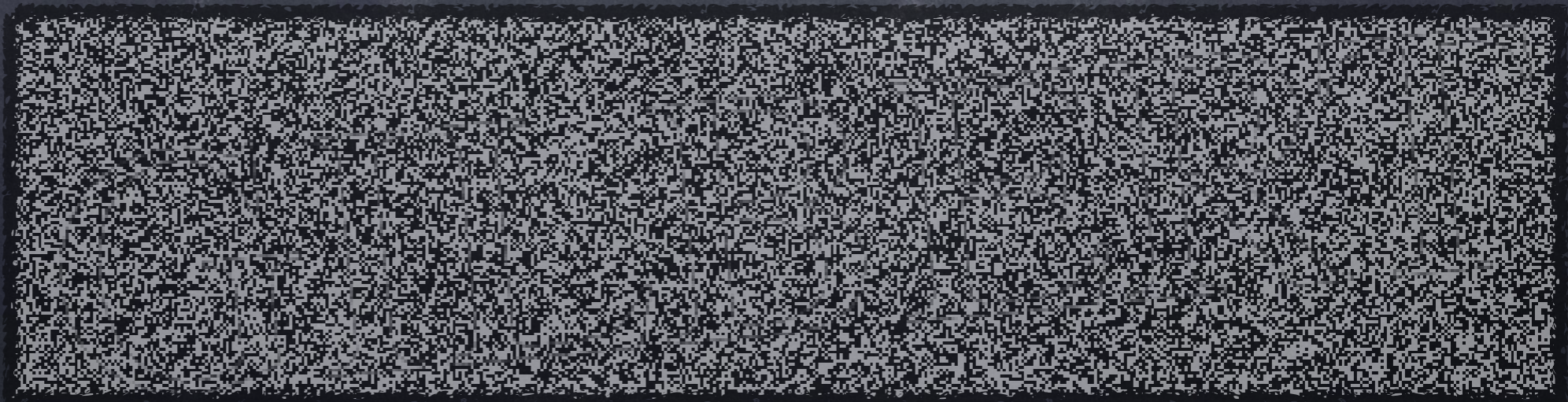
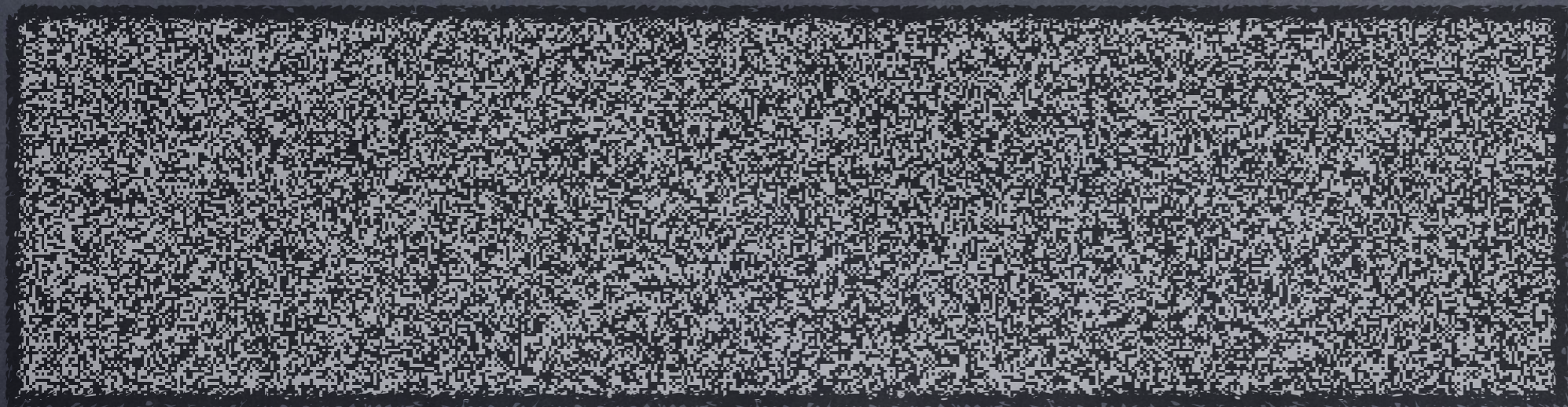


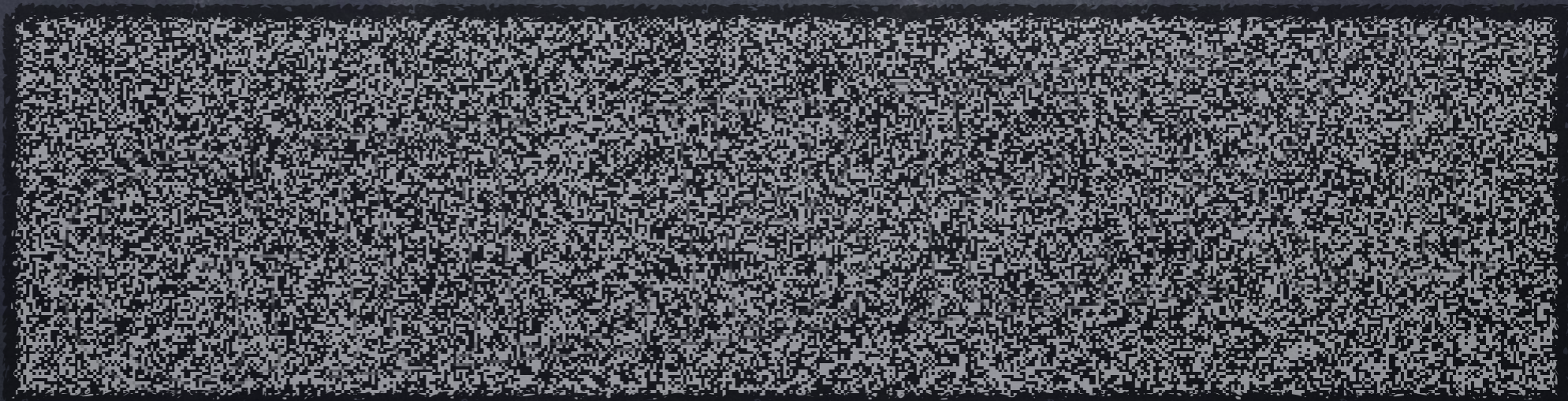
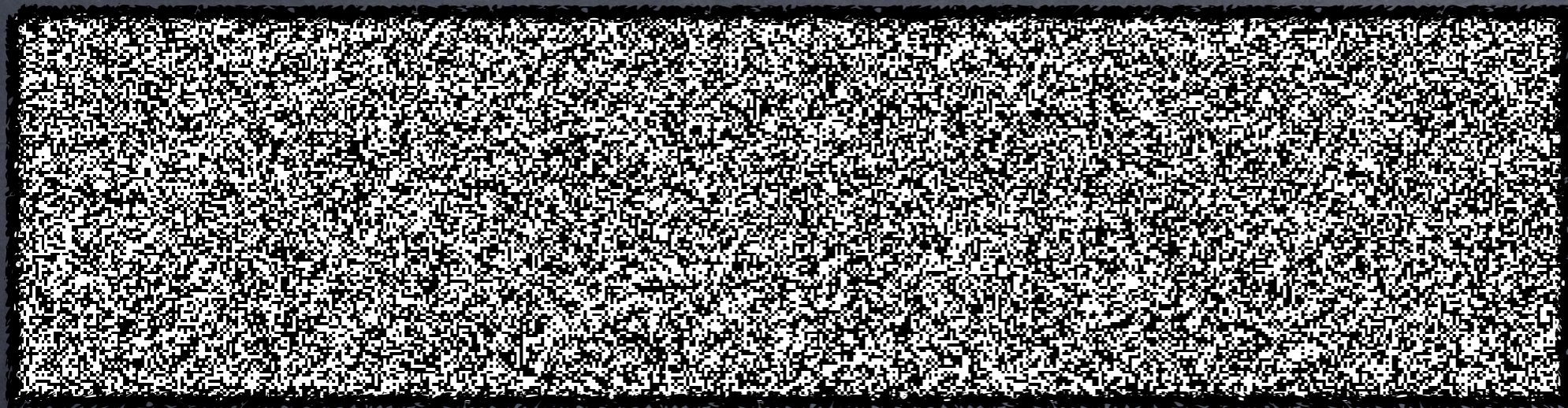


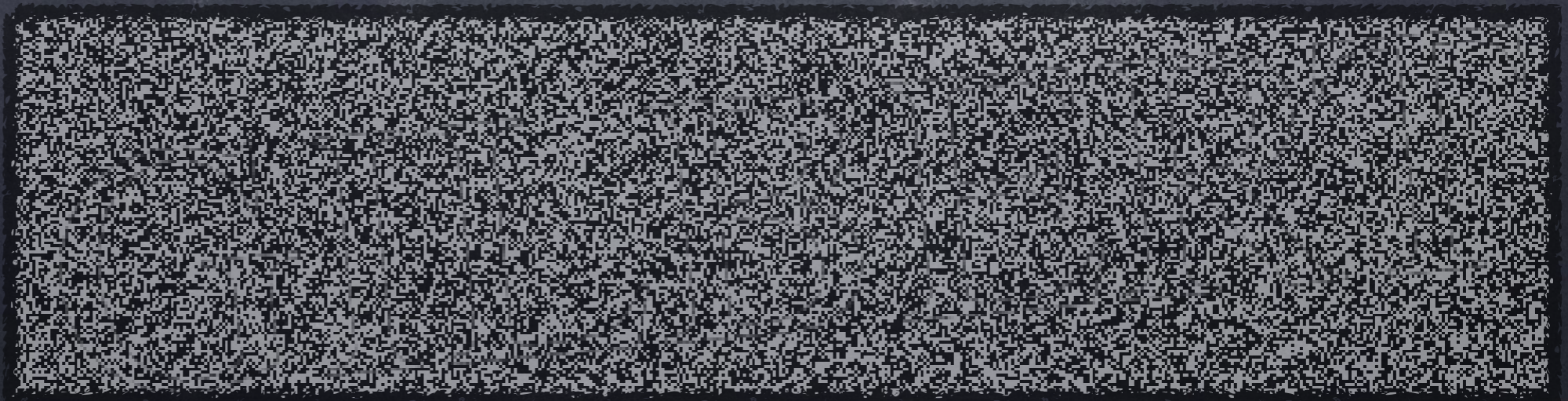
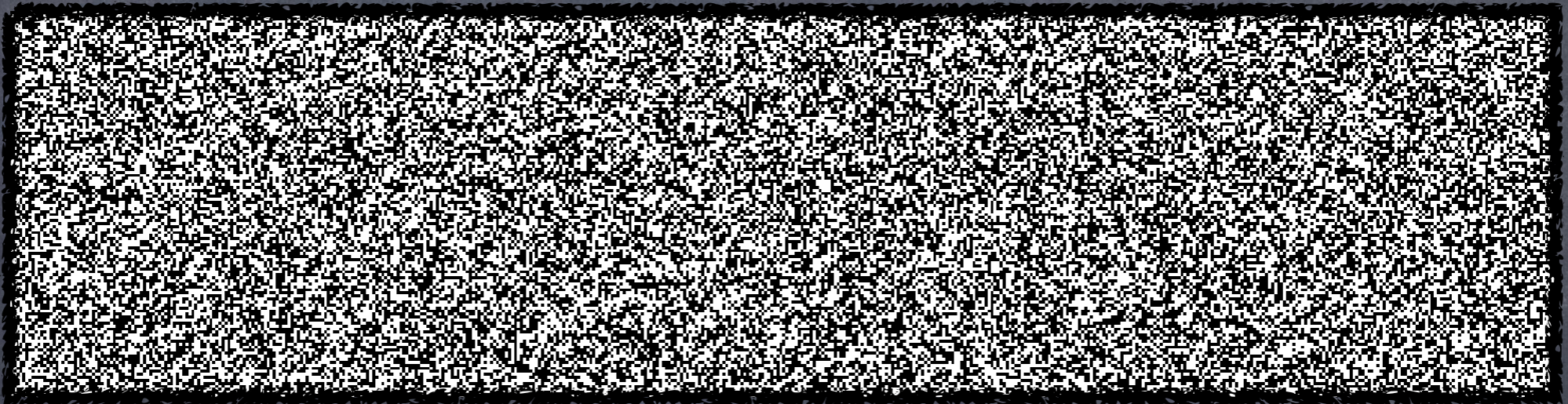
GILBERT









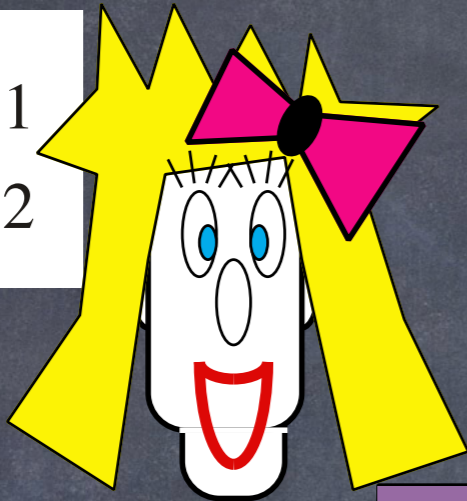


WILKES-BAWMI

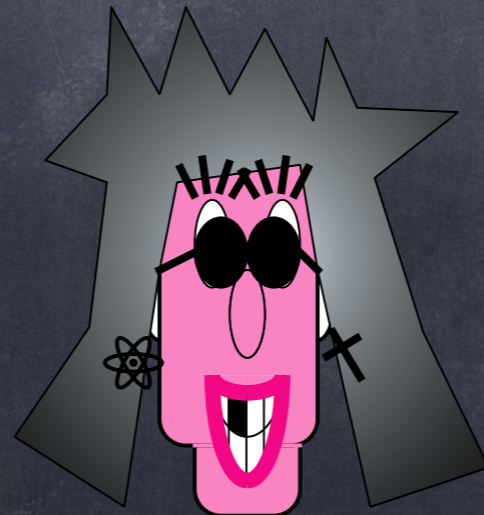
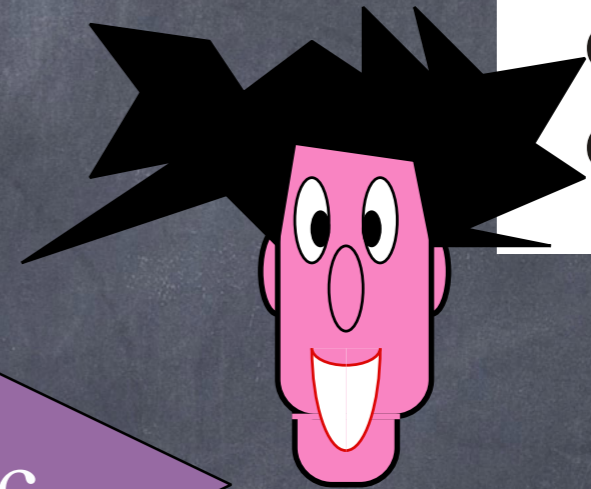


VERNAM's One-Time Pad

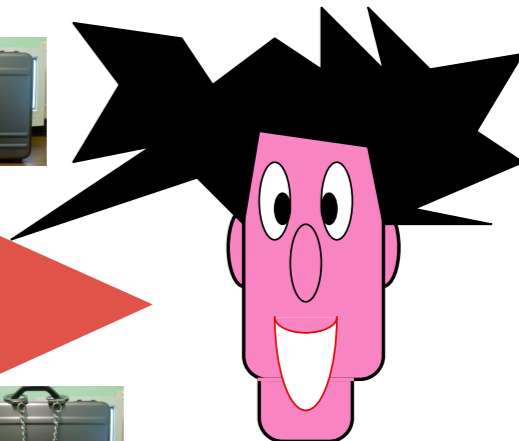
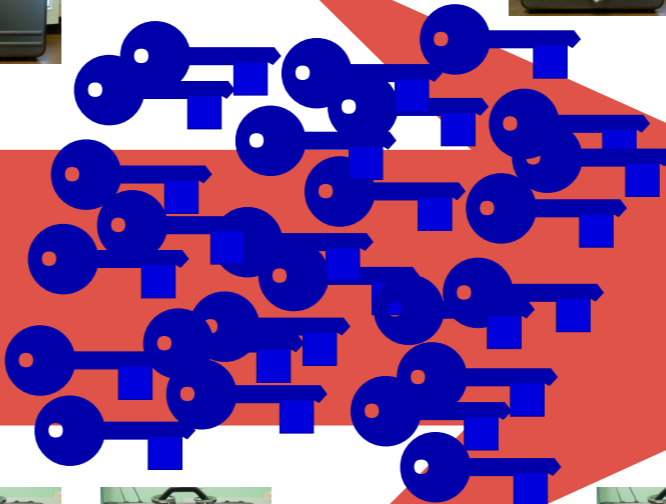
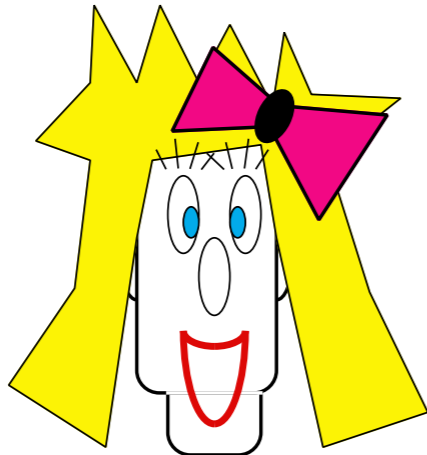
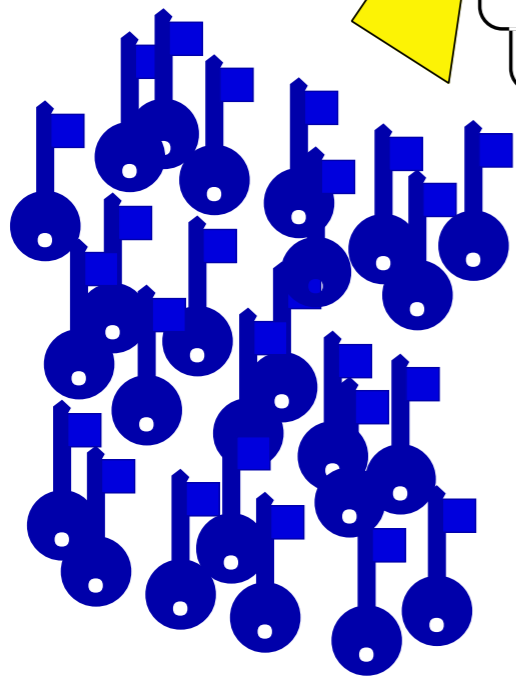
$$m_1 \oplus k = c_1$$
$$m_2 \oplus k = c_2$$



$$c_1 \oplus k = m_1$$
$$c_2 \oplus k = m_2$$



$$c_1 \oplus c_2 = m_1 \oplus m_2$$

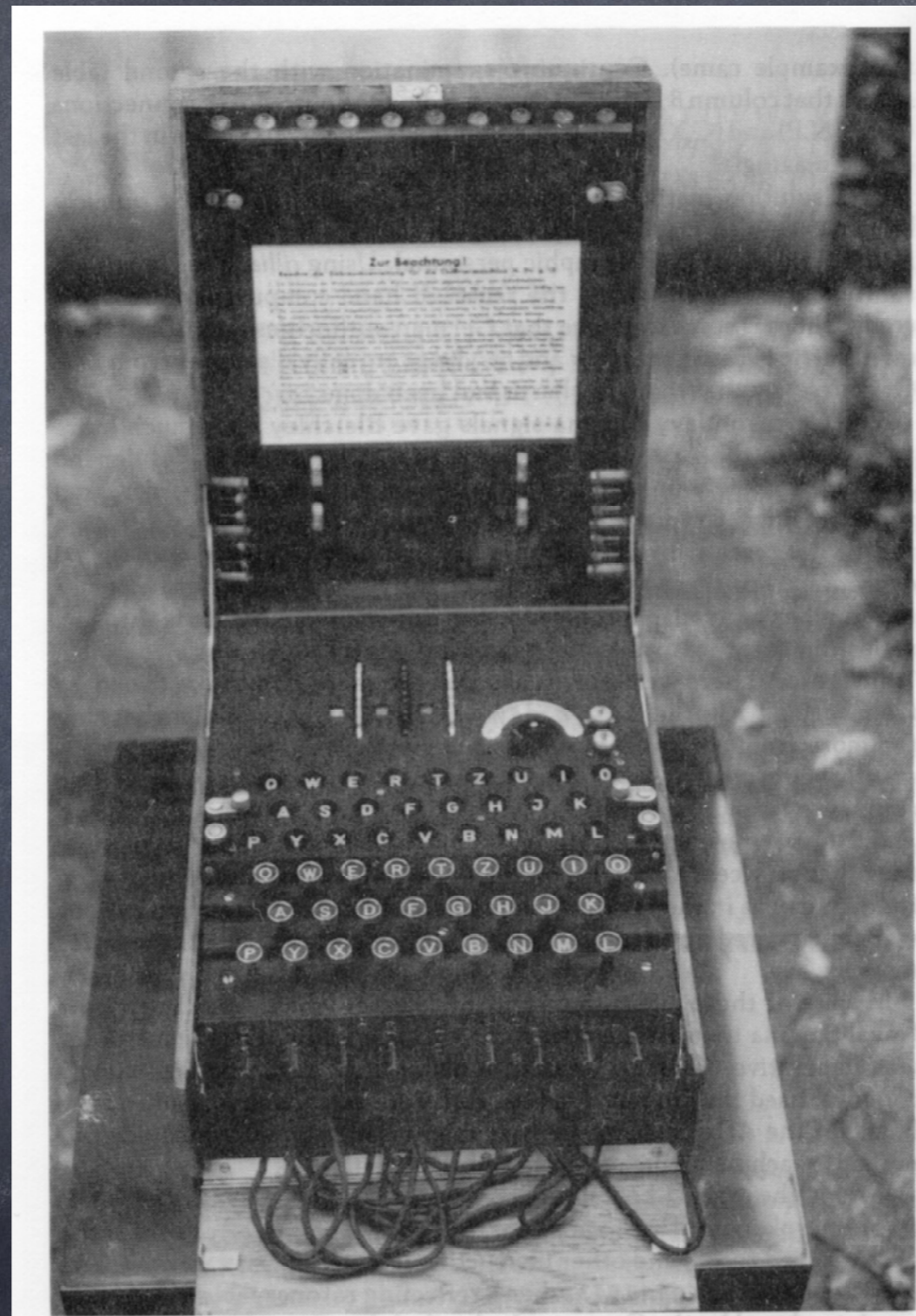


Complexity

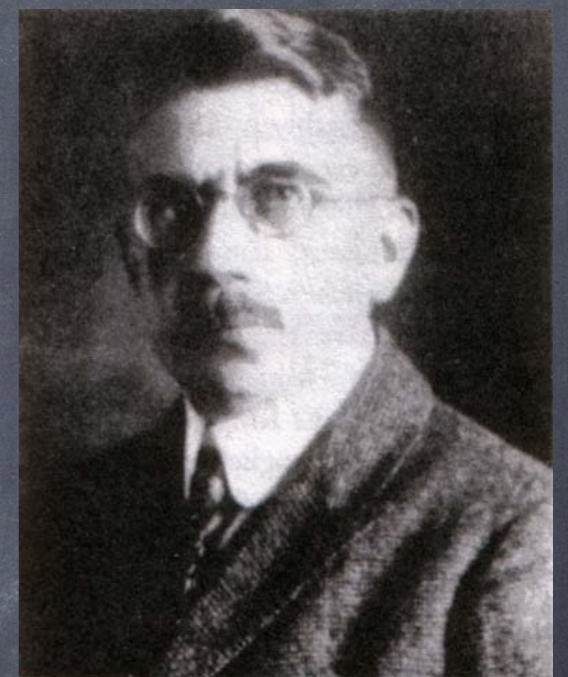
Theoretical

Cryptography

The Enigma Machine



GERMAN ARMY MILITARY ENIGMA. THIS MODEL WAS THE MOST WIDELY USED VERSION OF THE GERMAN WAR-TIME ENIGMAS.

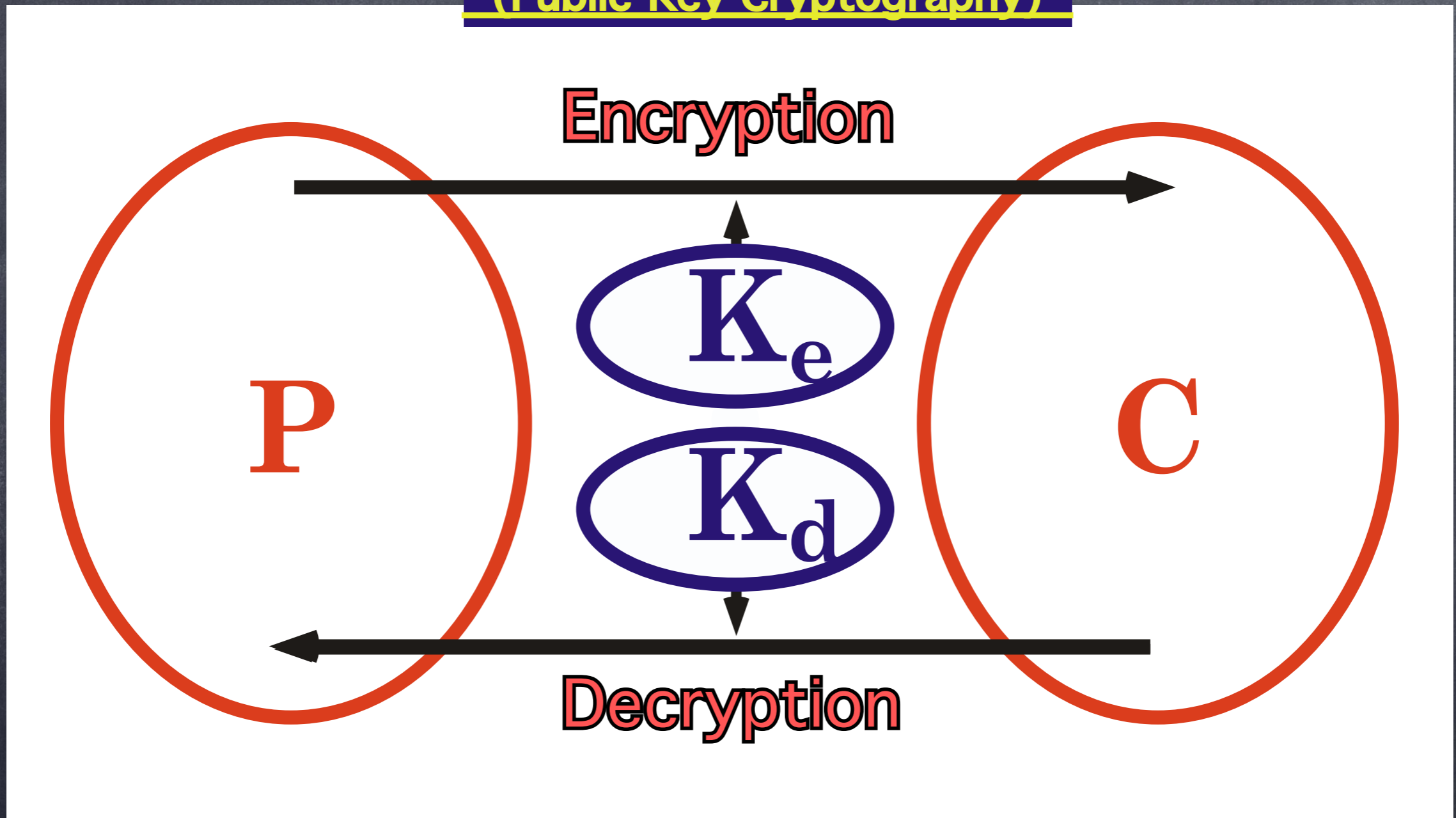


Arthur Scherbius

1930's

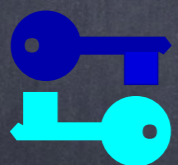
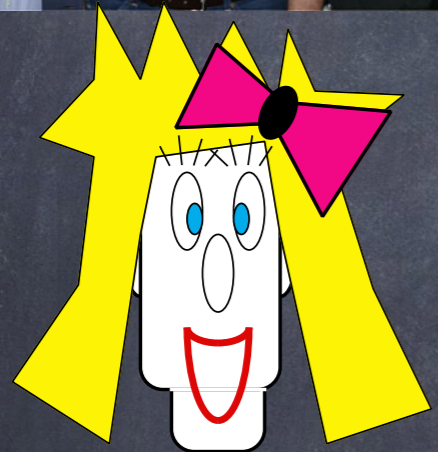
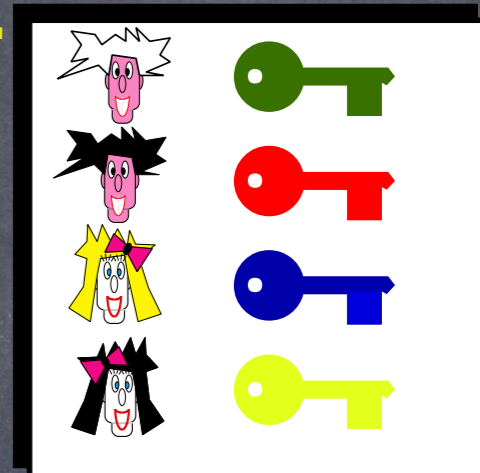
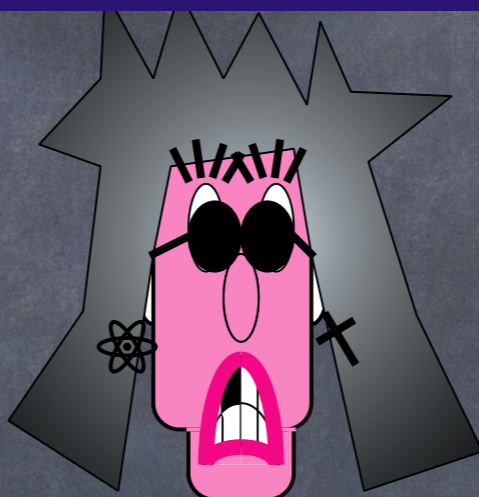
Asymmetric Encryption

(Public-Key Cryptography)

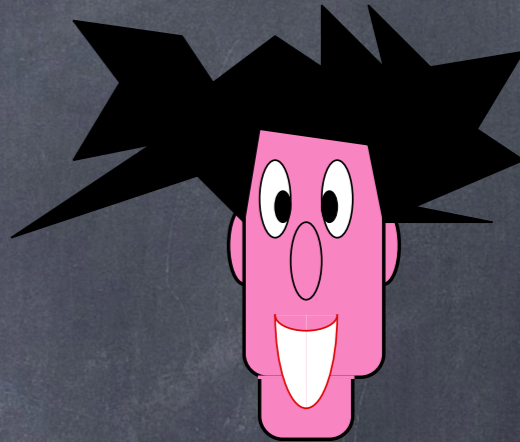


Complexity Theoretical Security

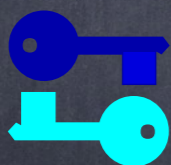
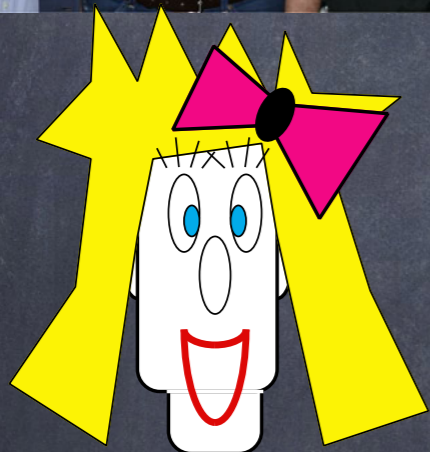
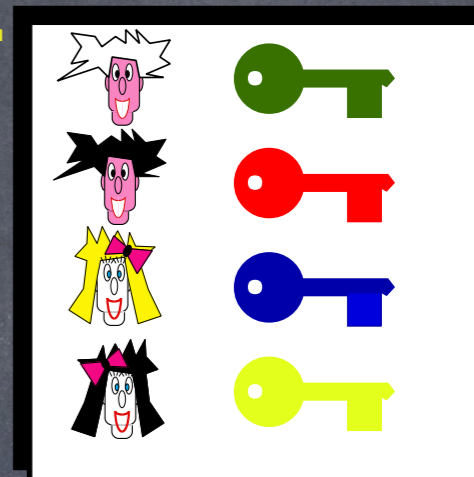
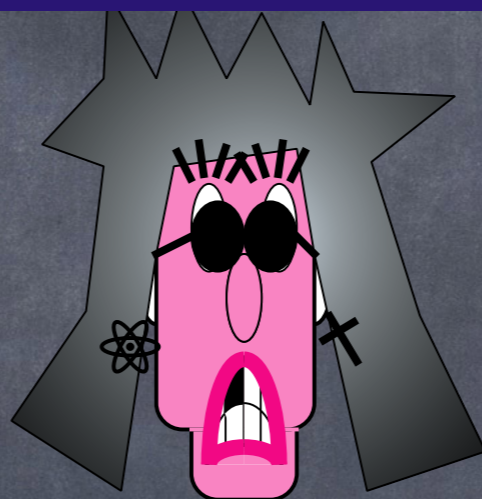
Public-Key Cryptography



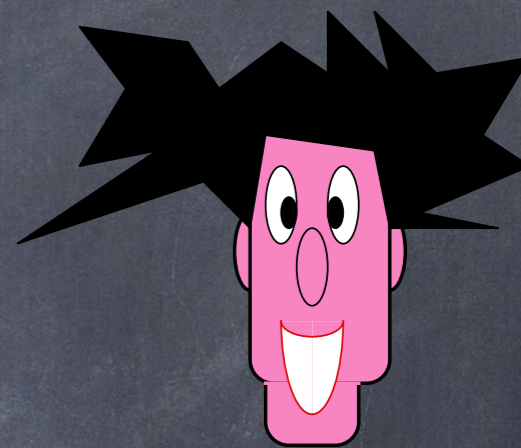
8RdewtU5qkLa\$es!T9@



Public-Key Cryptography



8RdewtU5qkLa\$es!T9@

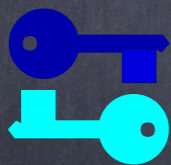
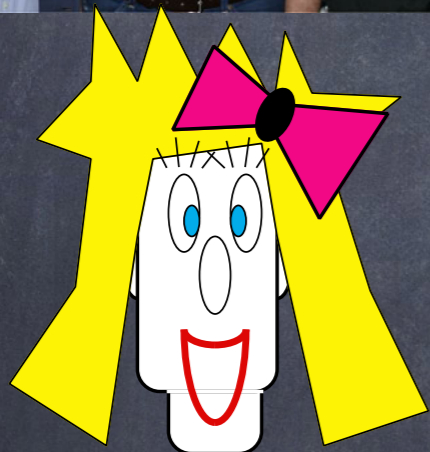
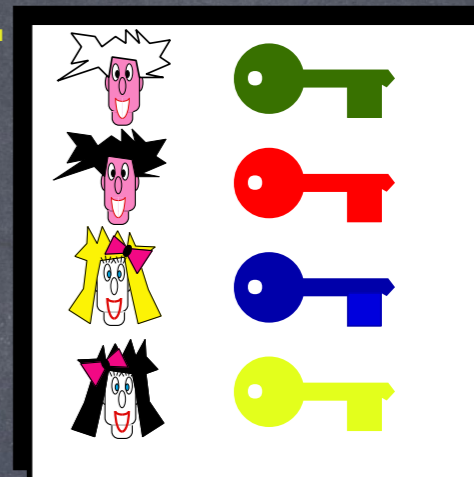
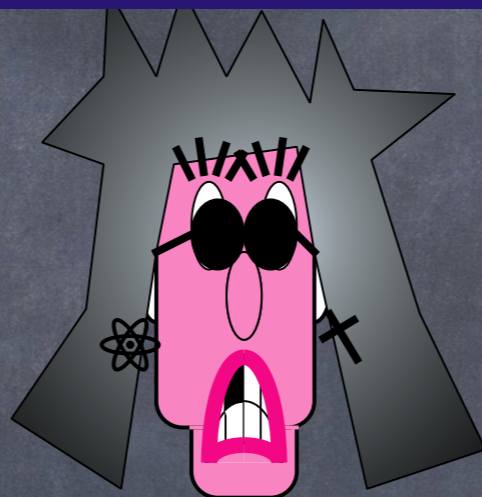


Encryption

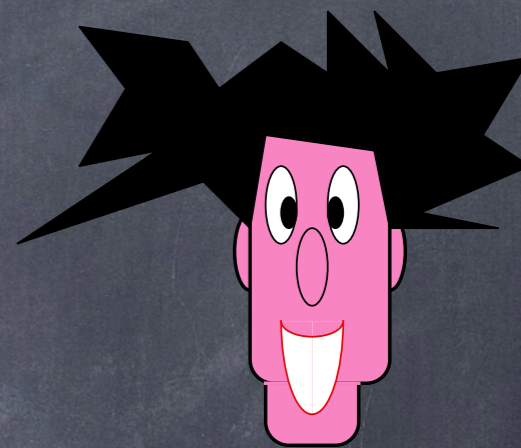


8RdewtU5qkLa\$  y me ?

Public-Key Cryptography



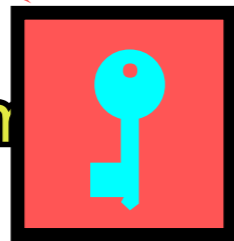
8RdewtU5qkLa\$es!T9@



Decryption



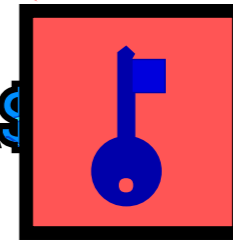
Will you marry me \$es!T9@



Encryption



8RdewtU5qkLa\$ me ?



Fast Exponentiation

- Input: base x and exponent e .

- Output: x^e . // ***** warning $|x^e| \sim e|x|$ *****

```
y = 1
```

```
WHILE e > 0 DO
```

```
  IF e % 2 = 1 THEN y = xy
```

```
  e = e / 2; x = x2
```

```
return y
```

- running time is $O(e|x|^2) = O(2^{|e|}|x|^2)$

Fast Modular Exponentiation

- Input: base x , modulus N and exponent e .
- Output: $x^e \% N$.

$y = 1$

WHILE $e > 0$ **DO**

IF $e \% 2 = 1$ **THEN** $y = xy \% N$

$e = e / 2; x = x^2 \% N$

return y

- running time is $O(|e| * |x|^2) = O(|x|^3)$

Example

Example

$$5^{13} \pmod{7}$$

Example

$$\begin{aligned} &5^{13} \pmod{7} \\ &= 5^{8+4+1} \pmod{7} \end{aligned}$$

Example

$$5^{13} \bmod 7$$

$$= 5^{8+4+1} \bmod 7$$

$$= 5^8 \times 5^4 \times 5^1 \bmod 7$$

Example

$$5^{13} \bmod 7$$

$$= 5^{8+4+1} \bmod 7$$

$$= 5^8 \times 5^4 \times 5^1 \bmod 7$$

$$5^0, 5^1, 5^2, 5^4, 5^8$$

Example

$$5^{13} \pmod{7}$$

$$= 5^{8+4+1} \pmod{7}$$

$$= 5^8 \times 5^4 \times 5^1 \pmod{7}$$

$$5^0, 5^1, 5^2, 5^4, 5^8$$

$$\equiv 1, 5, 4, 2, 4 \pmod{7}$$

Example

$$5^{13} \pmod{7}$$

$$= 5^{8+4+1} \pmod{7}$$

$$= 5^8 \times 5^4 \times 5^1 \pmod{7}$$

$$5^0, 5^1, 5^2, 5^4, 5^8$$

$$\equiv 1, 5, 4, 2, 4 \pmod{7}$$

$$= 4 \times 2 \times 5 \pmod{7}$$

Example

$$5^{13} \pmod{7}$$

$$= 5^{8+4+1} \pmod{7}$$

$$= 5^8 \times 5^4 \times 5^1 \pmod{7}$$

$$5^0, 5^1, 5^2, 5^4, 5^8$$

$$\equiv 1, 5, 4, 2, 4 \pmod{7}$$

$$= 4 \times 2 \times 5 \pmod{7}$$

$$= 1 \times 5 \pmod{7}$$

Example

$$5^{13} \pmod{7}$$

$$= 5^{8+4+1} \pmod{7}$$

$$= 5^8 \times 5^4 \times 5^1 \pmod{7}$$

$$5^0, 5^1, 5^2, 5^4, 5^8$$

$$\equiv 1, 5, 4, 2, 4 \pmod{7}$$

$$= 4 \times 2 \times 5 \pmod{7}$$

$$= 1 \times 5 \pmod{7}$$

$$= 5$$

Euclid's Algorithm



Note. $\gcd(a, b) = g \rightarrow \exists x, y \in \mathbb{Z}$ such that $g = ax + by$. The following recursive definition is based on the property $\gcd(a, b) = \gcd(a, b - a)$.

$$\gcd(a, b) = \begin{cases} a & \text{if } b = 0 \\ \gcd(b, a \bmod b) & \text{otherwise} \end{cases}$$

Euclidian Algorithm

- Input: integers a, b .
- Output: g, x, y such that $g = \text{GCD}(a, b)$.

$g = a; g' = b;$

WHILE $g' > 0$ **DO**

$k = g/g'$

$g'' = g - kg';$

$g = g';$

$g' = g'';$

return g

// $g'' = g \% g'$

- running time is $O(|a| * |b|)$

EXAMPLE

EXAMPLE

$\text{gcd}(7,19)$

EXAMPLE

$$\begin{aligned} &\text{gcd}(7,19) \\ &= \text{gcd}(19,7) \end{aligned}$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(19,7) \\ &= \gcd(7,19 \bmod 7) \end{aligned}$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(19,7) \\ &= \gcd(7,19 \bmod 7) \\ &= \gcd(7,5) \end{aligned}$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(19,7) \\ &= \gcd(7,19 \bmod 7) \\ &= \gcd(7,5) \\ &= \gcd(5,7 \bmod 5) \end{aligned}$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(19,7) \\ &= \gcd(7,19 \bmod 7) \\ &= \gcd(7,5) \\ &= \gcd(5,7 \bmod 5) \\ &= \gcd(5,2) \end{aligned}$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(19,7) \\ &= \gcd(7,19 \bmod 7) \\ &= \gcd(7,5) \\ &= \gcd(5,7 \bmod 5) \\ &= \gcd(5,2) \\ &= \gcd(2,5 \bmod 2) \end{aligned}$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(19,7) \\ &= \gcd(7,19 \bmod 7) \\ &= \gcd(7,5) \\ &= \gcd(5,7 \bmod 5) \\ &= \gcd(5,2) \\ &= \gcd(2,5 \bmod 2) \\ &= \gcd(2,1) \end{aligned}$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(19,7) \\ &= \gcd(7,19 \bmod 7) \\ &= \gcd(7,5) \\ &= \gcd(5,7 \bmod 5) \\ &= \gcd(5,2) \\ &= \gcd(2,5 \bmod 2) \\ &= \gcd(2,1) \\ &= \gcd(1,2 \bmod 1) = \gcd(1,0) = 1 \end{aligned}$$

Euclid's Extended Algorithm



The idea behind the following iterative algorithm is to maintain in each iteration the relations $g = ax + by$ and $g' = ax' + by'$ while reducing the value of g .

At the end of the algorithm, the value of g is $\gcd(a, b)$. The final value of x is such that $ax \equiv g \pmod{b}$ and by symmetry, the final value of y is such that $by \equiv g \pmod{a}$. When $\gcd(a, b) = 1$, we find that x is the multiplicative inverse of a modulo b and that y is the multiplicative inverse of b modulo a .

Extended Euclidian Algorithm

- Input: integers a, b .
- Output: g, x, y such that $g = \text{GCD}(a, b)$ and $g = ax + by$.

$g = a; g' = b; x = 1; y = 0; x' = 0; y' = 1;$

WHILE $g' > 0$ **DO**

$k = g/g'$

$g'' = g - kg'; x'' = x - kx'; y'' = y - ky';$ // $g'' = g \% g'$

$g = g'; x = x'; y = y';$

$g' = g''; x' = x''; y' = y'';$

return g, x, y

- running time is $O(|a| * |b|)$

EXAMPLE

EXAMPLE

$\gcd(7,19)$

$$x=1, y=0, x'=0, y'=1$$

EXAMPLE

$$\begin{aligned} &\gcd(7,19) \\ &= \gcd(19,7) \end{aligned}$$

$$x=1, y=0, x'=0, y'=1$$

$$x=0, y=1, x'=1, y'=0$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(19,7) \\ &= \gcd(7,19 \bmod 7) \end{aligned}$$

$$x=1, y=0, x'=0, y'=1$$

$$x=0, y=1, x'=1, y'=0$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(19,7) \\ &= \gcd(7,19 \bmod 7) \\ &= \gcd(7,5) \end{aligned}$$

$$x=1, y=0, x'=0, y'=1$$

$$x=0, y=1, x'=1, y'=0$$

$$x=1, y=0, x'=-2, y'=1$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(19,7) \\ &= \gcd(7,19 \bmod 7) \\ &= \gcd(7,5) \\ &= \gcd(5,7 \bmod 5) \end{aligned}$$

$$x=1, y=0, x'=0, y'=1$$

$$x=0, y=1, x'=1, y'=0$$

$$x=1, y=0, x'=-2, y'=1$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(19,7) \\ &= \gcd(7,19 \bmod 7) \\ &= \gcd(7,5) \\ &= \gcd(5,7 \bmod 5) \\ &= \gcd(5,2) \end{aligned}$$

$$x=1, y=0, x'=0, y'=1$$

$$x=0, y=1, x'=1, y'=0$$

$$x=1, y=0, x'=-2, y'=1$$

$$x=-2, y=1, x'=3, y'=-1$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(19,7) \\ &= \gcd(7,19 \bmod 7) \\ &= \gcd(7,5) \\ &= \gcd(5,7 \bmod 5) \\ &= \gcd(5,2) \\ &= \gcd(2,5 \bmod 2) \end{aligned}$$

$$x=1, y=0, x'=0, y'=1$$

$$x=0, y=1, x'=1, y'=0$$

$$x=1, y=0, x'=-2, y'=1$$

$$x=-2, y=1, x'=3, y'=-1$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(19,7) \\ &= \gcd(7,19 \bmod 7) \\ &= \gcd(7,5) \\ &= \gcd(5,7 \bmod 5) \\ &= \gcd(5,2) \\ &= \gcd(2,5 \bmod 2) \\ &= \gcd(2,1) \end{aligned}$$

$$x=1, y=0, x'=0, y'=1$$

$$x=0, y=1, x'=1, y'=0$$

$$x=1, y=0, x'=-2, y'=1$$

$$x=-2, y=1, x'=3, y'=-1$$

$$x=3, y=-1, x'=-8, y'=3$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(19,7) \\ &= \gcd(7,19 \bmod 7) \\ &= \gcd(7,5) \\ &= \gcd(5,7 \bmod 5) \\ &= \gcd(5,2) \\ &= \gcd(2,5 \bmod 2) \\ &= \gcd(2,1) \\ &= \gcd(1,2 \bmod 1) \end{aligned}$$

$$x=1, y=0, x'=0, y'=1$$

$$x=0, y=1, x'=1, y'=0$$

$$x=1, y=0, x'=-2, y'=1$$

$$x=-2, y=1, x'=3, y'=-1$$

$$x=3, y=-1, x'=-8, y'=3$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(19,7) \\ &= \gcd(7,19 \bmod 7) \\ &= \gcd(7,5) \\ &= \gcd(5,7 \bmod 5) \\ &= \gcd(5,2) \\ &= \gcd(2,5 \bmod 2) \\ &= \gcd(2,1) \\ &= \gcd(1,2 \bmod 1) \\ &= \gcd(1,0) = 1 \end{aligned}$$

$$x=1, y=0, x'=0, y'=1$$

$$x=0, y=1, x'=1, y'=0$$

$$x=1, y=0, x'=-2, y'=1$$

$$x=-2, y=1, x'=3, y'=-1$$

$$x=3, y=-1, x'=-8, y'=3$$

$$x=-8, y=3, x'=19, y'=-7$$

EXAMPLE

EXAMPLE

$\text{gcd}(7,19)$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(1,0) = 1 \end{aligned}$$

$$x=-8, y=3, x'=19, y'=-7$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(1,0) = 1 \end{aligned}$$

$$x=-8, y=3, x'=19, y'=-7$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(1,0) = 1 \end{aligned}$$

$$x=-8, y=3, x'=19, y'=-7$$

thus

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(1,0) = 1 \end{aligned}$$

$$x=-8, y=3, x'=19, y'=-7$$

thus

$$1 = -8 \times 7 + 3 \times 19$$

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(1,0) = 1 \end{aligned}$$

$$x=-8, y=3, x'=19, y'=-7$$

thus

$$1 = -8 \times 7 + 3 \times 19$$

and

EXAMPLE

$$\begin{aligned} & \gcd(7,19) \\ &= \gcd(1,0) = 1 \end{aligned}$$

$$x=-8, y=3, x'=19, y'=-7$$

thus

$$1 = -8 \times 7 + 3 \times 19$$

and

$$7^{-1} \bmod 19 = 11 = -8 \bmod 19$$

EXAMPLE

$$\begin{aligned} \gcd(7,19) \\ = \gcd(1,0) = 1 \end{aligned} \quad x=-8, y=3, x'=19, y'=-7$$

thus

$$1 = -8 \times 7 + 3 \times 19$$

and

$$7^{-1} \bmod 19 = 11 = -8 \bmod 19$$

$$19^{-1} \bmod 7 = 3 = 3 \bmod 7$$



Primality Testing

- Input: base a , modulus N .
- Output: Is N a base- a pseudo-prime? .

IF $\text{GCD}(a, N) > 1$ **THEN** return False

set $s \geq 0$ and t (odd) s.t. $N-1 = t2^s$

$x = a^2 \% N; y = N-1$

FOR $i = 1$ **TO** s

IF $x = 1$ **AND** $y = N-1$ **THEN** return True

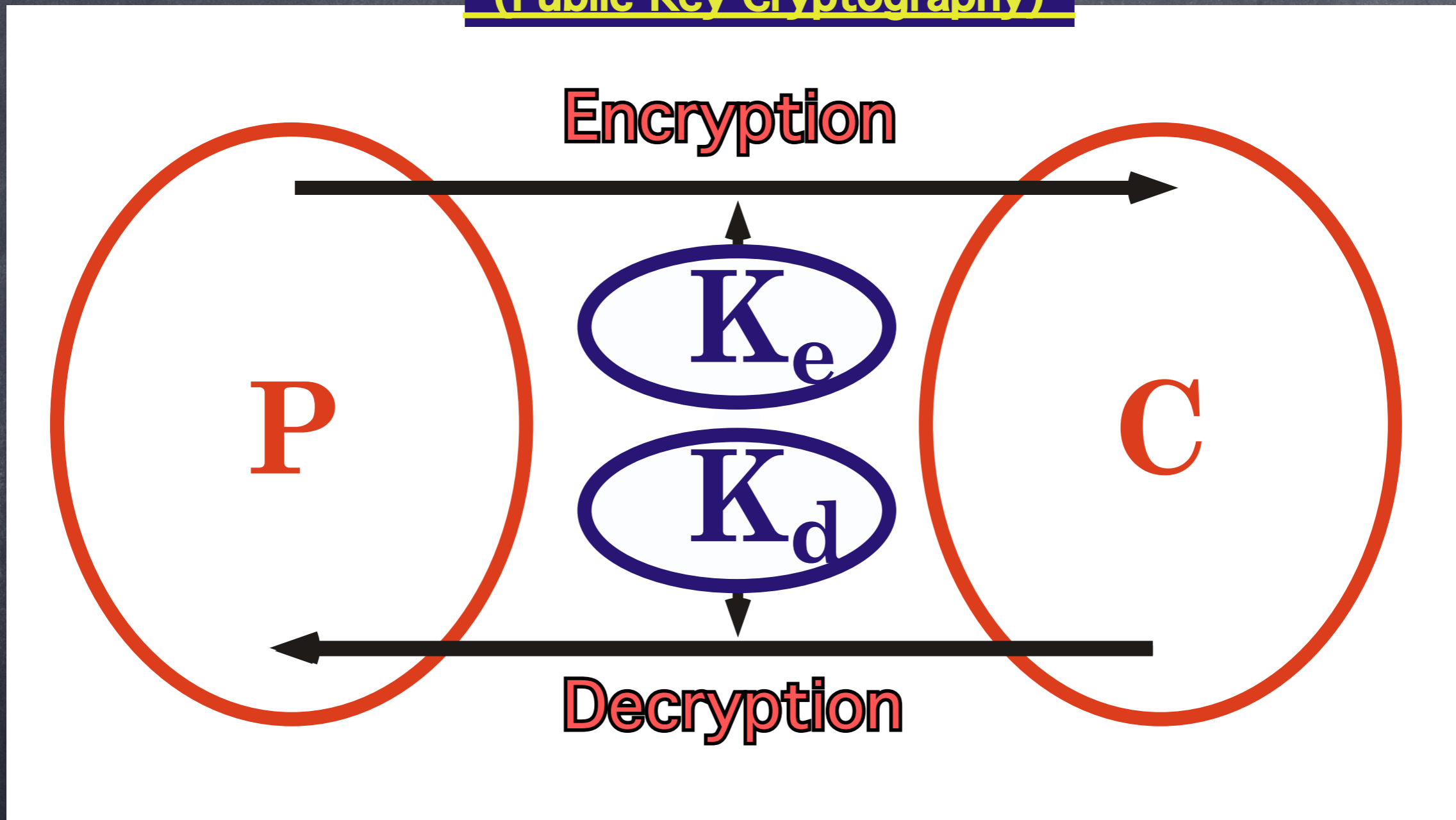
$y = x; x = x^2 \% N$

return False

- running time is $O(|N|^4)$

Asymmetric Encryption

(Public-Key Cryptography)



Complexity Theoretical Security

Definitions

DEFINITION A public-key encryption scheme is a tuple of PPT algorithms $(\text{Gen}, \text{Enc}, \text{Dec})$ s.t.:

1. The key generation algorithm Gen takes as input the security parameter 1^n and outputs a pair of keys (pk, sk) . We refer to the first of these as the public key and the second as the private key. We assume for convenience that pk and sk each have length at least n , and that n can be determined from pk, sk .

Definitions

2. The encryption algorithm **Enc** takes as input a public key pk and a message m from some underlying plaintext space. It outputs a ciphertext c , and we write this as

$$c \leftarrow \text{Enc}_{pk}(m).$$

3. The decryption algorithm **Dec** takes as input a private key sk and a ciphertext c , and outputs a message m or a special symbol \perp denoting failure. We assume without loss of generality that **Dec** is deterministic, and write this as

$$m := \text{Dec}_{sk}(c).$$

Definitions

It is required that there exists a negligible function negl such that for every n , every (pk, sk) output by $\text{Gen}(1^n)$, and every message m in the appropriate underlying plaintext space, it holds that

$$\Pr[\text{Dec}_{sk}(\text{Enc}_{pk}(m)) \neq m] \leq \text{negl}(n).$$

RSA Encryption

Public inventors



Private inventors



Ellis,



Cocks,



Williamson

1970's

RSA Key Generation

- Input: Security parameter 1^n .
- Output: N, e, d of size n .

$N \leftarrow p * q$ // for some large random primes p, q
 $\varphi(N) = (p-1) * (q-1)$
choose randomly e such that $\text{GCD}(e, \varphi(N)) = 1$
compute $d = e^{-1} \text{ mod } \varphi(N)$
return (N, e, d)

**In Cocks' variation, $e=N$ and
therefore $d=N^{-1} \text{ mod } \varphi(N)$.**

RSA Encryption

- **Gen:** on input 1^n run $\text{GenRSA}(1^n)$ and obtain (N,e,d) .
Let $\langle N,e \rangle$ be the public-key and $\langle d \rangle$ the private key.
- **Enc:** on input $\langle N,e \rangle$ and a message $0 < m < N$ compute
$$c = m^e \bmod N$$
- **Dec:** on input $\langle d \rangle$ and a ciphertext $0 < c < N$ compute
$$m = c^d \bmod N$$

The RSA Assumption

- The RSA problem can be described informally as follows: given a modulus N , an integer (exponent) $e > 0$ that is relatively prime to $\varphi(N)$, and an element $y \in \mathbb{Z}_N^*$, compute $e\sqrt{y} \bmod N$;

Given N, e, y find x such that $x^e = y \bmod N$.

The RSA Assumption

The RSA experiment $\text{RSA-inv}_{A, \text{GenRSA}}(n)$:

1. Run $\text{GenRSA}(1^n)$ to obtain (N, e, d) .

2. Choose $y \leftarrow \mathbb{Z}_N^*$.

3. A is given N, e, y , and outputs $x \in \mathbb{Z}_N^*$.

4. The output of the experiment is defined to be 1 if $y = x^e \pmod N$, and 0 otherwise.

The RSA Assumption

DEFINITION We say that the RSA problem is hard relative to **GenRSA** if for all probabilistic polynomial-time algorithms A there exists a negligible function negl such that

$$\Pr[\text{RSA-inv}_{A, \text{GenRSA}}(n) = 1] \leq \text{negl}(n).$$

Quantum Factoring



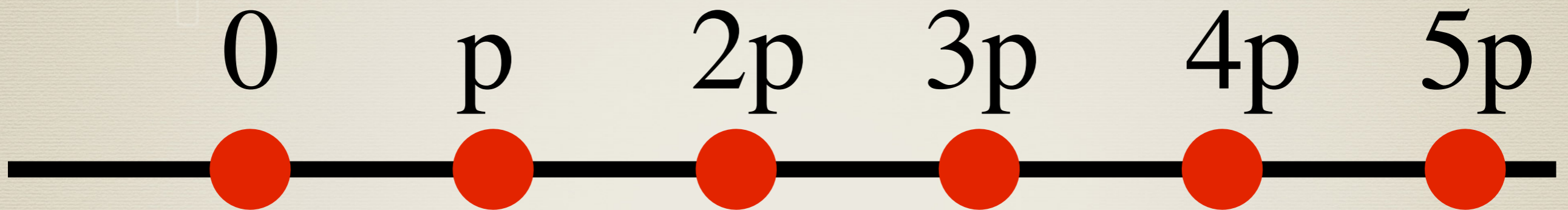
1990's

Approximate Integer GCD Based Cryptography

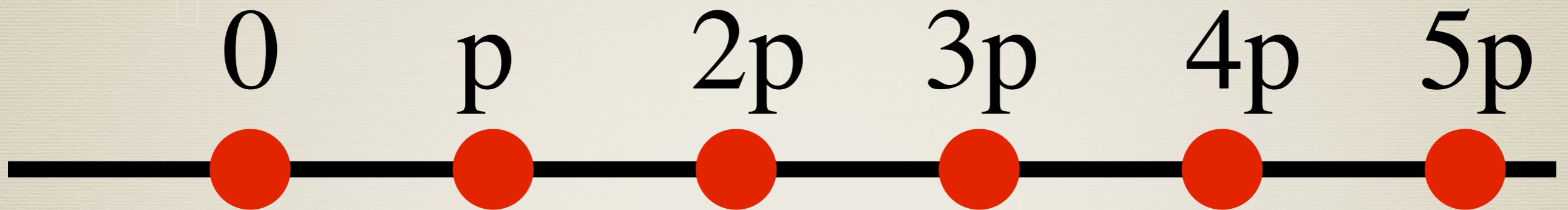
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2010's

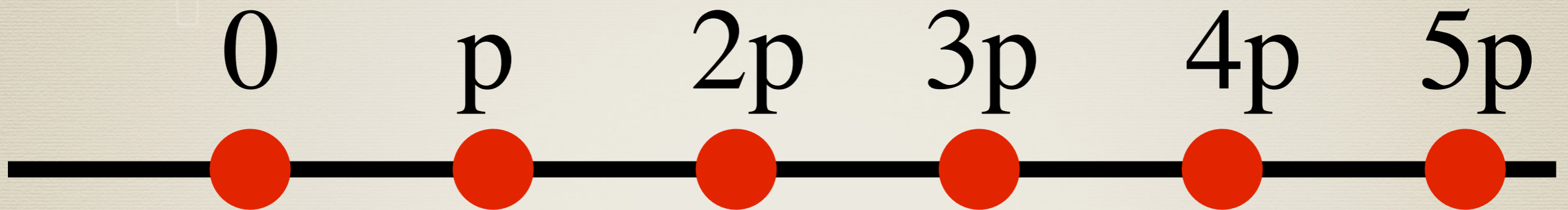
Approximate Integer GCD



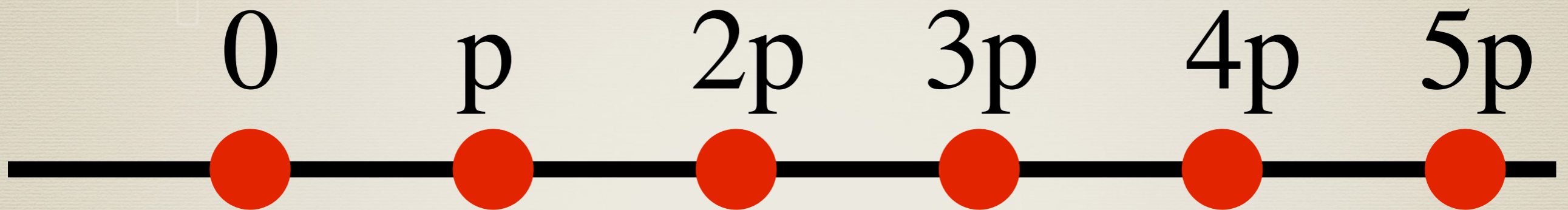
Approximate Integer GCD



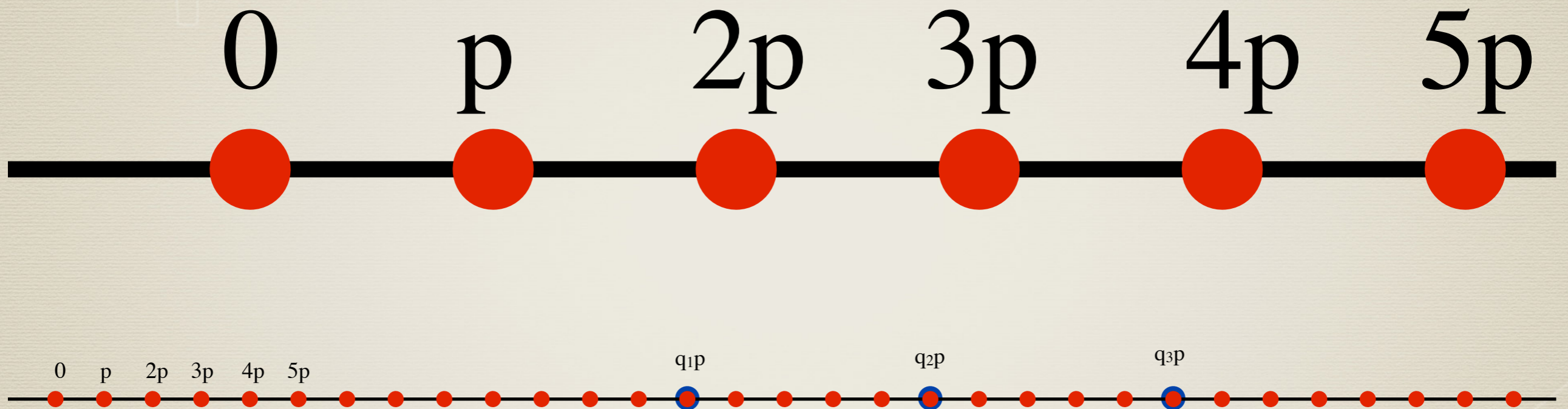
Approximate Integer GCD



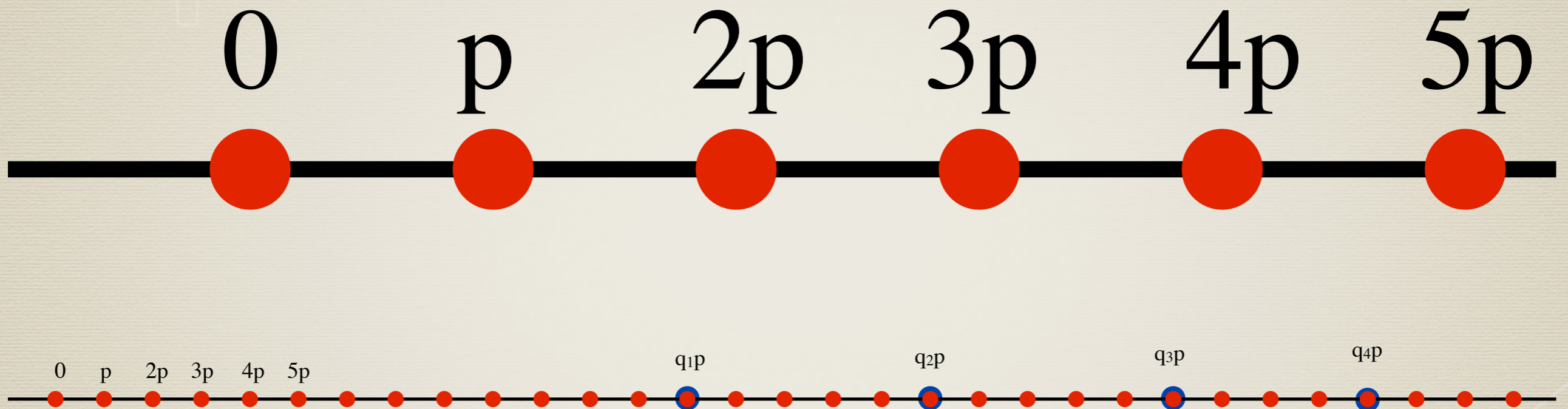
Approximate Integer GCD



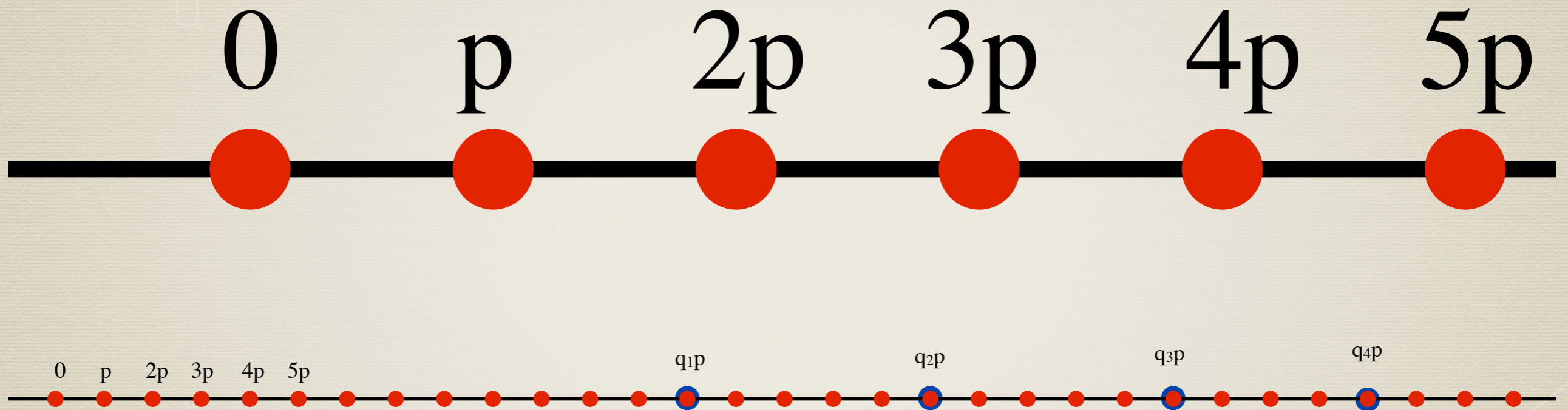
Approximate Integer GCD



Approximate Integer GCD



Approximate Integer GCD



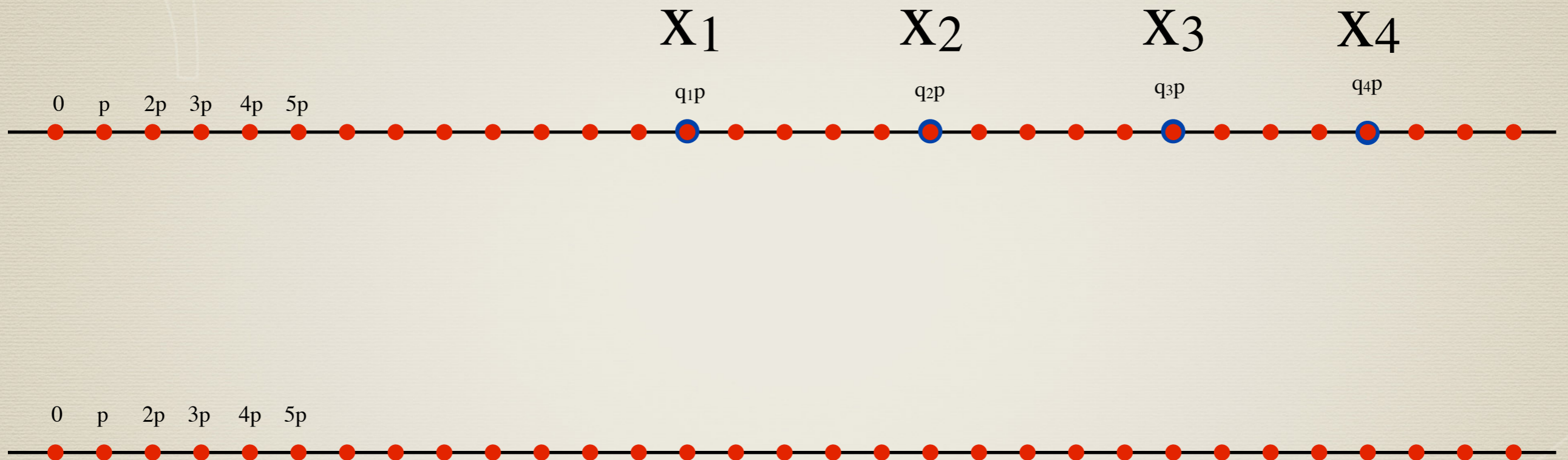
$$\text{GCD}(q_1p, q_2p, q_3p, q_4p) = p$$

Approximate Integer GCD



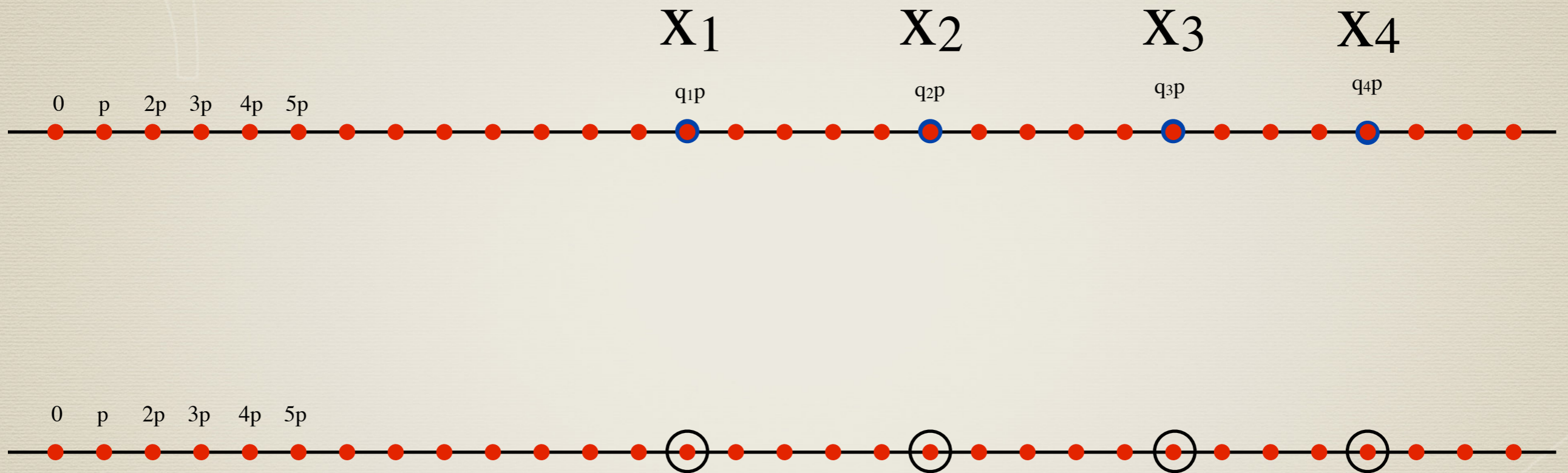
$$\text{GCD}(X_1, X_2, X_3, X_4) = p$$

Approximate Integer GCD



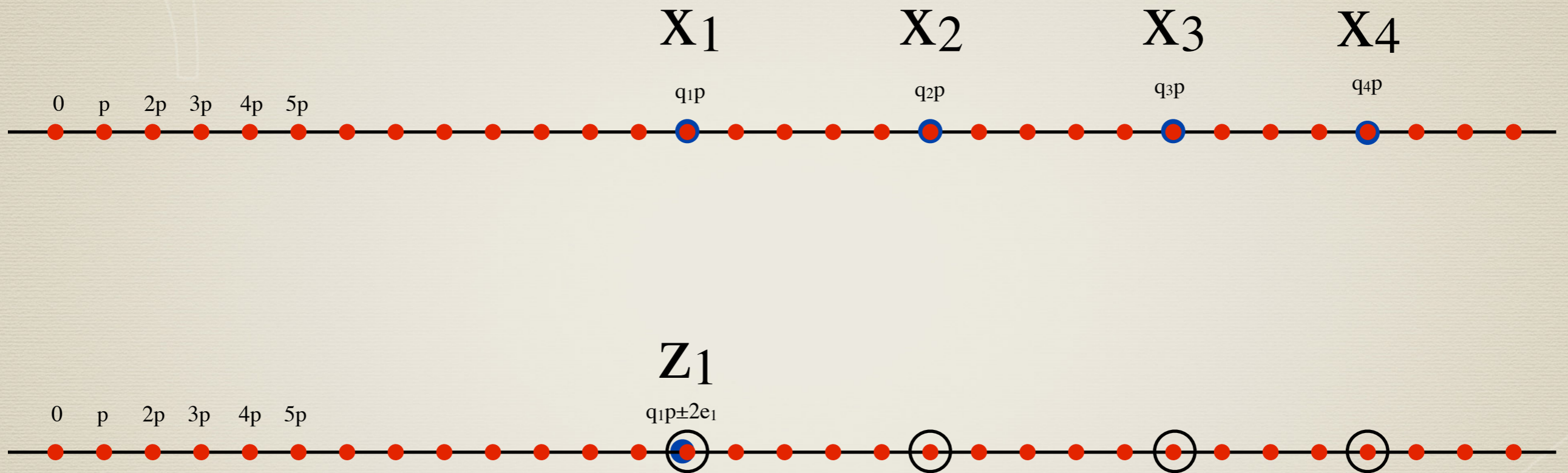
$$\text{GCD}(X_1, X_2, X_3, X_4) = p$$

Approximate Integer GCD



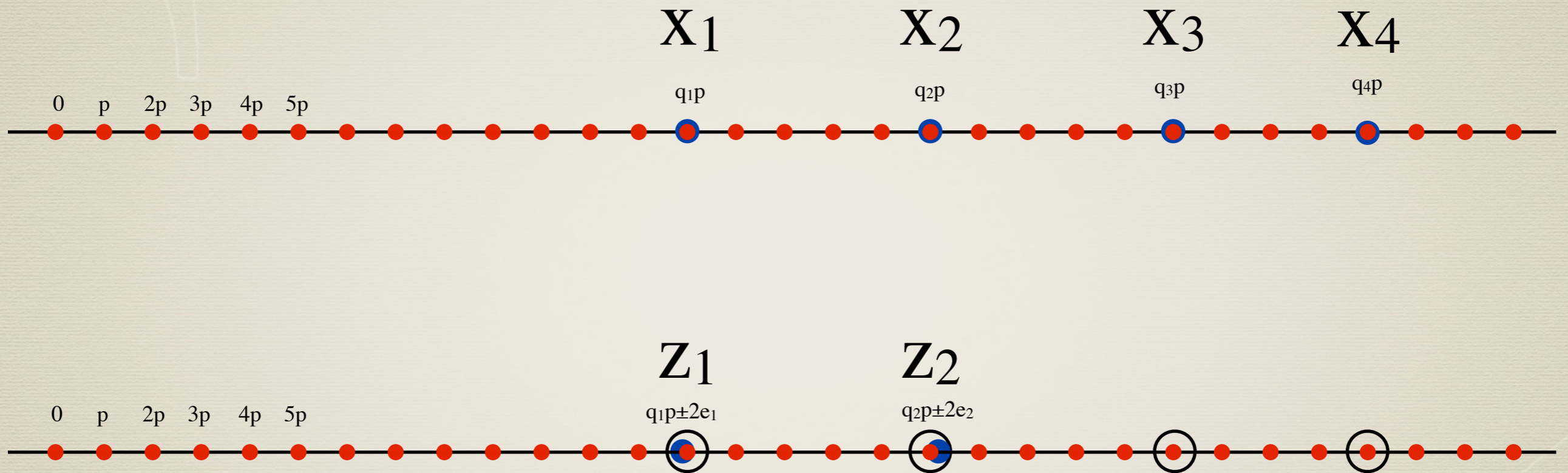
$$\text{GCD}(X_1, X_2, X_3, X_4) = p$$

Approximate Integer GCD



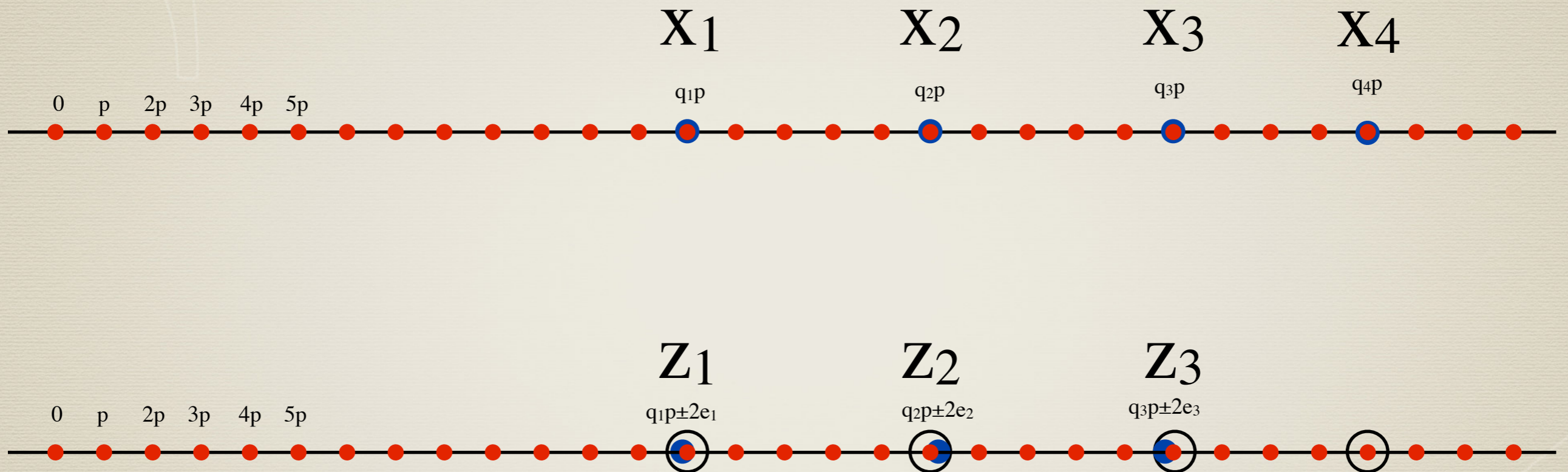
$$\text{GCD}(X_1, X_2, X_3, X_4) = p$$

Approximate Integer GCD



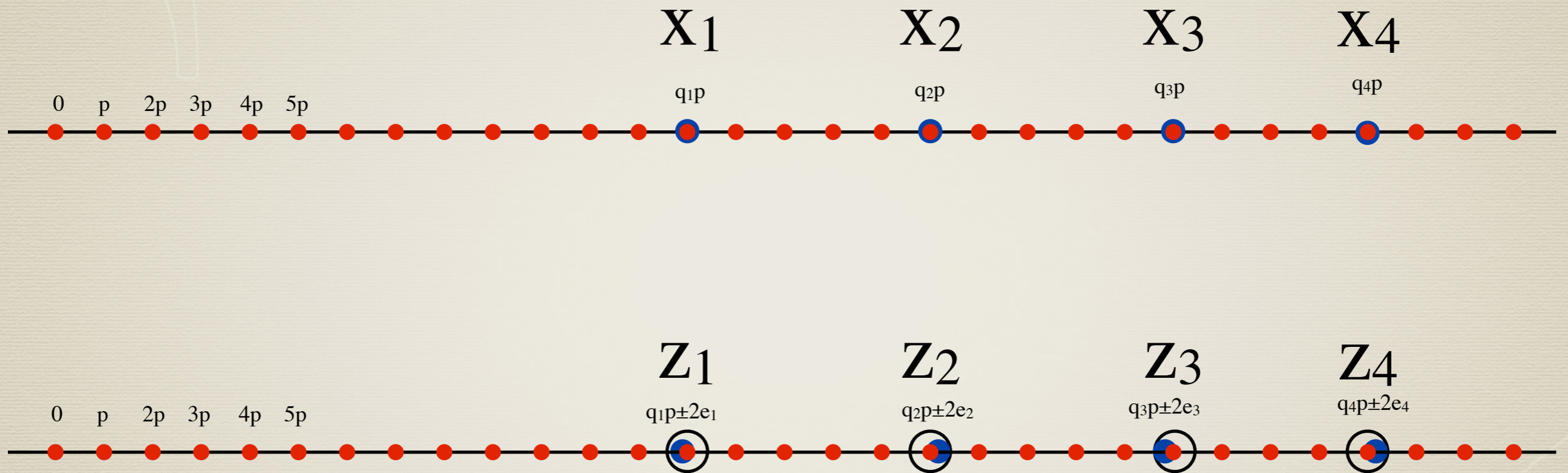
$$\text{GCD}(X_1, X_2, X_3, X_4) = p$$

Approximate Integer GCD



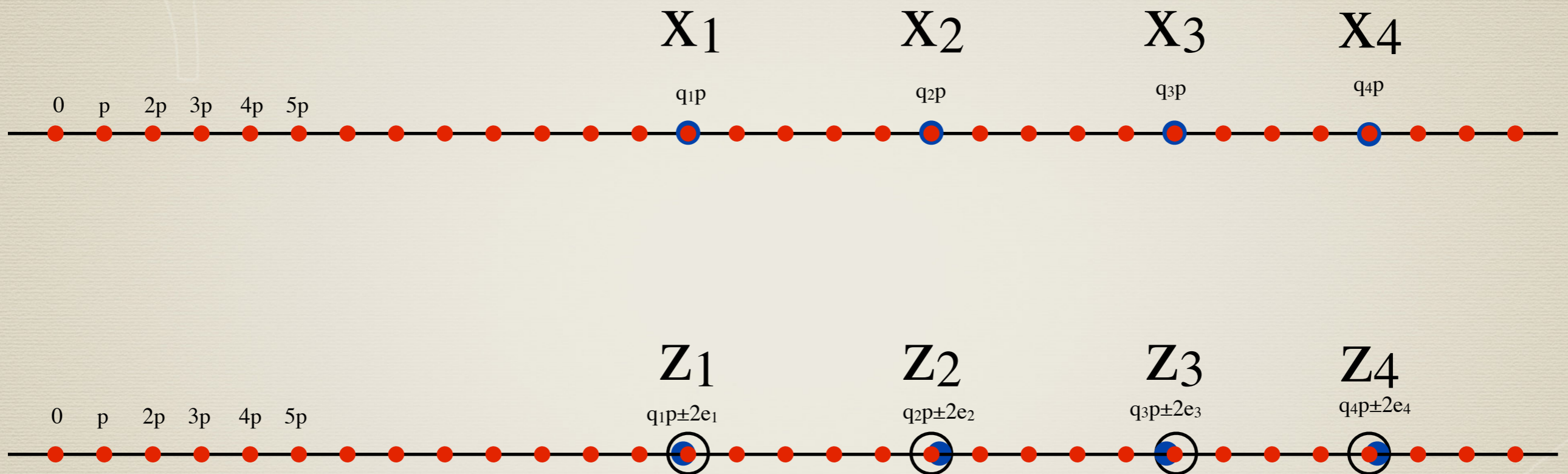
$$\text{GCD}(X_1, X_2, X_3, X_4) = p$$

Approximate Integer GCD



$$\text{GCD}(X_1, X_2, X_3, X_4) = p$$

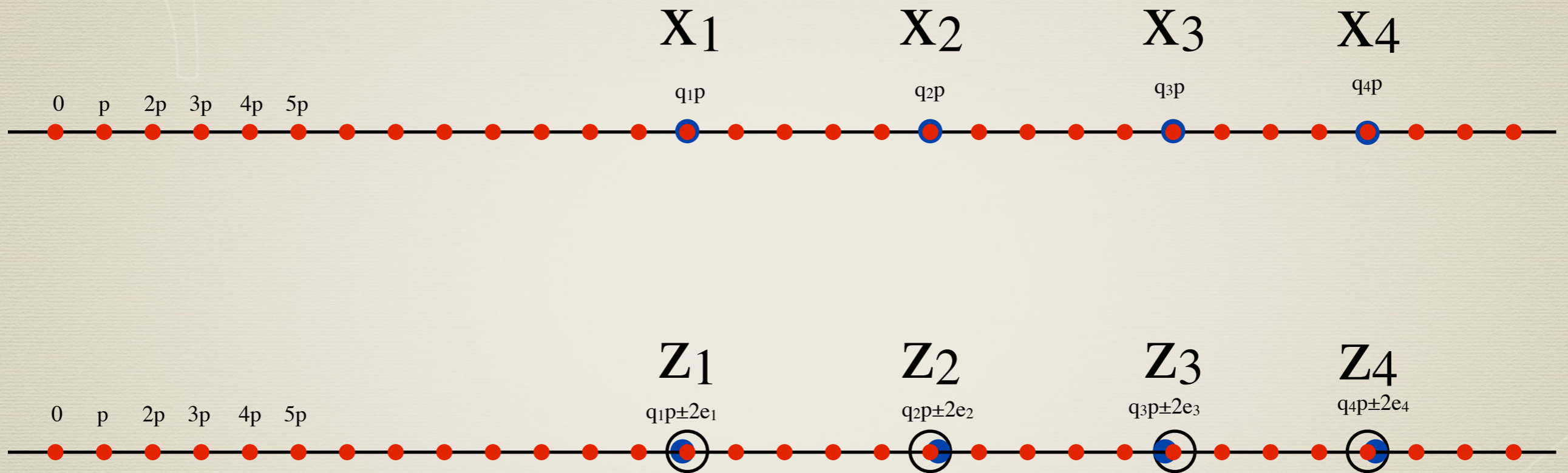
Approximate Integer GCD



$$\text{GCD}(X_1, X_2, X_3, X_4) = p$$

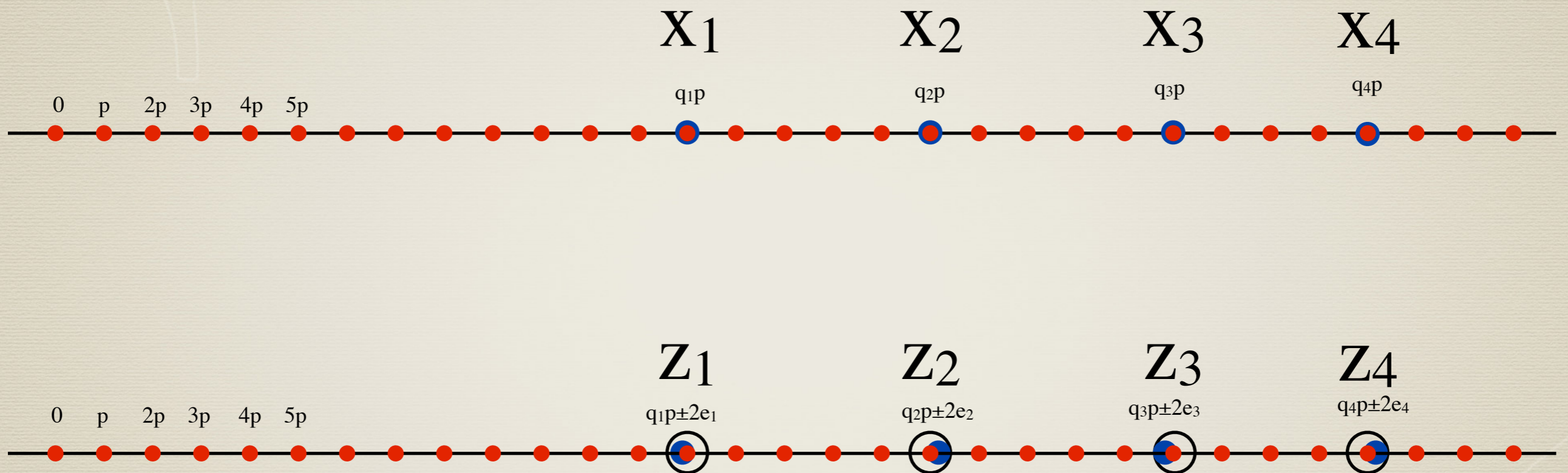
$$\text{GCD}(Z_1, Z_2, Z_3, Z_4) = 1$$

Approximate Integer GCD



$$\text{GCD}(X_1, X_2, X_3, X_4) = p$$

Approximate Integer GCD



$$\text{GCD}(X_1, X_2, X_3, X_4) = p$$

AIGCD : find p from Z_1, Z_2, Z_3, Z_4 ?

Approximate Integer GCD

z_1 z_2 z_3

\cdot \cdot \cdot

z_{k-1} z_k

z_0

0 1 2 3 4 5



Approximate Integer GCD

z_1 z_2 z_3

· · ·

z_{k-1} z_k

z_0

0 1 2 3 4 5



$$\sum_{1 \leq i \leq k} s_i z_i \pmod{z_0}$$

Approximate Integer GCD

$z_1 \quad z_2 \quad z_3 \quad \dots \quad z_{k-1} \quad z_k \quad z_0$



$$\sum_{1 \leq i \leq k} s_i z_i \pmod{z_0}$$

$$s_i \in \{0, 1\}$$

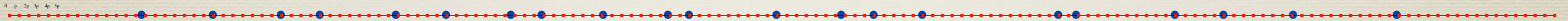
Approximate Integer GCD

z_1 z_2 z_3

\dots

z_{k-1} z_k

z_0

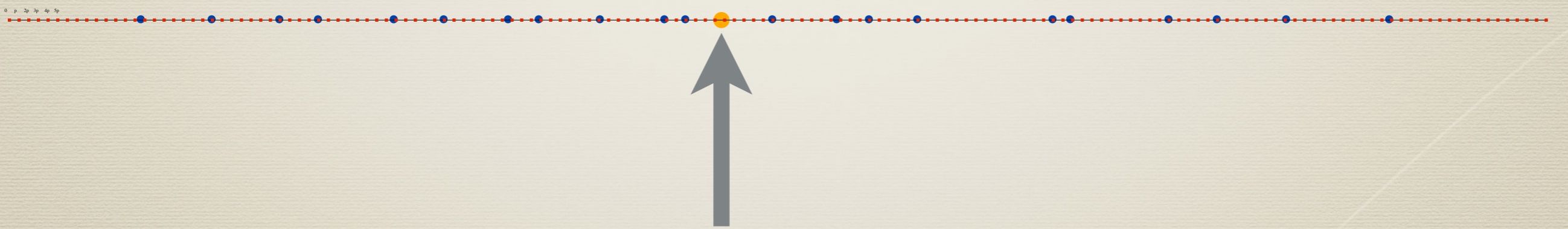


$$\sum_{1 \leq i \leq k} s_i z_i \pmod{z_0}$$

$$s_i \in \{0, 1\}$$

Approximate Integer GCD

z_1 z_2 z_3 . . . z_{k-1} z_k z_0

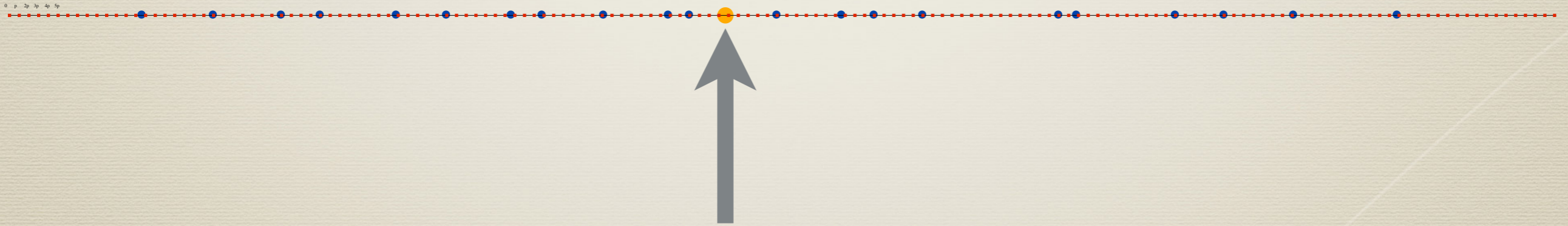


$$\sum_{1 \leq i \leq k} s_i z_i \pmod{z_0}$$

$$s_i \in \{0, 1\}$$

Approximate Integer GCD

$z_1 \quad z_2 \quad z_3 \quad \dots \quad z_{k-1} \quad z_k \quad z_0$

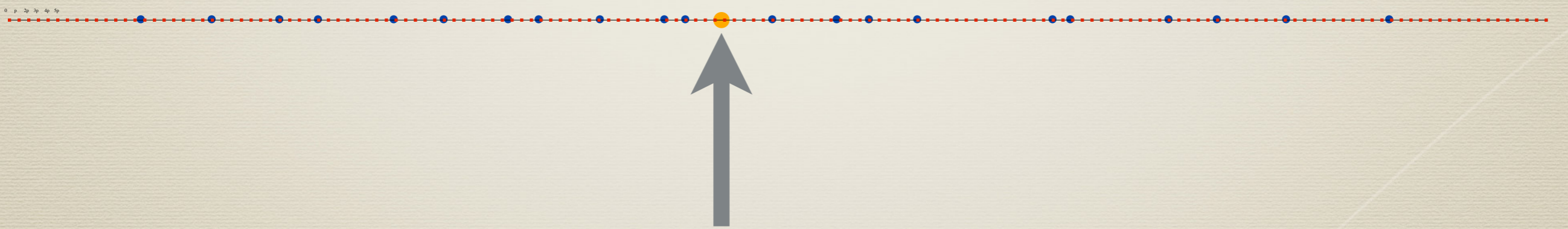


$$\sum_{1 \leq i \leq k} s_i z_i \pmod{z_0} \approx \sum_{1 \leq i \leq k} s_i x_i \pmod{x_0}$$

$$s_i \in \{0, 1\}$$

Approximate Integer GCD

z_1 z_2 z_3 . . . z_{k-1} z_k z_0



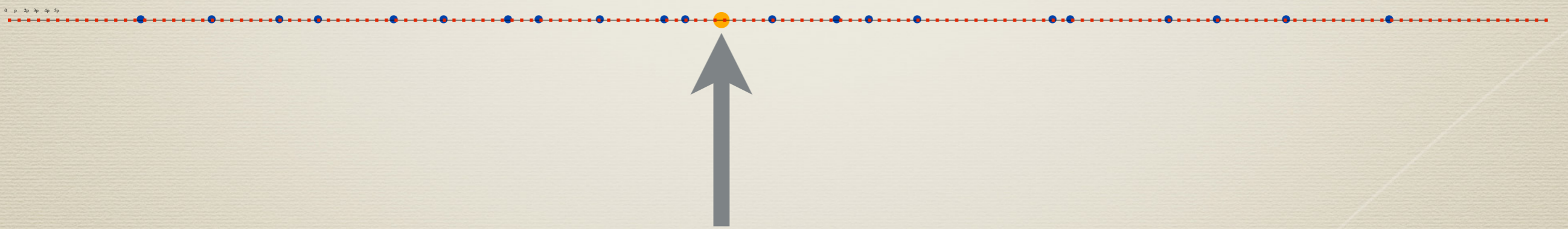
$$\sum_{1 \leq i \leq k} s_i z_i \bmod z_0 \approx \sum_{1 \leq i \leq k} s_i x_i \bmod x_0$$

$$s_i \in \{0, 1\}$$

$$\pm 2(k e_0 + \sum_{1 \leq i \leq k} e_i)$$

Approximate Integer GCD

z_1 z_2 z_3 . . . z_{k-1} z_k z_0

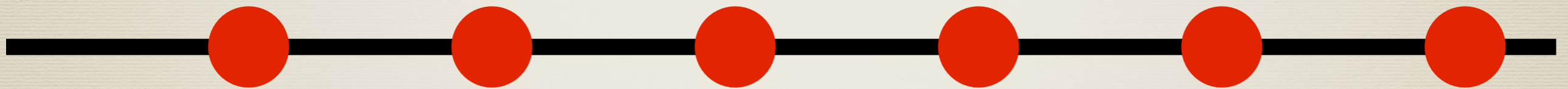


$$\left| \sum_{1 \leq i \leq k} s_i z_i \bmod z_0 - \left(\sum_{1 \leq i \leq k} s_i q_i \bmod q_0 \right) \times p \right| \leq 4k l e_{\max}$$

$s_i \in \{0, 1\}$

Approximate Integer GCD

Approximate Integer GCD



Approximate Integer GCD

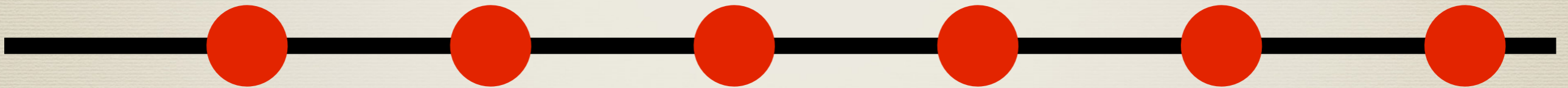
$$\Omega(s) = \sum_{1 \leq i \leq k} s_i z_i \pmod{z_0}$$



Approximate Integer GCD

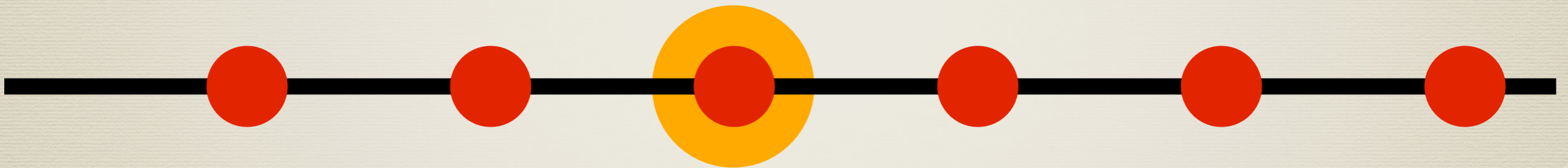
$$\Omega(s) = \sum_{1 \leq i \leq k} s_i z_i \pmod{z_0}$$

$$s \in \{0, 1\}^n$$



Approximate Integer GCD

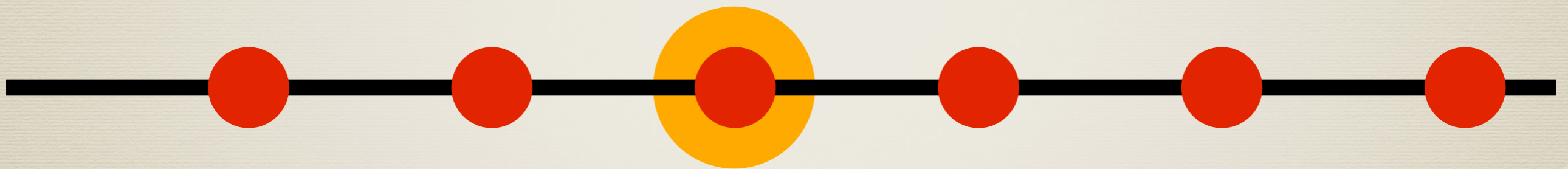
$$\Omega(s) = \sum_{1 \leq i \leq k} s_i z_i \pmod{z_0}$$
$$s \in \{0, 1\}^n$$



$$|e_{\max}| \leq \delta \ll p/8k$$

Approximate Integer GCD

$$\Omega(s) = \sum_{1 \leq i \leq k} s_i z_i \pmod{z_0}$$
$$s \in \{0, 1\}^n$$



$$|e_{\max}| \leq \delta \ll p/8k$$

$$\Omega(s) - p[\Omega(s)/p] = \text{small even error}$$

AIGCD encryption

AIGCD encryption

SK : p

AIGCD encryption

SK : p

PK : $z_0, z_1, z_2, \dots, z_k, \delta \ll p/8k \ll \delta' \ll p/2$

AIGCD encryption

SK : p

PK : $z_0, z_1, z_2, \dots, z_k, \partial \ll p/8k \ll \partial' \ll p/2$
 $e_i \in_U [-\partial \dots +\partial]$

AIGCD encryption

SK : p

PK : $z_0, z_1, z_2, \dots, z_k, \partial \ll p/8k \ll \partial' \ll p/2$

$e_i \in_U [-\partial \dots +\partial]$

$$\text{enc}(b) = \Omega(s) + 2e + b$$

$$s \in_U \{0, 1\}^n$$

$$e \in_U [-\partial' \dots +\partial']$$

AIGCD encryption

SK : p

PK : $z_0, z_1, z_2, \dots, z_k, \partial \ll p/8k \ll \partial' \ll p/2$

$e_i \in_U [-\partial \dots +\partial]$

$$\text{enc}(b) = \Omega(s) + 2e + b$$

$$s \in_U \{0, 1\}^n$$

$$e \in_U [-\partial' \dots +\partial']$$

$$\begin{aligned} \text{dec}(c) &= c - p[c/p] \bmod 2 \\ &= \text{parity of error} \end{aligned}$$



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SEARCH

Cryptography Today

In the current global environment, rapid and secure information sharing is important to protect our Nation, its citizens and its interests. Strong cryptographic algorithms and secure protocol standards are vital tools that contribute to our national security and help address the ubiquitous need for secure, interoperable communications.

Currently, Suite B cryptographic algorithms are specified by the National Institute of Standards and Technology (NIST) and are used by NSA's Information Assurance Directorate in solutions approved for protecting classified and unclassified National Security Systems (NSS). Below, we announce preliminary plans for transitioning to quantum resistant algorithms.

2015



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The Seventh International Conference on Post-Quantum Cryptography

Fukuoka, Japan, February 24-26, 2016

[NSA announced preliminary plans for transitioning to quantum resistant algorithms.](#)

[NIST published a draft of the report on post-quantum cryptography NISTIR 8105.](#)

Updates

February 24, 2016: The slides of NIST Announcement

["Post-Quantum Cryptography: NIST's Plan for the Future"](#)

by Dustin Moody are now available.

February 21, 2016: Useful information concerning the winter school and the conference can be found in the **public webfolder**: <https://goo.gl/UGwjua> (access is read-only)

February 15, 2016: Due to high attendance, for participants, who register **on site**, only access to an **electronic version** (instead of a hardcopy) of the proceedings will be provided.

February 10, 2016: Updated programs of the **conference** (with **hot topic session** included) and the **winter school** are now available.

February 9, 2016: Deadline for online registration has passed.

On-site registration will be available at the venue from February 22.

Post-Quantum Cryptography: NIST's Plan for the Future

Dustin Moody
Post Quantum Cryptography Team
National Institute of Standards and Technology (NIST)

24 Feb 2016

Timeline

- ▶ Fall 2016 – formal Call For Proposals
- ▶ Nov 2017 – Deadline for submissions
- ▶ 3–5 years – Analysis phase
 - NIST will report its findings
- ▶ 2 years later – Draft standards ready

- ▶ Workshops
 - Early 2018 – submitter's presentations
 - One or two during the analysis phase

Winter 2016
COMP-250: Introduction
to Computer Science

Lecture 14, February 25, 2016